पुना International School

Shree Swaminarayan Gurukul, Zundal

Grade – 10 MATHS Specimen copy Year 21-22

INDEX

CHAPTER NO. – 8

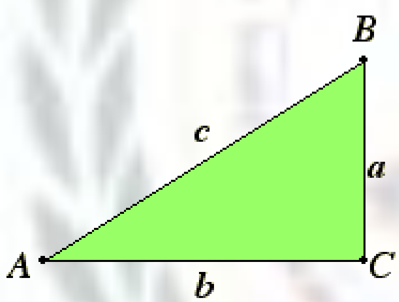
CHAPTER NAME – iNTRODUCTION TO TRIGONOMETRY

KEY POINTS TO REMEMBER –

Notes

- **Trigonometry** literally means measurement of sides and angles of a triangle.
- **Positive and Negative angles**: Angles in anti-clockwise direction are taken as positive angles and angles in clockwise direction are taken as negative angles.
- **Trigonometric Ratios of an acute angle of a right angled triangle**:

1. In a right triangle ABC, right-angled at B,



Solving right triangles

We can use the Pythagorean Theorem and properties of sines, cosines, and tangents to solve the triangle, that is, to find unknown parts in terms of known parts.

- Pythagorean Theorem: $a^2 + b^2 = c^2$.

- Sines: $\sin A = a/c$, $\sin B = b/c$.

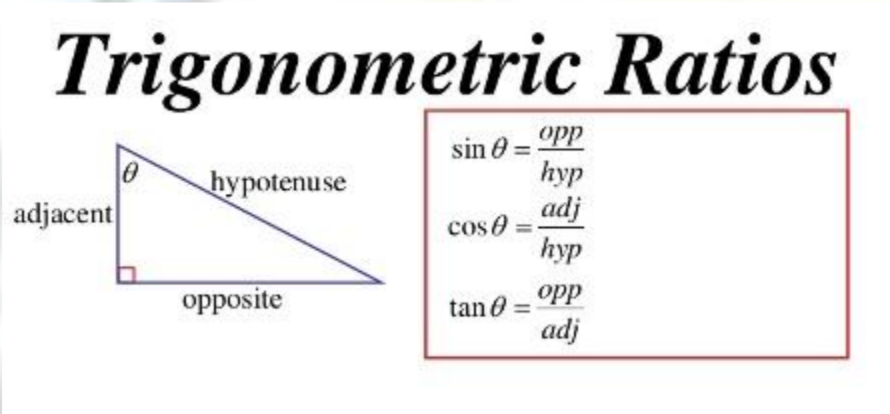
$$\sin A = \frac{\text{side opposite to angle A}}{\text{hypotenuse}}$$

- Cosines: $\cos A = b/c$, $\cos B = a/c$.

cosA = $\frac{\text{side adjacent to angle A}}{hypotenuse}$

- Tangents: tan A = a/b, tan B = b/a.

tanA = $\frac{\text{side opposite to angle A}}{side\ adjacent\ to\ angle\ A}$



- Cosec A = $\frac{\text{hypotenuse}}{side\ 0pposite\ to\ angle\ A}$
- secA = $\frac{\text{hypotenuse}}{side\ adjacent\ to\ angle\ A}$
- cot A= $\frac{\text{side opposite to angle A}}{side\ adjacent\ to\ angle\ A}$

Opposite of Sin: Cosecant

Opposite of Cos: Secant

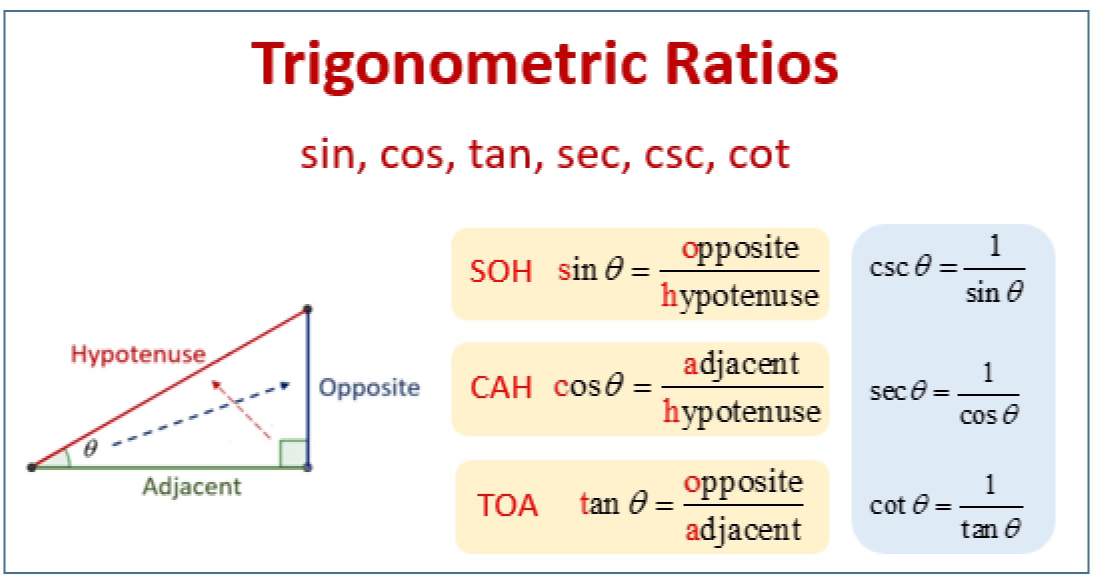
Opposite of Tan: Cotangent

Opposite of Cosecant: Sin

Opposite of Cotangent: Tan

Opposite of Secant: Cosecant

Sin ϴ (sin ϴ)	$\frac{\text{Perpendicular}}{\text{Hypotenuse}}$	$\frac{y}{r}$
Cosine ϴ (cos ϴ)	$\frac{\text{Base}}{\text{Hypotenuse}}$	$\frac{x}{r}$
Tangent ϴ (tan ϴ)	$\frac{\text{Perpendicular}}{\text{Base}}$	$\frac{y}{x}$
Cosecant ϴ (cossec ϴ)	$\frac{\text{Hypotenuse}}{\text{Perpendicular}}$	$\frac{r}{y}$
Secant ϴ (sec ϴ)	$\frac{\text{Hypotenuse}}{\text{Base}}$	$\frac{r}{x}$
Cotangent ϴ (cot ϴ)	$\frac{\text{Base}}{\text{Perpendicular}}$	$\frac{x}{y}$



- if one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of the angle can be easily determined.

(a) Find the sides of the right triangle in terms of k.

(b) Use Pythagoras Theorem and find the third side of the right triangle.

(c) Use definitions of t-ratios and substitute the values of sides.

(d) k is cancelled from numerator and denominator and the value of t-ratio is obtained.

- **Trigonometric Ratios of some specified angles**:

The values of trigonometric ratios for angles 0°, 30°, 45°, 60° and 90°

- The value of sin A or cos A never exceeds 1, whereas the value of sec A or cosec A is always greater than or equal to 1.

ratio \ angle θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
$\operatorname{cosec}\theta$	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
$\cot\theta$	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

- **Trigonometric Ratios of Complementary Angles**:

sin(90° – A) = cos A, cos(90° – A) = sinA;

tan (90° – A) = cot A, cot (90° – A) = tan A;

sec (90° – A) = cosec A,   cosec (90° – A) = sec A.

- **Trigonometric Identities**:

1. $\sin^2 A + \cos^2 A = 1$
2. $\sec^2 A - \tan^2 A = 1$ for $0° \le A < 90°$,
3. $\text{cosec}^2 A - \cot^2 A = 1$ for $0° < A \le 90°$

CHAPTER 8

INTRODUCTION TO TRIGONOMETRY

(Ex. 8.1)

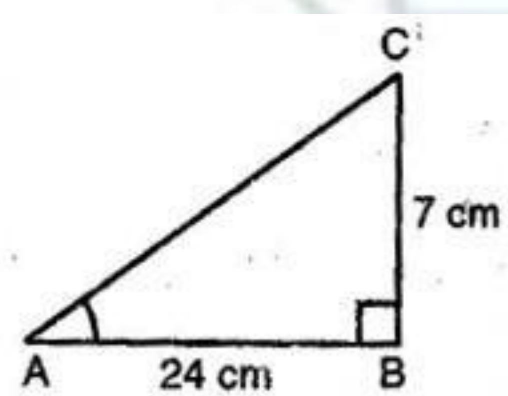
1. In ΔABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine:

(i) $\sin A \cos A$

(ii) $\sin C \cos C$

Ans. Let us draw a right angled triangle ABC, right angled at B. Using

Pythagoras theorem,



Let AC = 24k and BC = 7k

Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

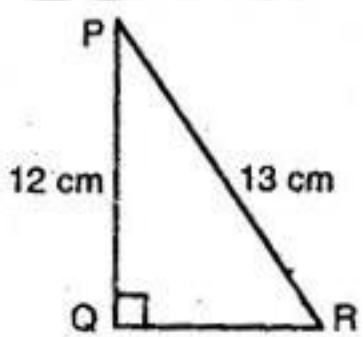
$= (24)^2 + (7)^2 = 576 + 49 = 625$

$\Rightarrow$ AC = 25 cm

(i) $\sin A = \frac{P}{H} = \frac{BC}{AC} = \frac{7}{25}$, $\cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{24}{25}$

