Example 8 (Method 2)

Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.

2	6
3	3
	1

2	72
2	36
2	18
3	9
3	3
	1

2	120
2	60
2	30
3	15
5	5
	1

$$6 = \boxed{2} \times 3$$

$$72 = \boxed{2} \times 2 \times 2 \times \boxed{3} \times 3$$

$$120 = \boxed{2} \times 2 \times 2 \times \boxed{3} \times 5$$

$$\text{H.C.F} = 2 \times 3$$

$$= 6$$

Finding LCM

2	6	72	120
2	3	36	60
2	3	18	30
3	3	9	15
3	1	3	5
5	1	1	5
	1	1	1

L.C.M = $2 \times 2 \times 2 \times 3 \times 3 \times 5$

= 360

THEOREM 1.3:-

Let p be a prime number. If p divides A^2, then p divides a, where a is positive integer.

Let a=p1.p2.p3.p4.p5…..pn where, p1, p2, p3, …, pn are prime numbers which are necessarily not distinct.⇒A^2=(p1.p2.p3.p4.p5…..pn).(p1.p2.p3.p4.p5……pn) It is given that p divides A^2. From the Fundamental theorem of Arithmetic, we know that every composite number can be expressed as product of unique prime numbers. This means that p is one of the numbers from (p1.p2.p3.p4.p5……pn). We have a=(p1.p2.p3.p4.p5..pn) and p is one of the numbers from (p1.p2.p3.p4.p5……pn).It means that p also divides a. Hence, it is proved that if p divides A^2 then it also divides a.

THEOREM 1.3:-

$\sqrt{2}$ is irrational.

Proof: We shall start by assuming √2 as rational. In other words, we need to find integers x and y such that √2 = x/y.

(i) Let x and y have a common factor other than 1, and so we can divide by that common factor and assume that x and y are coprime. So, y√2 = x.

(ii) Squaring both side, we get, $2y^2 = x^2$.

(iii) Thus, 2 divides x^2. and by theorem we can say that 2 divides x.

(iv) Hence, x = 2z for some integer z.

(v) Substituting x, we get, $2x^2 = 4z^2$e. $y^2 = 4z^2$; which means y^2 is divisible by 2, and so y will also be divisible by 2.

(vi) Now, from theorem, x and y will have 2 as a common factor. But, it is opposite to fact that x and y are co-prime.

(vii) Hence, we can conclude √2 is irrational.

Example 9

Prove that $\sqrt{3}$ is irrational.

We have to prove $\sqrt{3}$ is irrational

Let us assume the opposite,

i.e., $\sqrt{3}$ is rational

Hence, $\sqrt{3}$ can be written in the form $\frac{a}{b}$

where a and b ($b \neq 0$) are co-prime (no common factor other than 1)

Hence, $\sqrt{3} = \frac{a}{b}$

$\sqrt{3}\,b = a$

Squaring both sides

$(\sqrt{3}b)^2 = a^2$

$3b^2 = a^2$

$$\frac{a^2}{3} = b^2$$

Hence, 3 divides a^2

***By theorem**: If p is a prime number, and p divides a^2, then p divides a , where a is a positive number*

So, 3 shall divide a also ...(1)

Hence, we can say

$\frac{a}{3} = c$ where c is some integer

So, $a = 3c$

Now we know that

$3b^2 = a^2$

Putting a = 3c

$3b^2 = (3c)^2$

$3b^2 = 9c^2$

$b^2 = \frac{1}{3} \times 9c^2$

$b^2 = 3c^2$

$\frac{b^2}{3} = c^2$

Hence 3 divides b^2

> ***By theorem:*** *If p is a prime number, and p divides a^2, then p divides a , where a is a positive number*

So, 3 divides b also ...(2)

By (1) and (2)

3 divides both a & b

Hence 3 is a factor of a and b

So, a & b have a factor 3

Therefore, a & b are not co-prime.

