पु॰ना International School

Shree Swaminarayan Gurukul, Zundal

Grade VII

MATHEMATICS

Specimen Copy

2021-22

[Chap. No	Name
1	**INTEGERS**
2	**FRACTIONS AND DECIMALS** APRIL & May
3	**DATA HANDLING** JUNE
4	**SIMPLE EQUATION**
5	**LINES AND ANGLES** JUly
6	**THE TRIANGLE AND ITS PROPERTIES**
7	**CONGRUENCE OF TRIAGLES** August
8	**COMPARING QUANTITIES**
9	**RATIONAL NUMBERS** **September …Revision**
10	**PRACTICAL GEOMETERY** October
11	**PEREMETER AND AREA**
12	**ALGEBRIC EXPRESSION** November
13	**EXPONENTS AND POWERS**
14	**SYMMETRY** December
15	**VISUALISING SOLID SHAPES**

CHAPTER : 3

NAME: DATA HANDLING

Key Points To Remember

1) Average is a number that represents or shows the central tendency of a group of observations of data.

2)The arithmetic mean (AM) or simply mean is defined as follows:

$$\text{Arithematic mean} = \frac{\text{sum of all obsrvations}}{\text{number of all obsrvations}}$$

3) Range is the difference between the highest and the lowest observation of the data. i.e.
Range = Highest observation – Lowest observation

4) Mode of a set of observation is the observation that occurs, the most often e.g.
2 is the mode of a set of numbers 1, 1, 2, 4, 3, 2, 1, 2, 2, 4.

5) Median refers to the value which lies in the middle of the data with half of the observations above it and the other half below it.
e.g. 24, 36,46,17,18, 25, 35 is given data.
Firstly, data is to arranged in ascending order i.e. 17,18, 24,25, 35,36, 46.
Since the median is the middle observation, therefore 25 is the median.

- **If the data has an odd number of items, then the median is the middle number.**
- **If the data has an even number of items, then the median is mean of two middle numbers.**

6) A bar graph is a representation of numbers using bars of uniform widths.

7) Mode of the data is the longest bar if the bar represents frequency.

8) Double bar graphs help to compare two collections of data at a glance.

9) Collection of Data

A given collection of data may not give us a piece of the specific information related to that data. Before collecting data, we need to know what we would use it for.

Exercise 3.1

1.Find the range of heights of any ten students of your class.

Solution :

Let the heights of 10 students are as follows:
140 cm, 141.5 cm, 138 cm, 150 cm, 161 cm,
138 cm, 140.5 cm, 135.5 cm, 160 cm, 158 cm
Here, minimum height = 135.5 cm
Maximum height =161 cm
∴ Range = Maximum height – Minimum height
= 161 cm — 135.5 cm = 25.5 cm
Hence, the required range = 25.5 cm.

2.Organise the following marks in a class assessment in a tabular form.
4, 6, 7, 5, 3, 5, 4, 5, 2, 6, 2, 5, 1, 9, 6, 5, 8, 4, 6, 7
(i) Which number is the highest?
(ii) Which number is the lowest?
(iii) What is the range of the data?
(iv) Find the arithmetic mean.

Solution:

Forming frequency table

Marks (x_i)	Tally marks	Frequency (f_i)	$f_i x_i$
1	|	1	1
2	||	2	4
3	|	1	3
4	|||	3	12
5	~~||||~~	5	25
6	||||	4	24
7	||	2	14
8	|	1	8
9	|	1	9
		20	100

(i) 9 is the highest marks.
(ii) 1 is the lowest marks.
(iii) Range = Max. marks – Min. marks
= 9 – 1 = 8
(iv) Arithmetic mean = $\frac{\Sigma fixi}{\Sigma fi} = \frac{100}{20} = 5$

3. Find the mean of first five whole numbers.

Solution

First 5 whole numbers are 0, 1, 2, 3, 4

$\therefore$ **Mean** $= \dfrac{0+1+2+3+4}{5} = \dfrac{10}{5} = 2$

Answer : The required mean =2

4. A cricketer scores the following runs in eight innings:

58, 76, 40, 35, 46, 45, 0, 100

Find the mean score.

Solution:

Following are the scores of the runs in eight innings:

58, 76, 40, 35, 46, 45, 0, 100

$\therefore$ **Mean** $= \dfrac{\text{Sum of all runs}}{\text{number of innings}}$

$= \dfrac{58+76+40+35+46+45+0+100}{8}$

$= \dfrac{400}{8} = 50$

Answer : The required mean = 50

5. Following table shows the points of each player scored in four games:

Player	Game 1	Game 2	Game 3	Game 4
A	10	16	10	10
B	0	8	6	4
C	8	11	Did not play	13

Now answer the following questions:

(i) Find the mean to determine A's average number of points scored per game.

(ii) To find the mean number of points per game for C, would you divide the total points by 3 or by 4? Why?

(iii) B played in all the four games. How would you find the mean?

(iv) Who is the best performer?

Solution

i) Number of points scored by A in all games are
Game 1 – 14, Game 2 = 16, Game 3 = 10, Game 4 = 10

∴ Average score = $\frac{\mathbf{14+16+10+10}}{\mathbf{4}}$

$= \frac{\mathbf{50}}{\mathbf{4}} = \mathbf{12.5}$

ii) Since, C did not play Game 3, he played only 3 games. So, the total will be divided by 3.

(iii) Number of points scored by B in all the games are Game 1 = 0, Game 2 = 8, Game 3 = 6, Game 4 = 4

∴ Average score = $\frac{\mathbf{0+8+6+4}}{\mathbf{4}} = \frac{\mathbf{18}}{\mathbf{4}} = \mathbf{4.5}$

(iv) Mean score of C = $\frac{\mathbf{8+11+13}}{\mathbf{3}} = \frac{\mathbf{32}}{\mathbf{3}} = \mathbf{10.67}$

Mean score of C = 10.67
While mean score of A = 12.5
Clearly, A is the best performer.

6. The marks (out of 100) obtained by a group of students in a science test are 85, 76, 90, 85, 39, 48, 56, 95, 81 and 75. Find the
(i) highest and the lowest marks obtained by the students.
(ii) range of the marks obtained.
(iii) mean marks obtained by the group.

Solution

Marks obtained are:
85, 76, 90, 85, 39, 48, 56, 95, 81 and 75

(i) Highest marks = 95
Lowest marks = 39

(ii) Range of the marks
= Highest marks – Lowest marks

= 95 – 39 = 56

(iii) Mean marks = $\frac{\boldsymbol{sum\ of\ all\ marks\ obtained}}{\boldsymbol{number\ of\ students}}$

$$= \frac{85+76+90+85+39+48+56+95+81+75}{10}$$

$$= \frac{730}{10} = 73$$

7. The enrolment in a school during six consecutive years was as follows:
1555, 1670, 1750, 2013, 2540, 2820
Find the mean enrolment of the school for this period.

Solution:

Mean enrolment

$$= \frac{sum\ of\ th\ enrolment\ of\ all\ th\ years}{number\ of\ years}$$

$$= \frac{1555+1670+1750+2013+2540+2820}{6}$$

$$= \frac{12348}{6} = 2058$$

The required mean is 2058

8 The rainfall (in mm) in a city on 7 days of a certain week was recorded as follows:

Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Rainfall (in mm)	0.0	12.2	2.1	0.0	20.5	5.5	1.0

(i) Find the range of the rainfall in the above data.
(ii) Find the mean rainfall for the week.
(iii) On how many days was the rainfall less than the mean rainfall?

Solution

(i) Maximum rainfall = 20.5 mm
Minimum rainfall = 0.0 mm
∴ Range = Maximum rainfall – Minimum rainfall
= 20.5 mm – 0.0 mm = 20.5 mm

(ii) Mean rainfall

$= \frac{sum\ of\ rainfalls(\ in\ mm)}{number\ of\ days}$

$= \frac{0.0+12.2+2.1+0.0+20.5+5.5+1.0}{7}$

$= \frac{41.3}{7}$ mm=5.9mm

(iii) Number of days on which the rainfall was less than the mean rainfall = Monday, Wednesday, Thursday, Saturday, Sunday = 5 days.

9The heights of 10 girls were measured in cm and the results are as follows:
135, 150, 139, 128, 151, 132, 146, 149, 143, 141
(i) What is the height of the tallest girl?
(ii) What is the height of the shortest girl?
(Hi) What is the range of the data?
(iv) What is the mean height of the girls?
(v) How many girls have heights more than the mean height?

Solution

i) Height of the tallest girl = 151 cm.
(ii) Height of the shortest girl = 128 cm.
(iii) Range = Height of tallest girl – Height of the shortest girl
= 151 cm – 128 cm = 23 cm.

Mean height = $\frac{sum\ of\ all\ heights}{number\ of\ girls}$

$= \frac{135+150+139+128+151+132+146+149+143+141}{10}$

$= \frac{1414}{10}$ **=141.4 cm**

(v) Number of girls having more height than the mean height
= 150, 151, 146, 149 and 143 = 5 girls

Exercise 3.2

A) Write whether the following statement is true or false

i) The mode is always one of the number in a data.
(ii) The mean is one of the numbers in a data.

(iii) The median is always one of the numbers in a data.
(iv) The data 6, 4, 3, 8, 9, 12, 13, 9 has mean 9.

Answers

(i) True
(ii) False
(iii) True
(iv) False

1. The scores in mathematics test (out of 25) of 15 students is as follows:
19, 25, 23, 20, 9, 20, 15, 10, 5, 16, 25, 20, 24, 12, 20
Find the mode and median of this data. Are they same?
Solution:
Given data:
19, 25, 23, 20, 9, 20, 15, 10, 5, 16, 25, 20, 24, 12, 20
Let us arrange the given data in increasing order

$5, 9, 10, 12, 15, 16, 19, \underbrace{20, 20, 20, 20}, 23, 24,$
$25, 25$

Since 20 occurs 4 times (highest)
∴ Mode = 20
n = 15 (odd)

∴ Median = $\frac{n+1}{2}$ **th term** $= \frac{15+1}{2}$

$=$ **8^{th} term =20**

Thus, median = 20 and mode = 20
∴ Mode and median are same.

2. The runs scored in a cricket match by 11 players is as follows:
6, 15, 120, 50, 100, 80, 10, 15, 8, 10, 15
Find the mean, mode and median of this data. Are the three same?

Solution

Given data:
6, 15, 120, 50, 100, 80, 10, 15, 8, 10, 15

Mean = $\frac{sum\ of\ all\ the\ numbers}{number\ of\ terms}$

$= \dfrac{6+15+120+50+100+80+10+15+8+10+15}{11}$

$= \dfrac{429}{11} = 39$

Arranging the given data in increasing order, we get

$6, 8, 10, 10, \underbrace{15, 15, 15}, 50, 80, 100, 120$

ere, 15 occurs 3 times (highest)
∴ Mode = 15
n = 11 (odd)

∴ Median = $(\dfrac{11+1}{2})$**th term** = **6^{th} term = 15**

Thus mean = 39, mode = 15 and median = 15
No, they are not same.

3 The weights (in kg) of 15 students of a class are:
38, 42, 35, 37, 45, 50, 32, 43, 43, 40, 36, 38, 43, 38, 47
(i) Find the mode and median of this data.
(ii) Is there more than one mode?

Solution

Given data: 38, 42, 35, 37,45, 50, 32,43, 43,40, 36, 38, 43, 38, 47
Arranging in increasing order, we get

$32, 35, 36, 37, \underbrace{38, 38, 38}, 40, 42, \underbrace{43, 43, 43}, 45,$
$47, 50$

(i) Here, 38 and 42 occur 3 times (highest)
Thus mode = 38 and 43
n = 15(odd)

Given data: 38, 42, 35, 37,45, 50, 32,43, 43,40, 36, 38, 43, 38, 47
Arranging in increasing order, we get

$32, 35, 36, 37, \underbrace{38, 38, 38}, 40, 42, \underbrace{43, 43, 43}, 45,$
$47, 50$

(i) Here, 38 and 42 occur 3 times (highest)
Thus mode = 38 and 43
n = 15(odd)

Median = $\dfrac{n+1}{2}$ th term = $\dfrac{15+1}{2}$ th term

8th term = 40
Thus mode 38 and 43 and median = 40
(ii) Yes, the given data has two modes i.e. 38 and 43.

4. Find the mode and median of the data:
13, 16, 12, 14, 19, 12, 14, 13, 14

Solution:
Arranging the given data in increasing order, we get

$$12, 12, 13, 13, \underbrace{14, 14, 14}, 16, 19$$

Here, 14 occur 3 times (highest)

thus, mode = 14
n = 9(odd)

∴ Median $= \frac{n+1}{2}$ th term $\frac{9+1}{2}$ th term

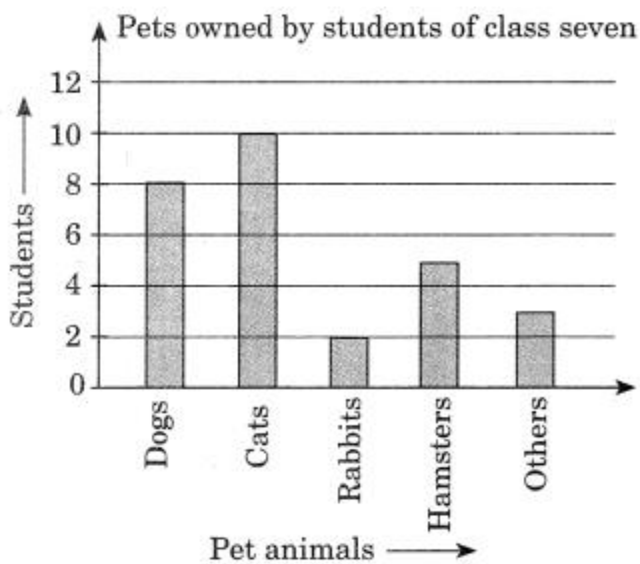
= 5th term = 14
Hence, mode = 14 and median = 14.

Exercise 3.3

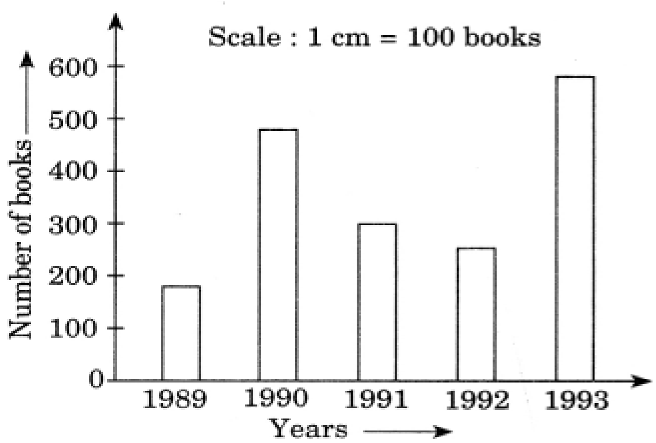
1 Use the bar graph to answer the following questions.
(a) Which is the most popular pet?
(b) How many students have dog as a pet?

Solution

From the given bar graph in figure, we have
(a) Cats are the most popular pet among the students.
(b) 8 students have dog as a pet animal.



2 Read the bar graph which shows the number of books sold by a bookstore during five consecutive years and answer the following questions:
(i) About how many books were sold in 1989, 1990, 1992?
(ii) In which year were about 475 books sold? About 225 books sold?
(iii) In which year were fewer than 250 books sold?
(iv) Can you explain how you would estimate the number of books sold in 1989?



Solution

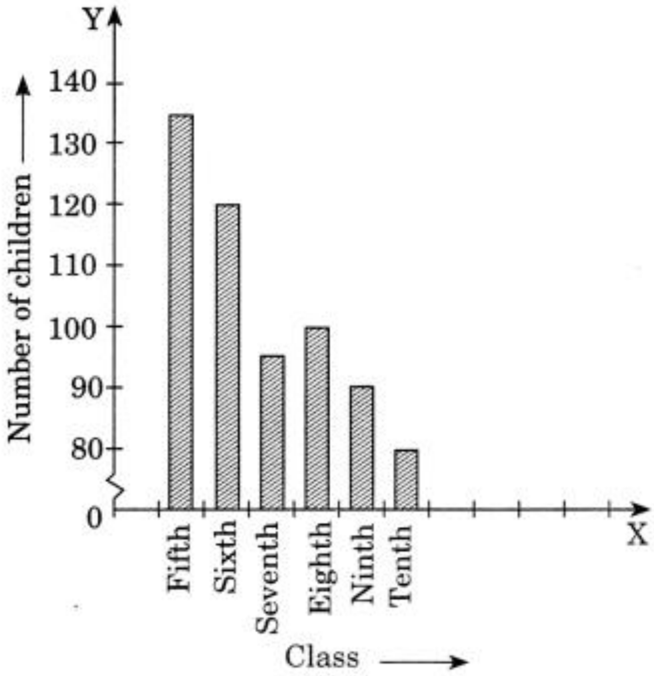
From the given bar graph, we have
(i) Number of books sold in the year 1989 is about 180, in 1990 is about 490 and in 1992 is about 295.
(ii) About 475 books were sold in 1990. About 225 books were sold in the year 1992.
(iii) Fewer than 250 books were sold in the years 1989 and 1992.
(iv) On y-axis, the line is divided into 10 small parts of 10 books each. So, we can estimate the number of books sold in 1989 is about 180.

3 Number of children in six different classes are given below. Represent the data on a bar graph.

Class	Fifth	Sixth	Seventh	Eighth	Ninth	Tenth
No. of children	135	120	95	100	90	80

(a) How would you choose a scale?
(b) Answer the following questions:

- Which class has the maximum number of children? And the minimum?
- Find the ratio of students of class sixth to £he students of class eighth.



(a) Scale on y-axis is 1 cm = 10 students
(b)

- **Fifth class has the maximum number of children i.e., 135.**
  Tenth class has the minimum number of children i.e., 80.
- **Number of children in class eight = 100**
  ∴ Ratio of class sixth to the students of class

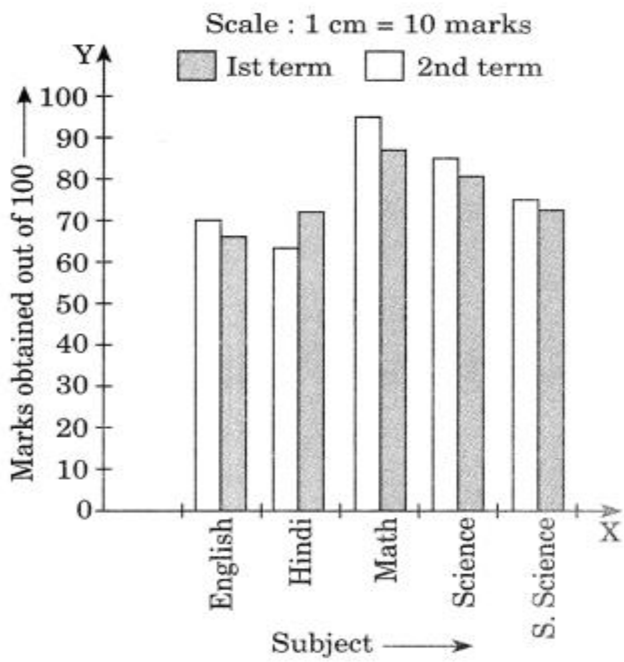
eighth = $\frac{\cancel{120}^{6}}{\cancel{100}_{5}} = \frac{6}{5}$, i.e., 6 : 5

4 The performance of a student in 1st term and 2nd term is given. Draw a double bar graph choosing appropriate scale and answer the following:

Subject	English	Hindi	Maths	Science	S.Science
1^{st} Term (M.M.100)	67	72	88	81	73
2^{nd} Term (M.M.10)	70	65	95	85	75

(i) In which subject, has the child improved his performance the most?
(ii) In which subject is the improvement the least?
(iii) Has the performance gone down in any subject?

Solution



i) **In Math, the performance of the students improved the most.**
(ii) In social science, the performance of the students improved the least.
(iii) Yes, in Hindi the performance of the students has gone down.

5 Consider this data collected from survey of a colony.

Favourite Sport	Cricket	Basket ball	Swimming	Hockey	Athletics
Watching	1240	470	510	430	250
Participating	620	320	320	250	105

i) Draw a double bar graph choosing an appropriate scale. What do you infer from the bar

graph?
(ii) Which sport is most popular?
(iii) Which is more preferred, watching or participating in sports?

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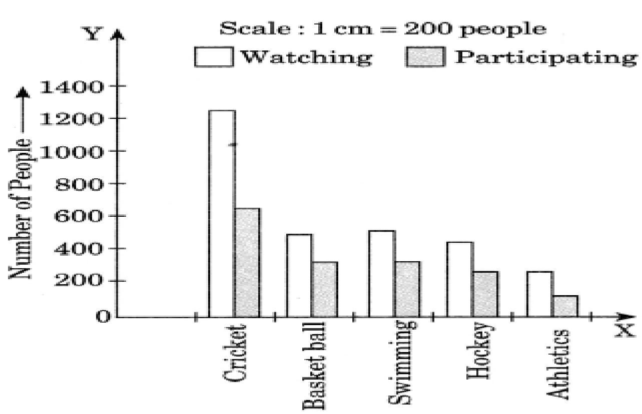


6Take the data giving the minimum and the maximum temperature of various cities given in the beginning of this chapter. Plot a double bar graph using the data and answer the following:

(i) Which city has the largest difference in the minimum and maximum temperature on the given date?

(ii) Which is the hottest city and which is the coldest city?

(iii) Name two cities where maximum temperature of one was less than the minimum temperature of the other.