(ii) $\sin C = \frac{P}{H} = \frac{AB}{AC} = \frac{24}{25}$, $\cos C = \frac{B}{H} = \frac{BC}{AC} = \frac{7}{25}$

2. In adjoining figure, find $\tan P - \cot R$**:**



Ans. In triangle PQR, Using Pythagoras theorem,

$PR^2 = PQ^2 + QR^2$

$\Rightarrow (13)^2 = (12)^2 + QR^2$

$\Rightarrow QR^2 = 169 - 144 = 25$

$\Rightarrow$ QR = 5 cm

$\therefore \tan P - \cot R = \frac{P}{B} - \frac{B}{P} = \frac{QR}{PQ} - \frac{QR}{PQ} = \frac{5}{13} - \frac{5}{13} = 0$

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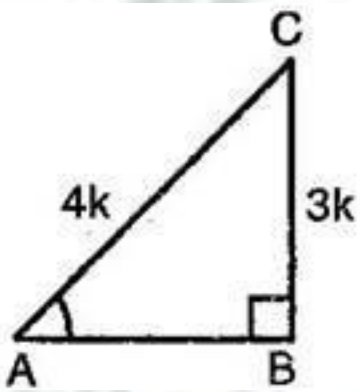
3. If $\sin A = \frac{3}{4}$, **calculate** $\cos A$ **and** $\tan A$.

Ans. Given: A triangle ABC in which $\angle B = 90^\circ$



Then, Using Pythagoras theorem,

AB= $\sqrt{(AC)^2 - (BC)^2} = \sqrt{(4k)^2 - (3k)^2}$

$= \sqrt{16k^2 - 9k^2} = k\sqrt{7}$

$\therefore \cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{k\sqrt{7}}{4k} = \frac{\sqrt{7}}{4}$

$\tan A = \frac{P}{B} = \frac{BC}{AB} = \frac{3k}{k\sqrt{7}} = \frac{3}{\sqrt{7}}$

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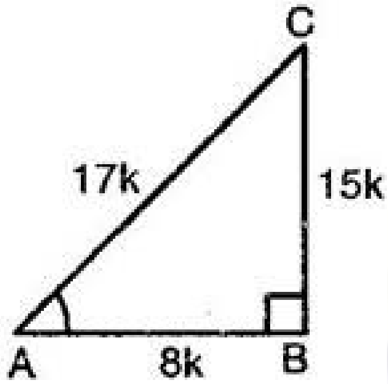
4. Given $15 \cot A = 8$, **find** $\sin A$ **and** $\sec A$

Ans. Given: A triangle ABC in which $\angle B = 90^\circ$

$15 \cot A = 8$

$\Rightarrow \cot A = \frac{8}{15}$



Let AB = $8k$ and BC = $15k$

Then using Pythagoras theorem,

AC = $\sqrt{(AB)^2 + (BC)^2}$

= $\sqrt{(8k)^2 + (15k)^2}$

= $\sqrt{64k^2 + 225k^2}$

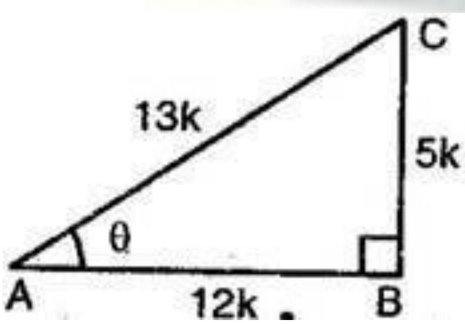
= $\sqrt{289k^2}$ = $17k$

$\therefore \sin A = \frac{P}{H} = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

$\sec A = \frac{H}{B} = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$

5. Given $\sec\theta = \frac{13}{12}$, **calculate all other trigonometric ratios.**

Ans. Consider a triangle ABC in which $\angle$A = θ and $\angle$B = 90°



Let AB = $12k$ and BC = $5k$

Then, using Pythagoras theorem,

BC = $\sqrt{(AC)^2-(AB)^2}$

= $\sqrt{(13k)^2-(12k)^2}$

= $\sqrt{169k^2-144k^2}$

= $\sqrt{25k^2}$ = $5k$

$$\therefore \sin\theta = \frac{P}{H} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos\theta = \frac{B}{H} = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

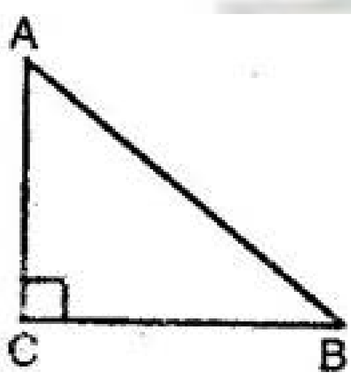
$$\tan\theta = \frac{P}{B} = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$$

$$\cot\theta = \frac{B}{P} = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$$

$$\cos ec\theta = \frac{H}{P} = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$$

6. If $\angle$ And $\angle$B are acute angles such that $\cos A = \cos B$, then show that $\angle$A = $\angle$B.

Ans. In right triangle ABC,



$\cos A = \frac{AC}{AB}$ $\cos B = \frac{BC}{AB}$

But $\cos A = \cos B$ [Given]

$\Rightarrow \frac{AC}{AB} = \frac{BC}{AB}$

$\Rightarrow$ AC=BC

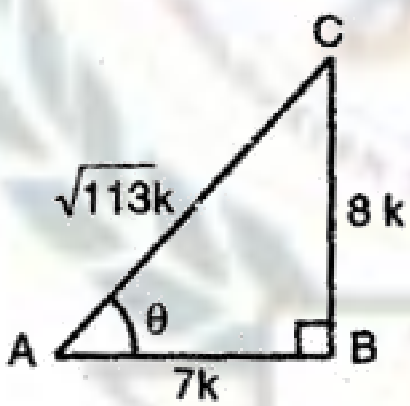
$\Rightarrow \angle A = \angle B$ [Angles opposite to equal sides are equal]

7. If $\cot\theta = \frac{7}{8}$, evaluate:

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\cot^2\theta$

Ans. Consider a triangle ABC in which $\angle A = \theta$ and $\angle B = 90^\circ$



Let AB = $7k$ and BC = $8k$

Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2 + (AB)^2}$

$= \sqrt{(8k)^2 + (7k)^2}$

$= \sqrt{64k^2 + 49k^2}$

$= \sqrt{113k^2}$ $= \sqrt{113}k$

$$\therefore \sin\theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

(i) $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \dfrac{1-\sin^2\theta}{1-\cos^2\theta}$

$$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}} = \frac{113-64}{113-49} = \frac{49}{64}$$

(ii) $\cot^2\theta = \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{49/113}{64/113} = \dfrac{49}{64}$

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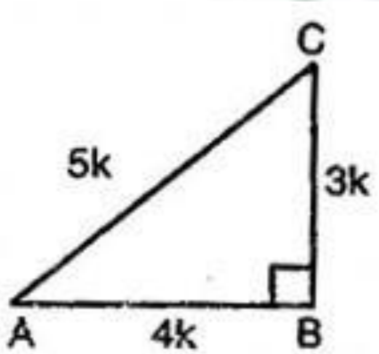
8. If $3\cot A = 4,$ **check whether** $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ **or not.**

Ans. Consider a triangle ABC in which $\angle$ B = 90°.



And $3\cot A = 4$

$\Rightarrow \cot A = \frac{4}{3}$

Let AB = $4k$ and BC = $3k$.

Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2 + (AB)^2}$

= $\sqrt{(3k)^2 + (4k)^2}$

= $\sqrt{16k^2 + 9k^2}$

= $\sqrt{25k^2}$ = $5k$

Sin A = BC / AC = 3k / 5k = 3/5

$$\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

And tan A = BC / AB =3k / 4k = 3/4

Now, L.H.S. $\frac{1-\tan^2 A}{1+\tan^2 A}$ = $\frac{1-\frac{9}{16}}{1+\frac{9}{16}}$

= $\frac{16-9}{16+9}$= $\frac{7}{25}$

R.H.S. $\cos^2 A - \sin^2 A$ = $\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

= $\frac{16}{25} - \frac{9}{25}$ = $\frac{7}{25}$

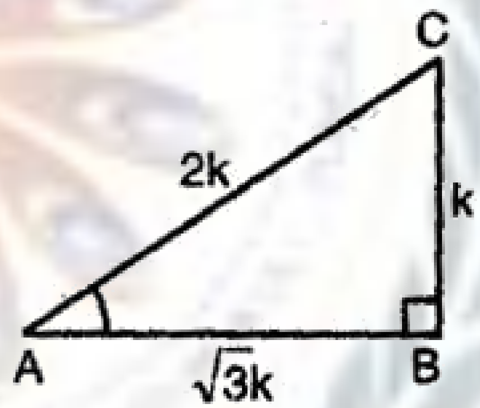
$\because$ L.H.S. = R.H.S.

