



MATHEMATICS Specimen Copy 2021-22

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CHAPTER NAME: UNDERSTANDING QUADRILATERALS

Curve: A figure formed on a plane surface by joining a number of points without lifting a pencil is called a curve.

Open Curve: A curve which does not end at the same starting point or which does not cut itself is called an open curve.

Closed Curve: A curve which cut itself or which starts and ends at the same point is called a closed curve.

Simple Closed Curve: A closed curve called a simple closed curve which does not intersect itself.

Polygon: A polygon is a closed figure bounded by three or more line segments such that each line segment intersects exactly two other points (vertices) as shown in the following figures.



Quadrilateral: A simple closed figure bounded by four line segments is called a quadrilateral, it has four sides i.e., AB, BC, CD and AD and four vertices as A, B, C and D and the sum of all angles of a quadrilateral is 360°.



Parallelogram: A quadrilateral in which opposite sides are parallel and equal is called parallelogram; written as || gm. and the diagonals of a parallelogram bisect each other.

Properties:

- Opposite sides are equal and parallel.
- Opposite angles are equal.
- Diagonals bisect each other.



Rectangle: A parallelogram each of whose angle is 90° and diagonals are equal, is called a rectangle. **Properties:**

- Opposite sides are equal and parallel.
- Each angle is a right angle.
- Diagonals are equal.
- Diagonals bisect each other.



Square: A quadrilateral in which all sides and angles are equal, is called a square. **Properties:**

- All the sides are equal and parallel.
- Each angle is a right angle.
- Diagonals are equal.
- Diagonals bisect each other at a right angle.



Rhombus: A parallelogram having all its sides equal, is called a rhombus. **Properties:**

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- All the side are equal.
- Opposite angles are equal.
- Diagonals bisect each other at a right angle.



Trapezium: A quadrilateral in which two opposite sides are parallel and the other two opposite sides are non-parallel, is called a trapezium.

If two non-parallel sides of a trapezium are equal, then it is called an isosceles trapezium.

The line segment joining the mid-points of non-parallel sides of a trapezium is called it's median.



Kite: A quadrilateral in which two pairs of adjacent sides are equal, is called a kite.

Properties:

Diagonals bisect each other at the right angle. In the figure, $m \angle B = m \angle D$, but $m \angle A \neq m \angle C$



Paper is a very common example of a plane surface. The curve obtained by joining a number of points consecutively without lifting the pencil from the paper is called a plane curve. A circle is a very common example of a plane curve.

A polygon is a simple closed curve formed of only line segments. A triangle is a very common example of a polygon.



Classification of Polygons

A polygon is said to be a triangle, quadrilateral, pentagon, hexagon, heptagon, octagon, nonagon, decagon,, n-gon according as its number of sides (or vertices) is 3, 4, 5, 6, 7, 8, 9, 10,, n respectively.

Diagonals

The line-segment joining any two non-consecutive vertices of a polygon is called its diagonal.

Convex and Concave Polygons

A polygon is said to be convex if it has no portion of its diagonals in its exterior otherwise it is said to be a concave polygon.

Regular and Irregular Polygons

A polygon which is both 'equiangular' (has all angles of equal measure) and 'equilateral' (has all sides of equal measure) is called a regular polygon, for example, a square, an equilateral triangle.

A polygonwhich is equiangular but not equilateral is called an irregular polygon. For example; a rectangle.

The sum of the measures of the three angles of a triangle is 180°.

The sum of the measures of the exterior angles of a polygon is 360°.

Kinds of Quadrilaterals

Kinds of Quadrilaterals

The important types of quadrilaterals are as follows:

- Trapezium
- Kite
- Parallelogram
- Rhombus
- Rectangle
- Square.
- Trapezium
- A quadrilateral which has only one pair of parallel sides is called a trapezium.

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- Kite
- A quadrilateral, which has exactly two pairs of equal consecutive sides, is called a kite.
- - Parallelogram A quadrilateral whose opposite sides are parallel is called a parallelogram.
- •
- Elements of a Parallelogram
 The elements of a parallelogram are as follows:

The elements of a parallelogram are as follows:

- two pairs of opposite sides
- four pairs of adjacent sides

- two pairs of opposite equal angles
- four pairs of adjacent angles.

The opposite sides of a parallelogram are of equal length.

The opposite angles of a parallelogram are of equal measure.

The diagonals of a parallelogram bisect each other at their point of intersection.

Rhombus

A quadrilateral whose all the four sides are of equal length is called a rhombus.

The diagonals of a rhombus are perpendicular bisectors of each other.

Rectangle

A rectangle is a parallelogram with equal angles.

The diagonals of a rectangle are of equal length.

Square

A square is a rectangle whose all the four sides are equal.

The diagonals of a square are perpendicular bisectors of each other.



Exercise 3.1

Q.1. Given here are some figures:



Classify each of the above figure on the basis of the following:

- (a) Simple curve
- (b) Simple closed curve
- (c) Polygon

(d) Convex polygon
(e) Concave polygon
Solution:
(a) Simple curve: (1), (2), (5), (6) and (7)
(b) Simple closed curve: (1), (2), (5), (6) and (7)
(c) Polygon: (1) and (2)
(d) Convex polygon: (2)
(e) Concave polygon: (1)

Q.2. How many diagonals does each of the following have?

(a) A convex quadrilateral

(b) A regular hexagon

(c) A triangle

Solution:

(a) In Fig. (i) ABCD is a convex quadrilateral which has two diagonals AC and BD.



b) In Fig. (ii) ABCDEF is a regular hexagon which has nine diagonals AE, AD, AC, BF, BE, BD, CF, CE and DF





c) In Fig. (iii) ABC is a triangle which has no diagonal.

Q.3.What is the sum of the measures of the angles of a convex quadrilateral? Will this property hold if the quadrilateral is not convex? (Make a non-convex quadrilateral and verify)

Solution:

In the given figure, we have a quadrilateral ABCD. Join AC diagonal which divides the quadrilateral into two triangles ABC and ADC.