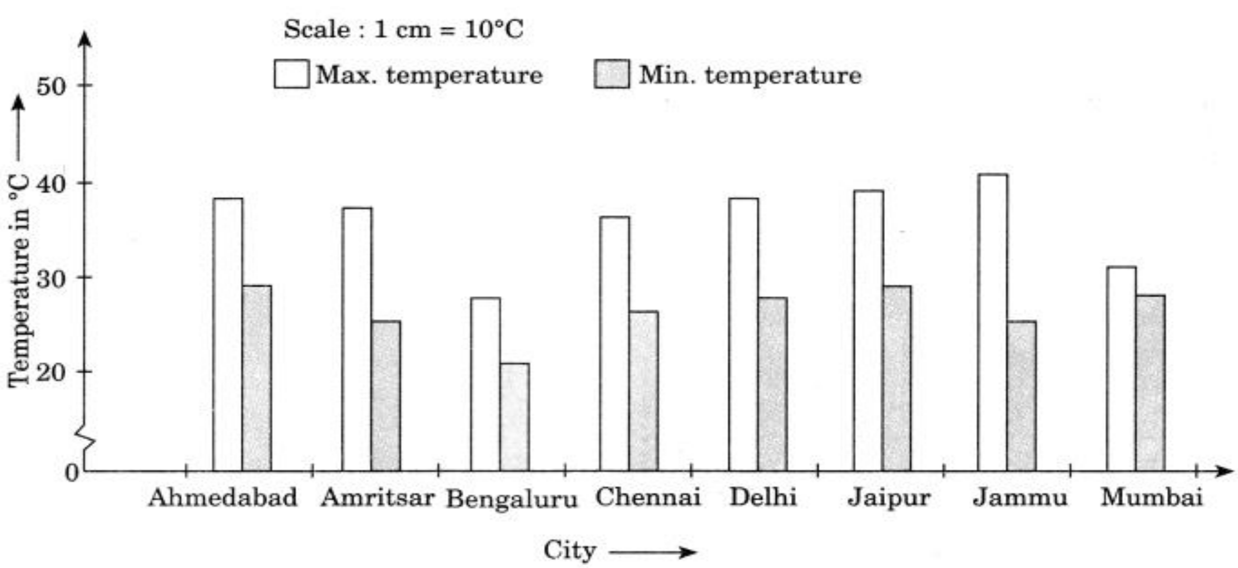
(iv) Name the city which has the least difference between its minimum and maximum temperature.

Temperature of Cities as on 20.6.2006

City	Maximum	Minimum
Ahmedabad	38^0	29^0
Amritsar	37^0	26^0
Bangalore	28^0	21^0
Chennai	36^0	27^0
Delhi	38^0	28^0
Jaipur	39^0	29^0
Jammu	41^0	26^0
Mumbai	32^0	27^0

Solution

Double bar graph



(i) Jammu has the largest difference between the maximum and minimum temperature i.e. 41°C – 26°C = 15°C
(ii) Hottest city is Jammu with 41°C temperature and coldest city is Bengaluru with 21°C temperature.
(iii)

- Bengaluru having its maximum temperature 28°C is less than the minimum temperature 29°C in Ahmedabad.
- Bengaluru having its maximum tem-perature 28°C is less than the maximum temperature 29°C in Jaipur.

(iv) Mumbai has the least difference between its minimum and maximum temperatures i.e. 32°C – 27°C = 5°C

Exercise 3.4

Tell whether the following situations are certain to happen, impossible to happen, can happen but not certain.
(i) You are older today than yesterday.
(ii) A tossed coin will land heads up.
(iii) A dice when tossed shall land up with 8 on top.
(iv) The next traffic light seen will be green.
(v) Tomorrow will be a cloudy day.

Solution

Event	Chance
(i) You are older today than yesterday.	Certain to happen
(ii) A tossed coin will land heads up.	Can happen but not certain
iii) A dice when tossed shall land up with 8 on top.	Impossible
(iv) The next traffic light seen will be green.	Can happen but not certain
(v) Tomorrow will be a cloudy day.	Can happen but not certain

2There are 6 marbles in a box with numbers from 1 to 6 marked on each of them.
(i) What is the probability of drawing a marble with number 2?
(ii) What is the probability of drawing a marble with number 5?

Solution

(i) Total number of marbles marked with the number from 1 to 6 = 6
∴ n(S) = 6

Number of marble marked with 2=1
∴ n(E) = 1

∴ Required probability =$\frac{n\ (E)}{n\ (S)}$ $= \frac{1}{6}$

(ii) Number of marble marked with 5 = 1
∴ n(E) = 1

∴ Required probability =$\frac{n\ (E)}{n\ (S)}$ $= \frac{1}{6}$

3. A coin is flipped to decide which team starts the game. What is the probability that your team will start?

Solution

Coin has 2 faces—Head (H) and Tail (T)
∴ Sample space S(n) = 2
Number of successful event n(E) = 1

∴ Required probability =$\frac{n\ (E)}{n\ (S)}$ $= \frac{1}{2}$

CHAPTER 4

SIMPLE EQUATION

1) What Equation Is?

An equation is a condition on a variable. The condition is that two expressions should have equal value. Note at least one of the two expressions must contain the variable.
An equation remains the same when the expressions on the left and on the right are interchanged. This property is often useful in solving equations.

2) Solving an Equation

For any balanced numerical equation, if we either:

- **add the same number to both sides,**
- **or subtract the same number from both sides,**
- **or multiply by the same number to both sides,**
- **or divide by the same number both its sides, the balance is undisturbed.**

3 More Equations

Transposing means moving to the other side. It has the same effect as adding the same number to (or subtracting the same number from) both sides of the equation.
When we transpose a number from one side of the equation to the other side, we change its sign.

4) Applications of Simple Equations to Practical Situations

Equations involving only a linear polynomial are called simple equations.
e.g. $4x + 5 = 65, 10y - 20 = 50$

- **In an equation, there is always an equality sign.**
- **A Simple Equation remains the same when the expression in the left and right are interchanged**.

The value of a variable, which makes the equation a true statement is called the solution of a linear equation.

e.g. $5x - 12 = -2$ is a equation
L.H.S = $5x - 12 = 5 \times 2 - 12 = 10 - 12 = -2$
L.H.S = R.H.S

5) In the case of the balanced equation, we have

- **add the same number to both the sides,**
- **subtract the same number from both the sides**
- **multiply both sides by the same number**
- **divide both sides by the same number, the balance remains undisturbed,**

ie. The value of the LHS remains equal to the value of the RHS.

6)Transposing means moving to the other side.

Transposition of a number has the same effect as adding the same number to (or subtracting the same number from) both sides of the equation. When you transpose a number from one side of the equation to the other side, you change its sign. For example, transposing +3 from the LHS to the RHS in equation $x + 3 = 8$ gives $x = 8 - 3 = 5$.

Short questions

a) Write equations for the following statements:

(i) The sum of numbers x and 4 is 9.
(ii) 2 subtracted from y is 8.
(iii) Ten times a is 70.
(iv) The number b divided by 5 gives 6.
(v) Three-fourth of t is 15.
(vi) Seven times m plus 7 gets you 77.
(vii) One-fourth of a number x minus 4 gives 4.

(viii) If you take away 6 from 6 times y, you get 60.
(ix) If you add 3 to one-third of z, you get 30.

Solution

Statements	Equations
(i) The sum of numbers x and 4 is 9.	$x + 4 = 9$
(ii) 2 subtracted from y is 8.	$y - 2 = 8$
(iii) Ten times a is 70.	$10a = 7$
(iv) The number b divided by 5 gives 6.	$\frac{b}{5} = 6$
(v) Three-fourth of t is 15.	$\frac{3}{4}t = 15$
(vi) Seven times m plus 7 gets you 77.	$7m + 7 = 77$
(vii) One-fourth of a number x minus 4 gives 4.	$\frac{1}{4}x - 4 = 4$
(viii) If you take away 6 from 6 times y, you get 60.	$6y - 6 = 60$
(ix) If you add 3 to one-third of z, you get 30.	$\frac{z}{3} + 3 = 30$

(v) $\frac{3m}{5} = 6$ (vi) $3p + 4 = 25$

(vii) $4p - 2 = 18$ (viii) $\frac{p}{2} + 2 = 8$

Solution

Equations	Statements
(i) $p + 4 = 15$	Sum of a number p and 4 gives 15.
(ii) $m - 7 = 3$	Seven subtracted from m gives 3.
(iii) $2m = 7$	Twice of m gives 7.
(iv) $\frac{m}{5} = 3$	m divided by 5 gives 3.
(v) $\frac{3m}{5} = 6$	Three times of m divided by 5 gives 6.
(vi) $3p + 4 = 25$	4 added to three times p gives 25.
(vii) $4p - 2 = 18$	2 subtracted from four times p gives 18.
(viii) $\frac{p}{2} + 2 = 8$	2 added to half of p gives 8.

1. Complete the last column of the table

S. No.	Equation	Value	Say, whether the equation is satisfied (Yes/No)
(i)	$x + 3 = 0$	$x = 3$	____
(ii)	$x + 3 = 0$	$x = 0$	____
(iii)	$x + 3 = 0$	$x = -3$	____
(iv)	$x - 7 = 1$	$x = 7$	____
(v)	$x - 7 = 1$	$x = 8$	____
(vi)	$5x = 25$	$x = 0$	____
(vii)	$5x = 25$	$x = 5$	____
(viii)	$5x = 25$	$x = -5$	____
(ix)	$\frac{m}{3} = 2$	$m = -6$	____
(x)	$\frac{m}{3} = 2$	$m = 0$	____
(xi)	$\frac{m}{3} = 2$	$m = 6$	____

Solution

S.No.	Equations	Value	Say, whether the equation is satisfied (Yes/No)
(i)	$x + 3 = 0$	$x = 3$	No
(ii)	$x + 3 = 0$	$x = 0$	No
(iii)	$x + 3 = 0$	$x = -3$	Yes
(iv)	$x - 7 = 1$	$x = 7$	No
(v)	$x - 7 = 1$	$x = 8$	Yes
(vi)	$5x = 25$	$x = 0$	No
(vii)	$5x = 25$	$x = 5$	Yes
(viii)	$5x = 25$	$x = -5$	No
(ix)	$\frac{m}{3} = 2$	$m = -6$	No
(x)	$\frac{m}{3} = 2$	$m = 0$	No
(xi)	$\frac{m}{3} = 2$	$m = 6$	Yes

2 Check whether the value given in the brackets is a solution to the given equation or not?

(a) n + 5 = 19; (n = 1)
(b) 7n + 5 = 19; in – -2)
(c) 7n + 5 = 19; (n = 2)

(d) 4p – 3 = 13; (p = 1)
(e) 4p – 3 = 13; (p = -4)
(f) 4p-3 = 13; (p = 0)

Solution

(a) $n + 5 = 19$ $(n = 1)$
Put $n = 1$ in LHS
$1 + 5 = 6 \neq 19$ (RHS)
Since LHS $\neq$ RHS
Thus $n = 1$ is not the solution of the given equation.

b) $7n + 5 = 19$; $(n = -2)$
Put $n = -2$ in LHS
$7 \times (-2) + 5 = -14 + 5 = -9 \neq 19$ (RHS)
Since LHS $\neq$ RHS
Thus, $n = -2$ is not the solution of the given equation.

c) $7n + 5 = 19$; $(n = 2)$
Put $n = 2$ in LHS
$7 \times 2 + 5 = 14 + 5 = 19 = 19$ (RHS)
Since LHS = RHS
Thus, $n – 2$ is the solution of the given equation.

(d) $4p – 3 = 13$; $(p = 1)$
Put $p = 1$ in LHS
$4 \times 1 – 3 = 4 – 3 = 1 \neq 13$ (RHS)
Since LHS $\neq$ RHS
Thus, $p = 1$ is not the solution of the given equation.

(e) $4p – 3 = 13$; $(p = -A)$
Put $p = -4$ in LHS
$4 \times (-4) – 3 = -16 – 3 = -19 \neq 13$ (RHS)
Since LHS $\neq$ RHS
Thus $p = -4$ is not the solution of the given equation.

f) $4p – 3 = 13$; $(p = 0)$
Put $p = 0$ in LHS
$4 \times (0) – 3 = 0 – 3 = -3 \neq 13$ (RHS)
Since LHS $\neq$ RHS
Thus $p – 0$ is not the solution of the given equation.

3 Solve the following equations by trial and error method:

(i) $5p + 2 = 17$
(ii) $3m – 14 = 4$

Solution:

(i) $5p + 2 = 17$
For $p = 1$, LHS
$= 5 \times 1 + 2 = 5 + 2 = 7 \neq 17$ (RHS)
For $p = 2$, LHS $= 5 \times 2 + 2$
$= 10 + 2 = 12 \neq 17$ (RHS)
For $p = 3$, LHS $= 5 \times 3 + 2$
$= 15 + 2 = 17 = 17$ (RHS)
Since the given equation is satisfied for $p = 3$ Thus, $p = 3$ is the required solution.

ii) $3m – 14 = 4$
For $m = 1$, LHS $= 3 \times 1 – 14$
$= 3 – 14 = -11 \neq 4$ (RHS)
For $m = 2$, LHS $= 3 \times 2 – 14 = 6 – 14$
$= -8 \neq 4$ (RHS)
For $m = 3$, LHS $= 3 \times 3 – 14 = 9 – 14$
$= -5 \neq 4$ (RHS)
Form $m = 4$, LHS $= 3 \times 4 – 14$
$= 12 – 14 = -2 \neq 4$ (RHS)
For $m = 5$, LHS $= 3 \times 5 – 14$
$= 15 – 14 = -1 \neq 4$ (RHS)
For $m = 6$, LHS $= 3 \times 6 – 14$
$= 18 – 14 = 4$ (=) 4 (RHS) .
Since, the given equation is satisfied for $m = 6$.
Thus, $m = 6$ is the required solution.

6. Set up an equation in the following cases:

(i) Irfan says that he has 7 marbles more than five times the marbles Parmit has. Irfan has 37 marbles. (Take m to be the number of Parmit's marbles)
(ii) Laxmi's father is 49 years old. He is 4 years older than three times Laxmi's age. (Take Laxmi's age to be y years)
(iii) The teacher tells the class that the highest marks obtained by a student in her class is twice the lowest marks plus 7. The highest score is 87. (Take the lowest score to be 1)
(iv) In an isosceles triangle, the vertex angle is twice either base angle. (Let the base angle be b in degrees. Remember that the sum of angles of a triangle is 180 degrees).

Solution

i) Let m be the Parmit's marbles.
$\therefore$ Irfan's marble $= 5m + 7$
Total number of Irfan's marble is given by 37.
Thus, the required equation is $5m + 7 = 37$

(ii) Let Laxmi's age bey years.
∴ Laxmi's father's age = 3y + 4
But the Laxmi's father age is given by 49
Thus the required equation is 3y + 4 = 49

(iii) Let the lowest score be l.
∴ The highest score = 2l + 1
But the highest score is given by 87.
Thus, the required equation is 2l + 1 = 87

iv) Let each base angle be 'b' degrees.
∴ Vertex angle of the triangle = 2b
Sum of the angles of a triangle = 180°
∴ Required equation is b + b + 2b = 180° or 4b = 180°

EXERCISE 4.2

Very short questions

1 Given first the step you will use to separate the variable and then solve the equation:

(a) x – 1 = 0
(b) x + 1 = 0
(c) x – 1 = 5
(d) x + 6 = 2
(e) y – 4 = -7
(f) y -4 = 4
(g) y + 4 = 4
(h) y + 4 = -4

Solution

a) x – 1 = 0
Adding 1 to both sides, we get
x – 1 + l = 0 + 1 ⇒ x = 1
Thus, x = 1 is the required solutions.
Check: Put x = 1 in the given equations
x – 1 = 0
1 – 1 = 0
0 = 0

LHS = RHS
Thus $x = 1$ is the correct solution.

(b) $x + 1 = 0$
Subtracting 1 from both sides, we get
$x + 1 – 1 = 0 – 1 \Rightarrow x = -1$
Thus $x = -1$ is the required solution.
Check: Put $x = -1$ in the given equation
$-1 + 1 = 0$
$0 = 0$
LHS = RHS
Thus $x = -1$ is the correct solution.

c) $x – 1 = 5$
Adding 1 to both sides, we get
$x – 1 + 1 = 5 + 1 \Rightarrow x = 6$
Thus $x = 6$ is the required solution.
Check: $x – 1 = 5$
Putting $x = 6$ in the given equation
$6 – 1 = 5 \Rightarrow 5 = 5$
LHS = RHS
Thus, $x = 6$ is the correct solution.

d) $x + 6 = 2$
Subtracting 6 from both sides, we get
$x + 6 – 6 = 2 – 6 \Rightarrow x = -4$
Thus, $x = -4$ is the required solution.
Check: $x + 6 = 2$
Putting $x = -4$, we get
$-4 + 6 = 2 \Rightarrow 2 = 2$ LHS = RHS
Thus $x = -4$ is the correct solution.

e) $y – 4 = -7$
Adding 4 to both sides, we get
$y – 4 + 4 = -7 + 4 \Rightarrow y = -3$
Thus, $y = -3$ is the required solution.
Check: $y – 4 = -7$
Putting $y = -3$, we get
$-3 – 4 = -7 \Rightarrow -7 = -7$
LHS = RHS
Thus, $y = -3$ is the correct solution.

(f) y – 4 = 4
Adding 4 to both sides, we get
y – 4 + 4 = 4 + 4 ⇒ y = 8
Thus, y = 8 is the required solution.
Check: y – 4 = 4
Putting y = 8, we get
8 – 4 = 4 ⇒ 4 = 4
LHS = RHS
Thus y = 8 is the correct solution.

g) y + 4 = 4
Subtracting 4 from both sides, we get
y + 4 – 4 = 4 – 4 ⇒ y = 0
Thus y = 0 is the required solution.
Check: y + 4 = 4
Putting y = 0, we get
0 + 4 = 4 ⇒ 4 = 4
LHS = RHS
Thus y = 0 is the correct solution.

(h) y + 4 =-4
Subtracting 4 from both sides, we get
y + 4 – 4 = -4 – 4 ⇒ y = -8
Thus, y = -8 is the required solution.
Check: y + 4 = -4
Putting y = -8, we get
-8 + 4 = -4 ⇒ -4 = -4
LHS = RHS
Thus, y = -8 is the correct solution.

2 Give first the step you will use to separate the variable and then solve the following equation:

(a) $3l = 42$ (b) $\frac{b}{2} = 6$
(c) $\frac{p}{7} = 4$ (d) $4x = 25$
(e) $8y = 36$ (f) $\frac{z}{3} = \frac{5}{4}$
(g) $\frac{a}{5} = \frac{7}{15}$ (h) $20t = -10$

Solution

(a) $3l = 42$

Dividing both sides by 3, we get

$$3l \div 3 = 42 \div 3 \quad \Rightarrow \quad l = \frac{\cancel{42}^{14}}{\cancel{3}} = 14$$

Thus $l = 14$

(b) $\frac{b}{2} = 6$

Multiplying both sides by 2, we get

$$\frac{b}{2} \times 2 = 6 \times 2 \quad \Rightarrow \quad b = 12$$

Thus, $b = 12$

(c) $\frac{p}{7} = 4$

Multiplying both sides by 7, we get

$$\frac{p}{7} \times 7 = 4 \times 7 \Rightarrow p = 28$$

Thus $p = 28$

(d) $4x = 25$

Dividing both sides by 4, we get

$$4x \div 4 = 25 \div 4$$

$$\Rightarrow \quad x = \frac{25}{4} = 6\frac{1}{4}$$

Thus $x = 6\frac{1}{4}$

(e) $8y = 36$

Dividing both sides by 8, we get

$$8y \div 8 = 36 \div 8$$

$$\Rightarrow \quad y = \frac{\cancel{36}^{9}}{\cancel{8}_{2}} = \frac{9}{2} = 4\frac{1}{2}$$

Thus, $y = 4\frac{1}{2}$

(f) $\frac{z}{3} = \frac{5}{4}$

Multiplying both sides by 3, we get

$$\frac{z}{3} \times 3 = \frac{5}{4} \times 3 \Rightarrow z = \frac{15}{4} = 3\frac{3}{4}$$

Thus, $z = 3\frac{3}{4}$

(g) $\frac{a}{5} = \frac{7}{15}$

Multiplying both sides by 5, we get

$$\frac{a}{5} \times 5 = \frac{7}{\cancel{15}_{3}} \times \cancel{5} = \frac{7}{3}$$

$$\Rightarrow \quad a = \frac{7}{3} = 2\frac{1}{3}$$

Thus, $a = 2\frac{1}{3}$

(h) $20t = -10$

Dividing both sides by 20, we get

$$20t \div 20 = -10 \div 20 \Rightarrow t = \frac{-10}{20} = \frac{-1}{2}$$

Thus $t = \frac{-1}{2}$

3 Give the steps you will use to separate the variables and then solve the equation:

a) 3n – 2 = 46
(b) 5m + 7 = 17
(c) 20p3=40
(d) 3p10=6

Solution

(a) 3n – 2 = 46
⇒ 3n- 2 + 2 = 46+ 2 (adding 2 to both sides)
⇒ 3n = 48
⇒ 3n + 3 = 48 ÷ 3

$$\Rightarrow \qquad n = \frac{\cancel{48}^{16}}{\cancel{3}} = 16$$

Thus $\quad n = 16$

(b) 5m + 7 = 17
⇒ 5m+ 7 – 7 = 17 – 7 (Subtracting 7 from both sides)
⇒ 5m = 10
⇒ 5m + 5 = 10 ÷ 5 (Dividing both sides by 5)

$$\Rightarrow \qquad m = \frac{\cancel{10}^{2}}{\cancel{5}} = 2$$

Thus $\quad m = 2$

c) $\mathbf{\frac{20\,p}{3} = 40}$

$$\Rightarrow \qquad \frac{20p}{3} \times 3 = 40 \times 3$$

(Multiplying both sides by 3)

$$\Rightarrow \qquad 20p = 120$$

$$\Rightarrow \qquad 20p \div 20 = 120 \div 20$$

(Dividing both sides by 20)

$$\Rightarrow \qquad p = \frac{\cancel{120}^{6}}{\cancel{20}} = 6$$

Thus $\quad p = 6$

d) $\mathbf{\frac{3\,p}{10} = 6}$

$\frac{3p}{10} \times 10 = 6 \times 10$ = (multiplying both sides by 10)