$\therefore \frac{1-\tan^2 A}{1+\tan^2 A}$ = $\cos^2 A - \sin^2 A$

9. In ΔABC right angles at B, if $\tan A = \frac{1}{\sqrt{3}}$, **find value of:**

(i) $\sin A \cos C + \cos A \sin C$

(ii) cos A cos C – sin A sin C



Ans. Consider a triangle ABC in which $\angle$ B = 90°.
Let BC = k and AB = $\sqrt{3}k$

Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2 + (AB)^2}$

= $\sqrt{(k)^2 + \left(\sqrt{3}k\right)^2}$

= $\sqrt{k^2 + 3k^2}$ = $\sqrt{4k^2}$ = $2k$

$\therefore \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

For $\angle$C, Base = BC, Perpendicular = AB and Hypotenuse = AC

$$\therefore \sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

(i) $\sin A \cos C + \cos A \sin C = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

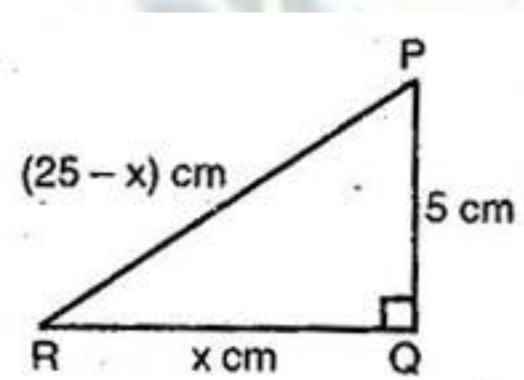
$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

(ii) $\cos A \cos C - \sin A \sin C =$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

10. In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of $\sin P, \cos P$ and $\tan P$.

Ans. In ΔPQR, right angled at Q.



PR + QR = 25 cm and PQ = 5 cm

Let QR = x cm, then PR = $(25-x)$ cm

Using Pythagoras theorem,

$$RP^2 = RQ^2 + QP^2$$

$$\Rightarrow (25-x)^2 = (x)^2 + (5)^2$$

$$\Rightarrow 625 - 50x + x^2 = x^2 + 25$$

$$\Rightarrow -50x = -600$$

$$\Rightarrow x = 12$$

$\therefore$ RQ = 12 cm and RP = 25 – 12 = 13 cm

$$\therefore \sin P = \frac{\text{RQ}}{\text{RP}} = \frac{12}{13}$$

$$\cos P = \frac{\text{PQ}}{\text{RP}} = \frac{5}{13}$$

$$\tan P = \frac{\text{RQ}}{\text{PQ}} = \frac{12}{5}$$

11. State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ **is always less than 1.**

(ii) $\sec A = \frac{12}{5}$ **for some value of angle A.**

(iii) $\cos A$ **is the abbreviation used for the cosecant of angle A.**

(iv) $\cot A$ **is the product of** $\cot$ **and A.**

(v) $\sin\theta = \frac{4}{3}$ **for some angle** θ.

Ans. (i) False because sides of a right triangle may have any length, so $\tan A$ may have any value.

(ii) True as $\sec A$ is always greater than 1.

(iii) False as $\cos A$ is the abbreviation of cosine A.

(iv) False as $\cot A$ is not the product of 'cot' and A. 'cot' is separated from A has no meaning.

(v) False as $\sin\theta$ cannot be > 1

(Ex. 8.2)

1. Evaluate:

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \cos ec 30^\circ}$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Ans. (i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$= \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2} + \dfrac{1}{2} \times \dfrac{1}{2}$

$= \dfrac{3}{4} + \dfrac{1}{4} = \dfrac{4}{4}$ =1

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$= 2(1)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2 - \left(\dfrac{\sqrt{3}}{2}\right)^2$

$= 2 + \dfrac{3}{4} - \dfrac{3}{4}$

= 2

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$= \dfrac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \dfrac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2+2\sqrt{3}} = \dfrac{\sqrt{3}}{\sqrt{2} \times 2\left(\sqrt{3}+1\right)}$

$= \dfrac{\sqrt{3}}{\sqrt{2}\times 2\left(\sqrt{3}+1\right)} \times \dfrac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \dfrac{\sqrt{3}\left(\sqrt{3}-1\right)}{\sqrt{2}\times 2\left(3-1\right)}$ [Since $(a+b)(a-b)=a^2-b^2$]

$= \dfrac{\sqrt{3}\left(\sqrt{3}-1\right)}{4\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{3\sqrt{2}-\sqrt{6}}{8}$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$= \dfrac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \dfrac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}} = \dfrac{3\sqrt{3}-4}{3\sqrt{3}+4}$

$= \dfrac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \dfrac{3\sqrt{3}-4}{3\sqrt{3}-4}$

$= \dfrac{27+16-24\sqrt{3}}{27-16}$ [Since $(a+b)(a-b)=a^2-b^2$]

$= \dfrac{43-24\sqrt{3}}{11}$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5 \times \frac{1}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{4}{4}}$$

$$= \frac{15+64-12}{12} = \frac{67}{12}$$

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2. Choose the correct option and justify:

(i) $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} =$

(A) $\sin 60^\circ$

(B) $\cos 60^\circ$

(C) $\tan 60^\circ$

(D) $\sin 30^\circ$

(ii) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} =$

(A) $\tan 90^\circ$

(B) 1

(C) $\sin 45^\circ$

(D) 0

(iii) $\sin 2A = 2\sin A$ **is true when A =**

(A) 0°

(B) 30°

(C) 45°

(D) 60°

(iv) $\dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ}$ =

(A) $\cos 60^\circ$

(B) $\sin 60^\circ$

(C) $\tan 60^\circ$

(D). None of these

Ans. (i) (A) $\dfrac{2\tan 30^\circ}{1+\tan^2 30^\circ}$

$= \dfrac{2 \times {}^{1}\!/\!{}_{\sqrt{3}}}{1+\left({}^{1}\!/\!{}_{\sqrt{3}}\right)^2}$

$= \dfrac{2}{\sqrt{3}} \times \dfrac{3}{3+1} = \dfrac{\sqrt{3}}{2} = \sin 60^\circ$

(ii) (D) $\dfrac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} = \dfrac{1-1}{1+1} = \dfrac{0}{2} = 0$

(iii). (A) Since A = 0, then

$\sin 2A = \sin 0^\circ = 0$ and

$2\sin A = 2\sin 0^\circ$ = $2 \times 0 = 0$

$\therefore \sin 2A = \sin A$ when A = 0

- **(iv).** $\dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ}$

= $\dfrac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$ = $\dfrac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$

= $\dfrac{2}{\sqrt{3}}\times\dfrac{3}{3-1}$ = $\sqrt{3} = \tan 60^\circ$

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3. If $\tan(A+B) = \sqrt{3}$ **and** $\tan(A-B) = \frac{1}{\sqrt{3}};\ \ 0^\circ < A+B \le 90^\circ;\ \ A > B,$ **find A and B.**

Ans. $\tan(A+B) = \sqrt{3}$

$\Rightarrow \tan(A+B) = \tan 60^\circ$

$\Rightarrow$ A + B = 60° ……….(i)

Also, $\tan(A-B) = \frac{1}{\sqrt{3}}$

$\Rightarrow$ A – B = 30° ……….(ii)

On adding eq. (i) and (ii), we get,

2A= $90^\circ \Rightarrow$ A= 45°

On Subtracting eq. (i) and eq. (ii), we get

2B= $30^\circ \Rightarrow$ B= 15°

7. State whether the following are true or false. Justify your answer.

(i) $\sin(A+B) = \sin A + \sin B$

(ii)The value of $\sin\theta$ **increases as** θ **increases.**

(iii)The value of $\cos\theta$ **increases as** θ **increases.**

(iv) $\sin\theta = \cos\theta$ **for all values of** $\theta.$

(v) $\cot A$ **is not defined for** $A = 0^\circ.$

Ans. (i) False, because, let A = 60° and B = 30°

Then, $\sin(A+B) = \sin(60^\circ + 30^\circ) = \sin 90^\circ$ = 1

And $\sin A + \sin B = \sin 60^\circ + \sin 30^\circ = \frac{\sqrt{3}}{2} + \frac{1}{2}$ = $\frac{\sqrt{3}+1}{2}$

$\therefore \sin(A+B) \neq \sin A + \sin B$

Ans
\
(ii)True, because it is clear from the table below

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

Therefore, it is clear, the value of $\sin\theta$ increases as θ increases.

(vi) False, because

θ	0°	30°	45°	60°	90°
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

It is clear, the value of $\cos\theta$ decreases as θ increases

(vi) False as it is only true for $\theta = 45^\circ$.

$$\Rightarrow \sin 45^\circ = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

(vii) True, because $\tan 0^\circ = 0$ and $\cot 0^\circ = \frac{1}{\tan 0^\circ}$

i.e $\frac{1}{0}$ undefined.