Hence, our assumption is wrong

∴ By contradiction,

$\sqrt{3}$ is irrational

Example 10

Show that $5 - \sqrt{3}$ is irrational.

We have to prove $5 - \sqrt{3}$ is irrational

Let us assume the opposite,

i.e., $5 - \sqrt{3}$ is rational

Hence, $5 - \sqrt{3}$ can be written in the form $\frac{a}{b}$

where a and b ($b \neq 0$) are co-prime (no common factor other than 1)

Hence, $5 - \sqrt{3} = \frac{a}{b}$

$$-\sqrt{3} = \frac{a}{b} - 5$$

$$-\sqrt{3} = \frac{a - 5b}{b}$$

$$-\sqrt{3} = \frac{a - 5b}{b}$$

$$\sqrt{3} = -\left(\frac{a - 5b}{b}\right)$$

$$\sqrt{3} = \frac{5b - a}{b}$$

Irrational **Rational**

Here, $\frac{-a + 5b}{b}$ is a rational number

But $\sqrt{3}$ is irrational

Since, Rational ≠ Irrational

This is a contradiction

∴ Our assumption is incorrect

Hence, $5 - \sqrt{3}$ is irrational

Hence proved.

Example 11

Show that $3\sqrt{2}$ is irrational.

We have to prove $3\sqrt{2}$ is irrational

Let us assume the opposite,

i.e., $3\sqrt{2}$ is rational

Hence, $3\sqrt{2}$ can be written in the form $\frac{a}{b}$

where a and b (b≠ 0) are co-prime (no common factor other than 1)

Hence, $3\sqrt{2} = \frac{a}{b}$

$$\sqrt{2} = \frac{1}{3} \times \frac{a}{b}$$

$$\sqrt{2} = \frac{a}{3b}$$

$$\sqrt{2} = \frac{a}{3b}$$

↓ **Irrational** ↘ **Rational**

Here, $\frac{a}{3b}$ is a rational number

But $\sqrt{2}$ is irrational

Since, Rational ≠ Irrational

This is a contradiction

∴ Our assumption is incorrect

Hence $3\sqrt{2}$ is irrational

Hence proved

Ex 1.3 , 1

Prove that $\sqrt{5}$ is irrational.

We have to prove $\sqrt{5}$ is irrational

Let us assume the opposite,

i.e., $\sqrt{5}$ is rational

Hence, $\sqrt{5}$ can be written in the form $\frac{a}{b}$

where a and b ($b \neq 0$) are co-prime (no common factor other than 1)

Hence, $\sqrt{5} = \frac{a}{b}$

$\sqrt{5}b = a$

Squaring both sides

$(\sqrt{5}b)^2 = a^2$

$5b^2 = a^2$

$$\frac{a^2}{5} = b^2$$

Hence, 5 divides a^2

> ***By theorem***: *If p is a prime number, and p divides a^2, then p divides a , where a is a positive number*

So, 5 shall divide a also ...(1)

Hence, we can say

$\frac{a}{5}$ = c where c is some integer

So, a = 5c

Now we know that

$$5b^2 = a^2$$

Putting a = 5c

$$5b^2 = (5c)^2$$

$$5b^2 = 25c^2$$

$5b^2 = 25c^2$

$b^2 = \frac{1}{5} \times 25c^2$

$b^2 = 5c^2$

$\frac{b^2}{5} = c^2$

Hence 5 divides b^2

***By theorem**: If p is a prime number, and p divides a^2, then p divides a , where a is a positive number*

So, 5 divides b also ...(2)

By (1) and (2)

5 divides both a & b

Hence 5 is a factor of a and b

So, a & b have a factor 5

Therefore, a & b are not co-prime.

Hence, our assumption is wrong

∴ By contradiction,

$\sqrt{5}$ is irrational

Ex 1.3 , 2

Prove that $3 + 2\sqrt{5}$ is irrational.