3p = 60

3p ÷ 3= 60 ÷ 3 (dividing both sides by 3)

$$\Rightarrow \qquad p = \frac{\cancel{60}^{20}}{\cancel{3}} = 20$$

Thus $\quad p = 20$

4 Solve the following equations:

(a) 10p = 100
(b) 10p + 10 = 100
(c) p4=5
(d) −p3=5
(e) 3p4=6
(f) 3s = -9
(g) 3s + 12 = 0
(h) 3s = 0
(i) 2q = 6
(j) 2q – 6 = 0
(k) 2q + 6 = 0
(l) 2q + 6 = 12

Solution

a) 10p = 100

⇒ 10p ÷ 10 = 100 ÷ 10 (Dividing both sides by 10)

$= p \; \frac{100}{10} = 10$

Thus p = 10

(b) 10p + 10 = 100

⇒ 10p + 10 – 10 = 100 -10 (Subtracting 10 from both sides)

⇒ 10p = 90

⇒ 10p ÷ 10 = 90 ÷ 10 (Dividing both side by 10)

$p = \frac{90}{10} = 9$

Thus p = 9

(c) $\frac{p}{4} = 5$

$\Rightarrow \quad \frac{p}{4} \times 4 = 5 \times 4$

(Multiplying both sides by 4)

$\Rightarrow \quad p = 20$

Thus $\quad p = 20$

(d) $\frac{-p}{3} = 5$

$\Rightarrow \quad \frac{-p}{3} \times 3 = 5 \times 3$

(Multiplying both sides by 3)

$\Rightarrow \quad -p = 15$

$\Rightarrow \quad -p \times (-1) = 15 \times (-1)$

[Multiplying both sides by (–1)]

$\Rightarrow \quad p = -15$

Thus $\quad p = -15$

(e) $\frac{3p}{4} = 6$

$\Rightarrow \quad \frac{3p}{4} \times 4 = 6 \times 4$

(Multiplying both sides by 4)

$\Rightarrow \quad 3p = 24$

$\Rightarrow \quad 3p \div 3 = 24 \div 3$

(Dividing both sides by 3)

$\Rightarrow \quad p = \frac{\cancel{24}^{8}}{\cancel{3}} = 8$

Thus $\quad p = 8$

(f) $3s = -9$

$\Rightarrow \quad 3s \div 3 = -9 \div 3$

y 3)

$\Rightarrow \quad s = -\frac{\cancel{9}^{3}}{\cancel{3}} = -3$

Thus $\quad s = -3$

(g) $3s + 12 = 0$

$\Rightarrow \quad 3s + 12 - 12 = 0 - 12$

(Subtracting 12 from both sides)

$\Rightarrow \quad 3s = -12$

$\Rightarrow \quad 3s \div 3 = -12 \div 3$

(Dividing both sides by 3)

$\Rightarrow \quad s = -\frac{\cancel{12}^{4}}{\cancel{3}} = -4$

Thus $\quad s = -4$

(h) $3s = 0$

$\Rightarrow \quad 3s \div 3 = 0 \div 3$

(Dividing both sides by 3)

$\Rightarrow \quad s = \frac{0}{3} = 0$

Thus, $\quad s = 0$

(*i*) $2q = 6$

$\Rightarrow \quad 2q \div 2 = 6 \div 2$

(Dividing both sides by 2)

$\Rightarrow \quad q = \frac{\cancel{6}^{3}}{\cancel{2}} = 3$

Thus $\quad q = 3$

(*j*) $2q - 6 = 0$

$\Rightarrow \quad 2q - 6 + 6 = 0 + 6$ (Adding 6 to both sides)

$\Rightarrow \quad 2q = 6$

$\Rightarrow \quad 2q \div 2 = 6 \div 2$ (Dividing both sides by 2)

$\Rightarrow \quad q = \frac{\cancel{6}^{3}}{\cancel{2}} = 3$

Thus $\quad q = 3$

(*k*) $2q + 6 = 0$

$\Rightarrow \quad 2q + 6 - 6 = 0 - 6$

(Subtracting 6 from each side)

$\Rightarrow \quad 2q = -6$

$\Rightarrow \quad 2q \div 2 = -6 \div 2$

(Dividing both sides by 2)

$\Rightarrow \quad q = -\frac{\cancel{6}^{3}}{\cancel{2}} = -3$

Thus $\quad q = -3$

(l) $2q + 6 = 12$

$\Rightarrow 2q + 6 - 6 = 12 - 6$ (Subtracting 6 from both sides)

$\Rightarrow 2q = 6$

$\Rightarrow \quad 2q \div 2 = 6 \div 2$

(Dividing both sides by 2)

$\Rightarrow \quad q = \frac{\cancel{6}^{3}}{\cancel{2}} = 3$

Thus $\quad q = 3$

EXERCISE 4.3

1Solve the following equation:

(a) $2y + \frac{5}{2} = \frac{37}{2}$ (b) $5t + 28 = 10$

(c) $\frac{a}{5} + 3 = 2$ (d) $\frac{q}{4} + 7 = 5$

(e) $\frac{5}{2}x = -10$ (f) $\frac{5}{2}x = \frac{25}{4}$

(g) $7m + \frac{19}{2} = 13$ (h) $6z + 10 = -2$

(i) $\frac{3l}{2} = \frac{2}{3}$ (j) $\frac{2b}{3} - 5 = 3$

Solution

(a) $2y + \frac{5}{2} = \frac{37}{2} \Rightarrow 2y = \frac{37}{2} - \frac{5}{2}$

(Transposing $\frac{5}{2}$ to RHS)

$\Rightarrow \quad 2y = \frac{37-5}{2}$

$= \frac{32}{2} = 16$

$\Rightarrow \quad 2y = 16$

$\Rightarrow \quad 2y \div 2 = 16 \div 2$

(Dividing both sides by 2)

$\Rightarrow \quad y = 8$

Check: Put $y = 8$ in LHS

$2 \times 8 + \frac{5}{2} = 16 + \frac{5}{2} = \frac{32+5}{2}$

$= \frac{37}{2}$ RHS as required.

(b) $5t + 28 = 10$

$\Rightarrow \quad 5t = 10 - 28$ (Transposing 28 to RHS)

$\Rightarrow \quad 5t = -18$

$\Rightarrow \quad 5t \div 5 = -18 \div 5$ (Dividing both sides by 5)

$\Rightarrow \quad t = \frac{-18}{5}$

Check: Put $t = \frac{-18}{5}$ in LHS

$5 \times \left(\frac{-18}{5}\right) + 28 = -18 + 28$

$= 10$ RHS as required

(c) $\frac{a}{5} + 3 = 2$

$\Rightarrow \quad \frac{a}{5} = 2 - 3$ (Transposing 3 to RHS)

$\Rightarrow \quad \frac{a}{5} = -1$

$\Rightarrow \quad \frac{a}{5} \times 5 = -1 \times 5$ (Multiplying both sides by 5)

$\Rightarrow \quad a = -5$

Check: Put $a = -5$ in LHS

$$\frac{-5}{5} + 3 = -1 + 3 = 2 \text{ RHS as required}$$

(*d*) $\frac{q}{4} + 7 = 5$

$\Rightarrow \quad \frac{q}{4} = 5 - 7$ (Transposing 7 to RHS)

$\Rightarrow \quad \frac{q}{4} = -2$

$\Rightarrow \quad \frac{q}{4} \times 4 = -2 \times 4$ (Multiplying both sides by 4)

$\Rightarrow \quad q = -8$

Check: Put $q = -8$ in LHS

$$\frac{-8}{4} + 7 = -2 + 7 = 5 \text{ RHS as required}$$

(*e*) $\frac{5}{2}x = -10$

$\Rightarrow \quad \frac{5}{2}x \times 2 = -10 \times 2$ (Multiplying both sides by 2)

$\Rightarrow \quad 5x = -20$

$\Rightarrow \quad 5x \div 5 = -20 \div 5$ (Dividing both sides by 5)

$\Rightarrow \quad x = -4$

Check: Put $x = -4$ in LHS

$$\frac{5}{2} \times (-4) = 5 \times (-2) = -10 \text{ RHS as required}$$

(*f*) $\frac{5}{2}x = \frac{25}{4}$

$\Rightarrow \quad \frac{5}{2}x \times 2 = \frac{25}{\cancel{4}_2} \times \cancel{2}$ (Multiplying both sides by 2)

$\Rightarrow \quad 5x = \frac{25}{2}$

$\Rightarrow \quad 5x \div 5 = \frac{25}{2} \div 5$ (Dividing both sides by 5)

$\Rightarrow \quad x = \frac{5}{2}$

Check: Put $x = \frac{5}{2}$ in LHS

$$\frac{5}{2} \times \frac{5}{2} = \frac{25}{4} \text{ RHS as required}$$

(*g*) $7m + \frac{19}{2} = 13$

$\Rightarrow \quad 7m = 13 - \frac{19}{2}$ $\left(\text{Transposing } \frac{19}{2} \text{ to RHS}\right)$

$\Rightarrow \quad 7m = \frac{26 - 19}{2} = \frac{7}{2} \quad \Rightarrow \quad 7m = \frac{7}{2}$

$\Rightarrow \quad 7m \div 7 = \frac{7}{2} \div 7$ (Dividing both sides by 7)

$m = \frac{1}{2}$

Check: Put $m = \frac{1}{2}$ in LHS

$7 \times \frac{1}{2} + \frac{19}{2} = \frac{7}{2} + \frac{19}{2}$

$= \frac{7+19}{2} = \frac{26}{2}$

$= 13$ RHS as required

(*h*) $6z + 10 = -2$

$\Rightarrow \quad 6z = -2 - 10$ (Transposing 10 to RHS)

$\Rightarrow \quad 6z = -12$

$\Rightarrow \quad 6z \div 6 = -12 \div 6$

(Dividing both sides by 6)

$\Rightarrow \quad z = -2$

Check: Put $z = -2$ in LHS

$6 \times (-2) + 10 = -12 + 10 = -2$ RHS as required.

(*i*) $\frac{3l}{2} = \frac{2}{3}$

$\Rightarrow \quad \frac{3l}{2} \times 2 = \frac{2}{3} \times 2$ (Multiplying both sides by 2)

$\Rightarrow \quad 3l = \frac{4}{3}$

$\Rightarrow \quad 3l \div 3 = \frac{4}{3} \div 3$ (Dividing both sides by 3)

$\Rightarrow \quad l = \frac{4}{3} \times \frac{1}{3} = \frac{4}{9}$

Check: Put $l = \frac{4}{9}$ in LHS

$\frac{\cancel{3}}{\cancel{2}} \times \frac{\cancel{4}^{2}}{\cancel{9}_{3}} = \frac{2}{3}$ RHS as required

(j) $\frac{2b}{3} - 5 = 3$

$\Rightarrow \quad \frac{2b}{3} = 5 + 3$ (Transposing 5 to RHS)

$\Rightarrow \quad \frac{2b}{3} = 8$

$\Rightarrow \quad \frac{2b}{3} \times 3 = 8 \times 3$ (Multiplying both sides by 3)

$\Rightarrow \quad 2b = 24$

$\Rightarrow \quad 2b \div 2 = 24 \div 2$ (Dividing both sides by 2)

$\Rightarrow \quad b = 12$

Check: Put $b = 12$ in LHS

$\frac{2}{\cancel{3}} \times \cancel{12}^{4} - 5 = 8 - 5 = 3$ RHS as required

2 Solve the following equations:

(a) 2(x + 4) = 12

(b) 3(n – 5) = 21

(c) 3(n – 5) = -21

(d) -4(2 + x) = 8

(e) 4(2 – x) = 8

Solution

(a) 2(x + 4) = 12

$\Rightarrow \frac{2(x+4)}{2} = \frac{12}{2}$ (Dividing both sides by 2)

⇒ x + 4 = 6

⇒ x = 6 – 4 (Transposing 4 to RHS)

⇒ x – 2

Check: Put x = 2 in LHS

2(2 + 4) = 2 × 6 = 12 RHS as required

b) 3(n – 5) = 21

⇒ $\frac{3(n-5)}{3} = \frac{21}{3}$ (Dividing both sides by 3)

⇒ n – 5 = 7

⇒ n = 7 + 5 (Transposing 5 to RHS)

n = 12

Check: Put n = 12 in LHS

3(12 – 5) = 3 × 7 = 21 RHS as required

(c) 3(n – 5) = -21

⇒ $\frac{3(n-5)}{3} = \frac{-21}{3}$ (Dividing both sides by 3)

⇒ n – 5 = -7

⇒ n = -7 + 5 (Transposing 5 to RHS)

⇒ n = -2

Check: Put n = -2 in LHS

3(-2 – 5) = 3 × -7

= -21 RHS as required

d) -4(2 + x) = 8

⇒ $\frac{-4(2+x)}{-4} = \frac{8}{-4}$ (Dividing both sides by -4)

⇒ 2 + x = -2

⇒ x = -2 – 2 (Transposing 2 to RHS)

⇒ x = —4

Check: Put x = -4 in LHS

-4(2 – 4) = -4 × -2 = 8 RHS as required

e) 4(2-x) = 8

⇒ $\frac{4(2+x)}{4} = \frac{8}{4}$ (Dividing both sides by 4)

⇒ 2 – x = 2 – 2 (Transposing 2 to RHS)

⇒ -x = 0

∴ x = 0 (Multiplying both sides by -1)

Check: Put x = 0 in LHS

4(2 – 0) = 4 × 2 = 8 RHS as required

3 Solve the following equations:

(a) 4 = 5(p- 2)

(b) -4 = 5(p – 2)
(c) 16 = 4 + 3 (t + 2)
(d) 4 + 5(p – 1) = 34
(e) 0 = 16 + 4(m – 6)
Solution:

(a) $4 = 5(p - 2)$

$\Rightarrow \quad \frac{4}{5} = \frac{5(p-2)}{5}$ (Dividing both sides by 5)

$\Rightarrow \quad \frac{4}{5} = p - 2$ (Transposing 2 to LHS)

$\Rightarrow \quad \frac{4}{5} + 2 = p$

$\Rightarrow \quad \frac{4+10}{5} = p \quad \Rightarrow \quad \frac{14}{5} = p$

$\Rightarrow \quad p = \frac{14}{5}$

Check: Put $p = \frac{14}{5}$ in RHS

$$5\left(\frac{14}{5} - 2\right) = 5\left(\frac{14-10}{5}\right) = \cancel{5} \times \frac{4}{\cancel{5}}$$

$= 4$ LHS as required

(b) $-4 = 5(p - 2)$

$\Rightarrow \quad -\frac{4}{5} = \frac{5(p-2)}{5}$ (Dividing both sides by 5)

$\Rightarrow \quad -\frac{4}{5} = p - 2$

$\Rightarrow \quad -\frac{4}{5} + 2 = p$ (Transposing 2 to LHS)

$\Rightarrow \quad \frac{-4+10}{5} = p \quad \Rightarrow \quad p = \frac{6}{5}$

Check: Put $p = \frac{6}{5}$ in RHS

$$5\left(\frac{6}{5} - 2\right) = 5\left(\frac{6-10}{5}\right) = \frac{\cancel{5} \times (-4)}{\cancel{5}}$$

$= -4$ LHS as required

c) 16 = 4 + 3 (t + 2)
⇒ 16 – 4 = 3(t + 2) (Transposing 4 to LHS)
⇒ 12 = 3 (t + 2)

$\Rightarrow \frac{\mathbf{12}}{\mathbf{3}} = \frac{3(t+2)}{43}$ (Dividing both sides by 3)

⇒ 4 = t + 2

⇒ 4 – 2 = t (Transposing 2 to LHS)

⇒ 2 = t or t = 2

Check: Put t = 2 in RHS

4 + 3(2 + 2) = 4 + 3 × 4 = 4 + 12

= 16 LHS as required

d) 4 + 5(p – 1) = 34

⇒ 5(p – 1) = 34 – 4(Transposing 4 to RHS)

⇒ 5(p – 1) = 30

$\Rightarrow \frac{5(p-1)}{5} = \frac{\mathbf{30}}{\mathbf{5}}$ (Dividing both sides by 5)

⇒ p – 1 = 6

⇒ P = 7

Check: Put p = 7 in LHS

4 + 5(7 – 1) = 4 + 5 × 6

= 4 + 30 = 34 RHS as required

e) 0 = 16 + 4(m – 6)

⇒ 0 — 16 = 4(m – 6) (Transposing 16 to LHS)

⇒ -16 = 4(m – 6)

$\Rightarrow \frac{\mathbf{-16}}{\mathbf{4}} = \frac{4(m-6)}{4}$ (Dividing both sides by 4)

⇒ -4 = m – 6

⇒ -4 + 6 = m (Transposing 6 to LHS)

⇒ 2 = m

or m = 2

Check: Put m = 2 in RHS

16 + 4(2 – 6) = 16 + 4 × (-4) = 16 – 16 = 0 LHS as required

EXERCISE 4.4

1 Set up equations and solve them to find the unknown numbers in the following cases:

(a) Add 4 to eight times a number; you get 60.
(b) One-fifth of a number minus 4 gives 3.
(c) If I take three-fourths of a number and add 3 to it, I get 21.
(d) When I subtracted 11 from twice a number, the result was 15.
e) Munna subtracts thrice the number of notebooks he has from 50, he finds the result to be 8.

(f) Ibenhal thinks of a number. If she adds 19 to it and divides the sum by 5, she will get 8.
(g) Anwar thinks of a number. If he takes away 7 from $\frac{5}{2}$ of the numbers, the result is 23.
Solution

(a) Let the required number be x.
Step I: 8x + 4
Step II: 8x + 4 = 60 is the required equation
Solving the equation, we have
8x + 4 = 60
⇒ 8x = 60 – 4 (Transposing 4 to RHS)
⇒ 8x = 56
⇒ $\frac{8x}{8} = \frac{56}{8}$ (Dividing both sides by 8)
⇒ x = 7
Thus, x – 7 is the required unknown number.

b) Let the required number be x.
Step I: $\frac{1}{5}$ x – 4
Step II: $\frac{1}{5}$ x – 4 = 3 is the required equation. 5
Solving the equation, we get
$\frac{1}{5}$ x – 4 = 3
⇒ $\frac{1}{5}$ x =4 + 3 (Transposing 4 to RHS)
⇒ $\frac{1}{5}$ x=7
⇒ $\frac{1}{5}$ x×5=7×5 (Multiplying both sides by 5)
⇒ x = 35 is the required unknown number,

(c) Let the required number be x.
Step I: $\frac{1}{5}$ x + 3

Step II: $\frac{1}{5}$ x + 3 = 21 is the required equation.
Solving the equation, we have

$$\frac{3}{4}x + 3 = 21$$
$$\Rightarrow \quad \frac{3}{4}x = 21 - 3 \text{(Transposing 3 to RHS)}$$
$$\Rightarrow \quad \frac{3}{4}x = 18$$
$$\Rightarrow \quad \frac{3}{4}x \times 4 = 18 \times 4 \quad \text{(Multiplying both sides by 4)}$$
$$\Rightarrow \quad 3x = 72$$
$$\Rightarrow \quad \frac{3x}{3} = \frac{72}{3} \quad \text{(Dividing both sides by 3)}$$
⇒ x = 24 is the required unknown number.

d) Let the required unknown number be x.
Step I: 2x – 11
Step II: 2x -11 = 15 is the required equations.
Solving the equation, we have
2x – 11= 15
⇒ 2x = 15 + 13 (Transposing 11 to RHS)
⇒ 2x = 28
⇒ $\frac{2x}{2} = \frac{28}{2}$ (Dividing both sides by 2)
⇒ x = 14 is the required unknown number,

(e) Let the required number be x.
Step I: 50 – 3x
Step II: 50 – 3x = 8 is the required equations.
Solving the equation, we have
50 – 3x = 8
⇒ -3x = 8 – 50 (Transposing 50 to RHS)
⇒ -3x = -42
⇒ $\frac{-3x}{-3} = \frac{-42}{-3}$ (Dividing both sides by -3)
⇒ x = 14 is the required unknown number.

(f) Let the required number be x.
Step I: x + 19
Step II: $\frac{x+19}{5}$
Step III: $\frac{x+19}{5}$ = 8 is the required equation.
Solving the equation, we have
$\frac{x+19}{5} = 8$
⇒ $\frac{x+19}{5} \times 5 = 8 \times 5$ (Multiplying both sides by 5)
⇒ x + 19 = 40
⇒ x = 40 – 19 (Transposing 19 to RHS)
∴ x = 21 is the required unknown number.

(g) Let the required number be x.
Step I: $\frac{5}{2}$ x – 7
Step II: $\frac{5}{5}$ – 7 = 23 is the required equation.
Solving the equation, we have

$$\frac{5}{2}x - 7 = 23$$

$$\Rightarrow \quad \frac{5}{2}x = 7 + 23 \quad \text{(Transposing 7 to RHS)}$$

$$\Rightarrow \quad \frac{5}{2}x = 30$$

$$\Rightarrow \quad \frac{5}{2}x \times 2 = 30 \times 2 \quad \text{(Multiplying both side of 2)}$$

$$\Rightarrow \quad 5x = 60$$

$$\Rightarrow \quad \frac{5x}{5} = \frac{60}{5} \quad \text{(Dividing both side by 5)}$$

⇒ x = 12 is the required unknown number.

2 Solve the following

(a) The teacher tells the class that the highest marks obtained by a student in her class is twice the lowest marks plus 7. The highest score is 87. What is the lowest score?

Solution

(a) Let the lowest score be x.
Step I: Highest marks obtained = 2x + 7
Step II: 2x + 7 = 87 is the required equation. Solving the equation, we have
2x + 7 = 87
⇒ 2x = 87 – 7 (Transposing 7 to RHS)
⇒ 2x = 80
⇒ $\frac{2x}{2}$ 2x2=802 (Dividing both sides by 2)
⇒ x = 40 is the required lowest marks.

(b) In an isosceles triangle, the base angle are equal. The vertex angle is 40°. What are the base angles of the triangle? (Remember, the sum of three angles of a triangle is 180°?)

