पुUना International School

Shree Swaminarayan Gurukul, Zundal

Grade – 9

MATHS

Specimen copy

Year 21-22

INDEX

CHAPTER 5

Introduction to Euclid's Geometry

(Ex. 5.1)

1. Which of the following statements are true and which are false? Give reasons for your answers.

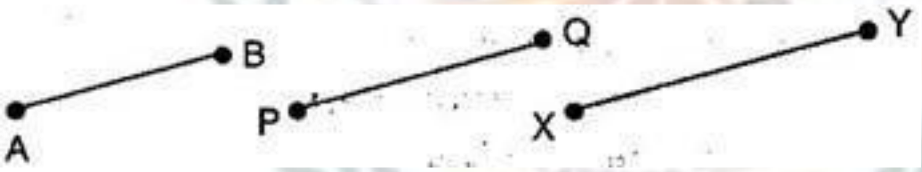
(i) **Only one line can pass through a single point.**

(ii) **There are infinite numbers of lines which pass through two distinct points.**

(iii) **A terminated line can be produced indefinitely on both the sides.**

(iv) **If two circles are equal, then their radii are equal.**

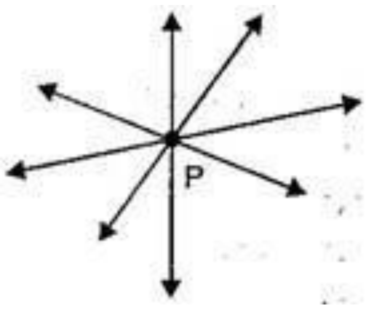
(v) **In Fig. 5.9, if AB = PQ and PQ = XY, then AB = XY**



Ans.(i) False

Correct statement: Infinite many lines can pass through a single point.

This is self-evident and can be seen visually by the student given below:



(ii) **False** because the given statement contradicts the postulate I of the Euclid that assures that there is a unique line that passes through two distinct points.

Through two points P and Q a unique line can be drawn.

(iii) True

Reason: We need to consider Euclid's Postulate 2: "A terminated line can be produced indefinitely."

(iv) True

Reason: Let us consider two circles with same radii.

We can conclude that, when we make the two circles overlap with each other, we will get a superimposed figure of the two circles.

Therefore, we can conclude that the radii of both the circles will also coincide and will be same.

(v) True

Reason: We are given that AB = PQ and PQ = XY.

By Euclid's axiom 1 i.e., things which are equal to the same thing are equal to one another.

Therefore, we can conclude that AB, PQ and XY are the lines with same dimensions, and hence if AB = PQ and PQ = XY, then AB = XY.

(iii) Give a definition for each of the following terms. Are there other terms that need to be defined first? What are they, and how might you define them?

(iii) parallel lines

(iv) perpendicular lines

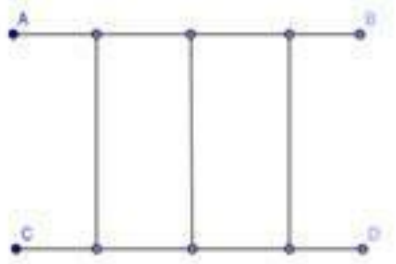
(v) line segment

(vi) radius of a circle

(v) Square

Ans. (i) Parallel lines

Two lines are said to be parallel, when the perpendicular distance between these lines is always constant or we can say that the lines that never intersect each other are called as parallel lines.



We need to define line first, in order to define parallel lines.

(i) Perpendicular lines

Two lines are said to be perpendicular lines, when angle between these two lines is 90°.

We need to define line and angle, in order to define perpendicular lines.

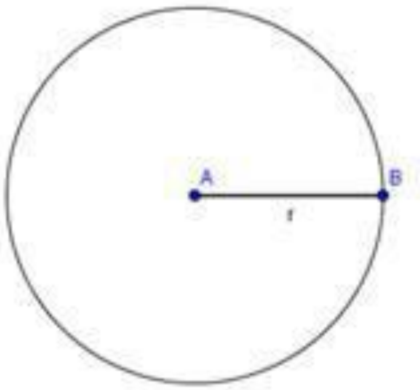
(ii) Line segment

A line of a fixed dimension between two given points is called as a line segment.

We need to define line and point, in order to define a line segment.

(i) Radius of a circle

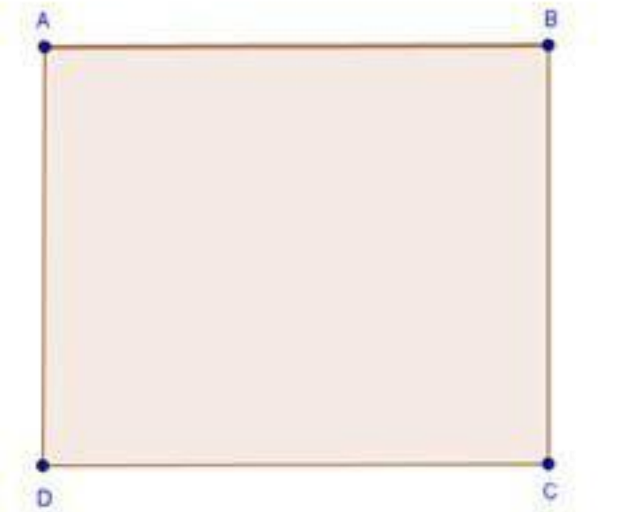
The distance of any point lying on the boundary of a circle from the center of the circle is called as radius of a circle.



We need to define circle and center of a circle, in order to define radius of a circle.

(ii) Square

A quadrilateral with all four sides equal and all four angles of 90° is called as a square.



We need to define quadrilateral and angle, in order to define a square.

3. Consider the two 'postulates' given below:

(iii) Given any two distinct points A and B, there exists a third point C, which is between A and B.

(iv) There exists at least three points that are not on the same line.

Do these postulates contain any undefined terms? Are these postulates consistent ? Do they follow from Euclid's postulates ? Explain.

Ans. We are given with following two postulates

(iv) Given any two distinct points A and B, there exists a third point C, which is between A and B.

(v) There exists at least three points that are not on the same line.

The undefined terms in the given postulates are point and line.

The two given postulates are consistent, as they do not refer to similar situations and they refer to two different situations.

We can also conclude that, it is impossible to derive at any conclusion or any statement that contradicts any well-known axiom and postulate.

The two given postulates do not follow from the postulates given by Euclid.

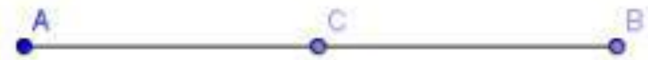
The two given postulates can be observed following from the axiom, "Given two distinct points, there is a unique line that passes through them".