In $\triangle ABC$, $\angle 3 + \angle 4 + \angle 6 = 180^{\circ}...(i)$ (angle sum property) In $\triangle ADC$, $\angle 1 + \angle 2 + \angle 5 = 180^{\circ}...(ii)$ (angle sum property) Adding, (i) and (ii) $\angle 1 + \angle 3 + \angle 2 + \angle 4 + \angle 5 + \angle 6 = 180^{\circ} + 180^{\circ}$ $\Rightarrow \angle A + \angle C + \angle D + \angle B = 360^{\circ}$ Hence, the sum of all the angles of a convex quadrilateral = 360°.

Let us draw a non-convex quadrilateral.

Yes, this property also holds true for a non-convex quadrilateral.



Q.4.Examine the table. (Each figure is divided into triangles and the sum of the angles reduced from that).

Figure	\square			
Side	3	4	5	6
Angle sum	180°	$2\times180^\circ=(4-2)\times180^\circ$	$3\times180^\circ=(5-2)\times180^\circ$	$4\times180^\circ=(6-2)\times180^\circ$

What can you say about the angle sum of a convex polygon with number of sides? (a) 7

(a) / (b) 8

(b) 0(c) 10

(**d**) n

Solution:

From the above table, we conclude that the sum of all the angles of a polygon of side 'n' = $(n - 2) \times 180^{\circ}$ (a) Number of sides = 7 Angles sum = $(7 - 2) \times 180^{\circ} = 5 \times 180^{\circ} = 900^{\circ}$ (b) Number of sides = 8 Angle sum = $(8 - 2) \times 180^{\circ} = 6 \times 180^{\circ} = 1080^{\circ}$ (c) Number of sides = 10 Angle sum = $(10 - 2) \times 180^{\circ} = 8 \times 180^{\circ} = 1440^{\circ}$ d) Number of sides = n Angle sum = $(n - 2) \times 180^{\circ}$

Q.5.What is a regular polygon? State the name of a regular polygon of
(i) 3 sides
(ii) 4 sides
(iii) 6 sides
Solution:
A polygon with equal sides and equal angles is called a regular polygon.
(i) Equilateral triangle



ii) Square



iii) Regular Hexagon



Q.6. Find the angle measure x in the following figures:



Solution:

(a) Angle sum of a quadrilateral = 360° $\Rightarrow 50^{\circ} + 130^{\circ} + 120^{\circ} + x = 360^{\circ}$ $\Rightarrow 300^{\circ} + x = 360^{\circ}$ $\Rightarrow x = 360^{\circ} - 300^{\circ} = 60^{\circ}$ (b) Angle sum of a quadrilateral = 360° $\Rightarrow x + 70^{\circ} + 60^{\circ} + 90^{\circ} = 360^{\circ} [:: 180^{\circ} - 90^{\circ} = 90^{\circ}]$ \Rightarrow x + 220° = 360° \Rightarrow x = 360° - 220° = 140° (c) Angle sum of a pentagon = 540° $\Rightarrow 30^{\circ} + x + 110^{\circ} + 120^{\circ} + x = 540^{\circ} [:: 180^{\circ} - 70^{\circ} = 110^{\circ}; 180^{\circ} - 60^{\circ} = 120^{\circ}]$ $\Rightarrow 2x + 260^{\circ} = 540^{\circ}$ $\Rightarrow 2x = 540^{\circ} - 260^{\circ}$ $\Rightarrow 2x = 280^{\circ}$ $\Rightarrow x = 140^{\circ}$ (d) Angle sum of a regular pentagon = 540° \Rightarrow x + x + x + x + x = 540° [All angles of a regular pentagon are equal] $\Rightarrow 5x = 540^{\circ}$ $\Rightarrow x = 108^{\circ}$

Q.7. (a) Find x + y + z



(b) Find x + y + z + w

(b) $\angle r + 120^\circ + 80^\circ + 60^\circ = 360^\circ$ [Angle sum property of a quadrilateral] $\angle r + 260^\circ = 360^\circ$ $\angle r = 360^\circ - 260^\circ = 100^\circ$



 $\Rightarrow y = 120^{\circ}$ and, $z + 30^{\circ} = 180^{\circ}$ [Linear pair] $\Rightarrow z = 180^{\circ} - 30^{\circ} = 150^{\circ}$ also, $x + 90^{\circ} = 180^{\circ}$ [Linear pair] $\Rightarrow x = 180^{\circ} - 90^{\circ} = 90^{\circ}$ Thus $x + y + z = 90^{\circ} + 120^{\circ} + 150^{\circ} = 360^{\circ}$ (b) $\angle r + 120^{\circ} + 80^{\circ} + 60^{\circ} = 360^{\circ}$ [Angle sum property of a quadrilateral]



 $\Rightarrow \angle a = 180^{\circ} - 120^{\circ} = 60^{\circ}$

 \Rightarrow y = 180° - 60°

Now, $y = 180^{\circ} - a$ (Linear pair)

Solution: (a) $\angle a + 30^\circ + 90^\circ = 180^\circ$ [Angle sum property] $\Rightarrow \angle a + 120^\circ = 180^\circ$







Now $x + 120^{\circ} = 180^{\circ}$ (Linear pair) $x = 180^{\circ} - 120^{\circ} = 60^{\circ}$ $y + 80^{\circ} = 180^{\circ}$ (Linear pair) $\Rightarrow y = 180^{\circ} - 80^{\circ} = 100^{\circ}$ $z + 60^{\circ} = 180^{\circ}$ (Linear pair) $\Rightarrow z = 180^{\circ} - 60^{\circ} = 120^{\circ}$ $w = 180^{\circ} - \angle r = 180^{\circ} - 100^{\circ} = 80^{\circ}$ (Linear pair) $x + y + z + w = 60^{\circ} + 100^{\circ} + 120^{\circ} + 80^{\circ} = 360^{\circ}$.

TEXTUAL EXERCISE 3.2

Q.1. Find x in the following figures.