We have to prove $3 + 2\sqrt{5}$ is irrational

Let us assume the opposite,

i.e., $3 + 2\sqrt{5}$ is rational

Hence, $3 + 2\sqrt{5}$ can be written in the form $\frac{a}{b}$

where a and b (b≠ 0) are co-prime (no common factor other than 1

Hence, $3 + 2\sqrt{5} = \frac{a}{b}$

$$2\sqrt{5} = \frac{a}{b} - 3$$

$$2\sqrt{5} = \frac{a - 3b}{b}$$

$$\sqrt{5} = \frac{1}{2} \times \frac{a - 3b}{b}$$

$$\sqrt{5} = \frac{a - 3b}{2b}$$

Irrational **Rational**

Here, $\frac{a - 3b}{2b}$ is a rational number

But $\sqrt{5}$ is irrational

Since, Rational ≠ Irrational

This is a contradiction

∴ Our assumption is incorrect

Hence $3 + 2\sqrt{5}$ is irrational

Hence proved

Ex 1.3 , 3

Prove that the following are irrationals :

(i) $\frac{1}{\sqrt{2}}$

We have to prove $\frac{1}{\sqrt{2}}$ is irrational

Let us assume the opposite,

i.e., $\frac{1}{\sqrt{2}}$ is rational

Hence, $\frac{1}{\sqrt{2}}$ can be written in the form $\frac{a}{b}$

where a and b (b≠ 0) are co-prime (no common factor other than 1)

Hence, $\frac{1}{\sqrt{2}} = \frac{a}{b}$

$$\frac{b}{a} = \sqrt{2}$$

Rational **Irrational**

Here, $\frac{b}{a}$ is a rational number

But $\sqrt{2}$ is irrational

Since, Rational ≠ Irrational

This is a contradiction

∴ Our assumption is incorrect

Hence $\frac{1}{\sqrt{2}}$ is irrational

Hence proved

Ex 1.3 , 3

Prove that the following are irrationals :

(ii) $7\sqrt{5}$

We have to prove $7\sqrt{5}$ is irrational

Let us assume the opposite,

i.e., $7\sqrt{5}$ is rational

Hence, $7\sqrt{5}$ can be written in the form $\frac{a}{b}$

where a and b (b≠ 0) are co-prime (no common factor other than 1)

Hence, $7\sqrt{5} = \frac{a}{b}$

$$\sqrt{5} = \frac{1}{7} \times \frac{a}{b}$$

$$\sqrt{5} = \frac{a}{7b}$$

↓ ↘

Irrational **Rational**

Here, $\frac{a}{7b}$ is a rational number

But $\sqrt{5}$ is irrational

Since, Rational ≠ Irrational

This is a contradiction

∴ Our assumption is incorrect

Hence $7\sqrt{5}$ is irrational

Hence proved

Ex 1.3 , 3

Prove that the following are irrationals :

(iii) $6 + \sqrt{2}$

We have to prove $6 + \sqrt{2}$ is irrational

Let us assume the opposite,

i.e., $6 + \sqrt{2}$ is rational

Hence, $6 + \sqrt{2}$ can be written in the form $\frac{a}{b}$

where a and b (b≠ 0) are co-prime (no common factor other than 1)

Hence, $6 + \sqrt{2} = \frac{a}{b}$

$$\sqrt{2} = \frac{a}{b} - 6$$

$$\sqrt{2} = \frac{a \; - 6b}{b}$$

↙ ↘

Irrational **Rational**

Here, $\frac{a \; - 6b}{b}$ is a rational number

But $\sqrt{5}$ is irrational

Since, Rational ≠ Irrational

This is a contradiction

∴ Our assumption is incorrect

Hence, $6 + \sqrt{2}$ is irrational

Hence proved.

$$\sqrt{2} = \frac{a - 6b}{b}$$

Irrational **Rational**

Here, $\frac{a - 6b}{b}$ is a rational number

But $\sqrt{5}$ is irrational

Since, Rational ≠ Irrational

This is a contradiction

∴ Our assumption is incorrect

Hence, $6 + \sqrt{2}$ is irrational

Hence proved.