Solution Let each base angle be x degrees.
Step I: Sum of all angles of the triangle (x + x + 40) degrees.
Step II: x + x + 40 = 180°
⇒ 2x + 40° = 180°
Solving the equation, we have
2x + 40° = 180°
2x = 180° – 40° (Transposing 40° to RHS)
2x = 140°
⇒ : $\frac{2x}{2}$ = : $\frac{140}{2}$ (Dividing both sides by 2)
⇒ x = 70°
Thus the required each base angle = 70°

c) Sachin scored twice as many runs as Rahul. Together, their runs fell two short of a double century. How many runs did each one score?

Solution

Let the runs scored by Rahul = x
Runs scored by Sachin = 2x
Step I: x + 2x = 3x
Step II: 3x + 2 = 200
Solving the equation, we have
3x + 2 = 200
⇒ 3x = 200 – 2 (Transposing 2 to RHS)
⇒ 3x = 198
⇒ $\frac{3x}{3} = \frac{198}{3}$ (Dividing both sides by 3)
⇒ x = 66
Thus, the runs scored by Rahul is 66 and the runs scored by Sachin = 2 × 66 = 132

4 Solve the following
(i) Irfan says that he has 7 marbles more than five times the marbles Parmit has. Irfan has 37 marbles. How many marbles does Parmit have?

(i) Let the number of marbles with Parmit be
Step I: Number of marbles that Irfan has = 5x + 7
Step II: 5x + 7 = 37 Solving the equation, we have 5x + 7 = 37
⇒ 5x = 37 – 7 (Transposing 7 to RHS)
⇒ 5x = 30
⇒ $\frac{5x}{5} = \frac{30}{5}$ (Dividing both sides by 5)
⇒ x = 6
Thus, the required number of marbles = 6.

(ii) Laxmi's father is 49 years old. He is 4 years older than three times Laxmi's age. What is Laxmi's age?

ii) Let Laxmi's age be x years.
Step I: Father's age = 3x + 4
Step II: 3x + 4 = 49
Solving the equation, we get
3x + 4-= 49
⇒ 3x = 49 – 4 (Transposing to RHS)
⇒ 3x = 45
⇒ $\frac{3x}{3} = \frac{45}{3}$ (Dividing both sides by 3)

⇒ x = 15
Thus, the age of Laxmi = 15 years

(iii) People of Sundargram planted trees in a village garden. Some of the trees were fruit trees. The number of non-fruit trees were two more than three times the number of fruit trees. What was the number of fruit trees planted if the number of non-fruit trees planted was 77?

Solution Let the number of planted fruit tree be x.
Step I: Number of non-fruit trees = 3x + 2
Step II: 3x + 2 = 77
Solving the equation, we have
3x + 2 = 77
⇒ 3x = 77 – 2 (Transposing 2 to RHS)
⇒ 3x = 75
⇒ $\frac{3x}{3} = \frac{75}{3}$ (Dividing both sides by 3)
⇒ x = 25
Thus, the required number of fruit tree planted = 25

4 Solve the following riddle:
I am a number,
Tell my identity!
Take me seven times over
And add a fifty!
To reach a triple century
You still need forty!

Solution

Suppose my identity number is x.
Step I: 7 + 50
Step II: 7x + 50 + 40 = 300
or 7x + 90 = 300
Solving the equation, we have
7x + 90 = 300
⇒ 7x = 300 – 90 (Transforming 90 to RHS)
⇒ 7x = 210

⇒ 7x7=2107 (Dividing both sides by 7)
⇒ x = 30
Thus, my identity is 30.

HOTS QUESTIONS

1 Each of the 2 equal sides of an isosceles triangle is twice as large as the third side. If the perimeter of the triangle is 30 cm, find the length of each side of the triangle.

Solution

Let the length of the third side be x cm.
Each equal side = 2x cm.
As per the condition of the question, we have
Perimeter = x + 2x + 2x = 30
⇒ 5x = 30
⇒ x = 6
Thus, the third side of the triangle = 6 cm
and other two equal sides are 2 × 6 = 12 cm each

2 A man travelled two-fifth of his journey by train, one-third by bus, one-fourth by car and the remaining 3 km on foot. What is the length of his total journey? [NCERT Exemplar]

Solution

Let the total length of total journey be x km.
Distance travelled by train = $\frac{2}{5}$ **x km**
Distance travelled by bus = = $\frac{1}{3}$ **x km**
Distance travelled by car = $\frac{1}{4}$ **x km**
Remaining distance = 3 km
As per the question, we have

$$x = \frac{2}{5}x + \frac{1}{3}x + \frac{1}{4}x + 3$$

$$\Rightarrow \quad x - \frac{2}{5}x - \frac{1}{3}x - \frac{1}{4}x = 3$$

$$\Rightarrow \quad \frac{60x - 24x - 20x - 15x}{60} = 3$$

[LCM of 5, 3 and 4 = 60]

$$\Rightarrow \quad \frac{60x - 59x}{60} = 3$$

$$\Rightarrow \quad \frac{x}{60} = 3$$

$$\Rightarrow \quad x = 3 \times 60 = 180 \text{ km}$$

Thus, the required journey = 180 km

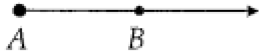
CHAPTER 5

LINES AND ANGLES

KEY POINTS TO REMEMBER

A line segment has two endpoints and it is denoted by $\overline{AB}$.

A ray has only one end point (its initial point) and it is denoted by $\overrightarrow{AB}$.

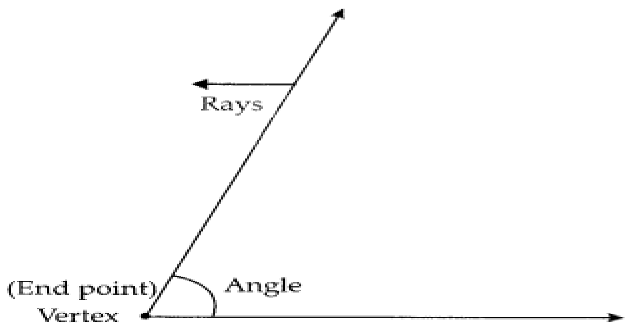


A line has no end points on either side and it is denoted by $\overleftrightarrow{AB}$.

If a single straight line passes through three or more points, the points are said to be collinear.

A, B, C are collinear points.

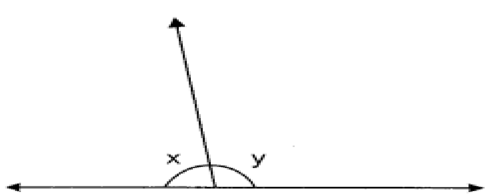
An angle is formed when two lines (or rays or line – segments) meet.

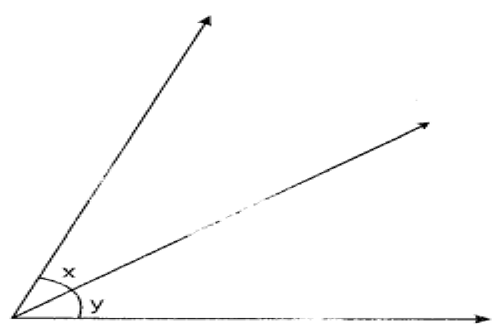


If the sum of measures of two angles is 90°, they are called complementary angles.
e.g. $\angle A = 60°$, $\angle B = 30°$
$\angle A + \angle B = 60° + 30° = 90°$

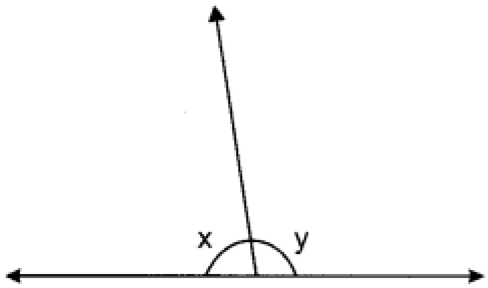
If the sum of measures of two angles is 180°, they are called supplementary angles.
e.g. $\angle A = 150°$, $\angle B = 30°$
$\angle A + \angle B = 150° + 30° = 180°$

If two angles have a common vertex and a common arm but no common interior, they are said to adjacent angles.





If two angles are adjacent and supplementary, they are said to be linear pair.



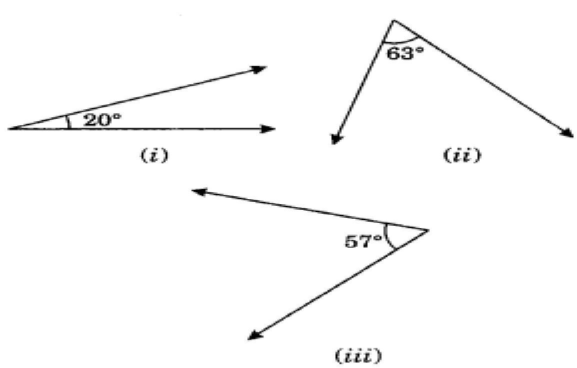
Fill in the blanks:

(i) If two angles are complementary, then the sum of their measures is ______ .
(ii) If two angles are supplementary, then the sum of their measures is ______ .
(iii) Two angles forming a linear pair are ______ .
(iv) If two adjacent angles are supplementary, they form a ______ .
(v) If two lines intersect at a point, then the vertically opposite angles are always ______ .
(vi) If two lines intersect at a point, and if one pair of vertically opposite angles are acute angles, then the other pair of vertically opposite angles are ______ .

Solution

i) 90°
(ii) 180°
(iii) Supplementary
(iv) Linear pair
(v) Equal
(vi) Obtuse angle

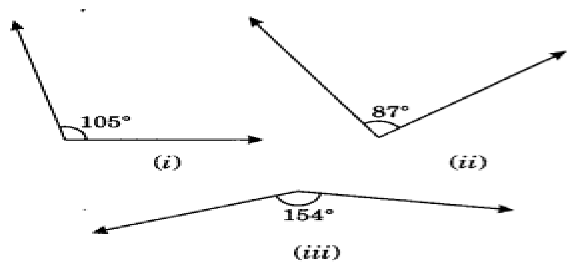
1 Find the complement of each of the following angles:



Solution

(i) Complement of 20° = 90° – 20° = 70°
(ii) Complement of 63° = 90° – 63° = 27°
(iii) Complement of 57° = 90° – 57° = 33

2 Find the supplement of each of the following angles:



2 Solution:

(i) Supplement of 105° = 180° – 105° = 75°
(ii) Supplement of 87° = 180° – 87° = 93°
(iii) Supplement of 154° = 180° – 154° = 26°

3 Identify which of the following pairs of angles are complementary and which are supplementary?

(i) 65°, 115° (ii) 63°, 27° (iii) 112°, 68°
(iv) 130°, 50° (v) 45°, 45° (vi) 80°, 10°

Solution:

(i) 65° (+) 115° = 180°
They are supplementary angles.
(ii) 63° (+) 27° = 90°
They are complementary angles.
(iii) 112° (+) 68° = 180°
They are supplementary angles.
(iv) 130° (+) 50° = 180°
They are supplementary angles.
(v) 45° (+) 45° = 90°
They are complementary angles.
(vi) 80° (+) 10° = 90°

4 Find the angle which equal to its complement

Solution:
Let the required angle be x°.

its complement = (90 – x)°
Now, re = 90 – x ⇒ x + x = 90
⇒ 2x = 90 ∴ x = $\frac{90°}{2}$ = 45°
Thus the required angles are 45°.

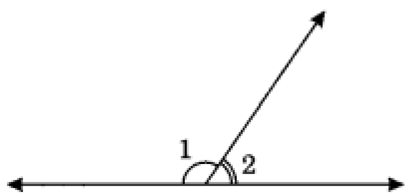
5. Find the angle which is equal to its supplement.

Let the required angle be x°.
∴ it supplement = (180 – x)°
Now, x = 180 – x
⇒ x + x = 180
⇒ 2x = 180°
∴ $x = \frac{180°}{2}$ = 90°.

Thus, the required angle is 90°.

6 In the given figure, ∠1 and ∠2 are supplementary angles.
If ∠1 is decreased, what changes should take place in∠2 so that both the angles

still remain supplementary.



Solution

∠1 + ∠2 = 180° (given)
If ∠1 is decreased by some degrees, then ∠2 will also be increased by the same degree so that the two angles still remain supplementary.

7 Can two angles be supplementary if both of them are:
(i) acute?
(ii) obtuse?
(iii) right?
(ii) Since, acute angle < 90°

∴ Acute angle + acute angle < 90° + 90° < 180°

Thus, the two acute angles cannot be supplementary angles.

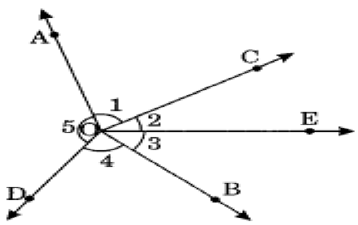
(ii) Since, obtuse angle > 90°
∴ Obtuse angle + obtuse angle > 90° + 90° > 180°

Thus, the two obtuse angles cannot be supplementary angles.
(iii) Since, right angle = 90°
∴ right angle + right angle = 90° + 90° = 180°
Thus, two right angles are supplementary angles.

8 An angle is greater than 45°. Is its complementary angle greater than 45° or equal to 45° or less than 45 °?

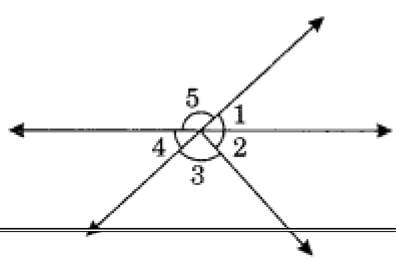
Solution:
Given angle is greater than 45°
Let the given angle be x°.
∴ x > 45
Complement of x° = 90° – x° < 45° [∵ x > 45°]
Thus the required angle is less than 45°.

9 In the following figure:
(i) Is ∠1 adjacent to ∠2?
(ii) Is ∠AOC adjacent to∠AOE?
(iii) Do ∠COE and ∠EOD form a linear pair?
(iv) Are ∠BOD and ∠DOA supplementary?
(v) Is ∠1 vertically opposite angle to ∠4?
(vi) What is the vertically opposite angle of ∠5?



Solution:
(i) Yes, ∠1 and ∠2 are adjacent angles.
(ii) No, ∠AOC is not adjacent to ∠AOE. [∵ OC and OE do not lie on either side of common arm OA] .
(iii) Yes, ∠COE and ∠EOD form a linear pair of angles.
(iv) Yes, ∠BOD and ∠DOA are supplementary. [∵ ∠BOD + ∠DOA = 180°]
(v) Yes, ∠1 is vertically opposite to ∠4.
(vi) Vertically opposite angle of ∠5 is ∠2 + ∠3 i.e. ∠BOC.

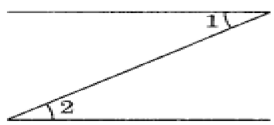
10. Indicate which pairs of angles are:
(i) Vertically opposite angles
(ii) Linear pairs
Solution:
(i) Vertically opposite angles are ∠1 and ∠4, ∠5 and (∠2 + ∠3)



(ii) Linear pairs are
∠1 and ∠5, ∠5 and ∠4

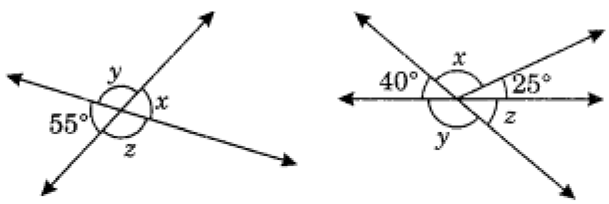
11 In the following figure, is ∠1 adjacent to ∠2 ? Give reasons.
Solution:

No, ∠1 and∠2 are not adjacent angles.



Reasons:
(i) ∠1 + ∠2 ≠ 180°
(ii) They have no common vertex.

12 Find the values of the angles x, y and z in each of the following:



(i) (ii)

From Fig. 1. we have
∠x = ∠55° (Vertically opposite angles)
∠x + ∠y = 180° (Adjacent angles)
55° + ∠y = 180° (Linear pair angles)
∴ ∠y = 180° – 55° = 125°
∠y = ∠z (Vertically opposite angles)
125° = ∠z
Hence, ∠x = 55°, ∠y = 125° and ∠z = 125°

ii) 25° + x + 40° = 180° (Sum of adjacent angles on straight line)
65° + x = 180°
∴ x = 180° – 65° = 115°
40° + y = 180° (Linear pairs)
∴ y = 180° – 40° = 140°
y + z = 180° (Linear pairs)

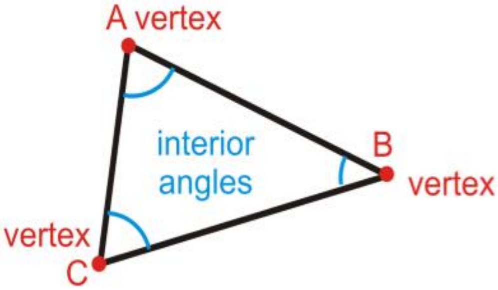
$140° + z = 180°$
$\therefore z = 180° – 140° = 40°$

Hence, x – 115°, y = 140° and z – 40°

CHAPTER 6

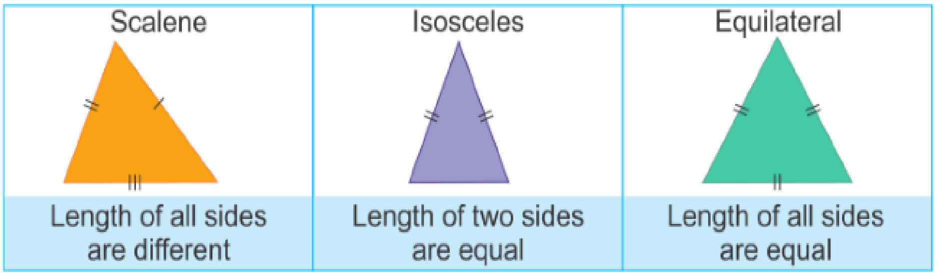
THE TRIANGLES AND ITS PROPERTIES

KEY POINTS TO REMEMBER

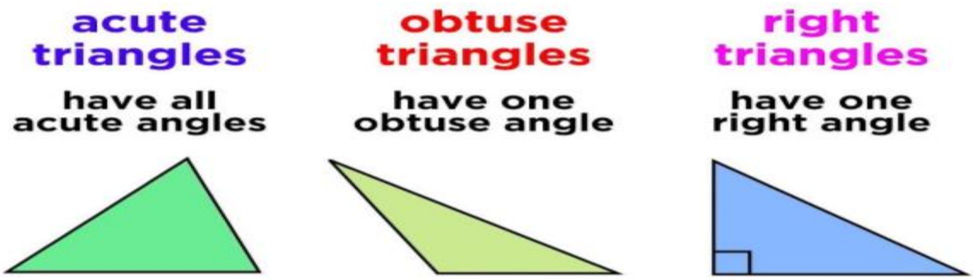


Here, in ∆ABC,

- AB, BC and CA are the three sides.
- A, B and C are three vertices.
- ∠A, ∠B and ∠C are the three angles.

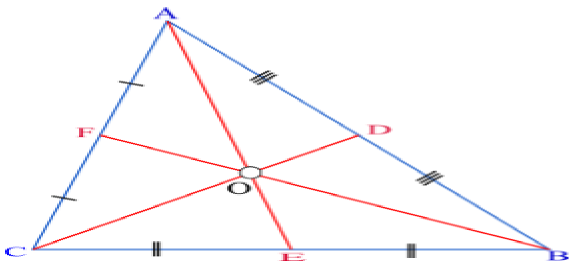


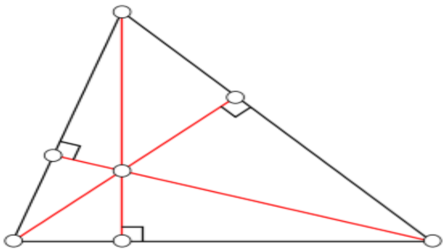
Types of Triangle on the basis of angles



Medians of a Triangle

Median is the line segment which made by joining any vertex of the triangle with the midpoint of its opposite side. Median divides the side into two equal parts.



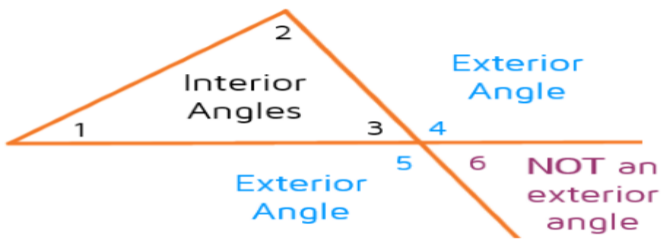


- The altitude form angle of 90°.
- There are three altitudes possible in a triangle.
- The point of intersection of all the three altitudes is called **Orthocenter**.

The Exterior Angle of a Triangle

If we extend any side of the triangle then we get an exterior angle.

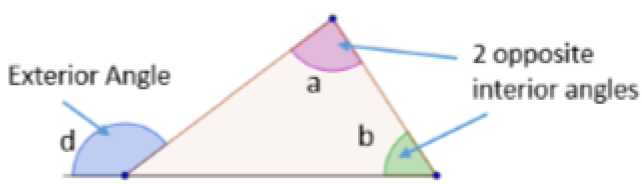
- An exterior angle must form a linear pair with one of the interior angles of the triangle.
- There are only two exterior angles possible at each of the vertices.



Here ∠4 and ∠5 are the exterior angles of the vertex but ∠6 is not the exterior angle as it is not adjacent to any of the interior angles of the triangle.

Exterior Angle Property of the Triangle

An Exterior angle of a triangle will always be equal to the sum of the two opposite interior angles of the triangle.

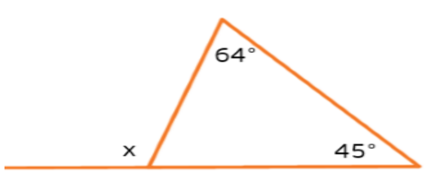


Here, ∠d = ∠a + ∠b

This is called the Exterior angle property of a triangle.

Example

Find the value of “x”.