Solution:

(a) We know that the sum of all the exterior angles of a polygon = 360° $125^{\circ} + 125^{\circ} + x = 360^{\circ}$ $\Rightarrow 250^{\circ} + x = 360^{\circ}$



x = $360^{\circ} - 250^{\circ} = 110^{\circ}$ Hence x = 110° (b) Here ∠y = $180^{\circ} - 90^{\circ} = 90^{\circ}$



and $\angle z = 90^{\circ}$ (given) $x + y + 60^{\circ} + z + 70^{\circ} = 360^{\circ}$ [: Sum of all the exterior angles of a polygon = 360°] $\Rightarrow x + 90^{\circ} + 60^{\circ} + 90^{\circ} + 70^{\circ} = 360^{\circ}$ $\Rightarrow x + 310^{\circ} = 360^{\circ}$ $\Rightarrow x = 360^{\circ} - 310^{\circ} = 50^{\circ}$ Hence $x = 50^{\circ}$

Q.2. Find the measure of each exterior angle of a regular polygon of
(i) 9 sides
(ii) 15 sides
Solution:
(i) We know the sum of all the exterior angles of polygon = 360°
Measure of each angle of 9 sided regular polygon = 3609 = 40°

(ii) Sum of all the exterior angles of a polygon = 360° Measure of each angle of 15 sided regular polygon = $36015 = 24^{\circ}$

Q.3.How many sides does a regular polygon have if the measure of an exterior angle is 24°?

Solution: Sum of all exterior angles of a regular polygon = 360° Number of sides

 $= \frac{360^{\circ}}{\text{measure of an angle}}$ $= \frac{360^{\circ}}{24} = 15$

Hence, the number of sides = 15

Q4 How many sides does a regular polygon have if each of its interior angles is 165°? Solution:

Let re be the number of sides of a regular polygon. Sum of all interior angles = $(n - 2) \times 180^{\circ}$ and, measure of its each angle

 $= \frac{(n-2) \times 180^{\circ}}{n}$ So, $\frac{(n-2) \times 180}{n} = 165$ $\Rightarrow 180n - 2 \times 180 = 165n$ $\Rightarrow 180n - 360 = 165n$ $\Rightarrow 180n - 165n = 360$ $\Rightarrow 15n = 360$ $\therefore \qquad n = \frac{360}{15} = 24.$

Hence, the number of sides = 24

Q.5. (a) Is it possible to have a regular polygon with measure of each exterior angle a is 22° ?

(b) Can it be an interior angle of a regular polygon? Why? Solution:

(a) Since, the sum of all the exterior angles of a regular polygon = 360° which is not divisible by 22° .

It is not possible that a regular polygon must have its exterior angle 22°.

(b) Sum of all interior angles of a regular polygon of side $n = (n - 2) \times 180^{\circ}$

: Measure of each interior angle

$$= \frac{(n-2) \times 180}{n}$$
$$\frac{(n-2) \times 180^{\circ}}{n} = 22^{\circ}$$
$$\Rightarrow 180n - 2 \times 180 = 22n$$
$$\Rightarrow 180n - 22n = 360$$
$$\Rightarrow 158n = 360$$
$$\therefore \qquad n = \frac{360}{158} = \frac{180}{79}$$

not a whole number.

Since number of sides cannot be in fractions.

It is not possible for a regular polygon to have its interior angle = 22° .

Q.6. (a) What is the minimum interior angle possible for a regular polygon? Why?(b) What is the maximum exterior angle possible for a regular polygon?Solution:

(a) Sum of all interior angles of a regular polygon of side $n = (n - 2) \times 180^{\circ}$ The measure of each interior angle

$$=\frac{(n-2)\times 180^{\circ}}{n}$$

For minimum possible interior angle $\frac{(n-2)\times 180^\circ}{n} > 0$

 \Rightarrow n > 2

The minimum measure the angle of an equilateral triangle $(n = 3) = 60^{\circ}$.

(b) From part (a) we can conclude that the maximum exterior angle of a regular polygon = $180^{\circ} - 60^{\circ} = 120^{\circ}$.

Textual Exercise 3.3

Q.1. Given a parallelogram ABCD. Complete each statement along with the definition or property used.



Solution:

(i) AD = BC [Opposite sides of a parallelogram are equal]

(ii) $\angle DCB = \angle DAB$ [Opposite angles of a parallelogram are equal]

- (iii) OC = OA [Diagonals of a parallelogram bisect each other]
- (iv) $m \angle DAB + m \angle CDA = 180^{\circ}$ [Adjacent angles of a parallelogram are supplementary]

Q.2.Consider the following parallelograms. Find the values of the unknowns x, y, z.





Solution:

(i) ABCD is a parallelogram.



 $\angle B = \angle D$ [Opposite angles of a parallelogram are equal]

 $\angle D = 100^{\circ}$

 \Rightarrow y = 100°

 $\angle A + \angle B = 180^{\circ}$ [Adjacent angles of a parallelogram are supplementary]

 \Rightarrow z + 100° = 180°

 \Rightarrow z = 180° - 100° = 80°

 $\angle A = \angle C$ [Opposite angles of a ||gm]

 $x = 80^{\circ}$ Hence $x = 80^{\circ}$, $y = 100^{\circ}$ and $z = 80^{\circ}$

(ii) PQRS is a parallelogram.



 $\angle P + \angle S = 180^{\circ}$ [Adjacent angles of parallelogram] $\Rightarrow x + 50^{\circ} = 180^{\circ}$ $x = 180^{\circ} - 50^{\circ} = 130^{\circ}$ Now, $\angle P = \angle R$ [Opposite angles are equal] $\Rightarrow x = y$ $\Rightarrow y = 130^{\circ}$ Also, y = z [Alternate angles] $z = 130^{\circ}$ Hence, $x = 130^{\circ}$, $y = 130^{\circ}$ and $z = 130^{\circ}$

(iii) ABCD is a rhombus.

[∵ Diagonals intersect at 90°]



x = 90° Now in △OCB, x + y + 30° = 180° (Angle sum property) ⇒ 90° + y + 30° = 180° ⇒ y + 120° = 180° ⇒ y = 180° - 120° = 60° y = z (Alternate angles) ⇒ z = 60° Hence, x = 90°, y = 60° and z = 60°. (iv) ABCD is a parallelogram



 $\angle A + \angle B = 180^{\circ}$ (Adjacent angles of a parallelogram are supplementary) $\Rightarrow x + 80^{\circ} = 180^{\circ}$ $\Rightarrow x = 180^{\circ} - 80^{\circ} = 100^{\circ}$ Now, $\angle D = \angle B$ [Opposite angles of a |jgm] $\Rightarrow y = 80^{\circ}$ Also, $z = \angle B = 80^{\circ}$ (Alternate angles) Hence $x = 100^{\circ}$, $y = 80^{\circ}$ and $z = 80^{\circ}$

(v) ABCD is a parallelogram.