Theorem 1.5 >

Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form $\frac{p}{q}$, where p and q are coprime, and the prime factorization of q is of the form $2^n 5^m$, where n, m are non-negative integers.

$2^n \cdot 5^m$

Ex. $0.375 \rightarrow \frac{375}{1000}$

$\rightarrow \frac{3}{2^3}$ $\quad 2^3 \cdot 5^0$

Ex 1.4 , 1

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$

$\frac{p}{q}$ is terminating if

(i) p & q are co-prime

and

(ii) q is of the form $2^n \, 5^m$

where n & m are non-negative integers

<u>Checking co-prime</u>

13 & 3125 have no common factors,

So, 13 & 3125 are co-prime

For Denominator (3125)

5	3125
5	625
5	125
5	25
5	5
	1

Hence $3125 = 5 \times 5 \times 5 \times 5 \times 5$

$= 5^5$

Denominator $= 5^5$

$= 1 \times 5^5$

$= 2^0 \times 5^5$

So , denominator is of the form $2^n 5^m$

where $n = 0$, $m = 5$

Thus , $\frac{13}{3125}$ is a terminating decimal

Ex 1.4 , 1

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(ii) $\frac{17}{8}$

$\frac{p}{q}$ is terminating if

(i) p & q are co-prime

and

(ii) q is of the form $2^n 5^m$

where n & m are non-negative integers

<u>Checking co-prime</u>

17 & 8 have no common factors,

So, 17 & 8 are co-prime

For Denominator (8)

$$
\begin{array}{c|l}
2 & 8 \\ \hline
2 & 4 \\ \hline
2 & 2 \\ \hline
 & 1
\end{array}
$$

Hence $8 = 2 \times 2 \times 2$

$= 2^3$

Denominator $= 2^3$

$= 1 \times 2^3$

$= 5^0 \times 2^3$

So , denominator is of the form $2^n\ 5^m$

where $n = 3$, $m = 0$

Thus, $\frac{17}{8}$ is a terminating decimal

Ex 1.4 , 1

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(iii) $\frac{64}{455}$

$\frac{p}{q}$ is terminating if

(i) p & q are co-prime

and

(ii) q is of the form $2^n 5^m$

where n & m are non-negative integers

<u>Checking co-prime</u>

64 & 455 have no common factors,

So, 64 & 455 are co-prime

Ex 1.4 , 1

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(iv) $\frac{15}{1600}$

$$\frac{15}{1600} = \frac{3}{320}$$

$\frac{p}{q}$ is terminating if

(i) p & q are co-prime

and

(ii) q is of the form $2^n\ 5^m$

where n & m are non-negative integers

<u>Checking co-prime</u>

3 & 320 have no common factors,

So, 3 & 320 are co-prime

For Denominator (320)

2	320
2	160
2	80
2	40
2	20
2	10
5	5
	1

Hence,

$320 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5$

$= 2^6 \times 5^1$

Denominator = $2^6 \times 5^1$

So , denominator is of the form $2^n \, 5^m$

where $n = 6$, $m = 1$

Thus, $\frac{3}{320}$ i.e. $\frac{15}{1600}$ is a terminating decimal

Ex 1.4 , 1

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(v) $\frac{29}{343}$

$\frac{p}{q}$ is terminating if

(i) p & q are co-prime

and

(ii) q is of the form $2^n 5^m$

where n & m are non-negative integers

<u>Checking co-prime</u>

29 & 343 have no common factors,

So, 29 & 343 are co-prime

For Denominator (343)