Solution

x is the exterior angle of the triangle and the two given angles are the opposite interior angles.

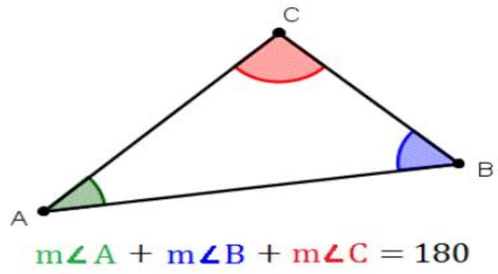
Hence,

x = 64°+ 45°

x = 109°

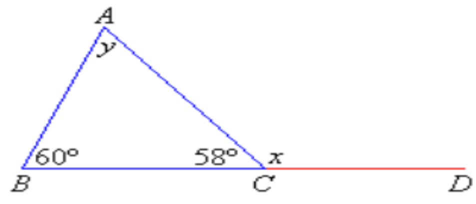
Angle Sum Property of a Triangle

This property says that the sum of all the interior angles of a triangle is 180°.



Example

Find the value of x and y in the given triangle.



Solution

x + 58° = 180° (linear pair)

x = 180° - 58°

x = 122°

We can find the value of y by two properties-

1. Angle sum property

60° + 58° + y = 180°

y = 180°- (60° + 58)

y = 62°

Exterior angle property

x = 60°+ y

122° = 60° + y

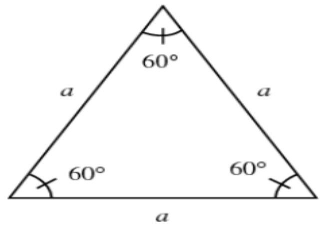
y = 122° - 60°

y = 62°

Two Special Triangles

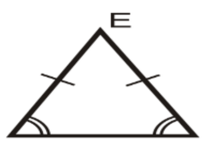
1. Equilateral Triangle

It is a triangle in which all the three sides and angles are equal.



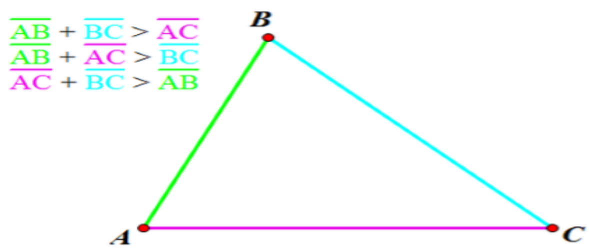
2. Isosceles Triangle

It is a triangle in which two sides are equal and the base angles opposite to the equal sides are also equal.



Sum of the length of the two sides of a triangle

Sum of the length of the two sides of a triangle will always be greater than the third side, whether it is an equilateral, isosceles or scalene triangle.



Example

Check whether it is possible to make a triangle using these measurements or not?

1. 3 cm, 4 cm, 7 cm

We have to check whether the sum of two sides is greater than the third side or not.

4 + 7 = 11

3 + 7 = 10

3 + 4 =7

Here the sum of the two sides is equal to the third side so the triangle is not possible with these measurements.

Here the sum of the two sides is equal to the third side so the triangle is not possible with these measurements.

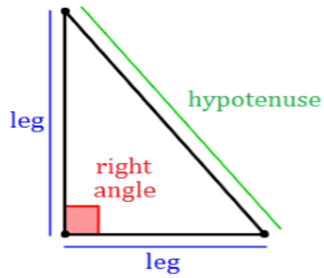
2. 2 cm, 5 cm, 6 cm

2 + 5 = 7

6 + 5 =11

6 + 2 = 8

Here the sum of the two sides is greater than the third side so the triangle could be made with these measurements.



Pythagoras theorem

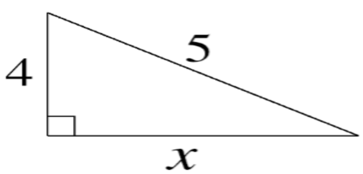
In a right angle triangle,

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$

The reverse of Pythagoras theorem is also applicable, i.e. if the Pythagoras property holds in a triangle then it must be a right-angled triangle.

Example

Find the value of x in the given triangle if the hypotenuse is 5 cm and height is 4 cm.



Solution

Given:

Hypotenuse = 5 cm

Height = 4 cm

Base = x cm

$(\text{Hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$

$5^2 = x^2 + 4^2$

$x^2 = 5^2 - 4^2$

$x^2 = 25 – 16$

x = 9

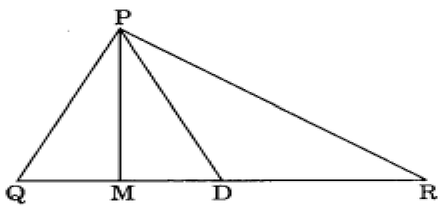
x = 3 cm

EXERCISE 6.1

1 In ΔPQR, D is the mid-point of $\overline{QR}$,

$\overline{PM}$ is

$\overline{PD}$ is



If QM = MR?

Solution:

$\overline{PM}$ is altitude.

$\overline{PD}$ is median.

No, QM ≠ MR.

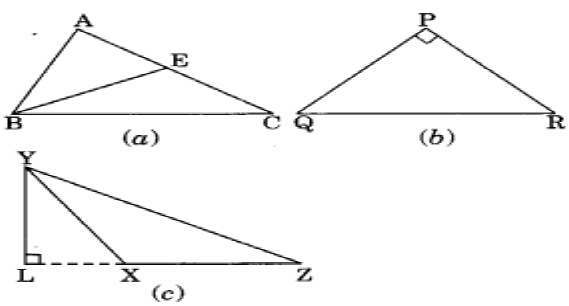
2 Draw rough sketches for the following:

(a) In ΔABC, BE is a median.

(ib) In ΔPQR, PQ and PR are altitudes of the triangle.

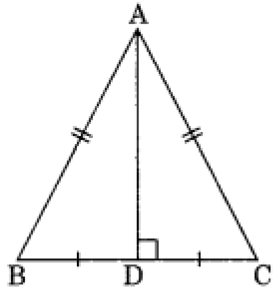
(c) In ΔXYZ, YL is an altitude in the exterior of the triangle.

Solution:



3 Verify by drawing a diagram if the median and altitude of an isosceles triangle can be same.

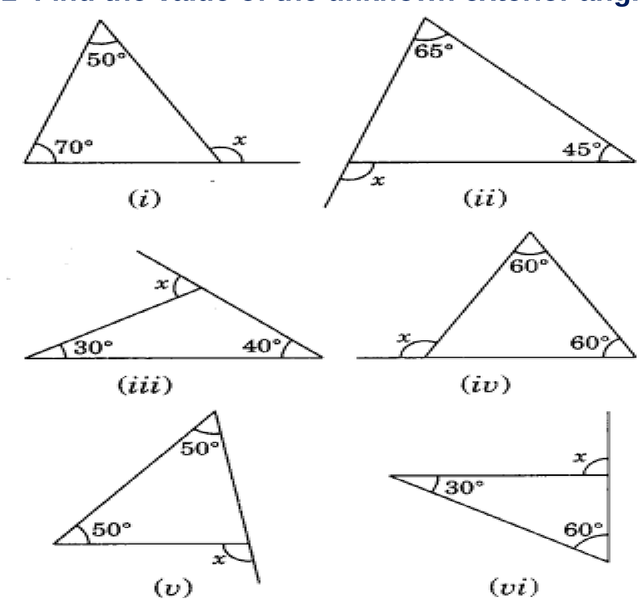
ΔABC is an isosceles triangle in which AB = AC



Draw AD as the median of the triangle.
Measure the angle ADC with the help of protractor we find, $\angle ADC = 90°$
Thus, AD is the median as well as the altitude of the ΔABC. Hence Verified.

EXERCISE 6.2

1 Find the value of the unknown exterior angle x in the following diagrams:



Solution:

(i) ∠x = 50° + 70° = 120° (Exterior angle is equal to sum of its interior opposite angles)

(ii) ∠x = 65°+ 45° = 110° (Exterior angle is equal to sum of its interior opposite angles)

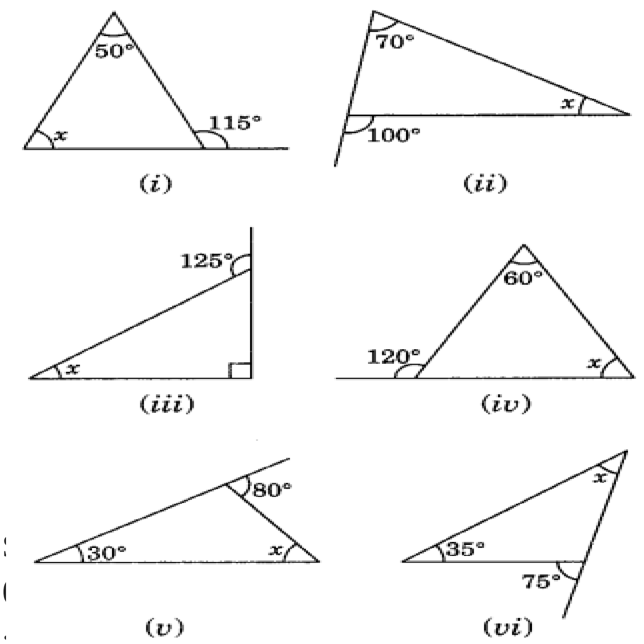
(iii) ∠x = 30° + 40° = 70° (Exterior angle is equal to sum of its interior opposite angles)

(iv) ∠x = 60° + 60° = 120° (Exterior angle is equal to sum of its interior opposite angles)

(v) ∠x = 50° + 50° =100° (Exterior angle is equal to sum of its interior opposite angles)

(vi) ∠x = 30° + 60° = 90° (Exterior angle is equal to sum of its interior opposite angle)

2. Find the value of the unknown interior angle x in the following figures:



(ii) ∠x + 70° = 110° (Exterior angle of a triangle)

∴ ∠x = 110° – 70° = 40°

(iii) ∠ x + 90° = 125° (Exterior angle of a right triangle)

∴ ∠x = 125° – 90° = 35°

(iv) ∠x + 60° = 120° (Exterior angle of a triangle)

∴ ∠x = 120° – 60° = 60°

(v)∠ X + 30° = 80° (Exterior angle of a triangle)

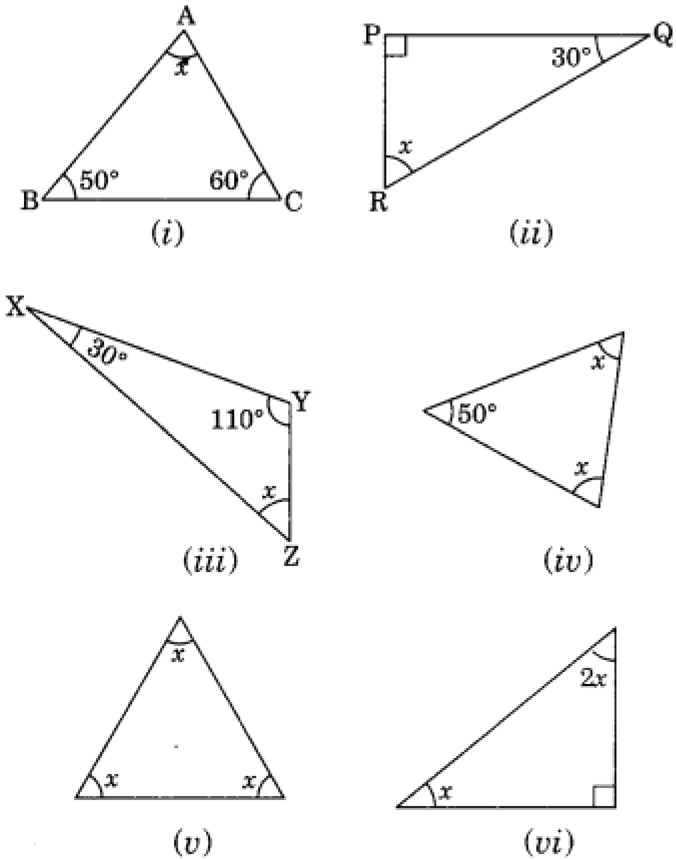
∴ ∠x = 80° – 30° = 50°

(vi) ∠ x + 35° = 75° (Exterior angle of a triangle)

∴ ∠ x = 75° – 35° = 40°

EXRCISE 6.3

1Find the value of the unknown x in the following diagrams:



Solution:

(i) By angle sum property of a triangle, we have
$\angle x + 50° + 60° = 180°$
$\Rightarrow \angle x + 110° = 180°$
$\therefore \angle x = 180° – 110° = 70°$

(ii) By angle sum property of a triangle, we have
$\angle x + 90° + 30 = 180°$ [Δ is right angled triangle]
$\Rightarrow \angle x + 120° = 180°$
$\therefore \angle x – 180° – 120° = 60°$

(iii) By angle sum property of a triangle, we have
$\angle x + 30° + 110° – 180°$
$\Rightarrow \angle x + 140° = 180°$
$\therefore \angle x = 180° – 140° = 40°$

(iv) By angle sum property of a triangle, we have
$\angle x + \angle x + 50° = 180°$
$\Rightarrow 2x + 50° = 180°$
$\Rightarrow 2x = 180° – 50°$

⇒ 2x = 130°

∴ x= $\frac{130°}{2}$=65∘

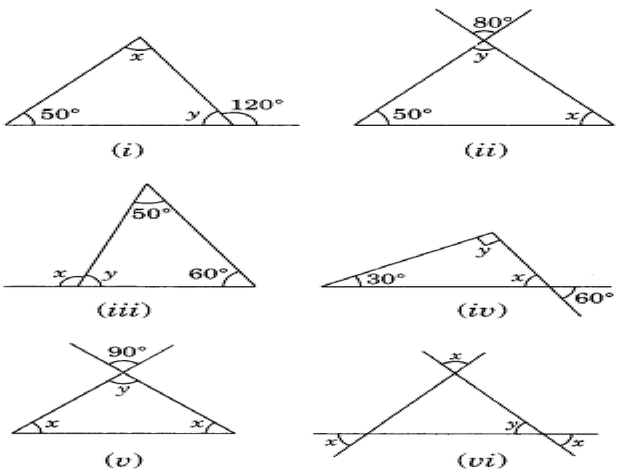
(v) By angle sum property of a triangle, we have
∠x + ∠x +∠x =180°
⇒ 3 ∠x = 180°

∴ ∠x= $\frac{180°}{3}$= =60∘

(vi) By angle sum property of a triangle, we have
x + 2 x + 90° = 180° (Δ is right angled triangle)
⇒ 3x + 90° = 180°
⇒ 3x = 180° – 90°
⇒ 3x = 90°

∴ x= $\frac{90°}{3}$ =30⁰

2 Find the values of the unknowns x and y in the following diagrams:



(i) ∠x + 50° = 120° (Exterior angle of a triangle)

∴ ∠x = 120°- 50° = 70°

∠x + ∠y + 50° = 180° (Angle sum property of a triangle)

70° + ∠y + 50° = 180°

∠y + 120° = 180°

∠y = 180° – 120°

∴ ∠y = 60°

Thus ∠x = 70 and ∠y – 60°

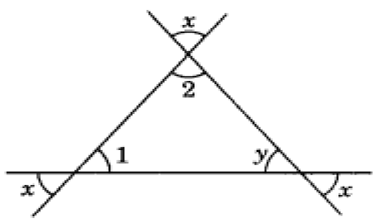
ii) ∠y = 80° (Vertically opposite angles are same)
∠x + ∠y + 50° = 180° (Angle sum property of a triangle)
⇒ ∠x + 80° + 50° = 180°
⇒ ∠x + 130° = 180°
∴ ∠x = 180° – 130° = 50°
Thus, ∠x = 50° and ∠y = 80°

(iii) ∠y + 50° + 60° = 180° (Angle sum property of a triangle)
∠y + 110° = 180°
∴ ∠y = 180°- 110° = 70°
∠x + ∠y = 180° (Linear pairs)
⇒ ∠x + 70° = 180°
∴ ∠x = 180° – 70° = 110°
Thus, ∠x = 110° and y = 70°

iv) ∠x = 60° (Vertically opposite angles)
∠x + ∠y + 30° = 180° (Angle sum property of a triangle)
⇒ 60° + ∠y + 30° = 180°
⇒ ∠y + 90° = 180°
⇒ ∠y = 180° – 90° = 90°
Thus, ∠x = 60° and ∠y = 90°

(v) ∠y = 90° (Vertically opposite angles)
∠x + ∠x + ∠y = 180° (Angle sum property of a triangle)
⇒ 2 ∠x + 90° = 180°
⇒ 2∠x = 180° – 90°
⇒ 2∠x = 90°
∴ $\angle x = \frac{90^\circ}{2} = 45^\circ$
Thus, ∠x = 45° and ∠y = 90°

$$\left.\begin{array}{l}\angle y = \angle x \\ \angle 1 = \angle x \\ \angle 2 = \angle x\end{array}\right\}$$



1 Is it possible to have a triangle with the following sides?
(i) 2 cm, 3 cm, 5 cm
(ii) 3 cm, 6 cm, 7 cm
(iii) 6 cm, 3 cm, 2 cm
Solution:

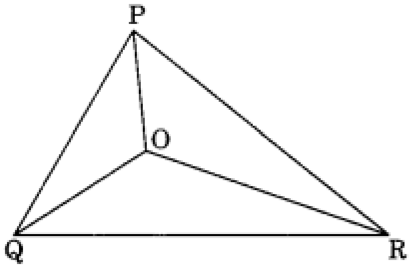
We know that for a triangle, the sum of any two sides must be greater than the third side.
(i) Given sides are 2 cm, 3 cm, 5 cm
Sum of the two sides = 2 cm + 3 cm = 5 cm Third side = 5 cm
We have, Sum of any two sides = the third side i.e. 5 cm = 5 cm
Hence, the triangle is not possible.

(ii) Given sides are 3 cm, 6 cm, 7 cm
Sum of the two sides = 3 cm + 6 cm = 9 cm Third side = 7 cm
We have sum of any two sides > the third

side. i.e. 9 cm > 7 cm
Hence, the triangle is possible.

(iii) Given sides are 6 cm, 3 cm, 2 cm
Sum of the two sides = 3 cm + 2 cm – 5 cm Third side = 6 cm
We have, sum of any two sides < the third sid6 i.e. 5 cm < 6 cm Hence, the triangle is not possible.

2 Take any point O in the interior of a triangle PQR . Is

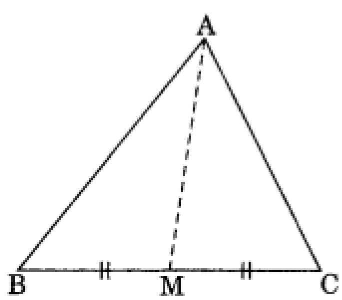


(i) OP + OQ > PQ?
(ii) OQ + OR > QR?
(iii) OR + OP > RP?

Solution:
(i) Yes, In ΔOPQ, we have
OP + OQ > PQ
[Sum of any two sides of a triangle is greater than the third side]
(ii) Yes, In ΔOQR, we have OQ + OR > QR
[Sum of any two sides of a triangle is greater than the third side]
(iii) Yes, In ΔOPR, we have OR + OP > RP
[Sum of any two sides of a triangle is greater than the third side]

3 AM is a median of a triangle ABC.

Is AB + BC + CA > 2AM ?
(Consider the sides of triangles ΔABM and ΔAMC)
Solution:
Yes, In AABM, we have

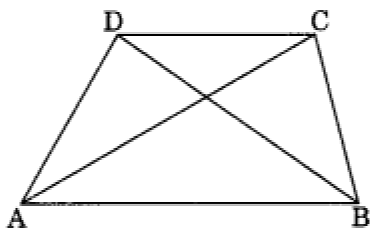


Yes, In AABM, we have
AB + BM > AM …(i)
[Sum of any two sides of a triangle is greater than the third side]

In ΔAMC, we have
AC + CM > AM …(ii)
[Sum of any two sides of a triangle is greater than the third side]
Adding eq (i) and (ii), we have
AB + AC + BM + CM > 2AM
AB + AC + BC + > 2AM
AB + BC + CA > 2AM
Hence, proved.

4 BCD is a quadrilateral.
Is AB + BC + CD + DA > AC + BD?

Solution:
Yes, Given ABCD in which AC and BD are its diagonals.

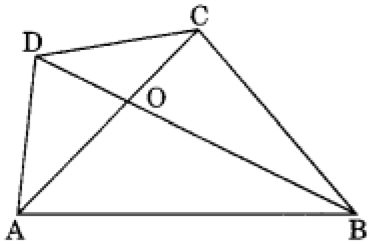


In ΔABC, we have
AB + BC > AC …(i)
[Sum of any two sides is greater than the third side]
In ΔBDC, we have
BC + CD > BD …(ii)
[Sum of any two sides is greater than the third side]
In ΔADC, we have
CD + DA > AC …(iii)
[Sum of any two sides is greater than the third side]
In ΔDAB, we have
DA + AB > BD …(iv)
[Sum of any two sides is greater than the third side]
Adding eq. (i), (ii), (iii) and (iv), we get

2AB + 2BC + 2CD + 2DA > 2AC + 2BD or AB + BC + CD + DA > AC + BD [Dividing both sides by 2]
Hence, proved.

5 ABCD is a quadrilateral.
Is AB + BC + CD + DA < 2(AC + BD)?

Solution:
Yes, we have a quadrilateral ABCD.



In ΔAOB, we have AB < AO + BO …(i)
[Any side of a triangle is less than the sum of other two sides]
In ΔBOC, we have
BC < BO + CO …(ii)

[Any side of a quadrilateral is less than the sum of other two sides]
In ΔCOD, we have
CD < CO + DO …(iii)
[Any side of a triangle is less than the sum of other two sides]
In ΔAOD, we have
DA < DO + AO …(iv)
[Any side of a triangle is less than the sum of other two sides]
Adding eq. (i), (ii), (iii) and (iv), we have
AB + BC + CD + DA
∠2AO + ∠BO + ∠CO + ∠DO
∠2(AO + BO + CO + DO)
∠2 [(AO + CO) + (BO + DO)]
∠2(AC + BD)

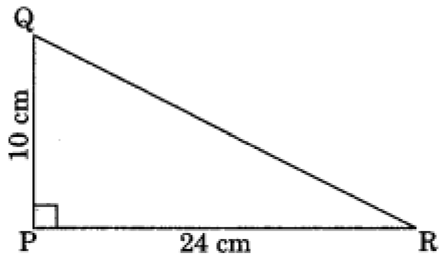
Thus, AB + BC + CD + DA < 2(AC + BD)
Hence, proved.