Ex – 8.3

1. **Evaluate:**

(i) $\frac{\sin 18^\circ}{\cos 72^\circ}$

(ii) $\frac{\tan 26^\circ}{\cot 64^\circ}$

(iii) $\cos 48^\circ - \sin 42^\circ$

(iv) $\operatorname{cosec} 31^\circ - \sec 59^\circ$

Ans. (i) $\dfrac{\sin 18^\circ}{\cos 72^\circ} = \dfrac{\sin(90^\circ - 72^\circ)}{\cos 72^\circ}$

$= \dfrac{\cos 72^\circ}{\cos 72^\circ}$ [Since $\sin(90^\circ - \theta) = \cos\theta$]

= 1

(iii) $\dfrac{\tan 26^\circ}{\cot 64^\circ} = \dfrac{\tan(90^\circ - 64^\circ)}{\cot 64^\circ}$

$= \dfrac{\cot 64^\circ}{\cot 64^\circ}$ [Since $\tan(90^\circ - \theta) = \cot\theta$]

= 1

(iii) $\cos 48^\circ - \sin 42^\circ$

$= \cos(90^\circ - 42^\circ) - \sin 42^\circ$

$= \sin 42^\circ - \sin 42^\circ$ [Since $\cos(90^\circ - \theta) = \sin\theta$]

= 0

(iv) $\operatorname{cosec} 31^\circ - \sec 59^\circ$

$= \operatorname{cosec}(90^\circ - 59^\circ) - \sec 59^\circ$

$= \sec 59^\circ - \sec 59^\circ$ [Since $\operatorname{cosec}(90^\circ - \theta) = \sec\theta$]

=0

2. **Show that:**

(i) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ = 1$

(ii) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ = 0$

Ans. (i) L.H.S. $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

= $\tan(90^\circ - 42^\circ)\tan(90^\circ - 67^\circ)\tan 42^\circ \tan 67^\circ$

= $\cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ$

= $\frac{1}{\tan 42^\circ} \cdot \frac{1}{\tan 67^\circ} . \tan 42^\circ . \tan 67^\circ$ = 1 = R.H.S.

.(ii) $\cos 38^\circ \cos 52^\circ - \sin 38^\circ \sin 52^\circ$

= $\cos(90^\circ - 52^\circ) . \cos(90^\circ - 38^\circ) - \sin 38^\circ . \sin 52^\circ$

= $\sin 52^\circ \sin 38^\circ - \sin 38^\circ \sin 52^\circ$ = 0 = R.H.S.

---

3. **If** $\tan 2A = \cot(A - 18^\circ)$, **where 2A is an acute angle, find the value of A. Ans.**

Given: $\tan 2A = \cot(A - 18^\circ)$

$\Rightarrow \cot(90^\circ - 2A) = \cot(A - 18^\circ)$ [Since $\tan(90^\circ - \theta) = \cot\theta$]

$90^0 - 2A = A - 18^0$

$90^0 + 18^0 = 2A + A$

$3A = 108^0$

$\Rightarrow$ A = 36°

---

4. **If** $\tan A = \cot B$,

prove that $A + B = 90^\circ$.

Ans. Given: $\tan A = \cot B$

$\Rightarrow \cot(90^\circ - A) = \cot B$

$\Rightarrow 90^\circ - A = B$

$\Rightarrow 90^\circ = A + B$

$\Rightarrow$ A+B= 90°

5. If $\sec 4A = \operatorname{cosec}(A - 20^\circ)$, where 4A is an acute angle, find the value of A

Ans. Given: $\sec 4A = \operatorname{cosec}(A - 20^\circ)$

$\Rightarrow \operatorname{cosec}(90^\circ - 4A) = \operatorname{cosec}(A - 20^\circ)$ [Since $\sec(90^\circ - \theta) = \operatorname{cosec}\theta$]

$\Rightarrow 90^\circ - 4A = A - 20^\circ$

$\Rightarrow -4A - A = -20^\circ - 90^\circ$

$\Rightarrow -5A = -110^\circ$

$\Rightarrow$ A = 22°

6. If A, B and C are interior angles of a ΔABC, then show that $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$.

Ans. Given: A, B and C are interior angles of a ΔABC.

$\therefore$ A + B + C = 180° [Triangle sum property]

Dividing both sides by 2, we get

$\Rightarrow \frac{A+B+C}{2} = 90^\circ$

$\Rightarrow \frac{A}{2} + \frac{B+C}{2} = 90^\circ$

$\Rightarrow \dfrac{B+C}{2} = 90^\circ - \dfrac{A}{2}$

$\Rightarrow$ [Since $\sin(90^\circ - \theta) = \cos\theta$]

7. Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45°.

Ans. $\sin 67^\circ + \cos 75^\circ$

$= \sin(90^\circ - 23^\circ) + \cos(90^\circ - 15^\circ)$ [Since $\sin(90^\circ - \theta) = \cos\theta$ and $\cos(90^\circ - \theta) = \sin\theta$]

$= \cos 23^\circ + \sin 15^\circ$

Ex. 8.4

1. **Express the trigonometric ratios $\sin A, \sec A$ and $\tan A$ in terms of $\cot A$**

   Ans. For $\sin A,$

By using identity $\operatorname{cosec}^2 A - \cot^2 A = 1$

$\Rightarrow \operatorname{cosec}^2 A = 1 + \cot^2 A$

$\Rightarrow \dfrac{1}{\sin^2 A} = 1 + \cot^2 A$

$$\Rightarrow \sin^2 A = \frac{1}{1+\cot^2 A}$$

For $\sec A,$

By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \sec^2 A = 1 + \tan^2 A$$

$$\sec^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

For $\tan A,$

$$\tan A = \frac{1}{\cot A}$$

(iv) Write the other trigonometric ratios of A in terms of $\sec A$ Ans.

For $\sin A,$

By using identity, $\sin^2 A + \cos^2 A = 1$

$$\Rightarrow \sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A} = \frac{\sec^2 A - 1}{\sec^2 A}$$

$$\Rightarrow \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

For $\cos A,$

$$\cos A = \frac{1}{\sec A}$$

For $\tan A,$

By using identity $\sec^2 A - \tan^2 A = 1$

$\Rightarrow \tan^2 A = \sec^2 A - 1$

$\Rightarrow \tan A = \sqrt{\sec^2 A - 1}$

For $\cos ecA,$

$$\cos ecA = \frac{1}{\sin A} \quad \frac{\frac{1}{\sqrt{\sec^2 A - 1}}}{\sec A}$$

For $\cot A,$

$$\cot A = \frac{1}{\tan A}$$

$$\Rightarrow \cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$

---

5. Evaluate:

(i) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

Ans. (i) $\dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

$= \dfrac{\sin^2 63^\circ + \sin^2(90^\circ - 63^\circ)}{\cos^2(90^\circ - 73^\circ) + \cos^2 73^\circ}$

$= \dfrac{\sin^2 63^\circ + \cos^2 63^\circ}{\sin^2 73^\circ + \cos^2 73^\circ}$

$\left[\because \sin(90^\circ - \theta) = \cos\theta, \cos(90^\circ - \theta) = \sin\theta\right]$

$= \dfrac{1}{1} = 1 \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

$= \sin 25^\circ.\cos(90^\circ - 25^\circ) + \cos 25^\circ.\sin(90^\circ - 25^\circ)$

$= \sin 25^\circ.\sin 25^\circ + \cos 25^\circ.\cos 25^\circ$

$= \sin^2 25^\circ + \cos^2 25^\circ = 1$

$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

---

4. **Choose the correct option. Justify your choice:**

(i) $9\sec^2 A - 9\tan^2 A$ =

(A) 1

(B) 9

(C) 8

(D) 0

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\cos ec\theta)$ =

(A) 0

(B) 1

(C) 2

(D) none of these

(iii) $(\sec A+\tan A)(1-\sin A)$ =

(A) $\sec A$

(B) $\sin A$

(C) $\cos ecA$ **(D).** $\cos A$

(iv) $\frac{1+\tan^2 A}{1+\cot^2 A}$ =

(A) $\sec^2 A$

(B) -1

(C) $\cot^2 A$

(D) none of these

Ans. (i) (B) $9\sec^2 A-9\tan^2 A$

= $9(\sec^2 A-\tan^2 A)$

= $9\times1=9$ [Since $\sec^2\theta - \tan^2\theta=1$]

(ii) (C) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\cos ec\theta)$

$= \left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$

$= \left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$

$= \frac{(\cos\theta+\sin\theta)^2-(1)^2}{\cos\theta.\sin\theta}$

$= \frac{\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$

$= \frac{1+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$

$\left[\because \sin^2\theta+\cos^2\theta=1\right]$

$=\frac{2\cos\theta\sin\theta}{\cos\theta.\sin\theta} = 2$

(iii)(D) $(\sec A+\tan A)(1-\sin A)$

$= \left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$

$= \left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$

$= \frac{1-\sin^2 A}{\cos A}$ [Since $(a+b)(a-b)=a^2-b^2$]

$= \frac{\cos^2 A}{\cos A}$

$\left[\because 1-\sin^2 A=\cos^2 A\right]$

$= \cos A$

(iv)(D) $\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A - \tan^2 A + \tan^2 A}{\operatorname{cosec}^2 A - \cot^2 A + \cot^2 A}$