7	343
7	49
7	7
	1

Hence $343 = 7 \times 7 \times 7$

$= 7^3$

Denominator $= 7 \times 7 \times 7$

So , denominator is **not** of the form $2^n\ 5^m$

Thus, $\frac{29}{343}$ has a non-terminating repeating decimal expansion

Ex 1.4 , 1

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(vi) $\frac{23}{2^3\ 5^2}$

$\frac{p}{q}$ is terminating if

(i) p & q are co-prime

and

(ii) q is of the form $2^n\ 5^m$

where n & m are non-negative integers

<u>Checking co-prime</u>

23 & $2^3\ 5^2$ have no common factors,

So, 23 & $2^3\ 5^2$ are co-prime

For Denominator ($2^3 5^2$)

Denominator = $2^3 \times 5^2$

So , denominator is of the form $2^n 5^m$

where n = 3 , m = 2

Thus , $\frac{23}{2^3 5^2}$ is a terminating decimal

Ex 1.4 , 1

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(vii) $\frac{129}{2^2\,5^7\,7^5}$

$\frac{p}{q}$ is terminating if

(i) p & q are co-prime

and

(ii) q is of the form $2^n\,5^m$

where n & m are non-negative integers

Checking co-prime

129 & $2^2\,5^7\,7^5$ have no common factors,

So, 129 & $2^2\,5^7\,7^5$ are co-prime

For Denominator ($2^2\ 5^7\ 7^5$)

Denominator = $2^2\ 5^7\ 7^5$

So , denominator is **not** of the form $2^n\ 5^m$

Thus, $\frac{129}{2^2\ 5^7\ 7^5}$ has a non-terminating repeating decimal expansion.

Ex 1.4 , 1

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(viii) $\frac{6}{15}$

$\frac{6}{15} = \frac{2}{5}$

$\frac{p}{q}$ is terminating if

(i) p & q are co-prime

and

(ii) q is of the form 2^n 5^m

where n & m are non-negative integers

<u>Checking co-prime</u>

2 & 5 have no common factors,

So, 2 & 5 are co-prime

For Denominator (5)

Denominator = 5

$= 5^1$

$= 1 \times 5^1$

$= 2^0 \times 5^1$

So , denominator is of the form $2^n \, 5^m$

where n = 0 , m = 1

Thus , $\frac{2}{5}$ i.e. $\frac{6}{15}$ is a terminating decimal

Ex 1.4 , 1

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(ix) $\frac{35}{50}$

$\frac{35}{50} = \frac{7}{10}$

$\frac{p}{q}$ is terminating if

(i) p & q are co-prime

and

(ii) q is of the form $2^n 5^m$

where n & m are non-negative integers

<u>Checking co-prime</u>

7 & 10 have no common factors,

So, 7 & 10 are co-prime

For Denominator (10)

Denominator = 10

$= 2 \times 5$

$= 2^1 \times 5^1$

So , denominator is of the form $2^n 5^m$

where n = 1 , m = 1

Thus, $\frac{7}{10}$ i.e. $\frac{35}{50}$ is a terminating decimal

Ex 1.4 , 1

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(x) $\frac{77}{210}$

$\frac{77}{210} = \frac{11}{30}$

$\frac{p}{q}$ is terminating if

(i) p & q are co-prime

and

(ii) q is of the form $2^n 5^m$

where n & m are non-negative integers

<u>Checking co-prime</u>

11 & 30 have no common factors,

So, 11 & 30 are co-prime

For Denominator (30)

Denominator = 30

$= 2 \times 3 \times 5$

So , denominator is **not** of the form $2^n 5^m$

Thus, $\frac{11}{30}$ i.e. $\frac{77}{210}$ is a non-terminating repeating decimal

Ex 1.4 , 3 (Method 1)

The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form , $\frac{p}{q}$ what can you say about the prime factors of q?

(i) 43.123456789

43.123456789 is terminating

So, it would be a rational number

$$43.123456789 = \frac{43123456789}{1000000000}$$

$$= \frac{43123456789}{(10)^9}$$

$$= \frac{43123456789}{(2 \times 5)^9}$$

$$= \frac{43123456789}{2^9 \times 5^9}$$

Hence 43.123456789 is now in the form of $\frac{p}{q}$

And the prime factors of q are in terms of 2 and 5