6 The length of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?

Solution:
Sum of two sides
= 12 cm + 15 cm = 27 cm
Difference of the two sides
= 15 cm – 12 cm = 3 cm
∴ The measure of third side should fall between 3 cm and 27 cm.

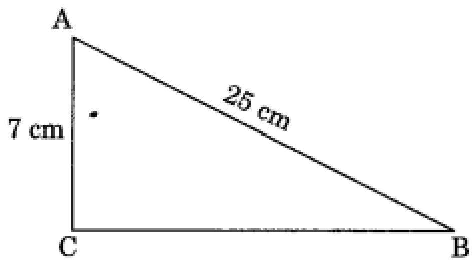
EXERCISE 6.5

PQR is a triangle, right angled at P. If PQ = 10 cm and PR = 24 cm, find QR.



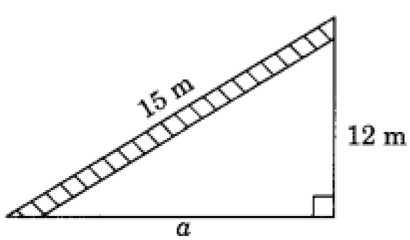
Solution:
In right angled triangle PQR, we have
$QR^2 = PQ^2 + PR^2$ From Pythagoras property)
$= (10)^2 + (24)^2$
$= 100 + 576 = 676$
$\therefore QR = \sqrt{676} = 26$ cm
The, the required length of QR = 26 cm.

ABC is a triangle, right angled at C. If AB = 25 cm and AC = 7 cm, find BC.



Solution:
In right angledΔABC, we have
$BC^2 + (7)^2 = (25)^2$ (By Pythagoras property)
$\Rightarrow BC^2 + 49 = 625$
$\Rightarrow BC^2 = 625 - 49$
$\Rightarrow BC^2 = 576$
$\therefore BC = \sqrt{576} = 24$ cm
Thus, the required length of BC = 24 cm.

3 A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance a. Find the distance of the foot of the ladder from the wall.



Solution:
Here, the ladder forms a right angled triangle.

∴ $a^2 + (12)^2 = (15)^2$ (By Pythagoras property)

⇒⇒ a2 = 225 – 144

⇒ $a^2 = 81$

∴ a = 81−−√ = 9 m

Thus, the distance of the foot from the ladder = 9m $a^2 + 144 = 225$

4 Which of the following can be the sides of a right triangle?

(i) 2.5 cm, 6.5 cm, 6 cm.

(ii) 2 cm, 2 cm, 5 cm.

(iii) 1.5 cm, 2 cm, 2.5 cm

Solution:

(i) Given sides are 2.5 cm, 6.5 cm, 6 cm.

Square of the longer side = $(6.5)^2 = 42.25$ cm.

Sum of the square of other two sides

= $(2.5)^2 + (6)^2 = 6.25 +$ **36Solution**:

(i) Given sides are 2.5 cm, 6.5 cm, 6 cm.

Square of the longer side = $(6.5)^2 = 42.25$ cm.

Sum of the square of other two sides

= 42.25 cm.

Since, the square of the longer side in a triangle is equal to the sum of the squares of other two sides.

∴ The given sides form a right triangle.= $(2.5)^2 + (6)^2 = 6.25 + 36$

(ii) Given sides are 2 cm, 2 cm, 5 cm .

Square of the longer side = $(5)^2$ = 25 cm Sum of the square of other two sides

= $(2)^2 + (2)^2$ =4 + 4 = 8 cm

Since 25 cm ≠ 8 cm

∴ The given sides do not form a right triangle.

(iii) Given sides are 1.5 cm, 2 cm, 2.5 cm

Square of the longer side = $(2.5)^2$ = 6.25 cm Sum of the square of other two sides

= $(1.5)^2 + (2)^2 = 2.25 + 4$

Since 6.25 cm = 6.25 cm = 6.25 cm

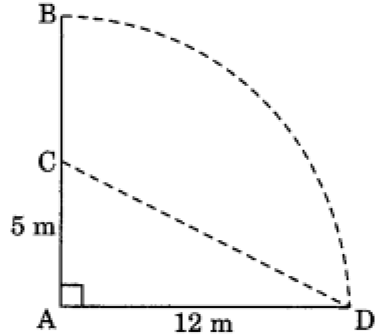
Since the square of longer side in a triangle is equal to the sum of square of other two sides.

∴ The given sides form a right triangle.

6 A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree . Find the original height of the tree.

Solution:

Let AB be the original height of the tree and broken at C touching the ground at D such that



AC = 5 m

and AD = 12 m

In right triangle ΔCAD,

$AD^2 + AC^2 = CD^2$ (By Pythagoras property)

$\Rightarrow (12)^2 + (5)^2 = CD^2$

$\Rightarrow 144 + 25 = CD^2$

$\Rightarrow 169 = CD^2$

$\therefore CD = \sqrt{169} = 13$ m

But CD = BC

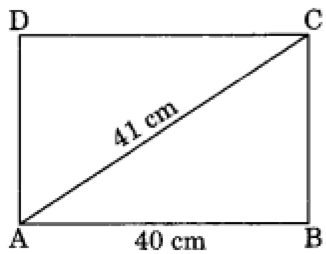
AC + CB = AB

5 m + 13 m = AB

∴ AB = 18 m .

Thus, the original height of the tree = 18 m.

7Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.



Solution:

Given: Length AB = 40 cm

Diagonal AC = 41 cm

In right triangle ABC, we have

$AB^2 + BC^2 = AC^2$ (By Pythagoras property)

$\Rightarrow (40)^2 + BC^2 = (41)^2$

$\Rightarrow 1600 + BC^2 = 1681$

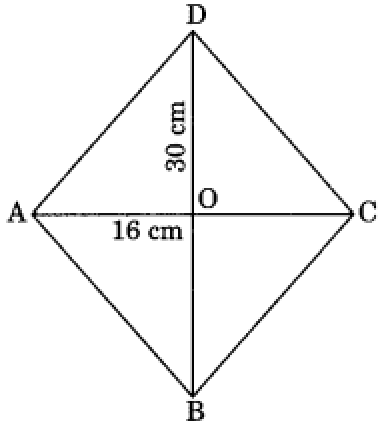
$\Rightarrow BC^2 = 1681 – 1600$

$\Rightarrow BC^2 = 81$

∴ BC = $\sqrt{81}$ = 9 cm
∴ AB = DC = 40 cm and BC = AD = 9 cm (Property of rectangle)

∴ The required perimeter
= AB + BC + CD + DA
= (40 + 9 + 40 + 9) cm
= 98 cm

8The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.



Solution:
Let ABCD be a rhombus whose diagonals intersect each other at O such that AC = 16 cm and BD = 30 cm
Since, the diagonals of a rhombus bisect each other at 90°.
∴ OA = OC = 8 cm and OB = OD = 15 cm
In right ΔOAB, $AB^2 = OA^2 + OB^2$ (By Pythagoras property)
= $(8)^2 + (15)^2$ = 64 + 225
= 289
∴ AB = $\sqrt{289}$ = 17 cm
Since AB = BC = CD = DA (Property of rhombus)
∴ Required perimeter of rhombus
= 4 × side = 4 × 17 = 68 cm.

CHAPTER 7

CONGRUENCE OF TRIANGLES

KEY POINTS TO REMEMBER

If two figures have exactly the same shape and size, then they are said to be congruent.
For congruence, we use the symbol '='

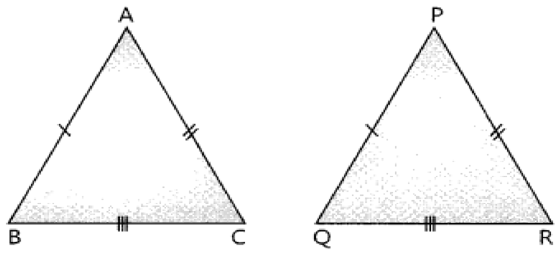
Two plane figures are congruent, if each when superposed on the other covers it exactly.

e.g. F_1 and F_2 are congruent if the trace copy of F_1 fits exactly on that F_2. We can write this as F_1 = F_2

Two line segments, $\overline{AB}$ and $\overline{CD}$ are congruent if they have equal lengths. We can write this as $\overline{AB}$ = $\overline{CD}$. However, it is common to write it as $\overline{AB}$ = $\overline{CD}$.

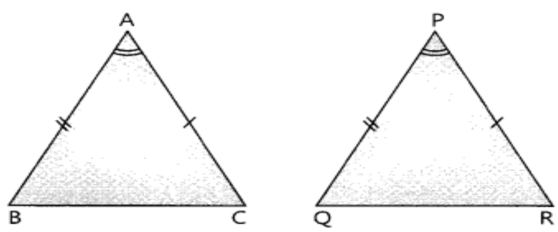
wo angles ∠ABC and ∠PQR, are congruent if their measures are equal. We can write this as ∠ABC = ∠PQR or m∠ABC = m∠PQR. Also, it is common to write it as ∠ABC = ∠PQR.

SSS Congruence of two triangles: Two triangles are congruent if the three sides of the one are equal to the three corresponding sides of the other.



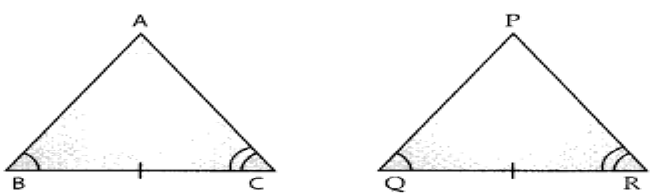
where, AB = PQ, AC = PR, BC = QR

SAS Congruence of two triangles: Two triangles are congruent if two sides and the angle included between them in one of the triangles are equal to the corresponding sides and the angle included between them of the other triangle

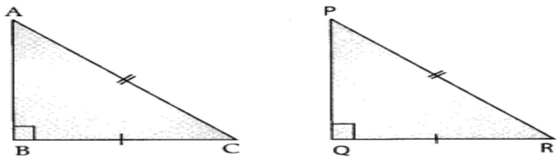


where, AB = PQ, AC = PR, ∠BAC = ∠QPR.

ASA congruence of two triangles: Two triangles are congruent if two angles and the side included between them in one of the triangles are equal to the corresponding angles and the side included between them of the other triangle.



RHS Congruence of two right-angled triangles: Two right-angled triangles are congruent if the hypotenuse and a leg of one of the triangles are equal to the hypotenuse and the corresponding leg of the other triangle



where, AC = PR, BC = QR, ∠ABC = ∠PQR = 90°.

Congruence of Plane Figures

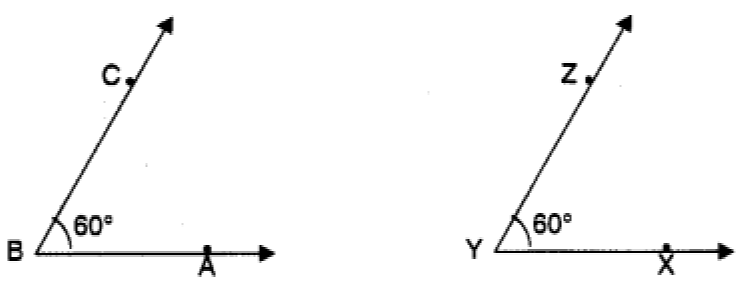
Two figures F_1 and F_2 are said to be congruent if they cover each other completely.
In this case, we write $F_1 = F_2$.

Congruence Among Line Segments

If two line segments have the same (i.e., equal) length, they are congruent. Conversely, if two line segments are congruent, they have the same length.
Line segments are congruent ⇔ their lengths are the same
If line segment AB is congruent to line segment CD, then we write AB = CD
Sometimes we also write AB = CD
and simply say that the line segments AB and CD are equal.

Congruence of Angles

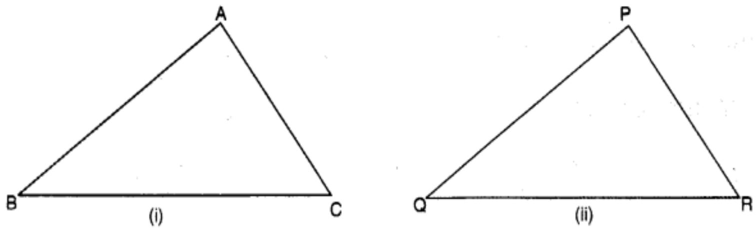
If two angles have the same measure, they are congruent.
Conversely, if two angles are congruent, their measures are the same.
Angles congruent ⇔ Angle measures same
or m∠ABC = m∠XYZ If ∠ABC is congruent to ∠XYZ, then we write ∠ABC = ∠XYZ
where m stands for the measure.



In the above case, the measure is 60°.

Congruence of Triangles

"Two triangles are congruent if they are copies of each other and when superposed, they cover each other exactly."



ΔABC and ΔPQR have the same size and shape. They are congruent. So we would express this as ΔABC = ΔPQR.

This means that, when we place ΔPQR on ΔABC, P falls on A, Q falls on B and R falls on C, also $\overline{PQ}$ falls along $\overline{AB}$, $\overline{QR}$ falls along $\overline{BC}$ and $\overline{PR}$ falls along $\overline{AC}$.

If under a given correspondence, two triangles are congruent, then their corresponding parts (i.e. angles and sides) that match one another are equal. Thus in these two congruent triangles, we have:
Corresponding vertices: A and P, B and Q; C and R.
Corresponding sides: $\overline{AB}$ and $\overline{PQ}$, $\overline{BC}$ and $\overline{QR}$; $\overline{AC}$ and $\overline{PR}$.
Corresponding angles: ∠A and ∠P, ∠B and ∠Q; ∠C and ∠R.

While talking about the congruence of triangles, not only the measures of angles and lengths of sides matter, but also the matching of vertices. In the above case, the correspondence is A ↔ P,

B ↔ Q, C ↔ R
We may write this as ABC ↔ PQR

Criteria for Congruence of Triangles
SSS Congruence Criterion
If under a given correspondence, the three sides of one triangle are respectively equal to the three sides of another triangle, then the triangles are congruent.

SAS Congruence Criterion
If under a corresponding two sides and the angle included between them of a triangle are respectively equal to two sides and the angle included between them of another triangle, then the triangles are congruent.

ASA Congruence Criterion
If under a correspondence two angles and the included side of a triangle are equal to two angles and the included side of another triangle, then the triangles are congruent.

Congruence Among Right-Angled Triangles
RHS Congruence Criterion
If under a correspondence, the hypotenuse and one side of a right-angled triangle are respectively equal to the hypotenuse and one side of another right-angled triangle, then the triangles are congruent.
We call this "RHS" congruence because R stands for Right-angle, H stands for Hypotenuse and S stands for Side.

1. Fill in the blanks

(a) Two line segments are congruent if _______ .
(b) Among the congruent angles, one has a measure of 70°, the measure of the other angle is _______ .
(c) When we write ∠A = ∠B, we actually mean
Solution:
(a) they have the same length
(b) 70°
(c) m∠A = m ∠B

EXERCISE 7.1

Very short answers

2. Give any two real life examples for congruent shapes.
Solution:

- Sharing blades of the same brand.
- Biscuits of the same packets.

3. If ΔABC ≅ ΔFED under the correspondence ABC ⟷ FED, write all the corresponding congruent parts of the triangles.
Solution:
Given: ΔABC = ΔFED
and ABC ⟷ FED

$$\therefore \begin{array}{l} \overline{AB} \leftrightarrow \overline{FE}, \overline{BC} \leftrightarrow \overline{ED}, \overline{AC} \leftrightarrow \overline{FD} \\ \angle A \leftrightarrow \angle F, \angle B \leftrightarrow \angle E, \angle C \leftrightarrow \angle D \end{array}$$

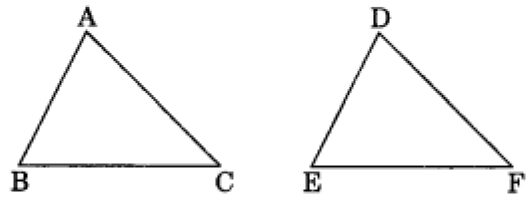
4. If ΔDEF ≅ ΔBCA. Write the part of ABCA that correspond to
(i) ∠E
(ii) $\overline{EF}$
(iii) ∠ZF
(iv) $\overline{DF}$
Solution:
Given: ΔDEF ≅ ΔBCA
(i) ∠E ⟷ ∠C
(ii) ∠F ⟷ ∠A
(iii) EF ⟷ ZA
(iv) DF ⟷ BA

EXERCISE 7.2

1 Which congruence criterion do you use in the following?
(a) Given:
AC = DF
AB = DE

BC = EF

So, ∆ABC = ∆DEF



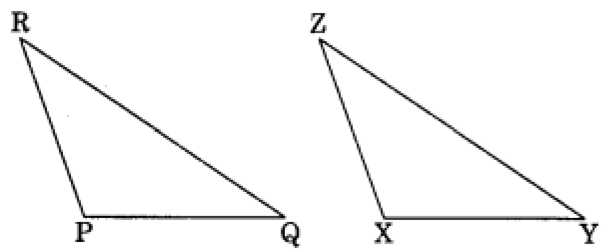
(b) Given:

ZX = RP

RQ = ZY

∠PRQ = ∠XZY

So, ∆PQR ≅ ∆XYZ

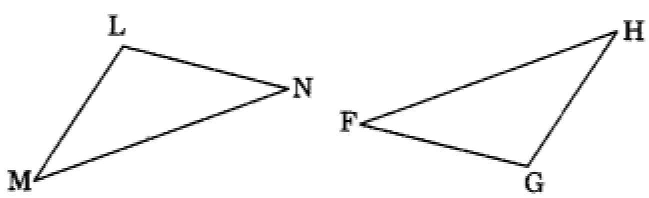


c) Given: ∠MLN = ∠FGH

∠NML = ∠GFH

ML = FG

So, ∆LMN = ∆GFH



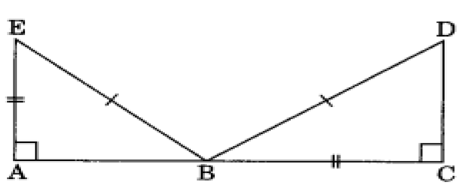
(d) Given:

EB = DB

AE = BC

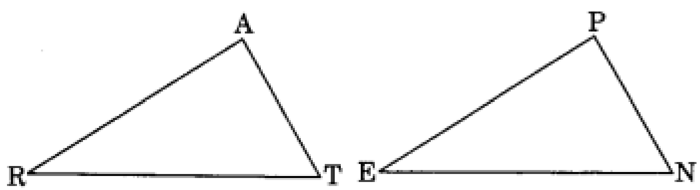
∠A = ∠C = 90°

∆ABE = ∆CDB



Solution:
(a) ΔABC ≅ ΔDEF (BY SSS rule)
(b) ΔPQR ≅ ΔXYZ (BY SAS rule)
(c) ΔLMN ≅ ΔGFH (BY ASA rule)
(d) ΔABE ≅ ΔCDB (BY RHS rule)

2 You want to show that ΔART = ΔPEN,
(a) If you have to use SSS criterion, then you need to show
(i) AR =
(ii) RT =
(iii) AT =

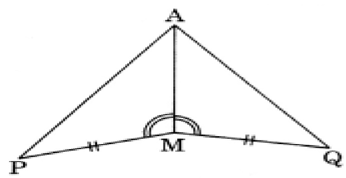


(b) If it is given that ∠T = ∠N and you are to use SAS criterion, you need to have
(i) RT = and
(ii) PN =
(c) If it is given that AT = PN and you are to use ASA criterion, you need to have
(i) ZA
(ii) ZT

Solution:
(a) For SSS criterion, we need
(i) AR = PE
(ii) RT = EN
(iii) AT = PN
(b) For SAS criterion, we need
(i) RT = EN and
(ii) PN = AT
(c) For ASA criterion, we need

(i) ∠A = ∠P
(ii) ∠T = ∠N

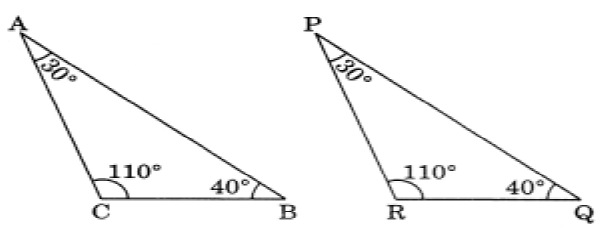
3 You have to show that ∆AMP ≅ ∆AMQ. In the fallowing proof, supply the missing reasons.



Solution

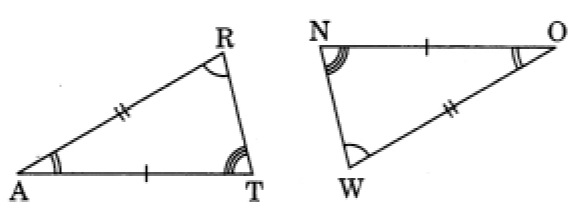
Steps	**Reasons**
(i) PM = QM	(i) Given
(ii) ∠PMA = ∠QMA	(ii) Given
(iii) AM = AM	(iii) Common
(iv) ∆AMP = ∆AMQ	(iv) SAS ru

4 In ∆ABC, ∠A = 30°, ∠B = 40° and ∠C = 110°
In ∆PQR, ∠P = 30°, ∠Q = 40° and ∠R = 110°.
A student says that ∆ABC = ∆PQR by AAA congruence criterion. Is he justified? Why or why not?Solution:
The student is not justified because there is not criterion for AAA congruence rule.