 $\Delta D = \Delta B \text{ [Opposite angles of a ||gm]}$ y = 112° x + y + 40° = 180° [Angle sum property] $\Rightarrow x + 112° + 40° = 180°$ $\Rightarrow x + 152° = 180°$ $\Rightarrow x = 180° - 152 = 28°$ z = x = 28° (Alternate angles) Hence x = 28°, y = 112°, z = 28°.

Q.3.Can a quadrilateral ABCD be a parallelogram if
(i) ∠D + ∠B = 180°?
(ii) AB = DC = 8 cm, AD = 4 cm and BC = 4.4 cm?
(iii) ∠A = 70° and ∠C = 65°?
Solution:
(i) For ∠D + ∠B = 180, quadrilateral ABCD may be a parallelogram if following conditions are also fulfilled.
(a) The sum of measures of adjacent angles should be 180°.
(b) Opposite angles should also be of same measures. So, ABCD can be but need not be a

(b) Opposite angles should also be of same measures. So, ABCD can be but need not be a parallelogram.

(ii) Given: AB = DC = 8 cm, AD = 4 cm, BC = 4.4 cmIn a parallelogram, opposite sides are equal. Here $AD \neq BC$ Thus, ABCD cannot be a parallelogram. (iii) $\angle A = 70^{\circ}$ and $\angle C = 65^{\circ}$ Since $\angle A \neq \angle C$ Opposite angles of quadrilateral are not equal. Hence, ABCD is not a parallelogram.

Q.4.Draw a rough figure of a quadrilateral that is not a parallelogram but has exactly two opposite angles of equal measure.

Solution:

ABCD is a rough figure of a quadrilateral in which $m \angle A = m \angle C$ but it is not a parallelogram. It is a kite



Q.5. The measures of two adjacent angles of a parallelogram are in the ratio 3 : 2. Find the measure of each of the angles of the parallelogram. Solution:

Let ABCD is parallelogram such that $m \angle B : m \angle C = 3 : 2$



Let $m \angle B = 3x^{\circ}$ and $m \angle C = 2x^{\circ}$ $m \angle B + m \angle C = 180^{\circ}$ (Sum of adjacent angles = 180°) $3x + 2x = 180^{\circ}$ $\Rightarrow 5x = 180^{\circ}$ $\Rightarrow x = 36^{\circ}$ Thus, $\angle B = 3 \times 36 = 108^{\circ}$ $\angle C = 2 \times 36^{\circ} = 72^{\circ}$ $\angle B = \angle D = 108^{\circ}$ and $\angle A = \angle C = 72^{\circ}$ Hence, the measures of the angles of the parallelogram are 108°, 72°, 108° and 72°.

Q.6. Two adjacent angles of a parallelogram have equal measure. Find the measure of each of the angles of the parallelogram.

Solution: Let ABCD be a parallelogram in which $\angle A = \angle B$



 $\angle A + \angle A = 180^{\circ}$ $\Rightarrow 2\angle A = 180^{\circ}$ $\Rightarrow \angle A = 90^{\circ}$ Thus, $\angle A = \angle C = 90^{\circ}$ and $\angle B = \angle D = 90^{\circ}$ [Opposite angles of a parallelogram are equal]

Q.7. The adjacent figure HOPE is a parallelogram. Find the angle measures x, y and z. State the properties you use to find them.



Solution: $\angle y = 40^{\circ}$ (Alternate angles) $\angle z + 40^{\circ} = 70^{\circ}$ (Exterior angle property) $\Rightarrow \angle z = 70^{\circ} - 40^{\circ} = 30^{\circ}$ $z = \angle EPH$ (Alternate angle) In $\triangle EPH$ $\angle x + 40^{\circ} + \angle z = 180^{\circ}$ (Adjacent angles) $\Rightarrow \angle x + 40^{\circ} + 30^{\circ} = 180^{\circ}$ $\Rightarrow \angle x + 70^{\circ} = 180^{\circ}$ $\Rightarrow \angle x = 180^{\circ} - 70^{\circ} = 110^{\circ}$ Hence x = 110°, y = 40° and z = 30°.

Q.8.The following figures GUNS and RUNS are parallelograms. Find x and y. (Lengths are in cm)



Q.9.In the above figure both RISK and CLUE are parallelograms. Find the value of x. Solution:

Here RISK and CLUE are two parallelograms. $\angle 1 = \angle L = 70^{\circ}$ (Opposite angles of a parallelogram) $\angle K + \angle 2 = 180^{\circ}$ Sum of adjacent angles is 180° $120^{\circ} + \angle 2 = 180^{\circ}$ $\angle 2 = 180^{\circ} - 120^{\circ} = 60^{\circ}$ In $\triangle OES$, $\angle x + \angle 1 + \angle 2 = 180^{\circ}$ (Angle sum property) $\Rightarrow \angle x + 70^{\circ} + 60^{\circ} = 180^{\circ}$ $\Rightarrow \angle x + 130^{\circ} = 180^{\circ}$ $\Rightarrow \angle x = 180^{\circ} - 130^{\circ} = 50^{\circ}$ Hence $x = 50^{\circ}$

Q.10 Explain how this figure is a trapezium. Which of its two sides are parallel?



Solution:

 $\angle M + \angle L = 100^{\circ} + 80^{\circ} = 180^{\circ}$ $\angle M$ and $\angle L$ are the adjacent angles, and sum of adjacent interior angles is 180° KL is parallel to NM Hence KLMN is a trapezium.

Q.11.Find m $\angle C$ in below figure if $\overline{AB} \parallel \overline{DC}$

Solution: Given that $\overline{AB} \parallel \overline{DC}$

 $m \angle B + m \angle C = 180^{\circ}$ (Sum of adjacent angles of a parallelogram is 180°)

D

 $120^{\circ} + m \angle C = 180^{\circ}$ $m \angle C = 180^{\circ} - 120^{\circ} = 60^{\circ}$ Hence m \angle C = 60^{\circ}

Q.12.Find the measure of $\angle P$ and $\angle S$ if $\overline{SP} \parallel \overline{RQ}$ in Fig 3.34 (If you find m $\angle R$, is there more than one method to find m $\angle P$?)