(v) If a point C lies between two points A and B such that AC = BC, then prove that $AC = \frac{1}{2}AB$ **. Explain by drawing the figure.**

Ans. We are given that a point *C* lies between two points *B* and *C*, such that $AC = BC$.

We need to prove that $AC = \frac{1}{2}AB$.

Let us consider the given below figure.



We are given that $AC = BC$.…(i)

An axiom of the Euclid says that "If equals are added to equals, the wholes are equal." Let us add *AC* to both sides of equation (i).

$AC + AC = BC + AC.$

An axiom of the Euclid says that “Things which coincide with one another are equal to one another.”

We can conclude that $BC + AC$ coincide with AB, or $AB =$

$BC + AC$.…(ii)

An axiom of the Euclid says that “Things which are equal to the same thing are equal to one another.”

From equations (i) and (ii), we can conclude that $AC +$

$AC = AB$, or$2AC = AB$.

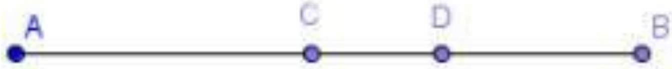
An axiom of the Euclid says that “Things which are halves of the same things are equal to one another.”

Therefore, we can conclude that $AC = \frac{1}{2} AB$.

5. In the above question, point C is called a mid-point of line segment AB, prove that every line segment has one and only one mid-point.

Ans. We need to prove that every line segment has one and only one mid-point.

Let us consider the given below line segment AB and assume that C and D are the mid-points of the line segment AB.



If C is the mid-point of line segment AB, then

$AC = CB$.

An axiom of the Euclid says that “If equals are added to equals, the wholes are equal.” $AC + AC =$

$CB + AC$.-----------(i)

From the figure, we can conclude that $CB + AC$ will coincide with AB.

An axiom of the Euclid says that "Things which coincide with one another are equal to one another."

$AC + AC = AB$.--------------------(ii)

An axiom of the Euclid says that "Things which are equal to the same thing are equal to one another."

Let us compare equations (i) and (ii), to get

$AC + AC = AB$, or $2AC = AB$.----------------(iii)

If D is the mid-point of line segment AB, then

$AD = DB$.

An axiom of the Euclid says that "If equals are added to equals, the wholes are equal."

$AD + AD = DB + AD$.----------------(iv)

From the figure, we can conclude that $DB + AD$ will coincide with AB.

An axiom of the Euclid says that "Things which coincide with one another are equal to one another."

$AD + AD = AB$.--------------------(v)

An axiom of the Euclid says that "Things which are equal to the same thing are equal to one another."

Let us compare equations (iv) and (v), to get

$AD + AD = AB$, or

$2AD = AB$.(vi)

An axiom of the Euclid says that "Things which are equal to the same thing are equal to one another."

Let us compare equations (iii) and (vi), to get

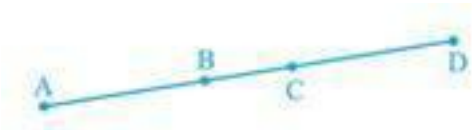
$2AC = 2AD$.

An axiom of the Euclid says that “Things which are halves of the same things are equal to one another.”

$AC = AD$.

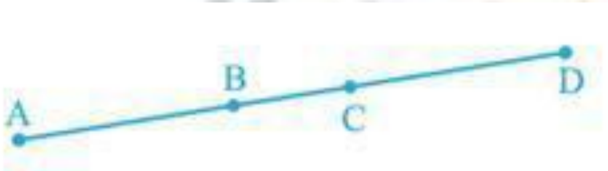
Therefore, we can conclude that the assumption that we made previously is false and a line segment has one and only one mid-point.

6. In the following figure, if AC = BD, then prove that AB = CD.



Ans. We are given that $AC = BD$.

We need to prove that $AB = CD$ in the figure given below.



From the figure, we can conclude that

$AC = AB + BC$, and

$BD=CD+BC$.

An axiom of the Euclid says that “Things which are equal to the same thing are equal to one another.”

$AB + BC = CD + BC$. -------------(i)

An axiom of the Euclid says that “when equals are subtracted from equals, the remainders are also equal.”

We need to subtract BC from equation (i), to get

$AB+BC-BC=CD+BC-BC$ $AB = CD$.

Therefore, we can conclude that the desired result is proved.

7. Why is axiom 5, in the list of Euclid's axioms, considered as a 'universal truth'? (Note that the question if not about fifth postulate)

Ans. We need to prove that Euclid's fifth axiom is considered as a universal truth.

Euclid's fifth axiom states that "the whole is greater than the part."

The above given axiom is a universal truth. We can apply the fifth axiom not only mathematically but also universally in daily life.

Mathematical proof:

Let us consider a quantity *z*, which has different parts as *a*, *b*, *x* and *y*.

$z = a + b + x + y$.

Therefore, we can conclude that *z* will always be greater than its corresponding parts *a*, *b*, *x* and *y*.

Universal proof:

We know that Mumbai is located in Maharashtra and Maharashtra is located in India.

In other words, we can conclude that Mumbai is a part of Maharashtra and Maharashtra is a part of India.

Therefore, we can conclude that whole India will be greater than Mumbai or Maharashtra or both.

Therefore, we can conclude that Euclid's fifth axiom is considered as a 'Universal truth'.

CHAPTER 5
Introduction to Euclid's Geometry

(Ex. 5.2)

1. **How would you rewrite Euclid's fifth postulate so that it would be easier to understand?**

Ans. We need to rewrite Euclid's fifth postulate so that it is easier to understand.

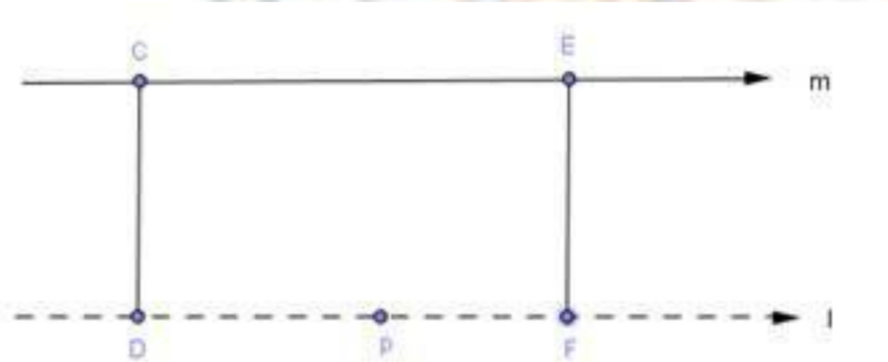
We know that Euclid's fifth postulate states that "No intersection of lines will take place when the sum of the measures of the interior angles on the same side of the falling line is exactly 180°."