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$$

$= \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$

$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$

5. Prove the following identities, where the angles involved are acute angles for which the expressions are defined:

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

(iii) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

(iv) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

(viii) $\left(\sin A + \cos ecA\right)^2 + \left(\cos A + \sec A\right)^2 = 7 + \tan^2 A + \cot^2 A$

(ix) $\left(\cos ecA - \sin A\right)\left(\sec A - \cos A\right) = \frac{1}{\tan A + \cot A}$

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

Ans. (i) L.H.S. $\left(\cos ec\theta - \cot\theta\right)^2$

= $\cos ec^2\theta + \cot^2\theta - 2\cos ec\theta\cot\theta$ [Since $(a - b)^2 = a^2 + b^2 - 2ab$]

= $\frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} - 2\times\frac{1}{\sin\theta}\cdot\frac{\cos\theta}{\sin\theta}$

= $\frac{1+\cos^2\theta - 2\cos\theta}{\sin^2\theta}$

= $\frac{\left(1-\cos\theta\right)^2}{\sin^2\theta}$ $\left[\because a^2 + b^2 - 2ab = \left(a-b\right)^2\right]$

= $\frac{\left(1-\cos\theta\right)\left(1-\cos\theta\right)}{1-\cos^2\theta}$

= $\frac{\left(1-\cos\theta\right)\left(1-\cos\theta\right)}{\left(1+\cos\theta\right)\left(1-\cos\theta\right)}$

$= \dfrac{1-\cos\theta}{1+\cos\theta}$

= R.H.S.

L.H.S. = $\dfrac{\cos A}{1+\sin A}+\dfrac{1+\sin A}{\cos A}$

$= \dfrac{\cos^2\theta+1+\sin^2\theta+2\sin A}{(1+\sin A)\cos A}$

$= \dfrac{\cos^2\theta+\sin^2\theta+1+2\sin A}{(1+\sin A)\cos A}$

$= \dfrac{1+1+2\sin A}{(1+\sin A)\cos A}\left[\because \sin^2\theta+\cos^2\theta=1\right]$

$= \dfrac{2+2\sin A}{(1+\sin A)\cos A} = \dfrac{2(1+\sin A)}{(1+\sin A)\cos A}$

$= \dfrac{2}{\cos A}$

= 2 sec A = RHS

(iii) L.H.S.

$= \dfrac{\dfrac{\sin\theta}{\cos\theta}}{1-\dfrac{\cos\theta}{\sin\theta}}+\dfrac{\dfrac{\cos\theta}{\sin\theta}}{1-\dfrac{\sin\theta}{\cos\theta}}$

$= \dfrac{\frac{\sin\theta}{\cos}}{\frac{\sin\theta-\cos\theta}{\sin\theta}}+\dfrac{\frac{\cos\theta}{\sin}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$= \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta - \cos\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta - \sin\theta}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta - \sin\theta)}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)}$

$= \frac{\sin^3\theta - \cos^3\theta}{\sin\theta\cos\theta(\sin\theta - \cos\theta)}$

$= \frac{(\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta - \cos\theta)}$

$\left[\because a^3 - b^3 = (a-b)(a^2 + b^2 + ab)\right]$

$= \frac{1 + \sin\theta\cos\theta}{\sin\theta\cos\theta} \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$= \frac{1}{\sin\theta\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta} + 1 = 1 + \frac{1}{\sin\theta\cos\theta}$

$= 1 + \sec\theta \operatorname{cosec}\theta$

(iv)

$\frac{1 + \sec A}{\sec A} = \frac{1 + \frac{1}{\cos A}}{\frac{1}{\cos A}}$ $\frac{\cos A}{}$

$$= \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}} = \frac{\cos A+1}{\cos A} \times \frac{\cos A}{1}$$

$$= 1+\cos A \times \frac{1-\cos A}{1-\cos A}$$

$$= \frac{1-\cos^2 A}{1-\cos A}$$ [Since $(a+b)(a-b)=a^2-b^2$]

$$= \frac{\sin^2 A}{1-\cos A}$$ = R.H.S.

(v) L.H.S. $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$

Dividing all terms by $\sin A$,

$$= \frac{\cot A-1+cosec\ A}{\cot A+1-cosec\ A} = \frac{\cot A+cosec\ A-1}{\cot A-cosec\ A+1}$$

$$= \frac{\cot A+cosec\ A-(cosec^2 A-cot^2 A)}{\cot A+1-cosec\ A}$$

$$= \frac{(\cot A+cosec\ A)-(cosec\ A+\cot A)(\mathrm{cosec}\ A-cot\ A)}{\cot A+1-cosec\ A}$$

$$= \frac{(\cot A+cosec\ A)[1-(cosec\ A-\cot A)]}{\cot A+1-cosec\ A}$$

$$= \frac{(\cot A+cosec\ A)(1-cosec\ A+\cot A\}}{(\cot A+1-cosec\ A)}$$

= cot A + cosec A

= R.H.S

(vi) L.H.S.

= $\sqrt{\frac{1+\sin A}{1-\sin A}} \times \sqrt{\frac{1+\sin A}{1+\sin A}}$

= $\sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$ $\left[\because (a+b)(a-b)=a^2-b^2\right]$

= $\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$ $\left[\because 1-\sin^2\theta=\cos^2\theta\right]$

= $\frac{1+\sin A}{\cos A}$ = $\frac{1}{\cos A}+\frac{\sin A}{\cos A}$

= $\sec A+\tan A$ = R.H.S.

(vii) L.H.S. $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}$

= $\frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta-1\right)}$

= $\frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left[2\left(1-\sin^2\theta\right)-1\right]}$

$\left[\because 1-\sin^2\theta=\cos^2\theta\right]$

$\frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2-2\sin^2\theta-1\right)}$

=

$$= \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(1-2\sin^2\theta\right)} = \frac{\sin\theta}{\cos\theta}$$

= $\tan\theta$ = R.H.S

(viii) L.H.S. $\left(\sin A+\operatorname{cosec}A\right)^2+\left(\cos A+\sec A\right)^2$

$$= \left(\sin A+\frac{1}{\sin A}\right)^2+\left(\cos A+\frac{1}{\cos A}\right)^2$$

$$= \sin^2 A+\frac{1}{\sin^2 A}+2.\sin A.\frac{1}{\sin A}+\cos^2 A+\frac{1}{\cos^2 A}+2.\cos A.\frac{1}{\cos A}$$

$$= 2+2+\sin^2 A+\cos^2 A+\frac{1}{\sin^2 A}+\frac{1}{\cos^2 A}$$

$$= 4+1+\frac{1}{\sin^2 A}+\frac{1}{\cos^2 A}$$

$$= 5+\operatorname{cosec}^2 A+\sec^2 A$$

$$= 5+1+\cot^2 A+1+\tan^2 A$$

$$\left[\because \operatorname{cosec}^2\theta=1+\cot^2\theta, \sec^2\theta=1+\tan^2\theta\right]$$

$$= 7+\tan^2 A+\cot^2 A$$

= R.H.S.

(ix) L.H.S. $\left(\operatorname{cosec}A-\sin A\right)\left(\sec A-\cos A\right)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$

$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A} = \sin A.\cos A$

=

Dividing all the terms by $\sin A.\cos A$,

$= \frac{\frac{\sin A.\cos A}{\sin A.\cos A}}{\frac{\sin^2 A}{\sin A.\cos A} + \frac{\cos^2 A}{\sin A.\cos A}}$

$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$

$= \frac{1}{\tan A + \cot A}$ = R.H.S.

(x) L.H.S. $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \frac{\sec^2 A}{\cos ec^2 A}$

$\left[\because 1+\tan^2\theta = \sec^2\theta, 1+\cot^2\theta = \cos ec^2\theta\right]$

$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1} = \tan^2 A$ = R.H.S.

Now, Middle side = $\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$

$= \left(\frac{1-\tan A}{\frac{\tan A - 1}{\tan A}}\right)^2$

$$= \left(\frac{1 - tan\, A}{\frac{-(1 - tanA)}{tanA}} \right)^2$$

$= (- \tan A)^2$

$= \tan^2 A$ -= R.H.S

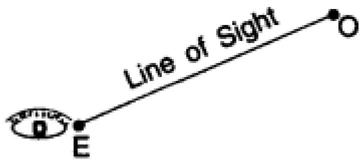
Notes

Chapter 9

Some Applications of Trigonometry

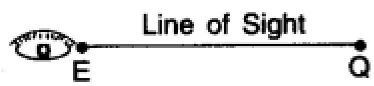
Line of Sight

When an observer looks from a point E (eye) at an object O then the straight line EO between the eye E and the object O is called the line of sight.



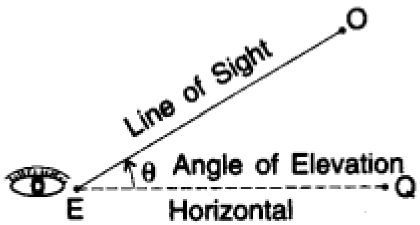
Horizontal

When an observer looks from a point E (eye) to another point Q which is horizontal to E, then the straight line, EQ between E and Q is called the horizontal line.