Example: In ∆ABC and ∆PQR, we have ∠A = 30°, ∠B = 40°, ∠C = 110°
∠P = 30°, ∠Q = 40°, ∠R = 110°
But ∆ABC is not congruent to ∆PQR.

5 In the figure, the two triangles are congruent. The corresponding parts are marked. We can write ∆RAT ≅ ?



Solution:
In ΔRAT and ΔWON
AT=ON (Given)
AR=OW(Given)
∠A = ∠O (Given)
∴ ΔRAT ≅ ΔWON (By SAS rule)

6. Complete the congruence statement:

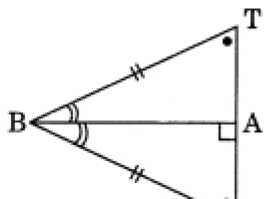


Fig. (*i*)
ΔBCA ≅ ?

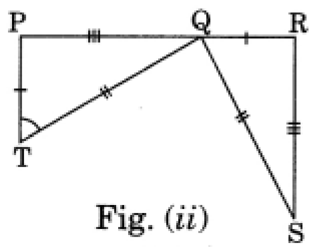


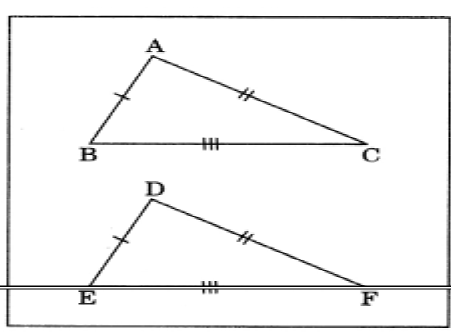
Fig. (*ii*)
ΔQRS ≅ ?

Solution:
Refer to Fig. (i)
In ΔBCA and ΔBTA
∠C = ∠T (Given)
BC=BT(Given)
∠BA = ∠TBA (Given)
∴ ΔBCA = ΔBTA (by ADA rule)
Refer to Fig. (ii)
In ΔQRS and ΔTPQ

RS=PQ (Given)
QS=TQ (Given)
∠RSQ = ∠PQT (Given)
∴ ΔQRS = ΔTPQ (by SAS rule)

7. In a squared sheet, draw two triangles of equal areas such that:
(i) the triangles are congruent.
(ii) the triangle are not congruent.
What can you say about their perimeters?

(i) On the given square sheet, we have draw two congruent triangles i.e.
ΔABC = ΔDEF



such that

$\overline{AB} = \overline{DE}, \overline{BC} = \overline{EF}$ and $\overline{AC} = \overline{DF}$

On adding, we get

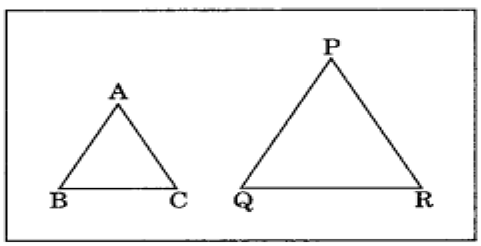
$\overline{AB} + \overrightarrow{BC} + \overrightarrow{AC} = \overline{DE} + \overline{EF} + \overline{DF}$

i.e. perimeters of ΔABC = Perimeter of ΔDEF

(ii) On the other square sheet, we have drawn two triangles ABC and PQR which are not congruent.

Such that

$$\overline{AB} \neq \overline{PQ}$$
$$\overline{BC} \neq \overline{QR}$$
and $$\overline{AC} \neq \overline{PR}$$

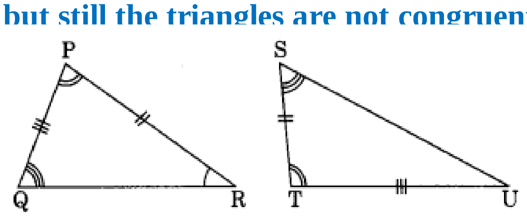


Adding both sides, we get

$\overline{AB} + \overline{BC} + \overline{AC} \neq \overline{PQ} + \overline{QR} + \overline{PR}$

i.e., perimeter of ΔABC ≠ the perimeter of ΔPQR.

8 Draw a rough sketch of two triangles, such that they have five pairs of congruent parts but still the triangles are not congruent.



Solution:

We have ΔPQR and ΔTSU

$\overline{PQ} = \overline{SU}$ (Given)

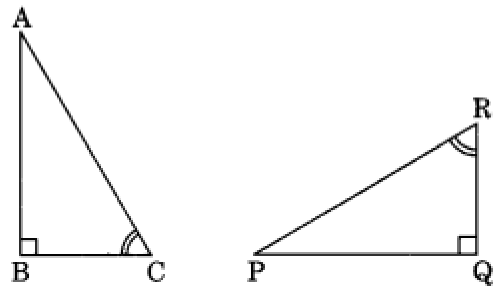
$\overline{PR} = \overline{ST}$ (Given)

∠Q = ∠S (Given)

∠P = ∠T (Given)

∠R = ∠U (Given)

9 If ΔABC and ΔPQR are to be congruent, name one additional pair of corresponding parts. What criterion did you use?

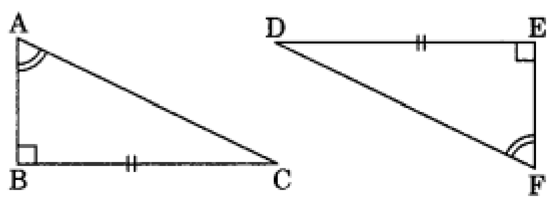


Solution:
In ΔABC and ΔPQR
∠B = ∠Q (Given)
∠C = ∠R (Given)
For ΔABC = ΔPQR

BC must equal to $\overline{QR}$ criterion that we used is ASA rule.

Hence, the additional pair of corresponding part is $\overline{BC} = \overline{QR}$.

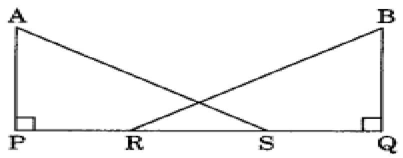
10. Explain, why ΔABC = ΔFED



Solution:
In ΔABC and ΔFED
∠B = ∠E = 90° (Given)
∠A = ∠F (Given)
∴ ∠A + ∠B = ∠E + ∠F
180° – ∠C = 180° – ∠D
[Angle sum property of triangles]
∴ ∠C =∠D
BC = ED (Given)
∴ ΔABC = ΔFED (By ASA rule)

HOTS QUESTIONS

1. In the given figure, AP = BQ, PR = QS. Show that ΔAPS = ΔBQR

Sc 

In ΔAPS and ΔBQR
AP = BQ (Given)
PR = QS (Given)
PR + RS = QS + RS (Adding RS to both sides)
PS = QR
∠APS = ∠BQR = 90° (Given)
ΔAPS = ΔBQR (by SAS rule)

2. Without drawing the figures of the triangles, write all six pairs of equal measures in each of the following pairs of congruent triangles.
(i) ΔABC = ADEF
(ii) ΔXYZ = ΔMLN
Solution:
(i) Given: ΔABC = ΔDEF
Here AB = DE
BC = EF
AC = DF
∠A = ∠D, ∠B = ∠E and ∠C = ∠F
(ii) Given ΔXYZ = ΔMLN
Here XY = ML
XZ = MN
∠X = ∠M, ∠Y = ∠L and ∠Z = ∠N

YZ = LN

3 Lengths of two sides of an isosceles triangle are 5 cm and 8 cm, find the perimeter of the triangle.

Solution:
Since the lengths of any two sides of an isosceles triangle are equal, then
Case I: The three sides of the triangle are 5 cm, 5 cm and 8 cm.
Perimeter of the triangle = 5 cm + 5 cm + 8 cm = 18 cm
Case II: The three sides of the triangle are 5 cm, 8 cm and 8 cm.
Perimeter of the triangle = 5 cm + 8 cm + 8 cm = 21 cm
Hence, the required perimeter is 18 cm or 21 cm.

CHAPTER 8

COMPARING QUANTITIES

KEY POINTS TO REMEMBER

To compare quantities, there are multiple methods, such as ratio and proportion, percentage, profit and loss, and simple interest.

The ratio of two quantities of the same kind and in the same unit is the fraction that one quantity is of the other.

The ratio a is to b as is the fraction $\frac{a}{b}$, and it is written as a : b.

In the ratio a : b, we call a as the first term or antecedent and b the second term or consequent.

To compare different ratios, firstly convert fractions into like fractions. If like fractions are equal, then the given ratios are said to be equivalent.
e.g. To check 1 : 2 and 2 : 3 are equivalent.

$$\frac{1}{2}=\frac{1\times 3}{2\times 3}=\frac{3}{6};\quad \frac{2}{3}=\frac{2\times 2}{3\times 2}=\frac{4}{6}$$

$$\therefore \frac{3}{6}<\frac{4}{6} \text{ which means } \frac{1}{2}<\frac{2}{3}$$

Therefore, the ratio 1 : 2 is not equivalent to the ratio 2 : 3.

To compare two quantities, units must be the same.

If the two ratios are equal, the four quantities are called in proportion.
a : b = c : d ⇒ a : b :: c : d.

Fractions are converted to percentages by multiply the fraction by 100 and write % sign
e.g. $\frac{1}{4} = \frac{1}{4} \times 100 = 25\%$

Decimals are converted to percentages by multiply the decimal number by 100 and shift the decimal point two places to the right side and write % sign.
e.g. 2.42 × 100 = 242 %

Profit = SP – CP [∵ SP > CP]
SP = Selling Price
CP = Cost Price

Loss = CP – SP [∵ CP > SP]
CP = Cost Price
SP = Selling Price

Profit % = $\frac{Profit}{C.P}$ × 100

Loss % = $\frac{Loss}{C.P}$ × 100

$$\text{Simple Interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$$

$$\text{Amount} = \text{Principal} + \text{Interest}$$

$$\text{Percentage Change} = \frac{\text{Amount of change}}{\text{Original Amount or Base}} \times 100$$

Equivalent Ratios

Different ratios are compared with each other to know whether they are equivalent or not. For this, we write the ratios in the form of fractions and then compare them by converting them into like fractions. If these like fractions are equal, we say that the given ratios are equivalent. Equivalent ratios are very important. Two ratios are said to be equivalent if when converted into like fractions, they are equal.

Unitary Method

In the unitary method, we first find the value of one unit and then the value of the required number of units.

Percentage-Another way of Comparing Quantities

Percentages are numerators of fractions with denominator 100. They are used for comparisons.

Meaning of Percentage

Percent means ‘per hundred’. It is represented by the symbol % and means hundredths too. Thus, 1% means 1 out of hundred or one-hundredths. It can be written as:

1% = $\frac{1}{100}$ = 0.01

Converting Fractional Numbers to Percentage

Fractional numbers can have different denominators. To compare fractional numbers we need a common denominator and it is more convenient to compare if the denominator is 100. So, we convert the fraction to percentages.

Percentages related to proper fractions are less than 100 whereas percentages related to improper fractions are more than 100.

Converting Decimals to Percentage

We multiply the decimal by 100 and affix percentage symbol.

Ratios to Percents

Sometimes, parts are given to us in the form of ratios and we need to convert those to percentage.

Increase or Decrease as Percent

There are times when we need to know what the increase in a certain quantity or decrease in it is as percent.
For example, if the population of a state is increased from 5,50,000 to 6,05,000, this could more clearly be understood if written as:
The population is increased by 10%.

$$\text{Percentage increase (or decrease)} = \frac{\text{Amount of change (increase or decrease)}}{\text{Original amount (or base)}} \times 100\%.$$

Prices Related to an Item on Buying and Selling
The buying price of an item is known as its Cost Price (CP).
The price at which we sell an item is known as its Selling Price (SP).

Profit or Loss as a Percentage

Cost Price: The buying price of an item is known as its cost price written in short as CP.
Selling Price: The price at which we sell an item is known as the selling price or in short SP.
Naturally, it is better if we sell the item at a higher price than our buying price.
Profit or Loss: We can decide whether the sale was profitable or not depending on the CP and SP.

If CP < SP then we have gained some amount, that is, we made a profit, profit = SP – CP
If CP = SP then we are in a no profit no loss situation
If CP > SP then we have lost some amount, Loss = CP – SP.
The profit or loss we find can be converted to a percentage. It is always calculated on the CP.

Note. If we are given any two of the three quantities related to price, that is, CP, SP, and Profit or Loss percent, we can find the third.

Charge has given on Borrowed Money or Simple Interest
Principal: The money borrowed is known as sum borrowed or principal.

Interest: We have to pay some extra money (or charge) to the bank for the money being used by us for some time. This is known as interest.

Amount: We can find the amount we have to pay at the end of the year by adding the above two. That is.
Amount = Principal + Interest.

Note: Interest is generally given in per cents for a period of one year. It is written as x percent per year or per annum or in short as x% p.a. (say 10 percent per year)
10% p.a. means on every ₹ 100 borrowed, ₹ 10 is the interest we have to pay for one year.

1 Find the ratio of:
(a) ₹ 5 to 50 paise
(b) 15 kg to 210 g
(c) 9 m to 27 cm
(d) 30 days to 36 hours

Solution

(a) ₹ 5 to 50 paise
Converting the given quantities into same units, we have
₹ 5 = 5 × 100 = 500 paise
∴ ₹ 5 : 50 paise
= 500 paise : 50 paise [∵ ₹ 1 = 100 paise]
= 10 : 1
So, required ratio is 10 : 1.

(b) 15 kg to 210 g
Converting the given quantities into same units, we have
15 kg = 15 × 1000
= 15000 g [∵ 1 kg = 1000 g]
∴ 15 kg : 210 g = 15000 g : 210 g
= 1500 : 21
= 500 : 7
So, the required ratio is 20 : 1.

So, the required ratio is 500 : 7.

2 In a computer lab, there are 3 computers for every 6 students. How many computers will be needed for 24 students?

Solution:
Using Unitary Method, we have
6 students require 3 computers
∴ 1 student will require = $\frac{3}{6}$ computers
∴ 24 students will require = $\frac{3}{6}$ x 24 computers

= 3 × 4 computers = 12 computers
Hence the number of computers required = 12.

3Population of Rajasthan = 570 lakhs and population of UP = 1660 lakhs.
Area of Rajasthan = 3 lakh km2 and area of UP = 2 lakh km2.

(i) How many people are there per km2 in both these States?
(ii) Which State is less populated?

Solution:
Given:
Population of Rajasthan = 570 lakhs
Population of UP = 1660 lakhs
Area of Rajasthan = 3 lakh km^2
Area of UP = 2 lakh km^2
(i) Number of people per km^2 of Rajasthan 5

$$= \frac{570 Lakhs}{3\ lakhs\ \text{km2}}$$

= 190 per km^2
Number of people in UP = 1660 lakhs
Area of UP = 2 lakh km^2
$= \frac{1660 Lakhs}{2\ lakhs\ \text{km2}}$ Number of people per km^2 of UP

Since 190 per km^2< 830 per km^2

(ii) Rajasthan is less populated state.

EXERCISE 8.2

1 Convert the given fractional numbers to per cents:

$(a)\ \frac{1}{8} \qquad (b)\ \frac{5}{4} \qquad (c)\ \frac{3}{40} \qquad (d)\ \frac{2}{7}$

$$(a)\ \frac{1}{8} = \frac{1 \times 100}{8 \times 100} = \frac{100}{8}\% = 12.5\% \text{ or } 12\frac{1}{2}\%$$

(b) $\frac{5}{4} = \frac{5 \times 100}{4 \times 100} = \frac{5}{4} \times 100\% = 125\%$

(c) $\frac{3}{40} = \frac{3}{40} \times \frac{100}{100} = \frac{3 \times 100}{40}\%$

$= \frac{15}{2}\% = 7.5\%$ or $7\frac{1}{2}\%$

2. Convert the given decimal fractions to per cents:

(a) 0.65

(b) 2.1

(c) 0.02

(d) 12.35

Solution

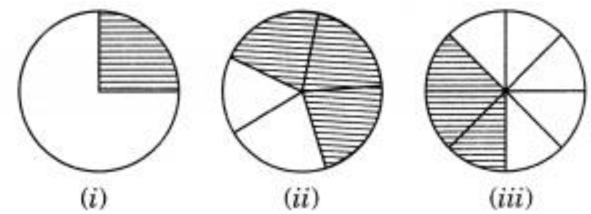
(a) $0.65 = \frac{0.651 \times 100}{100} = 0.65 \times 100\% = 65\%$

(b) $2.1 = \frac{2.1 \times 100}{100} = 2.1 \times 100\% = 210\%$

(c) $0.02 = \frac{0.02 \times 100}{100} = 0.02 \times 100\% = 2\%$

(d) $12.35 = \frac{12.35 \times 100}{100} = 12.35 \times 100\% = 1235\%$

3 Estimate what part of the figures is coloured and hence find the per cent which is coloured.



$= \frac{1}{4} \times \frac{100}{100} = \frac{100}{4}\% = 25\%$

$= \frac{3}{5} \times \frac{100}{100} = \frac{3}{5} \times 100\% = 60\%$

iii) Fraction of coloured part = 68

∴ Percentage of coloured parts

$= \frac{3}{8} \times \frac{100}{100} = \frac{3}{8} \times 100\% = 37.5\%$

4. Find:

(a) 15% of 250

(b) 1% of 1 hour

(c) 20% of ₹ 2500

(d) 75% of 1 kg

Solution

$$\doteq \frac{1}{100} \times 60 \text{ minutes} = \frac{3}{5} \text{ minutes}$$

$$= \frac{3}{5} \times 60 \text{ seconds} = 36 \text{ seconds}$$

(c) 20% of ₹ 2500 $= \frac{20}{100} \times$ ₹ 2500 = ₹ 500

(d) 75% of 1 kg = 75% of 1000 g

$$= \frac{75}{100} \times 1000 \; (\because \; 1 \text{ kg} = 1000 \text{ g})$$

$$= 750 \text{ g} = 0.75 \text{ kg}$$

5. Find the whole quantity if

(a) 5% of it is 600

(b) 12% of it is? 1080

(c) 40% of it is 500 km

(d) 70% of it is 14 minutes

(e) 8% of it is 40 litres

Solution:

Let the required whole quantity be x.

(a) 5% of x = 600

$$\Rightarrow \quad \frac{5}{100} \times x = 600$$

$$\Rightarrow \quad x = \frac{600 \times 100}{5} = 12000$$

$$\Rightarrow \quad \frac{12}{100} \times x = ₹\ 1080$$

$$\Rightarrow \quad x = ₹\ \frac{1080 \times 100}{12} = ₹\ 9{,}000$$

Thus, the required quantity is ₹ 9,000.

(c) 40% of x = 500 km

$$\Rightarrow \quad \frac{40}{100} \times x = 500 \text{ km}$$

$$\Rightarrow \quad x = \frac{500 \times 100}{40} \text{ km} = 1250 \text{ km}$$

Thus, the required quantity = 1250 km.

d) 70% of x = 14 minutes

$$\Rightarrow \quad \frac{70}{100} \times x = 14 \text{ minutes}$$

$$\Rightarrow \quad x = \frac{14 \times 100}{70} \text{ minutes}$$

$$= 20 \text{ minutes}$$

Thus, the required quantity = 20 minutes,

(e) 8% of x = 40 litre 8

$$\Rightarrow \quad \frac{8}{100} \times x = 40$$

$$\Rightarrow \quad x = \frac{\cancel{40}^{5} \times 100}{\cancel{8}} = 500 \text{ litre}$$

Thus, the required quantity = 500 litres

6 Convert given per cents to decimal fractions and also to fractions in simplest forms:
(a) 25%
(b) 150%
(c) 20%
(d) 5%

Per cent	Decimal form	Fraction form
(a) 25%	$\frac{25}{100} = 0.25$	$\frac{25}{100} = \frac{1}{4}$
(b) 150%	$\frac{150}{100} = 1.50$	$\frac{150}{100} = \frac{3}{2}$
(c) 20%	$\frac{20}{100} = 0.2$	$\frac{20}{100} = \frac{1}{5}$
(d) 5%	$\frac{5}{100} = 0.05$	$\frac{5}{100} = \frac{1}{20}$

7. In a city, 30% are females, 40% are males and remaining are children. What per cent

are children ?
Solution:
Given: 30% are females
40% are males
Total Percentage of females and males
= 30% + 40% = 70%
∴ Percentage of children
= (100 – 70)% = 30%are children?

8. Out of 15,000 voters in a constituency, 60% voted. Find the Percentage of voters who did not vote. Can you now find how many actually did not vote?

Solution:
Total number of voters = 15,000
Percentage of the voters who voted = 60%
∴ Percentage of the voters who did not vote
= (100 – 60)% = 40%
Actual number of voters who did not vote
= 40% of 15,000
$= \frac{40}{100} \times 15{,}000 = 6{,}000$

9. Meena saves ₹ 400 from her salary. If this is 10% of her salary. What is her salary?

Solution:
Let Meena's salary by ₹ x.
∴ 10% of x = ₹ 400

$$\Rightarrow \quad \frac{10}{100} \times x = ₹\ 400$$

$$\therefore \quad x = ₹\ \frac{400 \times 100}{10} = ₹\ 4000$$

Thus, her salary is ₹ 4000.

10 A local cricket team played 20 matches in one season. It won 25% of them. How many matches did they win?

Solution:
Number of matches played by the cricket team = 20
Percentage of the matches won by them = 25%
i.e. $\frac{25}{100} \times 20 = 5$ matches
Thus, the number of matches won by them = 5

CHAPTER 9

RATIONAL NUMBERS

KEY POINTS TO REMEMBER

The numbers used for counting objects are called counting numbers or **natural numbers**. These are: 1, 2, 3, 4, …

If we include 0 to natural numbers, we get whole numbers. Thus, 0, 1, 2, 3, 4, ….. are whole numbers.

If we include the negatives of natural numbers to the whole numbers, we get integers. Thus, ….., -3, -2, -1, 0, 1, 2, 3, …… are **integers**.

The numbers of the form $\frac{numerator}{denominator}$ where the numerator is either 0 or a positive integer and the denominator is a positive integer, are called fractions.