Solution: Given that $\angle Q = 130^\circ$ and $\angle R = 90^\circ$ $\overline{SP} \parallel \overline{RQ}$ (given)

 $\angle P + \angle Q = 180^{\circ} \text{ (Adjacent angles)}$ $\Rightarrow \angle P + 130^{\circ} = 180^{\circ}$ $\Rightarrow \angle P = 180^{\circ} - 130^{\circ} = 50^{\circ}$ and, $\angle S + \angle R = 180^{\circ} \text{ (Adjacent angles)}$ $\Rightarrow \angle S + 90^{\circ} = 180^{\circ}$ $\Rightarrow \angle S = 180^{\circ} - 90^{\circ} = 90^{\circ}$ Alternate Method: $\angle Q = 130^{\circ}, \ \angle R = 90^{\circ} \text{ and } \ \angle S = 90^{\circ}$ We know that $\angle P + \angle Q + \angle R + \angle Q = 360^{\circ} \text{ (Angle sum property of quadrilateral)}$ $\Rightarrow \angle P + 310^{\circ} = 360^{\circ}$ $\Rightarrow \angle P = 360^{\circ} - 310^{\circ} = 50^{\circ}$ Hence m $\angle P = 50^{\circ}$

Textual Exercise 3.4

Q.1.State whether True or False.

- (a) All rectangles are squares.
- (b) All rhombuses are parallelograms.
- (c) All squares are rhombuses and also rectangles.
- (d) All squares are not parallelograms.
- (e) All kites are rhombuses.
- (f) All rhombuses are kites.
- (g) All parallelograms are trapeziums.
- (h) All squares are trapeziums.

Solution:

- (a) False
- (b) True
- (c) True
- (d) False
- (e) False
- (f) True
- (g) True

Q.2. Identify all the quadrilaterals that have(a) four sides of equal length(b) four right angles

Solution:

- (a) Squares and rhombuses.
- (b) Rectangles and squares.

Q.3.Explain how a square is

(i) a quadrilateral(ii) a parallelogram

(iii) a rhombus

(iv) a rectangle

Solution:

(i) Square is a quadrilateral because it is closed with four line segments.

(ii) Square is a parallelogram due to the following properties:

(a) Opposite sides are equal and parallel.

(b) Opposite angles are equal.

(iii) Square is a rhombus because its all sides are equal and opposite sides are parallel.

(iv) Square is a rectangle because its opposite sides are equal and has equal diagonal.

Q.4.Name the quadrilaterals whose diagonals

(i) bisect each other

(ii) are perpendicular bisectors of each other

(iii) are equal

Solution:

(i) Parallelogram, rectangle, square and rhombus

(ii) Square and rhombus

(iii) Rectangle and square

Q.5. Explain why a rectangle is a convex quadrilateral.

Solution:

In a rectangle, both of its diagonal lie in its interior. Hence, it is a convex quadrilateral.

Q.6. ABC is a right-angled triangle and O is the midpoint of the side opposite to the right angle. Explain why O is equidistant from A, B and C. (The dotted lines are drawn additionally to help you).



Solution:

Since the right-angled triangle ABC makes a rectangle ABCD by the dotted lines. Therefore OA = OB = OC = OD [Diagonals of a rectangle are equal and bisect each other] Hence, O is equidistant from A, B and C.





Very Short Answer Type:

Q.1.In the given figure, ABCD is a parallelogram. Find x.



Solution:

AB = DC [Opposite sides of a parallelogram]

3x + 5 = 5x - 1

 \Rightarrow 3x - 5x = -1 - 5

 $\Rightarrow -2x = -6$ $\Rightarrow x = 3$

Solution:

Q.2.In the given figure find x + y + z.



We know that the sum of all the exterior

 $x + y + z = 360^{\circ}$

Question 3.

Q.3.In the given figure, find x.



Solution:

 $\angle A + \angle B + \angle C = 180^{\circ} \text{ [Angle sum property]}$ $(x + 10)^{\circ} + (3x + 5)^{\circ} + (2x + 15)^{\circ} = 180^{\circ}$ $\Rightarrow x + 10 + 3x + 5 + 2x + 15 = 180$ $\Rightarrow 6x + 30 = 180$ $\Rightarrow 6x = 180 - 30$ $\Rightarrow 6x = 150$ $\Rightarrow x = 25$ Question 4.

Q.4. The angles of a quadrilateral are in the ratio of 2 : 3 : 5 : 8. Find the measure of each angle.

Solution:

Sum of all interior angles of a quadrilateral = 360°

Let the angles of the quadrilateral be $2x^{\circ}$, $3x^{\circ}$, $5x^{\circ}$ and $8x^{\circ}$.

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2x + 3x + 5x + 8x = 360^{\circ}
\Rightarrow 18x = 360^{\circ}
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 \Rightarrow x = 20°

Hence the angles are

 $2 \times 20 = 40^{\circ}$,

 $3 \times 20 = 60^{\circ}$,

 $5 \times 20 = 100^{\circ}$

and $8 \times 20 = 160^{\circ}$.

Question 5.

Find the measure of an interior angle of a regular polygon of 9 sides.

Solution:

Measure of an interior angle of a regular polygon

of *n* sides = $\frac{(n-2) \times 180^{\circ}}{n}$ For *n* = 9, we have

$$\frac{(9-2)\times 180^{\circ}}{9} = \frac{7\times 180^{\circ}}{9}$$

$$= 7 \times 20^{\circ} = 140^{\circ}$$

Hence, the angle is 140°.

Question 6.

Length and breadth of a rectangular wire are 9 cm and 7 cm respectively. If the wire is bent into a square, find the length of its side.

Solution:

Perimeter of the rectangle = 2 [length + breadth] = $2[9 + 7] = 2 \times 16 = 32$ cm.



Now perimeter of the square = Perimeter of rectangle = 32 cm.

Side of the square $=\frac{32}{4}=8$ cm. Hence, the length of the side of square = 8 cm.

Question 7. In the given figure ABCD, find the value of x.



Solution:

Sum of all the exterior angles of a polygon = 360° x + 70° + 80° + 70° = 360°

 $\Rightarrow x + 220^{\circ} = 360^{\circ}$ $\Rightarrow x = 360^{\circ} - 220^{\circ} = 140^{\circ}$

Q.8.In the parallelogram given alongside if $m \angle Q = 110^{\circ}$, find all the other angles.