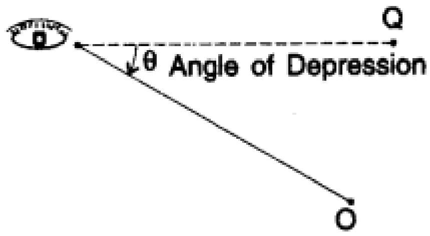
Angle of Elevation

When the eye is below the object, then the observer has to look up from the point E to the object O. The measure of this rotation (angle θ) from the horizontal line is called the angle of elevation.



Angle of Depression

When the eye is above the object, then the observer has to look down from the point E to the object. The horizontal line is now parallel to the ground. The measure of this rotation (angle θ) from the horizontal line is called the angle of depression.



How to convert the above figure into the right triangle.

Case I: Angle of Elevation is known

Draw OX perpendicular to EQ.

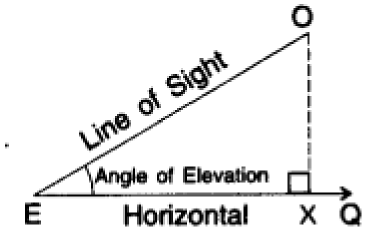
Now ∠OXE = 90°

ΔOXE is a rt. Δ, where

OE = hypotenuse

OX = opposite side (Perpendicular)

EX = adjacent side (Base)



Case II: Angle of Depression is known

(i) Draw OQ’parallel to EQ

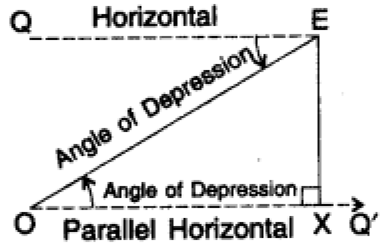
(ii) Draw perpendicular EX on OQ’.

(iii) Now ∠QEO = ∠EOX = Interior alternate angles

ΔEXO is an rt. Δ. where

EO = hypotenuse

OX = adjacent side (base) and EX = opposite side (Perpendicular)

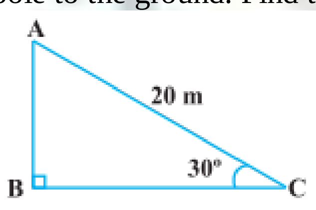


- Choose a trigonometric ratio in such a way that it considers the known side and the side that you wish to calculate.
- The eye is always considered at ground level unless the problem specifically gives the height of the observer.

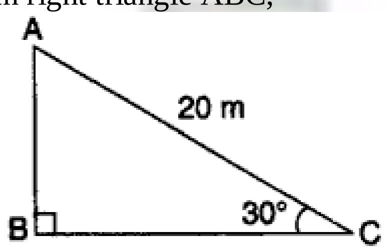
The object is always considered as a point.
Some **P**eople **H**ave
Sin θ = Perpendicular / Hypotenuse
Curly **B**lack **H**air
Cos θ = Base / Hypotenuse
Turning **P**ermanent **B**lack.
tan θ = Perpendicular / Base

Exercise 9.1

1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°.



Sol. In right triangle ABC,

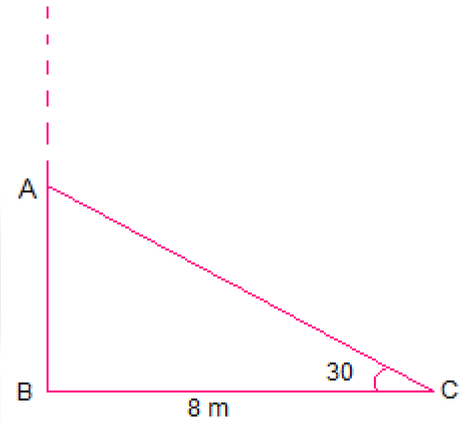


$\sin 30^0 = AB/AC \quad \Rightarrow 1/2 = AB/20$
$\Rightarrow AB = 10m$

Hence, the height of the pole is 10 m.

2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Sol.



Let AC be the broken part of the tree.

∴Total height of the tree = AB + AC

In right △△ABC,

cos 30° = BC / AC

⇒ √3 / 2 = 8/AC

⇒ AC = 16 /√3

Also,

tan 30° = AB / BC

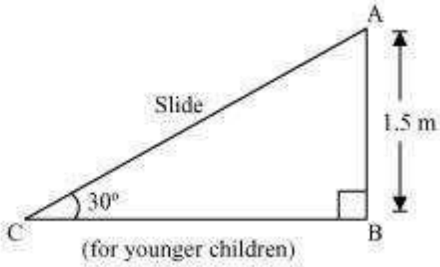
⇒⇒ 1/ √3 = AB / 8

⇒⇒ AB = 8 / √3

Total height of the tree = AB + AC = 16/√3 + 8/√3 = 24/√3 = $\frac{24 X\sqrt{3}}{\sqrt{3}X\sqrt{3}}$ = 8√3 m

3. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slides in each case?

Sol. In the first case:



Height of slide= 1.5 m and angle of elevation = 30°

Now,

$\sin\theta = p / h$

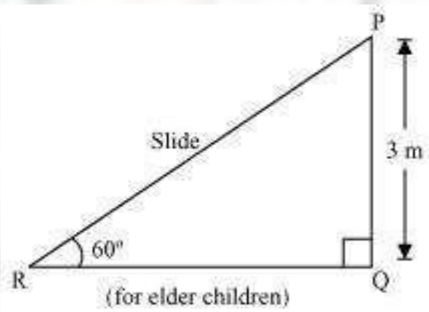
where p = perpendicular, i.e. height of the slide and h = hypotenuse, i.e. length of the slide and θ is the angle of elevation

$\sin 30^0 = 1.5 / h$

$1/ 2 = 1.5 / h$

Hence, h = 3 m

In the second case:



Height of slide, = 3 m, angle of elevation = 60°

$\sin\theta = p / h$

$\sin 60^0 = 3 / h$

$\sqrt{3} / 2 = 3/ h$

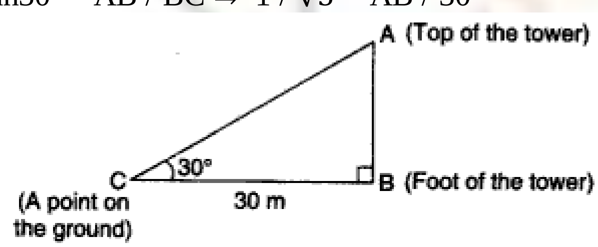
Hence, $h = 2\sqrt{3}$ m

Therefore, the length of the slide in the first and the second case are 3 m and $2\sqrt{3}$ m respectively.

4. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

Sol. In right triangle ABC,

$\tan 30^0 = AB / BC \Rightarrow 1 / \sqrt{3} = AB / 30$

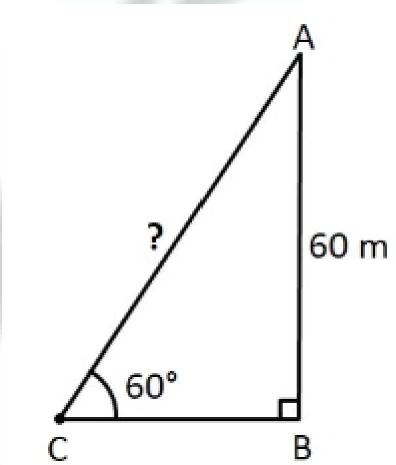


$AB = 30 / \sqrt{3} \Rightarrow AB = 10\sqrt{3}$ m

Hence, the height of the tower is 10√3 m.

5. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60^{0}. Find the length of the string, assuming that there is no slack in the string.

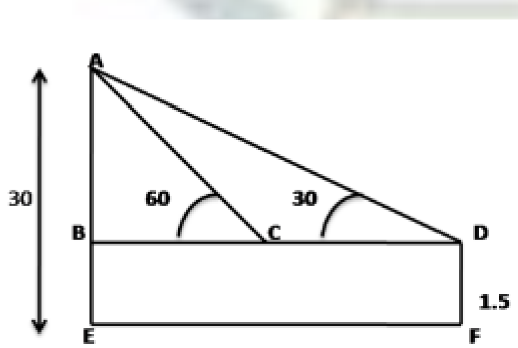
Sol.



In right triangle ABC,
$\sin 60^0 = AB / AC$
$\Rightarrow \sqrt{3} / 2 = 60 / AC$
$AC = 120 / \sqrt{3}$
Multiplying √3 in both numerator and denominator,
$AC = 120 X \sqrt{3} / \sqrt{3} X \sqrt{3}$
$AC = 40\sqrt{3}$m
Hence the length of the string is 40√3m.

6. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Sol.