We compare two fractions by finding their equivalent forms. We have studied all the four basic operations of addition, subtraction, multiplication, and division on them. In this chapter, we shall further extend the number system by introducing **rational numbers.**

What are Rational Numbers?

A number of the form $\frac{p}{q}$ where p and q (≠0) are integers, is called a rational number.

Numerator and Denominator

In $\frac{p}{q}$, the integer p is the numerator, and the integer q (≠0) is the denominator.

Thus in $\frac{-3}{7}$, the numerator is -3 and the denominator is 7.

Equivalent Rational Numbers

If we multiply the numerator and denominator of a rational number by the same non-zero integer, we obtain another rational number equivalent to the given rational number.

Positive and Negative Rational Numbers

A rational number whose numerator and denominator both are positive integers is called a positive rational number.

Rational Numbers on a Number Line

Positive rational numbers are marked on the right of 0 on the number line whereas negative rational

numbers are marked on the left of 0 on the number line.
The method of representation is the same as the method of representation of fractions on the number line

Rational Numbers in Standard Form

A rational number is said to be in the standard form if its denominator is a positive integer and the numerator and the denominator have no common factor other than 1. Note that the negative sign occurs only in the numerator.
A rational number in standard form is said to be in its lowest form.

Reduction of a Rational Number to its Lowest Form

To reduce a rational number to its standard form (or lowest form), we divide its numerator and denominator by their HCF ignoring the negative sign, if any.
However, if there is a negative sign in the denominator, we divide by -HCF'.

Comparison of Rational Numbers

Two positive rational numbers can be compared exactly as we compare two fractions.
Two negative rational numbers can be compared by ignoring their negative signs and then reversing the order.
Comparison of a negative and a positive rational number is obvious as a negative rational number is always less than a positive rational number.

Additive Inverse

The additive inverse of the rational number $\frac{p}{q}$ is - $\frac{p}{q}$

Subtraction

While subtracting two rational numbers, we add the additive inverse of the rational number to be subtracted to the other rational number.

Multiplication

Multiplication of a rational number by a positive integer:

While multiplying a rational number by a positive integer, we multiply the numerator by that integer, keeping the denominator unchanged.

Multiplication of rational number by a negative integer:

While multiplying a rational number by a negative integer, we multiply the numerator by that integer, keeping the denominator unchanged.

Multiplication of two rational numbers (none of which is an integer):

Based on the above observations,
So, as done in fractions we multiply two rational numbers as follows:

- Step 1. Multiply the numerators of the two rational numbers.
- Step 2. Multiply the denominators of the two rational numbers.
- Step 3. Write the product as $\frac{Result\ of\ step\ 1}{Result\ of\ step\ 2}$

Division

The reciprocal of the rational number $\frac{p}{q}$ is $\frac{q}{p}$

To divide one rational number by other rational number, we multiply one rational number by the reciprocal of the other.

Product of Reciprocals

The product of a rational number with its reciprocal is always 1

A rational number is defined as a number that can be expressed in the form $\frac{p}{q}$ where p and q are integers and q ≠ 0.

$$e.g.\ \frac{4}{5}, \frac{2}{3}, \frac{6}{13}, \text{ etc.}$$

EXERCISE 9.1

1.List five rational numbers between:

(i) -1 and 0

(ii) -2 and -1

(iii) $\frac{-4}{5}$ and $\frac{-2}{3}$

(iv) $\frac{1}{2}$ and $\frac{2}{3}$

Solution:

(i) -1 and 0

Converting each of rational numbers as a denominator 5 + 1 = 6, we have

$$-1 = \frac{-1 \times 6}{6} = \frac{-6}{6} \text{ and } \frac{0 \times 6}{6} = \frac{0}{6}$$

$$\text{So, } \frac{-6}{6} < \frac{-5}{6}, \frac{-4}{6}, \frac{-3}{6}, \frac{-2}{6}, \frac{-1}{6} < \frac{0}{6}$$

$$\text{or } -1 < \frac{-5}{6}, \frac{-2}{3}, \frac{-1}{2}, \frac{-1}{3}, \frac{-1}{6} < \frac{0}{6}$$

Hence, the required five rational numbers between -1 and 0 are $\frac{-5}{6}$, $\frac{-2}{3}$, $\frac{-1}{2}$, $\frac{-1}{3}$ **and** $\frac{-1}{6}$

(ii) -2 and -1

Converting each of rational numbers as a denominator 5 + 1 = 6, we have

$$-2 = \frac{-2 \times 6}{6} = \frac{-12}{6},$$

$$-1 = \frac{-1 \times 6}{6} = \frac{-6}{6}$$

$$\text{So, } \frac{-12}{6} < \frac{-11}{6} < \frac{-10}{6} < \frac{-9}{6} < \frac{-8}{6} < \frac{-7}{6} < \frac{-6}{6}$$

$$\text{or } -2 < \frac{-11}{6} < \frac{-5}{3} < \frac{-3}{2} < \frac{-4}{3} < \frac{-7}{6} < -1$$

Hence, the required rational numbers are

$$\frac{-11}{6}, \frac{-5}{3}, \frac{-3}{2}, \frac{-4}{3} \text{ and } \frac{-7}{6}.$$

iii) $\frac{-4}{5}$ and $\frac{-2}{3}$

Converting each of the rational numbers as a denominator 5 × 3 = 15, we have

$(\because \text{LCM of 5 and 3} = 15)$

$$-\frac{4}{5} = -\frac{4 \times 3}{5 \times 3} = -\frac{12}{15},$$

$$-\frac{2}{3} = \frac{-2 \times 5}{3 \times 5} = \frac{-10}{15}$$

Since there is only one integer i.e. -11 between -12 and -10, we have to find equivalent rational numbers.

$$\frac{-12}{15} = \frac{-12 \times 3}{15 \times 3} = \frac{-36}{45},$$

$$\frac{-10}{15} = \frac{-10 \times 3}{15 \times 3} = \frac{-30}{45}$$

$$\therefore \frac{-36}{45} < \frac{-35}{45} < \frac{-34}{45} - \frac{33}{45} < \frac{-32}{45} < \frac{-31}{45} < \frac{-30}{45}$$

$$\text{or } \frac{-4}{5} < \frac{-7}{9} < \frac{-34}{45} < \frac{-11}{15} < \frac{-32}{45} < \frac{-31}{45} < \frac{-2}{3}$$

Hence, the required rational numbers are

$$-\frac{7}{9}, \frac{-34}{45}, \frac{-11}{15}, \frac{-32}{45}, \frac{-31}{45}$$

iv) $\frac{1}{2}$ and $\frac{2}{3}$

Converting each of the rational numbers in their equivalent rational numbers, we have

$$\frac{2}{3} = \frac{2 \times 12}{3 \times 12} = \frac{24}{36}$$

$$\therefore \frac{18}{36} < \frac{19}{36} < \frac{20}{36} < \frac{21}{36} < \frac{22}{36} < \frac{23}{36} < \frac{24}{36}$$

$$\text{or } \frac{1}{2} < \frac{19}{36} < \frac{5}{9} < \frac{7}{12} < \frac{11}{18} < \frac{23}{36} < \frac{2}{3}$$

Hence, the required rational numbers are

$$\frac{19}{36}, \frac{5}{9}, \frac{7}{12}, \frac{11}{18} \text{ and } \frac{23}{36}.$$

2 Write four more rational numbers in each of the following patterns:

(i) $\frac{-3}{5}, \frac{-6}{10}, \frac{-9}{15}, \frac{-12}{20}, \ldots$

(ii) $\frac{-1}{4}, \frac{-2}{8}, \frac{-3}{12}, \ldots$

(iii) $\frac{-1}{6}, \frac{2}{-12}, \frac{3}{-18}, \frac{4}{-24}, \ldots$

(iv) $\frac{-2}{3}, \frac{2}{-3}, \frac{4}{-6}, \frac{6}{-9}, \ldots$

Solution

(i) Given pattern is

$$\frac{-3}{5} = \frac{-3 \times 1}{5 \times 1}$$
$$\frac{-6}{10} = \frac{-3 \times 2}{5 \times 2}$$
$$\frac{-9}{15} = \frac{-3 \times 3}{5 \times 3}$$
$$\frac{-12}{20} = \frac{-3 \times 4}{5 \times 4}$$

Proceeding with the same pattern, we have

$$\frac{-3 \times 5}{5 \times 5} = \frac{-15}{25}$$
$$\frac{-3 \times 6}{5 \times 6} = \frac{-18}{30}$$
$$\frac{-3 \times 7}{5 \times 7} = \frac{-21}{35}$$
$$\frac{-3 \times 8}{5 \times 8} = \frac{-24}{40}$$

Hence, the required rational numbers are

$\frac{-15}{25}, \frac{-18}{30}, \frac{-21}{35}$ and $\frac{-24}{40}$

(ii) Given pattern is:

$$\frac{-1}{4} = \frac{-1 \times 1}{4 \times 1}$$
$$\frac{-2}{8} = \frac{-1 \times 2}{4 \times 2}$$
$$\frac{-3}{12} = \frac{-1 \times 3}{4 \times 3}$$

Proceeding with the same pattern, we have

$$\frac{-1\times 4}{4\times 4} = \frac{-4}{16}$$

$$\frac{-1\times 5}{4\times 5} = \frac{-5}{20}$$

$$\frac{-1\times 6}{4\times 6} = \frac{-6}{24}$$

$$\frac{-1\times 7}{4\times 7} = \frac{-7}{28}$$

Hence, the required rational numbers is

$\frac{-4}{16}, \frac{-5}{20}, \frac{-6}{24}$ and $\frac{-7}{28}$

(*iii*) Given pattern is:

$$-\frac{1}{6} = -\frac{1\times 1}{6\times 1}$$

$$\frac{2}{-12} = \frac{-2}{12} = \frac{-1\times 2}{6\times 2}$$

$$\frac{3}{-18} = \frac{-3}{18} = \frac{-1\times 3}{6\times 3}$$

$$\frac{4}{-24} = \frac{-4}{24} = \frac{-1\times 4}{6\times 4}$$

Proceeding with the same pattern, we have

$$\frac{1\times 5}{-6\times 5} = \frac{5}{-30}$$

$$\frac{1}{-6}\times\frac{6}{6} = \frac{6}{-36}$$

$$\frac{1}{-6}\times\frac{7}{7} = \frac{7}{-42}$$

$$\frac{1}{-6}\times\frac{8}{8} = \frac{8}{-48}$$

Hence, the required rational numbers are

$\frac{5}{-30}, \frac{6}{-36}, \frac{7}{-42}$ and $\frac{8}{-48}$

(*iv*) The given pattern is:

$$\frac{-2}{3} = -\frac{2\times 1}{3\times 1}$$

$$\frac{2}{-3} = \frac{2}{-3} = \frac{2\times 1}{-3\times 1}$$

$$\frac{4}{-6} = \frac{-4}{6} = \frac{-2\times 2}{3\times 2}$$

$$\frac{6}{-9} = \frac{-6}{9} = \frac{-2\times 3}{3\times 3}$$

Proceeding with the same pattern, we have

$$\frac{-2\times 4}{3\times 4} = \frac{-8}{12} \text{ or } \frac{8}{-12}$$

$$\frac{-2\times 5}{3\times 5} = \frac{-10}{15} \text{ or } \frac{10}{-15}$$

$$\frac{-2\times 6}{3\times 6} = \frac{-12}{18} \text{ or } \frac{12}{-18}$$

$$\frac{-2\times 7}{3\times 7} = \frac{-14}{21} \text{ or } \frac{14}{-21}$$

Hence, the required rational numbers are

$\frac{8}{-12}, \frac{10}{-15}, \frac{12}{-18}, \frac{14}{-21}$

1 Find the sum:

(i) $\frac{5}{4}+\left(\frac{-11}{4}\right)$ (ii) $\frac{5}{3}+\frac{3}{5}$

(iii) $\frac{-9}{10}+\frac{22}{15}$ (iv) $\frac{-3}{-11}+\frac{5}{9}$

(v) $\frac{-8}{19}+\frac{(-2)}{57}$ (vi) $\frac{-2}{3}+0$

(vii) $-2\frac{1}{3}+4\frac{3}{5}$

Solution

(i) $\dfrac{5}{4}+\left(\dfrac{-11}{4}\right) = \dfrac{5-11}{4} = \dfrac{-6}{4} = \dfrac{-6\div 2}{4\div 2} = \dfrac{-3}{2}$

(ii) $\dfrac{5}{3}+\dfrac{3}{5} = \dfrac{5\times 5}{3\times 5}+\dfrac{3\times 3}{5\times 3}$

[$\because$ LCM of 3 and 5 = 15]

$= \dfrac{25}{15}+\dfrac{9}{15} = \dfrac{25+9}{15} = \dfrac{34}{15} = 2\dfrac{4}{15}$

(iii) $\dfrac{-9}{10}+\dfrac{22}{15} = \dfrac{-9\times 3}{10\times 3}+\dfrac{22\times 2}{15\times 2}$

[LCM of 10 and 15 = 30]

$= \dfrac{-27}{30}+\dfrac{44}{30} = \dfrac{-27+44}{30} = \dfrac{17}{30}$

(iv) $\dfrac{-3}{-11}+\dfrac{5}{9} = \dfrac{3}{11}+\dfrac{5}{9}$

$= \dfrac{3\times 9}{11\times 9}+\dfrac{5\times 11}{9\times 11}$ [LCM of 9 and 11 = 99]

$= \dfrac{27}{99}+\dfrac{55}{99} = \dfrac{27+55}{99} = \dfrac{82}{99}$

(v) $\dfrac{-8}{19}+\dfrac{(-2)}{57} = \dfrac{-8\times 3}{19\times 3}-\dfrac{2\times 1}{57\times 1}$

$= \dfrac{-24}{57}-\dfrac{2}{57} = \dfrac{-24-2}{57} = \dfrac{-26}{57}$

(vi) $\dfrac{-2}{3}+0 = \dfrac{-2}{3}+\dfrac{0}{3} = \dfrac{-2+0}{3} = \dfrac{-2}{3}$

(vii) $-2\dfrac{1}{3}+4\dfrac{3}{5} = \dfrac{-7}{3}+\dfrac{23}{5} = \dfrac{-7\times 5}{3\times 5}+\dfrac{23\times 3}{5\times 3}$

[LCM of 3 and 5 = 15]

$= \dfrac{-35}{15}+\dfrac{69}{15} = \dfrac{-35+69}{15} = \dfrac{34}{15} = 2\dfrac{4}{15}$

2. Find:

(i) $\dfrac{7}{24}-\dfrac{17}{36}$ (ii) $\dfrac{5}{63}-\left(-\dfrac{6}{21}\right)$

(iii) $\dfrac{-6}{13}-\left(-\dfrac{7}{15}\right)$ (iv) $\dfrac{-3}{8}-\dfrac{7}{11}$

(v) $-2\dfrac{1}{9}-6$

Solution

(i) $\dfrac{7}{24}-\dfrac{17}{36}$ LCM of 24 and 36 = 72

$$=\frac{7\times3}{24\times3}-\frac{17\times2}{36\times2}=\frac{21}{72}-\frac{34}{72}=\frac{-13}{72}$$

(ii) $\dfrac{5}{63}-\left(\dfrac{-6}{21}\right)=\dfrac{5}{63}+\dfrac{6}{21}$

LCM of 63 and 21 = 63

$$=\frac{5\times1}{63\times1}+\frac{6\times3}{21\times3}=\frac{5}{63}+\frac{18}{63}=\frac{5+18}{63}=\frac{23}{63}$$

(iii) $\dfrac{-6}{13}-\left(-\dfrac{7}{15}\right)=\dfrac{-6}{13}+\dfrac{7}{15}$

LCM of 13 and 15 = 195

$$=\frac{-6\times15}{13\times15}+\frac{7\times13}{15\times13}$$

$$=\frac{-90}{195}+\frac{91}{195}=\frac{-90+91}{195}=\frac{1}{195}$$

(iv) $\dfrac{-3}{8}-\dfrac{7}{11}$ LCM of 8 and 11 = 88

$$\frac{-3\times11}{8\times11}-\frac{7\times8}{11\times8}=\frac{-33}{88}-\frac{56}{88}$$

$$=\frac{-33-56}{88}=\frac{-89}{88}=-1\frac{1}{88}$$

(v) $-2\dfrac{1}{9}-6=-\dfrac{19}{9}-\dfrac{6}{1}$ [LCM of 9 and 1 = 9]

$$\therefore\ \frac{-19\times1}{9\times1}-\frac{6\times9}{1\times9}=\frac{-19}{9}-\frac{54}{9}=\frac{-19-54}{9}$$

$$=\frac{-73}{9}=-8\frac{1}{9}$$

3 Find the product:

(i) $\frac{9}{2} \times \left(\frac{-7}{4}\right)$ (ii) $\frac{3}{10} \times (-9)$

(iii) $\frac{-6}{5} \times \frac{9}{11}$ (iv) $\frac{3}{7} \times \left(\frac{-2}{5}\right)$

(v) $\frac{3}{11} \times \frac{2}{5}$ (vi) $\frac{3}{-5} \times \frac{-5}{3}$

Solution

(i) $\frac{9}{2} \times \left(\frac{-7}{4}\right) = \frac{9 \times (-7)}{2 \times 4} = \frac{-63}{8} = -7\frac{7}{8}$

(ii) $\frac{3}{10} \times (-9) = \frac{3}{10} \times \frac{-9}{1}$

$= \frac{3 \times (-9)}{10 \times 1} = \frac{-27}{10} = -2\frac{7}{10}$

(iii) $\frac{-6}{5} \times \frac{9}{11} = \frac{-6 \times 9}{5 \times 11} = \frac{-54}{55}$

(iv) $\frac{3}{7} \times \left(\frac{-2}{5}\right) = \frac{3 \times (-2)}{7 \times 5} = \frac{-6}{35}$

(v) $\frac{3}{11} \times \frac{2}{5} = \frac{3 \times 2}{11 \times 5} = \frac{6}{55}$

(vi) $\frac{3}{-5} \times \frac{-5}{3} = \frac{-3}{5} \times \frac{-5}{3} = \frac{(-3) \times (-5)}{5 \times 3} = \frac{15}{15} = 1$

4 Find the value of:

(i) $(-4) \div \frac{2}{3}$ (ii) $\frac{-3}{5} \div 2$

(iii) $\frac{-4}{5} \div (-3)$ (iv) $\frac{-1}{8} \div \frac{3}{4}$

(v) $\frac{-2}{13} \div \frac{1}{7}$ (vi) $\frac{-7}{12} \div \left(\frac{-2}{13}\right)$

(vii) $\frac{3}{13} \div \left(\frac{-4}{65}\right)$

Solution

(i) $(-4) \div \frac{2}{3} = -4 \times \frac{3}{2}$

(iii) $\frac{-4}{5} \div (-3) = \frac{-4}{5} \div \frac{-3}{1} = \frac{-4}{5} \times \frac{-1}{3}$

$= \frac{(-4) \times (-1)}{5 \times 3} = \frac{4}{15}$

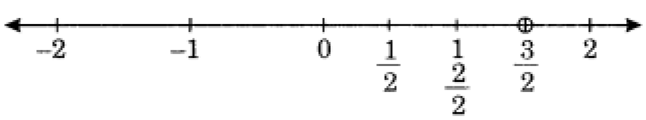
(iv) $\frac{-1}{8} \div \frac{3}{4} = \frac{-1}{8} \times \frac{4}{3}$

$= \frac{-1 \times 4}{8 \times 3} = \frac{-4}{24} = \frac{-4 \div 4}{24 \div 4} = \frac{-1}{6}$

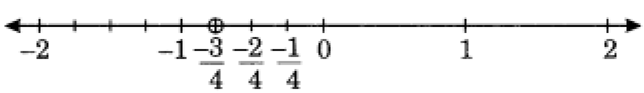
HOTS Questions

1.Represent $\frac{3}{2}$ and $\frac{-3}{4}$ on number lines.

(i) $\frac{3}{2}$



(ii) $-\frac{3}{4}$



2**. Insert five rational numbers between:**

(i) $\frac{-2}{3}$ and -1 (ii) $-\frac{1}{2}$ and $\frac{-3}{2}$

Solution

(i) $\frac{-2}{3}$ and $-1 \Rightarrow \frac{-2}{3}$ and $\frac{-1}{1}$

LCM of 3 and 1 = 3

$\therefore \frac{-2\times 1}{3\times 1} = \frac{-2}{3}$ and $\frac{-1\times 3}{1\times 3} = \frac{-3}{3}$

We know that there is no integer between -2 and -3.

$\therefore$ Multiplying and dividing by 5 + 1 = 6 to each of the rational numbers, we have

$\frac{-2\times 6}{3\times 6} = \frac{-12}{18}$ and $\frac{-3\times 6}{3\times 6} = \frac{-18}{18}$

Here, integers between -12 and -18 are -13, -14, -15, -16 and -17.

$\therefore$ The required rational numbers are

$\frac{-13}{18}, \frac{-14}{18}, \frac{-15}{18}, \frac{-16}{18}$ and $\frac{-17}{18}$

i.e., $\frac{-13}{18}, \frac{-7}{9}, \frac{-5}{6}, \frac{-8}{9}, \frac{-17}{18}$

(ii) $-\frac{1}{2}$ and $\frac{-3}{2}$

Since, the denominator are same and there is only one integer between -1 and -3.

$\therefore$ Multiplying and dividing by 5 + 1 = 6 to each of the rational numbers, we have

$\frac{-1\times 6}{2\times 6} = \frac{-6}{12}$ and $\frac{-3\times 6}{2\times 6} = \frac{-18}{12}$

Here, the integers between -6 and -18 are $-7, -8, -9, -10, -11$

$\therefore$ The required rational numbers are

$\frac{-7}{12}, \frac{-8}{12}, \frac{-9}{12}, \frac{-10}{12}, \frac{-11}{12}$

i.e., $\frac{-7}{12}, \frac{-2}{3}, \frac{-3}{4}, \frac{-5}{6}, \frac{-11}{12}$