Solution: Givenm $\angle Q = 110^{\circ}$ Thenm $\angle S = 110^{\circ}$ (Opposite angles are equal) Since $\angle P$ and $\angle Q$ are supplementary. Then m $\angle P + m\angle Q = 180^{\circ}$ $\Rightarrow m\angle P + 110^{\circ} = 180^{\circ}$ $\Rightarrow m\angle P = 180^{\circ} - 110^{\circ} = 70^{\circ}$ $\Rightarrow m\angle P = m\angle R = 70^{\circ}$ (Opposite angles) Hence m $\angle P = 70, m\angle R = 70^{\circ} \Rightarrow m\angle P = 180^{\circ} - 110^{\circ} = 70^{\circ}$ $\Rightarrow m\angle P = m\angle R = 70^{\circ}$ (Opposite angles) Hence m $\angle P = 70, m\angle R = 70^{\circ}$ and m $\angle S = 110^{\circ}$

Question 9. Q.9. In the given figure, ABCD is a rhombus. Find the values of x, y and z.



Solution: AB = BC (Sides of a rhombus) x = 13 cm. Since the diagonals of a rhombus bisect each other z = 5 and y = 12Hence, x = 13 cm, y = 12 cm and z = 5 cm.

Question 10.

Q.10. In the given figure, ABCD is a parallelogram. Find x, y and z.



Higher Order Thinking Skills (HOTS)

Question 17. The diagonal of a rectangle is thrice its smaller side. Find the ratio of its sides.



Solution: Let AD = x cmdiagonal BD = 3x cmIn right-angled triangle DAB, $AD^2 + AB^2 = BD^2$ (Using Pythagoras Theorem) $x^2 + AB^2 = (3x)^2$ $\Rightarrow x^2 + AB^2 = 9x^2$ $\Rightarrow AB^2 = 9x^2 - x^2$ $\Rightarrow AB^2 = 8x^2$ $\Rightarrow AB^2 = 8x^2$ $\Rightarrow AB = \sqrt{8x} = 2\sqrt{2x}$ Required ratio of AB : AD = $2\sqrt{2x} : x = 2\sqrt{2} : 1$

Question 18.

If AM and CN are perpendiculars on the diagonal BD of a parallelogram ABCD, Is $\triangle AMD = \triangle CNB$? Give reason.



In triangles AMD and CNB, AD = BC (opposite sides of parallelogram) $\angle AMB = \angle CNB = 90^{\circ}$ $\angle ADM = \angle NBC$ (AD || BC and BD is transversal.) So, $\triangle AMD = \triangle CNB$ (AAS)

Q. MULTIPLE CHOICE QUESTIONS:

Q.1.. Which one of the following is a regular quadrilateral?

a) Squareb) Trapeziumc) Kited) RectangleAnswer: (a)

2. The perimeter of a parallelogram whose parallel sides have lengths equal to 12 cm and 7cm is:a) 21cm

- b) 42 cm
- c) 19 cm

d) 38 cm Answer : (d)

3. If $\angle A$ and $\angle C$ are two opposite angles of a parallelogram, then:

a) $\angle A > \angle C$ b) $\angle A = \angle C$ c) $\angle A < \angle C$ d) None of the above Answer: (b)

4. If ∠A and ∠B are two adjacent angles of a parallelogram. If ∠A = 70°, then ∠B = ?
a) 70°
b) 90°
c) 110°
d) 180°
Answer: (c)

5. ABCD is a rectangle and AC & BD are its diagonals. If AC = 10cm, then BD is:

a) 10 cm b) 5 cm c) 15 cm d) 20 cm Answer: (a)

CHAPTER NO. – 4

CHAPTER NAME – Practical Geometry

KEY POINTS TO REMEMBER –

To construct a quadrilateral uniquely, it is necessary to have the knowledge of at least five of its parts.

Five necessary parts to construct a quadrilateral may be:

1. four sides and diagonal

Three sides and two diagonals

- 2. Four sides and an angle
- 3. Three sides and two included angles
- 4. Two adjacent sides and three angles

To construct a rhombus:

- 1. Draw a diagonal of a given length and draw perpendicular bisector of this diagonal base.
- 2. Take half of the given measurement of second diagonal and cut off arcs on either side of the perpendicular bisector. It will give two points of a rhombus.
- 3. Join these points with the points on the first diagonal. It will give the required rhombus.

Before constructing a quadrilateral, one must draw a rough sketch of given measurements.

Trapezium, rhombus, and square are different forms of a parallelogram.

Rhombus can be a parallelogram, but a parallelogram cannot be a rhombus.

A square and a rhombus both have equal sides and a square can be rhombus but a rhombus cannot be a square.

We have learned the procedure of drawing triangles in the preceding class. We know that three measurements (of sides and angles) are required to draw a unique triangle. Here, we shall investigate whether four measurements are sufficient to draw a quadrilateral or not.

Constructing A Quadrilateral

We shall learn how to draw a unique quadrilateral when the following measurements are given:

- four sides and one diagonal
- two diagonals and three sides
- two adjacent sides and three angles
- three sides and two included angles
- other special properties.

When the Lengths of Four Sides and a Diagonal are given

In this case we divide the quadrilateral into two triangles which can be easily drawn with the help of the given measurements.

Suppose that we are to construct the quadrilateral ABCD. Then two cases arise:

- Diagonal AC is given: Here we divide the quadrilateral ABCD into two triangles ABC and ADC which can be easily drawn.
- Diagonal BD is given: Here we divide the quadrilateral ABCD into two triangles ABD and BCD which can be easily drawn.

When Two Diagonals and Three Sides are given

In this case, we draw any one of the two diagonals. It determines two vertices of the quadrilateral. Then taking two suitable sides of the three sides, we locate the third vertex of the quadrilateral. Finally using the second diagonal and the remaining side, we locate the fourth vertex of the quadrilateral.

When Two Adjacent Sides and Three Angles are Known

We know that the sum of the angles of a quadrilateral is 360°. Since the three angles are given, therefore, if need be, the measure of the fourth angle can be readily obtained.