Let AE is the Length of the building.
So AE = 30

Again BE = DF = 1.5

AB = AE - BE = 30 - 1.5 = 28.5

Now in triangle ABC,

$\tan 60^0 = AB / BC$

$\Rightarrow \sqrt{3} = 28.5 / BC$

$\Rightarrow BC = 28.5/ \sqrt{3}$

Again in triangle ABD

$\tan 30 = AB / BD$

$1/ \sqrt{3} = 28.5 / BD$

$\Rightarrow BD = 28.5 \text{ X} \sqrt{3}$

$\Rightarrow BC + CD = 28.5\sqrt{3}$

$\Rightarrow 28.5/\sqrt{3} + CD = 28.5\sqrt{3}$

$\Rightarrow CD = 28.5 \sqrt{3} - 28.5 / \sqrt{3}$

(taking L.C.M)

$\Rightarrow CD = 28.5\times3 - 28.5 / \sqrt{3}$

$\Rightarrow CD = 28.5(3-1) / \sqrt{3}$

$\Rightarrow CD = (28.5\times2) / \sqrt{3}$

$\Rightarrow CD = (57) / \sqrt{3}$

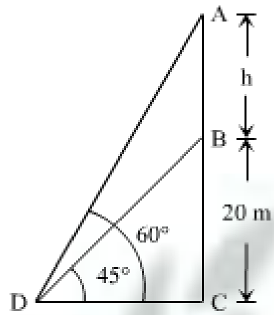
$\Rightarrow CD = (57) \text{ X } \sqrt{3} / \sqrt{3}\times \sqrt{3}$ (Multiply $\sqrt{3}$ in numerator and denominator)

$\Rightarrow CD = 57\sqrt{3}/ 3$

$\Rightarrow CD = 19\sqrt{3}$

The distance he walked towards the building is $19\sqrt{3}$ m

7. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower

Sol. 

Let BC be the building, AB be the transmission tower, and D be the point on the ground.

In ΔBCD,

tan45º = BC / CD

⇒1= 20 /CD

⇒ CD = 20

In ΔACD,

tan60º = AC / CD

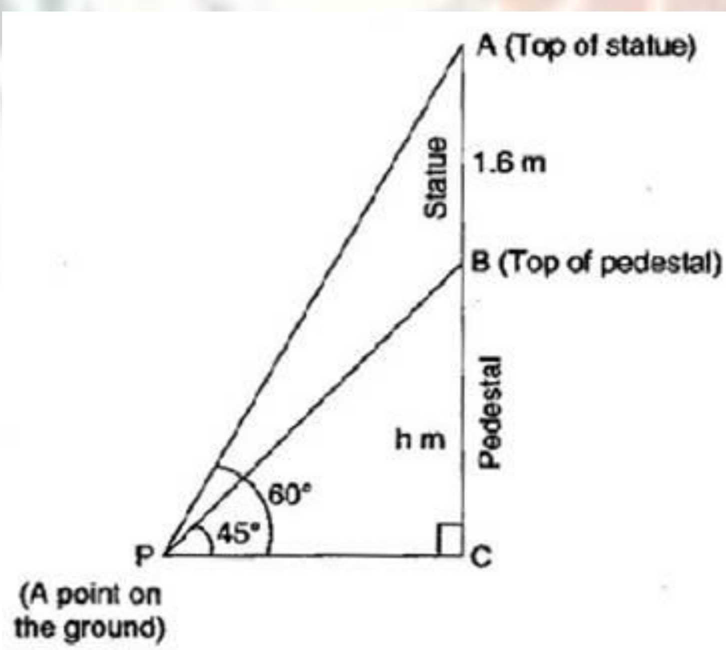
⇒√3 = AB+BC / CD

⇒√3 = AB+20 / 20

⇒AB + 20 = 20√3

⇒AB = 20(√3 − 1) m.

8. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60º and from the same point the angle of elevation of the top of the pedestal is 45º. Find the height of the pedestal.

Sol. 

Let the height of the pedestal be h m.

∴ BC = h m

In right triangle ACP,

$\tan 60^0 = AC / PC$

$\Rightarrow \sqrt{3} = AB+BC / PC$

$\Rightarrow \sqrt{3} = 1.6+h / PC$ ....(i)

In right triangle BCP,

$\tan 45^0 = BC / PC$

$\Rightarrow 1 = h / PC \Rightarrow PC = h$

$\therefore \sqrt{3} = 1.6 + h$ [From eq. (i)]

$\Rightarrow \sqrt{3} = 1.6 + h / h \Rightarrow \sqrt{3}\, h = 1.6 + \sqrt{3}$

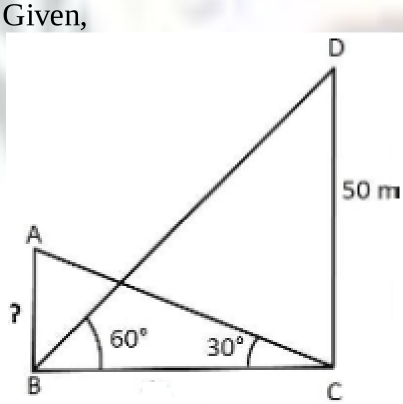
$\Rightarrow 1.6 = (\sqrt{3} - 1)\, h \Rightarrow h = 1.6 (\sqrt{3}+1) / (\sqrt{3}-1)(\sqrt{3}+1)$

$\Rightarrow h = 1.6 (\sqrt{3}+1) / 3-1 \Rightarrow h = 1.6 (\sqrt{3} + 1) / 2$

$\Rightarrow h = 0.8(\sqrt{3} + 1)$ m

9. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Sol. Given,



Let the height of building be AB and height of tower CD

Height of the tower (CD) = 50 m

Angle of elevation of top of building from foot of tower = 30^o

Hence, $\angle ACB = 30^0$

Angle of elevation of top of tower from foot of building = 60^o

Hence, $\angle DBC = 60^o$

$\angle ABC = 90^o$ & $\angle DCB = 90^o$

In a right angle triangle DBC,

$\tan B = DC / BC$

$\tan 60^0 = 50 / BC$

$BC = 50 / \sqrt{3}$

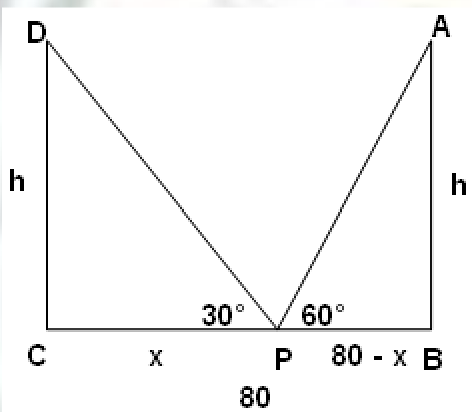
Similarly,

In a right angle triangle ABC,
tan c = AB / BC
$\tan 30^0$ = AB / BC
1/ √3 = AB / 50/ √3 = AB √3 / 50

1/√3 × 50 / √3 = AB
AB=1/√3 ×50 /√3
AB=50 / √3 m

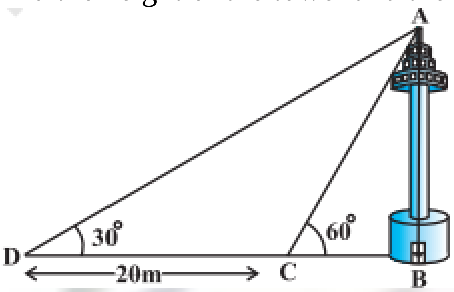
10. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60^o and 30^o respectively. Find the height of the poles and the distances of the point from the poles.

Sol.

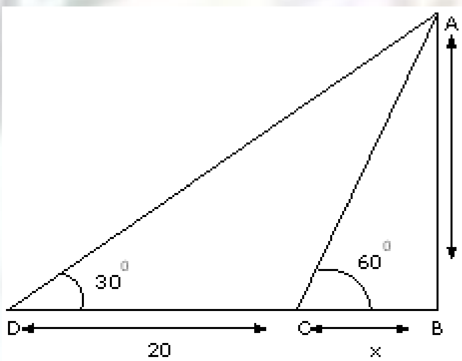


Suppose AB and CD are the two poles of equal height h m. BC be the 80 m wide road. P is any point on the road. Let CP be x m,
therefore BP = (80 – x) . Also, ∠APB = 60^o and ∠DPC =30^o
In right angled triangle DCP,
tan 30^o = CD / CP
⇒ h / x = 1/ √3
⇒ h = x / √3 ....(1)
In right angled triangle ABP,
Tan 60^o = AB / PB
⇒ √3 = h / 80−x
⇒ h = √3 (80−x)
⇒ x / √3 = √3(80−x)
⇒ x = 3(80 – x)
⇒ x = 240 – 3x
⇒ x + 3x = 240
⇒ 4x = 240
⇒ x = 60
Height of the pole, h = x/√3 = 60/√3 = 20√3.
Thus, the position of the point P is 60 m from C and the height of each pole is 20√3 m.

11. A TV tower stands vertically on a bank of a canal. From a point on the other bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60^o from a point 20 m away from this point on the same bank the angle of elevation of the top of the tower is 30^o. Find the height of the tower and the width of the canal.



Sol.