We know that Play fair's axiom states that "For every line *l* and for every point *P* not lying on l, there exists a unique line m passing through *P* and parallel to *l*".

The above mentioned Play fair's axiom is easier to understand in comparison to the Euclid's fifth postulate.

Let us consider a line *l* that passes through a point *p* and another line *m*. Let these lines be at a same plane.

Let us consider the perpendicular *CD* on *l* and *FE* on *m*.



From the above figure, we can conclude that *CD* = *EF*.

Therefore, we can conclude that the perpendicular distance between lines *m* and *l* will be constant throughout, and the lines *m* and *l* will never meet each other or in other words, we can say that the lines *m* and *l* are equidistant from each other.

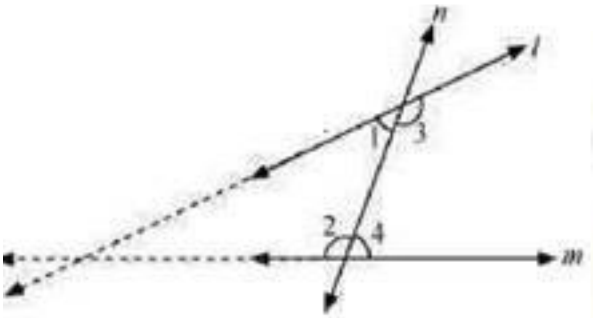
2. Does Euclid's fifth postulate imply the existence of parallel lines ? Explain.

Ans. We need to verify whether Euclid's fifth postulate imply the existence of parallel lines or not.

The answer to the above statement is Yes.

Let us consider two lines *m* and *l*.

In the figure given below, we can conclude that the lines *m* and *l* will intersect further.

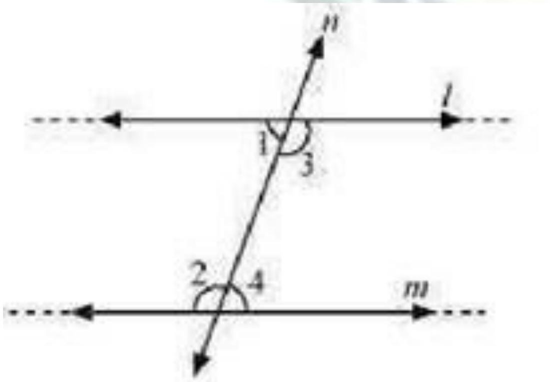


From the figure, we can conclude that

$\angle 1 + \angle 2 < 180^\circ$, and $\angle 3 + \angle 4 > 180^\circ$.

We know that Euclid's fifth postulate states that "No intersection of lines will take place when the sum of the measures of the interior angles on the same side of the falling line is exactly 180°."

Let us consider lines *l* and *m*.



From the above figure, we can conclude that lines *l* and *m* will never intersect from either side.

Therefore, we can conclude that the lines *l* and *m* are parallel.

WORK SHEET

CHAPTER – 5 Std -9th

Introduction to Euclid's Geometry

1.A surface is that which has

a. length and breadth b. length only c. breadth only d. length and height

2. The number of lines that can pass through a given point is

a. Two b. None c. only one d. Infinitely many

3. The number of dimensions, a solid has

a. 1 b. 2 c. 3 d. 0

4. Two plane intersect each other to form a

a. plane b. point c. straight line d. angle

5. Which of the following need a proof?

a. Axiom b. Theorem c. postulate d. Definition

6. Euclid's stated that all right angles are equal to each other in the form of:

a. an axiom b. a definition c. a postulate d. a proof

7. If the point F lies in between M and N and C is midpoint of MF then :

a. MC + FN=MN b. MF + CF=MN c. MC + CN=MN d. CF + CN=MN

8. The number of interwoven isosceles triangle in sriyantra (in the Atharvedas) is

a. 7 b. 8 c. 9 d. 11

9. If PQ is a line segment of length 12 cm and R is a point in its interior, then $PR^2 + QR^2 + 2PR.QR$ equal.

a. 12 b. 13 c. 144 d. 169

10. Greek's emphasized on.

a. inductive reasoning b. deductive reasoning

c. Both (a) and (b) d. practical use of geometry

Solve

11. Write first postulate 1.

12 Write first postulate 2

13 Write first postulate 3

14 Write first postulate 4

15 If a point C lies between two point A and B such that AB = BC , then prove that

AC = $\frac{1}{2}$ AC. Explain by drawing the figure.

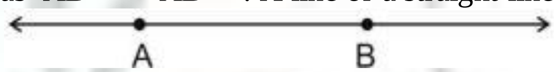
16 In figure, if AC = BD, then prove that AB = CD

Notes
CHAPTER – 6
LINES AND ANGLES

1. **Basic Terms and Definitions**
2. **Intersecting Lines and Non-intersecting Lines**
3. **Pairs of Angles**
4. **Parallel Lines and a Transversal**
5. **Lines Parallel to the same Line**
6. **Angle Sum Property of a Triangle**

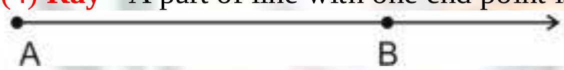
(1) **Point** - We often represent a point by a fine dot made with a fine sharpened pencil on a piece of paper.

(2) **Line** - A line is completely known if we are given any two distinct points. Line AB is represented by as AB←→ AB↔ . A line or a straight line extends indefinitely in both the directions.



(3) **Line segment** - A part (or portion) of a line with two end points is called a line segment.

(4) **Ray -** A part of line with one end point is called a ray.It usually denotes the direction of line



(5) **Collinear points** - If three or more points lie on the same line, they are called collinear points, otherwise they are called non-collinear points.

(6) **Angle** - An angle is the union of two non-collinear rays with a common initial point.

Types of Angles -

(1) **Acute angle** - An acute angle measure between 0o0o and 90o90o

(2) **Right angle** - A right angle is exactly equal to 90o90o

(3) **Obtuse angle** - An angle greater than 90o90o but less than 180o180o

(4) **Straight angle** - A straight angle is equal to 180o180o

(5) **Reflex angle** - An angle which is greater than 180o180o but less than 360o360o is called a reflex angle.

