Class 9 Chapter Name: NUMBER SYSTEM

Introduction to number system:

The collection of numbers is called the Number system.

These numbers are of different types such as natural numbers, whole numbers, integers, rational numbers and irrational numbers. Let us see the table below to understand with the examples.

Types	Symbols	Examples
Natural Numbers	N	1, 2, 3, 4, 5, ……
Whole Numbers	W	0,1, 2, 3, 4, 5….
Integers	Z	…., -3, -2, -1, 0, 1, 2, 3, …
Rational Numbers	Q	p/q form, where p and q are integers and q is not zero.
Irrational Numbers		Which can't be represented as rational numbers

Note: every natural number is an integer and 0 is a whole number which is not a whole number.

Natural Numbers

All the numbers starting from 1 till infinity are natural numbers, such as 1,2,3,4,5,6,7,8,…….infinity. These numbers lie on the right side of the number line and are positive.

Whole Numbers

All the numbers starting from 0 till infinity are whole numbers such as 0,1,2,3,4,5,6,7,8,9,…..infinity. These numbers lie on the right side of the number line from 0 and are positive.

Integers

Integers are the whole numbers which can be positive, negative or zero. They cover rational numbers also but not the irrational numbers.

Example: 2, 33, 0, -67, 9.777, are integers.

Rational Numbers

A number which can be represented in the form of p/q is called a rational number. For example, 1/2, 4/5, 26/8, etc.

Irrational Numbers

A number is called an irrational number if it can't be represented in a p/q form, where p and q are integers.

Example: √3, √5, √11, etc.

Real Numbers

The collection of all rational and irrational numbers is called real numbers. Real numbers are denoted by R.

Every real number is a unique point on the number line and also every point on the number line represents a unique real number.

Difference between Terminating and Recurring Decimals

Terminating Decimals	**Repeating Decimals**
If the decimal expression of a/b terminates. I.e comes to an end, then the decimal so obtained is called Terminating decimals.	A decimal in which a digit or a set of digits repeats repeatedly periodically is called a repeating decimal.
Example: ¼ =0.25	Example: ⅔ = 0.666…

Some Special Characteristics of Rational Numbers:

- Every Rational number is expressible either as a terminating decimal or as a repeating decimal.
- Every Terminating decimal is a rational number.
- Every repeating decimal is a rational number.

Irrational Numbers

- The non-terminating, non repeating decimals are irrational numbers.

  Example: 0.0100100001001…

- Similarly, if m is a positive number which is not a perfect square, then $\sqrt{m}$ is irrational. Example: $\sqrt{3}$

- If m is a positive integer which is not a perfect cube, then $\sqrt[3]{m}$ is irrational.

Example: $\sqrt[3]{2}$

Properties of Irrational Numbers

- These satisfy the commutative, associative and distributive laws for addition and multiplication.
- Sum of two irrationals need not be irrational.

Example: $(2 + \sqrt{3}) + (4 - \sqrt{3}) = 6$

- Difference of two irrationals need not be irrational.

Example: $(5 + \sqrt{2}) - (3 + \sqrt{2}) = 2$

- Product of two irrationals need not be irrational.

Example: $\sqrt{3} \times \sqrt{3} = 3$

- Sum of rational and irrational is irrational.
- The difference of a rational number and an irrational number is irrational.
- Product of rational and irrational is irrational.
- Quotient of rational and irrational is irrational.

Real Numbers

A number whose square is non-negative is called a real number.

- Real numbers follow Closure property, associative law, commutative law, the existence of an additive identity, existence of additive inverse for Addition.
- Real numbers follow Closure property, associative law, commutative law, the existence of a multiplicative identity, existence of multiplicative inverse, Distributive laws of multiplication over Addition for Multiplication.

Rationalisation

If we have an irrational number, then the process of converting the denominator to a rational number by multiplying the numerator and denominator by a suitable number, is called rationalisation.