Let 'h' (AB) be the height of tower and x be the width of the river

In ΔABC,

$\tan 60^0 = h / x$

$\Rightarrow h = \sqrt{3}\, x$ ....(i)

In ΔABD,

$\tan 30^0 = h / x+20$

$\Rightarrow h = x + 20 / \sqrt{3}$ ....(ii)

Equating (i) and (ii),

$\sqrt{3}\, x = x+ 20 / \sqrt{3}$

$\Rightarrow 3x = x + 20$

$\Rightarrow 2x = 20$

$\Rightarrow x = 10$ m

Put x = 10 in (i),

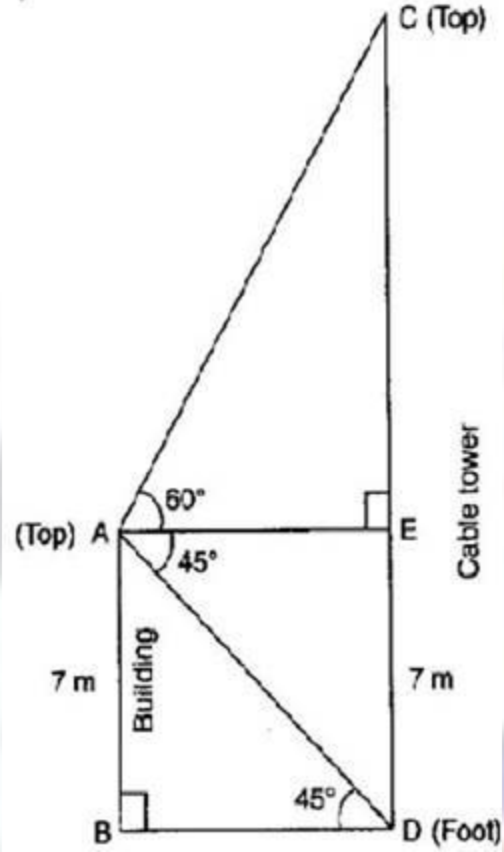
$h = \sqrt{3}\, x$

$\Rightarrow h = 10\sqrt{3}$ m

12. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60^o and the angle of depression of its foot is 45^o. Determine the height of the tower.

Sol. In right triangle ABD,

$\tan 45^0$ = AB / BD tan



⇒ 1 = 7/ BD

⇒ BD = 7 m

⇒ AE = 7 m

In right triangle AEC,

$\tan 60^0$ = CE / AE

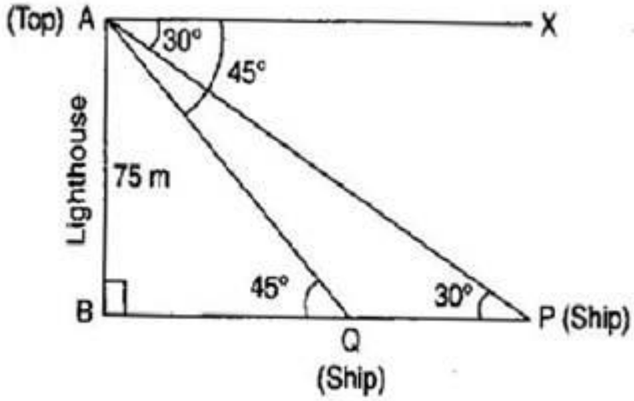
⇒$\sqrt{3}$ = CE / 7 ⇒ CE =7 $\sqrt{3}$ m

∴ CD = CE + ED = CE + AB = $7\sqrt{3} + 7 = 7(\sqrt{3}+1)$ m

Hence height of the tower is $7(\sqrt{3} + 1)$ m.

13. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30^o and 45^o. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between two ships.

Sol. In right triangle ABQ,



$\tan 45^0 = AB / BQ$

$\Rightarrow 1 = 75 / BQ$

$\Rightarrow BQ = 75$ m .... (i)

In right triangle ABP,

$\tan 30^0 = AB / BP$

$\Rightarrow 1/\sqrt{3} = AB / BQ + QP$

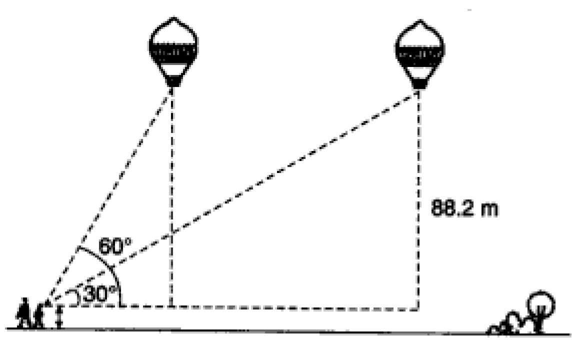
$\Rightarrow 1/\sqrt{3} = AB / 75 + QP$ [From eq. (i)]

$\Rightarrow 75 + QP = 75\sqrt{3}$

$QP = 75(\sqrt{3} - 1)$ m

Hence the distance between the two ships is $75(\sqrt{3}-1)$ m.

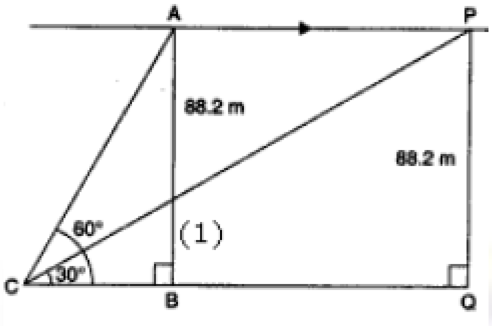
14. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance traveled by the balloon during the interval.



Sol. As per question:
$AB = PQ = 88.2 - 1.2$ m
In right triangle ABC,
$\tan 60^0 = AB / BC$

$\Rightarrow \sqrt{3} = 87 / BC$

$\Rightarrow BC = 87 / \sqrt{3} = 87 \times \sqrt{3} / \sqrt{3} \times \sqrt{3} = 29\sqrt{3}$ m



In right triangle PQC,

$\tan 30^0 = PQ / CO$

$\Rightarrow \tan 30^0 = PQ / CB+BQ \Rightarrow 1/\sqrt{3} = 87 / 29\sqrt{3} + BQ$ .... From (1)

$\Rightarrow 29\sqrt{3} + BQ = 87\sqrt{3}$

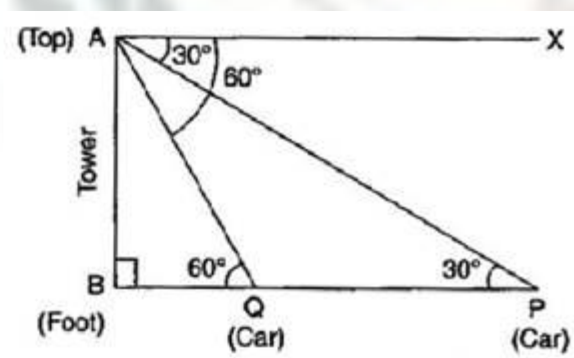
$\Rightarrow BQ = (87 – 29)\sqrt{3}$

$\Rightarrow BQ = 58\sqrt{3}$ m

Hence, the distance travelled by the balloon during the interval is $58\sqrt{3}$m.

15. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching to the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the further time taken by the car to reach the foot of the tower

Sol.



In right triangle ABP,

$\tan 30^0 = AB / BP$

$\Rightarrow 1/\sqrt{3} = AB / BP$

$BP = AB\sqrt{3}$ .... (i)

In right triangle ABQ,

$\tan 60^0 = AB / BQ$

$\Rightarrow \sqrt{3} = AB / BQ$

$\Rightarrow BQ = AB / \sqrt{3}$ .... (ii)

$\because$ PQ = BP - BQ

$\therefore PQ = AB\sqrt{3} – AB / \sqrt{3} = 3AB−AB / \sqrt{3} = 2 AB / \sqrt{3} = 2BQ$ [From eq. (ii)]

$\Rightarrow BQ = 1 / 2 PQ$

$\because$ Time taken by the car to travel a distance PQ = 6 seconds.

$\therefore$ Time taken by the car to travel a distance BQ, i.e. $1 / 2 PQ = 1 / 2 \times 6 = 3$ seconds.

Hence, the further time taken by the car to reach the foot of the tower is 3 seconds.

16. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Sol. Let $\angle APB = \theta$

Then, $\angle AQB = 90^0 – \theta$

$\because \angle APB$ and $\angle AQB$ are complementary

In right triangle ABP,

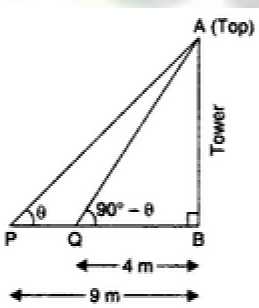
$\tan\theta = AB / PB$

$\Rightarrow \tan\theta = AB / 9$ ....(1)

In right triangle ABQ,

$\tan(90^0 – \theta) = AB / QB$

$\Rightarrow \cot \theta = AB / 4$ ....(2)



Multiplying (1) and (2), we get

$AB / 9 \times AB / 4 = \tan\theta \times \cot\theta$

$\Rightarrow AB^2 / 36 = 1$

$\Rightarrow AB^2 = 36 \Rightarrow \quad AB = \sqrt{36} = \quad 6m$

Hence, the height of the tower is 6 m.

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