(6) **Complementary angles -** Two angles whose sum is 90o90o are called complementary angles. Let one angle be x, then its complementary angle be (90∘−x).(90∘−x).

(7) **Supplementary angle** - Two angles whose sum is 180o180o are called supplementary angles. Let one angle be x, then its supplementary angle be (180∘−x).(180∘−x).

(8) **Adjacent angles** -Two **angles** are **Adjacent** when they have a common side and a common vertex (corner point) and don't overlap.

(9**) Linear pair** - A **linear pair** of angles is formed when two lines intersect. Two angles are said to be **linear** if they are adjacent angles formed by two intersecting lines. The measure of a straight angle is 180 degrees, so a **linear pair** of angles must add up to 180 degrees

(10**) Vertically opposite angles -** Vertically opposite angles are formed when two lines intersect each other at a point. Vertically opposite angles are always equal.

TRANSVERSAL - A line which intersects two or more given lines at distinct points, is called a transversal of the given line.

(a) Corresponding angles

(b) Alternate interior angles

(c) Alternate exterior angles

(d) Interior angles on the same side of the transversal.

- If a transversal intersects two parallel lines, then

(i) each pair of corresponding angles is equal.

(ii) each pair of alternate interior angles is equal.

(iii) each pair of interior angle on the same side of the transversal is supplementary.

- If a transversal interacts two lines such that, either

(i) any one pair of corresponding angles is equal, or

(ii) any one pair of alternate interior angles is equal or

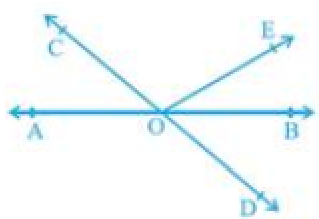
(iii) any one pair of interior angles on the same side of the transversal is supplementary ,then the lines are parallel.

- Lines which are parallel to a given line are parallel to each other.
- The sum of the three angles of a triangle is 180o180o
- The sum of all angles round a point is equal to 360∘.360∘.
- If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

- If two parallel lines are intersected by a transversal, the bisectors of any pair of alternate interior angles are parallel and vice-versa.
- If two parallel lines are intersected by a transversal, then bisectors of any two corresponding angles are parallel and vice-versa.
- If a line is perpendicular to one of the given parallel lines, then it is also perpendicular to the other line.

Ex: 6.1

1. In Figure lines AB and CD intersect at O.
If $\angle AOC+\angle BOE=70\circ\angle AOC+\angle BOE=70\circ$ and $\angle BOD=40\circ\angle BOD=40\circ$,find $\angle BOE\angle BOE$ and reflex $\angle COE\angle COE$



Sol. We are given that $\angle\angle AOC + \angle\angle BOE = 70°$ and $\angle\angle BOD = 40°$
We need to find $\angle\angle BOE$ and reflex $\angle\angle COE$
From the given figure, we can conclude that $\angle\angle AOE$ and $\angle\angle BOE$ form a linear pair.
We know that sum of the angles of a linear pair is 180°
$\therefore \angle AOE + \angle BOE = 180°$
$\because \angle AOE = \angle AOC + \angle\angle\ COE$
$\therefore \angle AOC + \angle COE + \angle\angle BOE = 180°$
$\therefore \angle AOC + \angle BOE + \angle\angle COE = 180°$
$\Rightarrow 70° + \angle COE = 180°$
$\Rightarrow \angle COE = 180° - 70°$
$= 110°$
Reflex $\angle COE = 360° - \angle COE$
$= 360° - 110°$
$= 250°$
$\angle AOC = \angle BOD$ (Vertically opposite angles), or

$\angle BOD + \angle BOE = 70$

But, we are given that $\angle\angle BOD = 40°$.

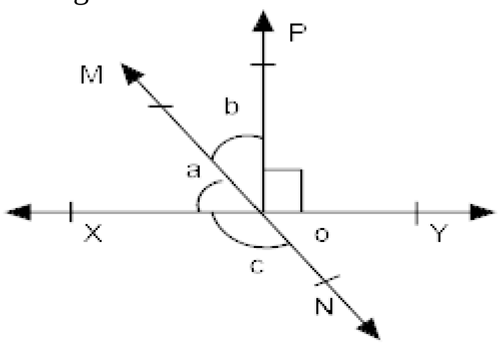
$40° + \angle\angle BOE = 70°$

$\angle BOE = 70° - 40°$

$= 30°$.

Therefore, we can conclude that Reflex $\angle\angle COE = 250°$ and $\angle\angle BOE = 30°$

2. In fig lines XY and MN intersect at O If $\angle POY = 90°$ and a:b = 2:3 find $\angle c$.



Sol. Lines XY and MN intersect at O.

$\therefore \angle C = \angle XON = \angle MOY$ [vertically opposite angle]

$= \angle b + \angle POY = \angle b + \angle POY$

But, $\angle\angle POY = 90°$

$\therefore \angle C = \angle b + 90°$ ...(i)

Also,

$\angle POX = 180° - \angle POY$

$= 180° - 90°$

$= 90°$

$\therefore\therefore a + b = 90°$

But,

a:b = 2:3 [Given]

$a=25\times900a=25\times900$

$= 36°$ ...(ii)

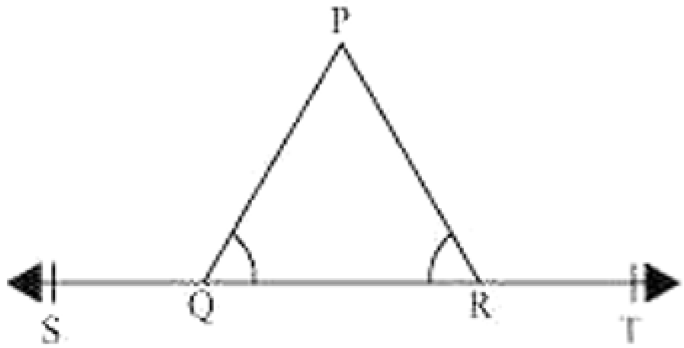
Thus, From (i) and (ii) we get

$b = 90° - 36° = 54°$

$\angle C = 54° + 90°$ (From (1))

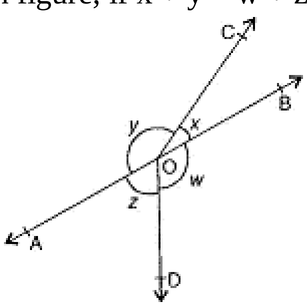
$= 144°$

3. In the given figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$



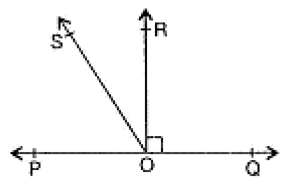
Sol. We need to prove that ∠∠PQS = ∠∠PRT
We are given that ∠∠PQR = ∠∠PRQ
From the given figure, we can conclude that ∠∠PQS and ∠∠PQR, and ∠∠PRQ and ∠∠PRT form a linear pair.
We know that sum of the angles of a linear pair is 180∘180∘
∴∠PQS+∠PQR=180∘,∴∠PQS+∠PQR=180∘, and ...(i)
∠PRQ+∠PRT=180∘.∠PRQ+∠PRT=180∘. ...(ii)
From equation (i) and (ii), we can conclude that
∠PQS+∠PQR=∠PRQ+∠PRT.∠PQS+∠PQR=∠PRQ+∠PRT.
But, ∠∠PQR = ∠∠PRQ
∴∴ ∠∠PQS = ∠∠PRT
Hence, proved.