Thus, the angle between two adjacent sides is known if it is not even already given. To construct the quadrilateral, we first draw two adjacent sides with the included angle between them. Then, at the other extremities of the adjacent sides, we draw rays at the given angles so as to intersect at a vertex (fourth) of the quadrilateral. Thus the quadrilateral can be drawn completely.

When Three Sides and Two included Angles are Given

We specifically note the three sides and the two included angles. First, we draw the common side of the two angles. At its extremities, we draw the other sides of the given angles with their given measures. In the last, we join the extremities of the other two sides. Thus, we can draw the quadrilateral completely.

Some Special Cases

There are certain specific quadrilaterals which can be constructed with less number (<5) of available measurements.

<u>Ex 4.1</u>

Quadrilateral ABCD AB = 4.5 cm, BC = 5.5 cm, CD = 4 cm, AD = 6 cm, AC = 7 cm (ii) Quadrilateral JUMP JU = 3.5 cm, UM = 4 cm, MP = 5 cm, PJ = 4.5 cm, PU = 6.5 cm (iii) Parallelogram MORE OR = 6 cm, RE = 4.5 cm, EO = 7.5 cm (iv) Rhombus BEST

Solution:

(i) We have to draw first rough sketch.



Construction:

Step I: Draw AB = 4.5 cm Step II: Draw an arc with centre B and radius 5.5 cm. Step III: Draw another arc with centre A and radius 7 cm to meet the previous arc at C.

Step IV: Draw an arc with centre C and radius 4 cm. Step V: Draw another arc with centre A and radius 6 cm to cut the former arc at D. Step VI: Join BC, AC, CD and AD.

120 357 34

(ii) We have to draw the first rough sketch.



Thus ABCD is the required quadrilateral. Construction:

Step I: Draw JU = 3.5 cm.

Step II: Draw an arc with centre J and radius 4.5 cm.

Step III: Draw another arc with centre U and radius 6.5 cm to meet the previous arc at P. Step IV: Join JP and UP.

Step V: Draw an arc with centre U and radius 4 cm.

Step VI: Draw another arc with centre P and radius 5 cm to meet the previous arc at M. Step VII: Join UM and PM.

Thus, JUMP is the required quadrilateral.

iii) We have to draw the first rough sketch.



Construction: (Opposite sides of a parallelogram are equal)

Step I: Draw OR = 6 cm.

Step II: Draw an arc with centre R and radius 4.5 cm.

Step III: Draw another arc with centre O and radius 7.5 cm to meet the previous arc at E. Step IV: Join RE and OE.

Step V: Draw an arc with centre E and radius 6 cm.

Step VI: Draw another arc with centre O and radius 4.5 cm to meet the former arc at M. Step VII: Join EM and OM.

Thus, MORE is the required parallelogram.

(iv)



Construction: (All sides of a rhombus are equal)

Step I: Draw BE = 4.5 cm

Step II: Draw an arc with centre B and radius 4.5 cm.

Step III: Draw another arc with centre E and radius 6 cm to meet the previous arc at T.

Step IV: Join BT and ET.

Step V: Draw two arcs with centres E and T with equal radii 4-5 cm to meet each other at S. . Step VI: Join ES and TS.

Thus, BEST is the required rhombus.

EXERCISE 4.2

Q.1. Construct the following quadrilaterals. (i) Quadrilateral LIFT LI = 4 cm IF = 3 cm TL = 2.5 cm LF = 4.5 cm IT = 4 cm(ii) Quadrilateral GOLD OL = 7.5 cm GL = 6 cm GD = 6 cm LD = 5 cm OD = 10 cm(ii) Rhombus BEND BN = 5.6 cm DE = 6.5 cm Solution: (i) Construction:



Step I: Draw LI = 4 cm.

Step II: Draw an arc with centre I and radius 3 cm.

-

Step III: Draw another arc with centre L and radius 4.5 cm to meet the former arc at F. Step IV: Join LF and IF.

Step V: Draw an arc with centre L and radius 2.5 cm.

Step VI: Draw another arc with centre I and radius 4 cm to meet the previous arc at T. Step VII: Join LT and IT.

Thus LIFT is the required quadrilateral.

(ii) Construction:



Step I: Draw OL = 7.5 cm

Step II: Draw an arc with centre O and radius 10 cm.

Step III: Draw another arc with centre L and radius 5 cm to meet the previous arc at D. Step IV: Join OD and LD.

Step V: Draw an arc with centre L and D with equal radii of 6 cm to meet each other at G. Step VI: Join LG and DG.

Thus GOLD is the required quadrilateral.

(iii) Construction: (The diagonals of a rhombus bisect each other at the right angle)



Step I: Draw BN = 5.6 cm. Step II: Draw the right bisector of BN at O. Step III: Draw two arcs with centre O and radius $12 \times DE$, i.e., $12 \times 6.5 = 3.25$ cm to meet the right bisector at D and E. $\frac{1}{2} \times DE$; i.e. $\frac{1}{2} \times 6.5 = 3.25$ cm to meet the right bisector at D and E. Step IV: Join BE, EN, ND and BD. Thus, BEND is the required rhombus.

EXERCISE 4.3

Construct the following quadrilaterals: (i) Quadrilateral MORE $MO = 6 \text{ cm}, \angle R = 105^{\circ}, OR = 4.5 \text{ cm}, \angle M = 60^{\circ}, \angle O = 105^{\circ}$ (ii) Quadrilateral PLAN $PL = 4 \text{ cm}, LA = 6.5 \text{ cm}, \angle P = 90^{\circ}, \angle A = 110^{\circ}, \angle N = 85^{\circ}$ (iii) Parallelogram HEAR $HE = 5 \text{ cm}, EA = 6 \text{ cm}, \angle R = 85^{\circ}$ (iv) Rectangle OKAY OK = 7 cm, KA = 5 cm

Solution: (i) Construction: Step I: Draw OR = 4.5 cmStep II: Draw two angles of 105° each at O and R with the help of protactor. Step III: Cut OM = 6 cm.



Step IV: Draw an angle of 60° at M to meet the angle line t

(i) Construction: Quadrilateral PLAN



Step I: Draw LA = 6.5 cm

Step II: Draw an angle of 75° at L and 110° at A with the help of a protractor. [:: $360^{\circ} - (110^{\circ} + 90^{\circ} + 85^{\circ}) = 75^{\circ}$]

Step III: Cut LP = 4 cm.