Laws of Radicals:

Let a>0 be a real number, and let p and q be rational numbers, then we have:

i) $(a^p \text{ x } a^q) = a^{(p+q)}$

ii) $(a^p)^q = a^{pq}$

iii)$a^p / a^q = a^{(p-q)}$

iv) $a^p \text{ x } b^p = (ab)^p$

Example : Simplify (i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{3}}$ (ii) $\left(3^{\frac{1}{5}}\right)^4$

(iii) $\dfrac{7^{\frac{1}{5}}}{7^{\frac{1}{3}}}$ (iv) $13^{\frac{1}{5}} \cdot 17^{\frac{1}{5}}$

Solution :

(i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{3}} = 2^{\left(\frac{2}{3}+\frac{1}{3}\right)} = 2^{\frac{3}{3}} = 2^1 = 2$

(ii) $\left(3^{\frac{1}{5}}\right)^4 - 3^{\frac{4}{5}}$

(iii) $\dfrac{7^{\frac{1}{5}}}{7^{\frac{1}{3}}} = 7^{\left(\frac{1}{5}-\frac{1}{3}\right)} = 7^{\frac{3-5}{15}} = 7^{\frac{-2}{15}}$

(iv) $13^{\frac{1}{5}} \cdot 17^{\frac{1}{5}} = (13 \times 17)^{\frac{1}{5}} = 221^{\frac{1}{5}}$

Let a and b be positive real numbers. Then

(i) $\sqrt{ab} = \sqrt{a}\sqrt{b}$

(ii) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

(iii) $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right) = a - b$

(iv) $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right) = a^2 - b$

(v) $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{c}+\sqrt{d}\right) = \sqrt{ac}+\sqrt{ad}+\sqrt{bc}+\sqrt{bd}$

(vi) $\left(\sqrt{a}+\sqrt{b}\right)^2 = a + 2\sqrt{ab} + b$

Example : Simplify the following expressions:

(i) $\left(5+\sqrt{7}\right)\left(2+\sqrt{5}\right)$

(ii) $\left(5+\sqrt{5}\right)\left(5-\sqrt{5}\right)$

(iii) $\left(\sqrt{3}+\sqrt{7}\right)^2$

(iv) $\left(\sqrt{11}-\sqrt{7}\right)\left(\sqrt{11}+\sqrt{7}\right)$

Solution : (i) $\left(5+\sqrt{7}\right)\left(2+\sqrt{5}\right) = 10 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{35}$

(ii) $\left(5+\sqrt{5}\right)\left(5-\sqrt{5}\right) = 5^2 - \left(\sqrt{5}\right)^2 = 25 - 5 = 20$

(iii) $\left(\sqrt{3}+\sqrt{7}\right)^2 = \left(\sqrt{3}\right)^2 + 2\sqrt{3}\sqrt{7} + \left(\sqrt{7}\right)^2 = 3 + 2\sqrt{21} + 7 = 10 + 2\sqrt{21}$

(iv) $\left(\sqrt{11}-\sqrt{7}\right)\left(\sqrt{11}+\sqrt{7}\right) = \left(\sqrt{11}\right)^2 - \left(\sqrt{7}\right)^2 = 11 - 7 = 4$

Worksheet

Practices at Home

Simplify each of the following expressions:

(i) $\left(3+\sqrt{3}\right)\left(2+\sqrt{2}\right)$ (ii) $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)$

(iii) $\left(\sqrt{5}+\sqrt{2}\right)^2$ (iv) $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

Find : (i) $64^{\frac{1}{2}}$ (ii) $32^{\frac{1}{5}}$ (iii) $125^{\frac{1}{3}}$

Find : (i) $9^{\frac{3}{2}}$ (ii) $32^{\frac{2}{5}}$ (iii) $16^{\frac{3}{4}}$ (iv) $125^{\frac{-1}{3}}$

Simplify : (i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$ (ii) $\left(\frac{1}{3^3}\right)^7$ (iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$ (iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$