4. In figure, if x + y = w + z, then prove that AOB is a line.

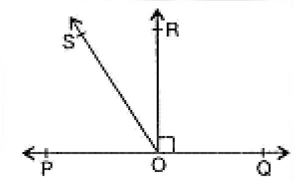


Sol. Given that, x + y = w + z ... (1)
As the sum of all angles round a point is equal to 360^o
∴∴ x + y + w + z = 360^o
∴∴ x + y + x + y = 360^o
∴∴ 2(x + y) = 360^o
∴∴ x+y=36002x+y=36002
∴∴ x + y = 180^o
∴∴ AOB is a line.

5. In the figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that ∠ROS=12(∠QOS−∠POS)∠ROS=12(∠QOS−∠POS)



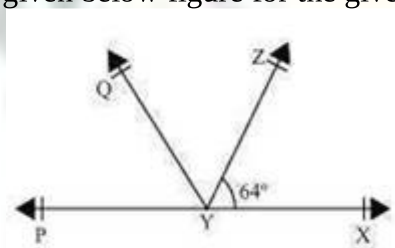
Sol.



Ray OR is perpendicular to line PQ
∴∴ ∠∠QOR = ∠∠POR = 90^o . . . .(1)
∠∠QOS = ∠∠QOR + ∠∠ROS. . . .(2)
∠∠POS = ∠∠POR – ∠∠ROS. . . .(3)
From (2) and (3),
∴∴ ∠∠QOS – ∠∠POS = (∠∠QOR – ∠∠POR) + 2∠∠ROS = 2∠∠ROS . . . [Using (1)]
∴∴ ∠ROS=12(∠QOS−∠POS)

6. If the ∠XYZ=64∘∠XYZ=64∘ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects ∠∠ZYP, find ∠∠ XYQ and reflex ∠∠QYP.

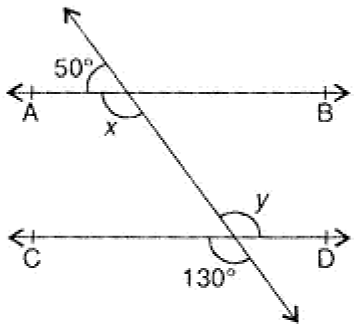
Sol. We are given that ∠∠XYZ = 64^o, XY is produced to P and YQ bisects ∠∠ZYP We can conclude the given below figure for the given situation:



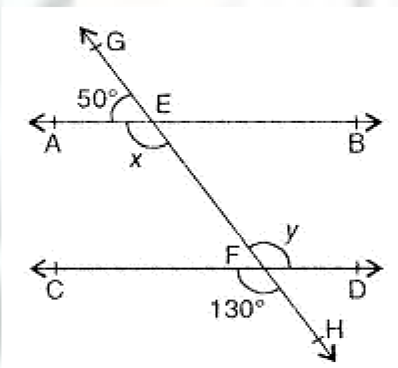
We need to find ∠∠XYQ and reflex ∠∠QYP
From the given figure, we can conclude that ∠∠XYZ and ∠∠ZYP form a linear pair.
We know that sum of the angles of a linear pair is 180^o.
∠∠XYZ + ∠∠ZYP = 180^o
But ∠∠XYZ = 64^o
⇒⇒ 64^o + ∠∠ZYP = 180^o
⇒⇒ ∠∠ZYP = 116^o
Ray YQ bisects ∠∠ZYP,or
∠∠QYZ = ∠∠QYP = 116o2116o2 = 58^o
∠∠XYQ = ∠∠QYZ + ∠∠XYZ
= 58^o + 64^0 = 122^o.
Reflex ∠∠QYP = 360∘360∘ - ∠∠QYP
= 360^0 - 58^0
= 302^0.
Therefore, we can conclude that ∠∠XYQ = 122^o and Reflex ∠∠QYP = 302^o

Ex: 6,2

1. In figure, find the values of x and y and then show that AB || CD.



Sol.



$\angle\angle AEG + \angle\angle AEH = 180^\circ$ . . . [Linear pair]

$\therefore\therefore$ $50^\circ + x = 180^\circ$

$\therefore\therefore$ $x = 180^\circ - 50^\circ = 130^\circ$ . . . (1)

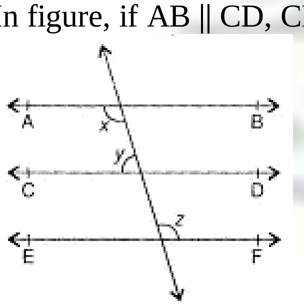
$y = 130^\circ$ . . . [Vertically opposite angles] . . . (2)

$x = y$ . . . . [From (1) and (2)]

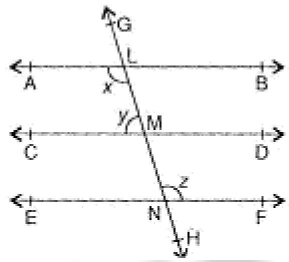
But x and y are alternate interior angles and they are equal.

$\therefore\therefore$ AB || CD

2. In figure, if AB || CD, CD || EF and y : z = 3 : 7, find x.



Sol.