Step IV: Draw an angle of 90° at P which meets the angle line through A at N. Thus PLAN is the required quadrilateral.

iii) Construction: Parallelogram HEAR

(Opposite sides of a parallelogram are equal)



Step I: Draw HE = 5 cm.

Step II: Draw an angle of 85° at E and cut EA = 6 cm.

Step III: Draw an arc with centre A and radius 5 cm.

Step IV: Draw another arc with centre H and radius 6 cm to meet the previous arc at R. Step V: Join HR and AR

Thus, HEAR is the required parallelogram.

(iv) Construction: Rectangle OKAY

(Each angle of a rectangle is 90° and opposite sides are equal.)



Step I: Draw OK = 7 cm.
Step II: Draw the angle of 90° at K and cut KA = 5 cm.
Step III: Draw an arc with centre O and radius 5 cm.
Step IV: Draw another arc with centre A and radius 7 cm to meet the previous arc at Y.
Step V: Join OY and AY.
Thus OKAY is the required rectangle.

EXERCISE 4.4

Construct the following quadrilaterals: (i) Quadrilateral DEAR DE = 4 cm, EA = 5 cm, AR = 4.5 cm, $\angle E = 60^{\circ}$, $\angle A = 90^{\circ}$ (ii) Quadrilateral TRUE TR = 3.5 cm, RU = 3 cm, UE = 4.5 cm, $\angle R = 75^{\circ}$, $\angle U = 120^{\circ}$

Solution: (i) Construction:



Step I: Draw DE = 4 cm.

Step II: Draw an angle of 60° at E.

Step III: Draw an arc with centre E and radius 5 cm to meet the angle line at A. Step IV: Draw an angle of 90° at A and cut AR = 4.5 cm.

Step V: Join DR. Thus, DEAR is the required quadrilateral.

(ii) Construction:



Step I: Draw TR = 3.5 cm Step II: Draw an angle of 75° at R and cut RU = 3 cm. Step III: Draw an angle of 120° at U and cut UE = 4.5 cm. Step IV: Join TE. Thus, TRUE is the required quadrilateral.

EXERCISE 4.5

Draw the following: **Q.1. The square READ with RE = 5.1 cm.** Solution: Construction:



Step I: Draw RE = 5.1 cm. Step II: Draw an angle of 90° at E and cut EA = 5.1 cm. Step III: Draw two arcs from A and R with radius 5.1 cm to cut each other at D. Step IV: Join RD and AD. Thus, READ is the required square.

Q.2. A rhombus whose diagonals are 5.2 cm and 6.4 cm long. Solution:

Construction:



Step I: Draw AC = 6.4 cm. Step II: Draw the right bisector of AC at E. Step III: Draw two arcs with centre E and radius $=\frac{5.2}{2} = 2.6$ cm to cut the previous diagonal at B and D. Step IV: Join AD, AB, BC and DC. Thus ABCD is the required rhombus.

Q.3 A rectangle with adjacent sides of lengths 5 cm and 4 cm.

Solution:

Construction: Let the two adjacent sides of a rectangle PQRS be PQ = 5 cm and QR = 4 cm.



Step I: Draw PQ = 5 cm.

Step II: Draw an angle of 90° at Q and cut QR = 4 cm.

Step III: Draw an arc with centre R and radius 5 cm.

Step IV: Draw another arc with centre P and radius 4 cm to meet the previous arc at S. Step V: Join RS and PS.

Thus, PQRS is the required rectangle.

Q.4.

A parallelogram OKAY where OK = 5.5 cm and KA = 4.2 cm. Is it unique? Solution: Construction: Step I: Draw OK = 5.5 cm. Step II: Draw an angle of any measure (say 60°) at K and cut KA = 4.2 cm.



Step III: Draw an arc with centre A and radius of 5.5 cm.

Step IV: Draw another arc with centre O and radius 4.2 cm to cut the previous arc at Y. Step V: Join AY and OY.

Thus, OKAY is the required parallelogram.No, it is not a unique parallelogram. The angle at K can be of measure other than 60° .

<u>Worksheet</u> <u>Class 08 - Mathematics (Practical Geometry)</u>

General Instructions: All questions are compulsory.

Q.1 to Q.2 carries one mark each.

Q.3 to Q.7 carries two marks each.

Q.8 and Q.9 carries three marks each.

Q.10 to Q.12 carries four marks each.

- 1. Name the special quadrilateral whose all angles and sides are equal?
- 2. When does a parallelogram become a square?
- 3. State True or False:
 - a. The diagonals of rhombus are perpendicular and bisect each other.
 - b. Opposite angles of rectangle formed at the point where diagonals meet are congruent.
 - c. A rhombus can be constructed if a side and a diagonal is known.
 - d. All sides of parallelogram are congruent.
 - e.

4. Fill in the blanks

- i. For any two rational numbers a and b, $a \times b =$ _____.
- ii. For any three rational numbers a, b and c, a + (b + c) =_____.
- iii. Reciprocal of a / b where $b \neq 0$ is _____
- iv. Every whole number can be written in a / b where b =_____.
- 5. Match the following:

	-	
Column A		Column B
a. Number of sides in decagon		Rhombus
b. Diagonals are perpendicular		Equilateral triangle
c. Equal sides triangle	iii.	Parallelogram
d. Adjacent angles are supplementary	iv.	Ten

- 6. Construct a parallelogram ABCD in which AB = 4 cm, BC = 5 cm and $\angle B = 60^{\circ}$.
- 7. Construct a parallelogram HOME with HO = 6 cm, HE = 4 cm and OE = 3 cm.
- 8. Construct a rhombus LEND where LN=5.6cm and DE=6.5cm.
- 9. Construct a rectangle MIST where MI=7cm, IS=5cm.
- 10. Construct a trapezium PQRS in which PQ||SR, $\angle P = 105^{\circ}$, PS = 3cm, PQ = 4cm, RQ = 4.5cm and RS = 8cm.
- 11. Construct Quadrilateral GOLD. OL = 7.5cm, GL = 6 cm, GD = 6 cm, LD = 5 cm, OD = 10 cm
- 12. Construct a quadrilateral ABCD, with AB = 4cm, BC = 5cm, CD = 6.5cm, $\angle B = 105^{\circ}$ and $\angle C = \angle 80^{\circ}$.