As AB || CD and CD || EF

∴∴ AB || EF

∴∴ x = z . . . . [Alternate interior angles] . . . (1)

x + y = 180^o . . . (2)

z + y = 180^o . . . [From (1) and (2)]

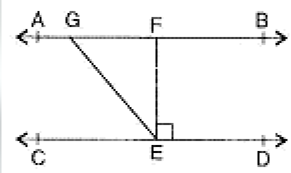
y : z = 3 : 7

Sum of the ratios = 3 + 7 = 10

∴∴ y=310×1800=540 and z=710×1800=1260y=310×1800=540 and z=710×1800=1260

∴∴ x = z = 126^o

3. In figure, if AB || CD, EF ⊥ CD and ∠∠GED = 126^o. Find ∠∠AGE, ∠∠GEF and ∠∠FGE.



Sol. ∠∠AGE = ∠∠GED = 126^o . . . [Alternate interior angles]

∠∠GED = 126^o

∴∴ ∠∠GEF + ∠∠FED = 126^o

∴∴ ∠∠GEF + 90^o = 126^o . . . [As EF ⊥ CD ∴∴ ∠∠FED = 90^o]

∴∴ ∠∠GEF = 126^o – 90^o = 36^o

∠∠GEC + ∠∠GEF + ∠∠FED = 180^o

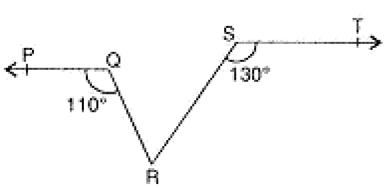
∠∠GEC + 36^o + 90^o = 180^o

∠∠GEC + 126^o = 180^o

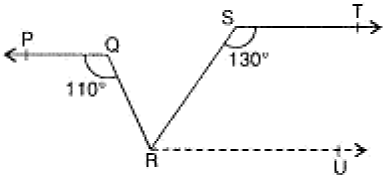
∠∠GEC = 180^o – 126^o = 54^o

∠∠FGE = ∠∠GEC = 54^o . . . . [Alternate interior angles]

4. In figure, if PQ || ST, ∠∠PQR = 110^o and ∠∠RST = 130^o. Find ∠∠QRS.

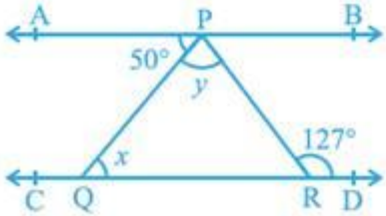


Sol. Draw a line RU parallel to ST through point R.



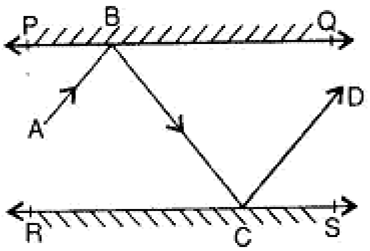
∠∠RST + ∠∠SRU = 180°
∴∴ 130° + ∠∠SRU = 180°
∴∴ ∠∠SRU = 180° – 130° = 50° . . . (1)
∠∠QRU = ∠∠PQR = 110° . . . . [Alternate interior angles]
∴∴ ∠∠QRS + ∠∠SRU = 110°
∴∴ ∠∠QRS + 50° = 110° . . . [Using (1)]
∴∴ ∠∠QRS = 110° – 50° = 60°

5. In the given figure, if AB || CD, ∠∠ APQ = 50° and ∠∠PRD = 127°, find x and y.

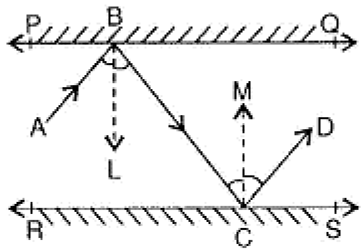


Sol. We are given that AB∥CD,∠APQ=50∘AB∥CD,∠APQ=50∘ and ∠∠PRD = 127°
We need to find the value of x and y in the figure.
∠∠APQ = x = 50° (Alternate interior angles)
∠∠PRD = ∠∠APR = 127° (Alternate interior angles)
∠∠APR = ∠∠QPR + ∠∠APQ.
127° = y + 50°
⇒⇒ y = 77°.
Therefore, we can conclude that x = 55° and y = 77°
Alternatively, 127° = y + x (because exterior angle is equal to the sum of interior opposite angles).
so, 127° = y + 50°
which gives, x = 50° and y = 77°

6. In figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B. The reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.



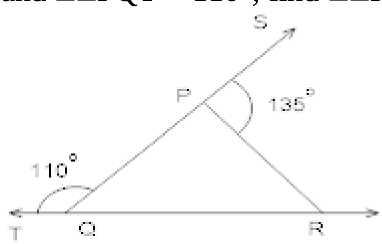
Sol. Draw ray BL ⊥⊥ PQ and ray CM ⊥⊥ RS.



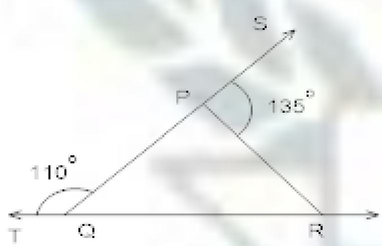
BL ⊥⊥ PQ, CM ⊥⊥ RS and PQ ∥∥ RS
BL || CM
∠∠LBC = ∠∠MCB . . . (1)
∠∠ABL = ∠∠LBC . . . [Angle of incident = Angle of reflection] . . . .(2)
∠∠MCB = ∠∠MCD . . . [Angle of incident = Angle of reflection] . . . . (3)
∠∠ABL = ∠∠MCD . . . [From (1), (2) and (3)]
∠∠LBC + ∠∠ABL = ∠∠MCB + ∠∠MCD . . . [Adding (1) and (4)]
∠∠ABC = ∠∠BCD are alternate interior angles and are equal.
∴∴ AB || CD

<u>Ex: 6.3</u>

1. In fig. sides QP and RQ of △△PQR are produced to points S and T respectively. If ∠∠SPR = 135° and ∠∠PQT = 110°, find ∠∠PRQ

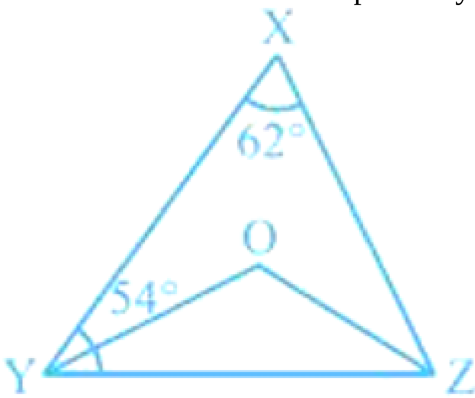


Sol.

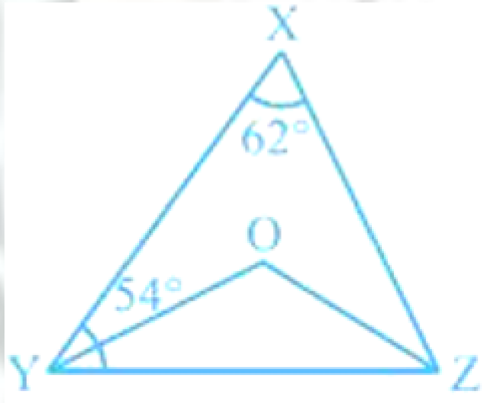


∠PQT+∠PQR=180o∠PQT+∠PQR=180o
110o+∠PQR=180o110o+∠PQR=180o
∠PQR=180o−110o∠PQR=180o−110o
∠PQR = 70o∠PQR = 70o
Also∠SPR = ∠PQR + ∠PRQ∠SPR = ∠PQR + ∠PRQ [Interior angle theorem]
135o=70o+∠PRQ135o=70o+∠PRQ
∠PRQ = 135o−70o∠PRQ = 135o−70o
∠PRQ = 65o

2. In the given figure, ∠∠X = 62°, ∠∠XYZ = 54°. If YO and ZO are the bisectors of ∠∠XYZ and ∠∠XZY respectively of ∆∆XYZ,f ind ∠∠OZY and ∠∠YOZ.

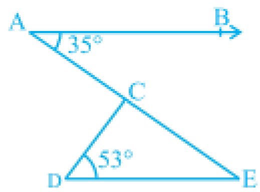


Sol. We are given that ∠X=62∘,∠XYZ=54∘∠X=62∘,∠XYZ=54∘ and YO and ZO are bisectors of ∠XYZ and ∠XZY∠XYZ and ∠XZY respectively.



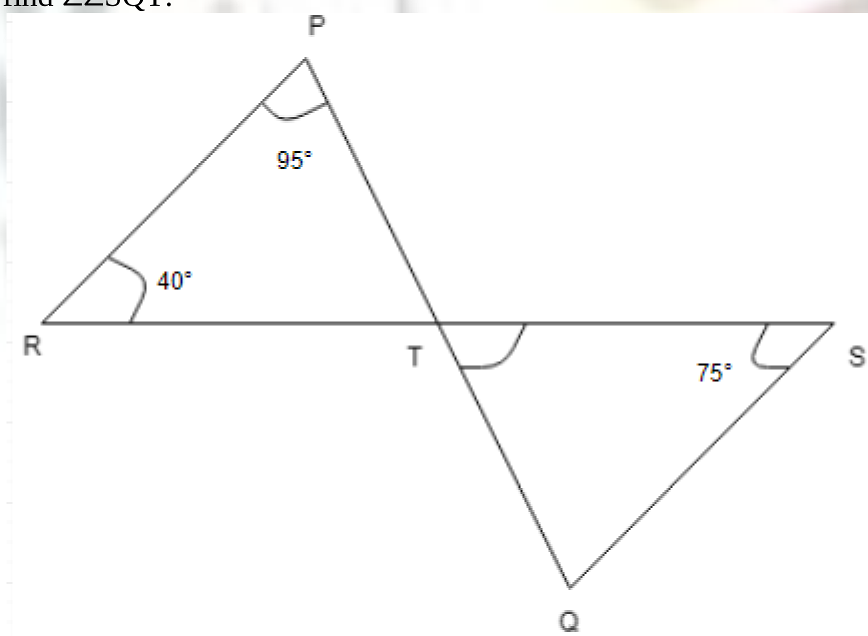
In triangle XYZ, ∠∠X + ∠∠Y + ∠∠Z = 180°
⇒⇒ 62° + 54° + ∠∠Z = 180°
⇒⇒ 116° + ∠∠Z = 180°
∠∠Z = 64°
Now, ∠OZY=12∠XZY∠OZY=12∠XZY
So, we get, ∠∠OZY = 32°
Also, ∠OYZ=12∠XYZ∠OYZ=12∠XYZ
Hence, ∠∠OYZ = 27°
Again, from the figure, we can conclude that in ∆∆OYZ
∠OYZ+∠OZY+∠YOZ=180∘.∠OYZ+∠OZY+∠YOZ=180∘. (Angle sum property)
27° + 32° + ∠∠YOZ = 180°
⇒⇒ 59° + ∠∠YOZ = 180°
⇒⇒ ∠∠YOZ = 121°.
Therefore, we can conclude that ∠∠YOZ = 121°. and ∠∠OZY = 32°

3. In the given figure, if AB || DE, ∠∠BAC = 35° and ∠∠CDE = 53° find ∠∠DCE.



Sol. We are given that AB∥DEAB∥DE, ∠∠BAC = 35° and ∠∠CDE = 53°
We need to find the value of ∠DCE∠DCE in the figure given below.
From the figure, we can conclude that ∠∠BAC = ∠∠CED = 35° (Alternate interior angles) From the figure, we can conclude that in △△DCE
∠DCE+∠CED+∠CDE=180∘.∠DCE+∠CED+∠CDE=180∘.(Angle sum property)
∠DCE+35∘+53∘=180∘∠DCE+35∘+53∘=180∘
⇒∠DCE+88∘=180∘⇒∠DCE+88∘=180∘
⇒∠DCE=92∘.⇒∠DCE=92∘.
Therefore, we can conclude that ∠DCE=92∘∠DCE=92∘

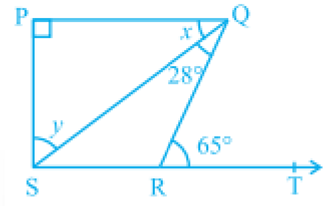
4. In figure if lines PQ and RS intersect at point T. Such that ∠∠PRT = 40° ∠∠RPT = 95° and ∠∠TSQ = 75°, find ∠∠SQT.



Sol. In △△PRT
∠P + ∠R + ∠1 = 1800∠P + ∠R + ∠1 = 1800 [By angle sum property]
950+400+∠1=1800950+400+∠1=1800
∠1 = 1800−1350∠1 = 1800−1350
∠1 = 450∠1 = 450
∠1 = ∠2∠1 = ∠2 [vertically opposite angle]
∠2 = ∠450∠2 = ∠450
In △△TQS ∠∠2 + ∠∠Q + ∠∠S = 180°
450+∠Q+750=1800450+∠Q+750=1800

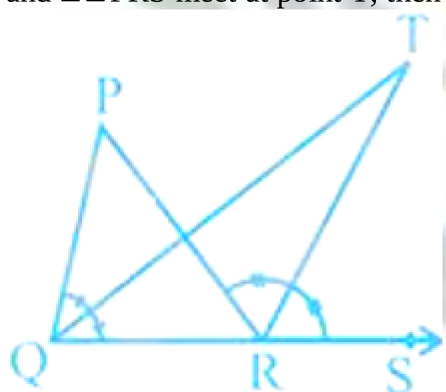
∠Q + 1200=1800∠Q + 1200=1800
∠Q = 1800−1200∠Q = 1800−1200
∠Q = 600∠Q = 600
∠SQT = 600

5. In figure, if PQ ⊥ PS, PQ || SR, ∠∠SQR = 28º and ∠∠QRT = 65º, then find the values of x and y.



Sol. ∠∠QRT = ∠∠RQS + ∠∠QSR . . . [Sum of two interior opposite angles equal to exterior angle]
∴∴ 65º = 28º + ∠∠QSR
∴∴ ∠∠QSR = 65º – 28º = 37º
∠∠QPS = 90º . . . [PQ ⊥ SP]
∠∠QPS = 90º
As PQ || SR
∴∴ ∠∠QPS + ∠∠PSR = 180º . . . [sum of consecutive interior angles on the same side of transversal]
∴∴ 90º + ∠∠PSR = 180º
∴∴ ∠∠PSR = 180º – 90º = 90º
∠∠PSR + ∠∠QSR = 90º
∴∴ y + 37º = 90º
∴∴ y = 90º – 37º = 53º
In PQS,
∠∠PQS + ∠∠QSP + ∠∠QPS = 180º . . . [Sum of all the angles of a triangle]
∴∴ x + y + 90º = 180º
∴∴ x + 53º + 90º = 180º
∴∴ x + 143º = 180º
∴∴ x = 180º – 143º = 37º

6. In the given figure, the side QR of ∆∆PQR is produced to a point S. If the bisectors of ∠∠PQR and ∠∠PRS meet at point T, then prove that ∠QTR=12∠QPR∠QTR=12∠QPR.

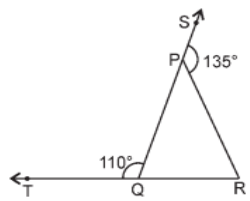


Sol. We need to prove that ∠QTR=12∠QPR∠QTR=12∠QPR in the figure given below.
We know that "If a side of a triangle is produced, then the exterior angle so formed is equal to the sum

of the two interior opposite angles.”
From the figure, we can conclude that in △△QTR, ∠∠TRS is an exterior angle
∠QTR+∠TQR=∠TRS, or∠QTR+∠TQR=∠TRS, or
∠QTR=∠TRS−∠TQR∠QTR=∠TRS−∠TQR ...(i)
Also, in △△PQR, ∠∠PRS is an exterior angle
∠QPR+∠PQR=∠PRS.∠QPR+∠PQR=∠PRS.
We are given that QT and RTQT and RT are angle bisectors of ∠PQR and ∠PRS.∠PQR and ∠PRS.
∠QPR+2∠TQR=2∠TRS∠QPR+2∠TQR=2∠TRS
∠QPR=2(∠TRS−∠TQR).∠QPR=2(∠TRS−∠TQR).
We need to substitute equation (i) in the above equation, to get
∠QPR=2∠QTR, or∠QPR=2∠QTR, or
∠QTR=12∠QPR.∠QTR=12∠QPR.
Therefore, we can conclude that the desired result is proved.

Work sheet
Chap 6

1. If angle is such that six times its compliment is 12° less than twice its supplement, then the value of angle is
   a. 38° b. 48° c. 58° d. 68°
2. If angles measures X and Y form a complimentary pair , then which of the following measures of angle will form a supplementary pair?
   a. (x + 47°), (y + 43°)
   b. (x - 23°), (y + 23°)
   c. (x - 47°), (y - 43°)
   d. No such pair is possible.
3. If one angle of a triangle is equal to the sum of the other two angles, then the triangle is.
   a. An isosceles triangle
   b. An obtuse triangle
   c. An equilateral triangle
   d. A right triangle
4. An exterior angle of a triangle is 105° and its two interior opposite angles are equal. Each of these equal angles is.
   a. 37 ½° b. 52½° c.72½° d.75°
5. Sides QP and RQ of triangle PQR are produced to point S and T respectively if angle SPR= 135° and angle PQT =110 ° find angle PRQ



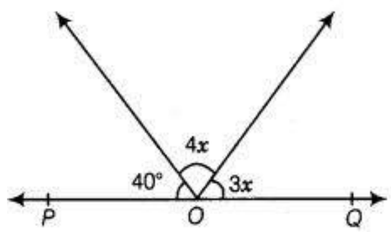
a. 35°

b. 45°

c. 55°

d. 65°

6. In the figure, POQ is a line. The value of x is.



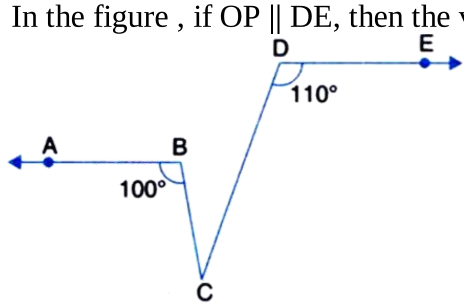
a. 20°

b. 25°

c. 30°

d. 35°

7. In the figure , if OP || DE, then the value of ∠BCD is



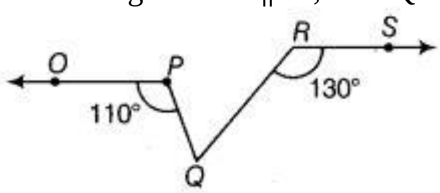
a. 30°

b. 45°

c. 55°

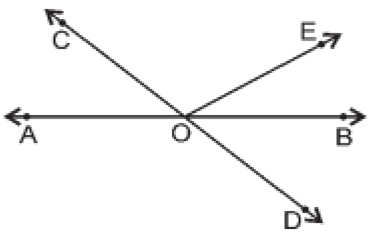
d. 65°

8. In the figure if OP||RS, ∠OPQ = 110° and ∠QRS = 130°, then ∠PQR is equal to.



a. 40° b. 50° c.60° d.70°

9. If one of a triangle is equal to the sum of the other two, then triangle is a / an
   a. Acute angle triangle
   b. Obtuse angle triangle
   c. Right angle triangle
   d. None of these
10. In the given figure, lines AB and CD intersect at O. If ∠AOC + ∠BOE = 80° and ∠BOD = 30°, then ∠BOE equals to



a. 30° b.40° c.50° d.60°