Notes
Chapter –12

Algebraic Expressions

Algebraic expressions are formed from **variables** and **constants**. We use the operations of **addition, subtraction, multiplication** and **division** on the variables and constants to form expressions. For example, the expression $4xy + 7$ is formed from the variables x and y and constants 4 and 7. The constant 4 and the variables x and y are multiplied to give the product $4xy$ and the constant 7 is added to this product to give the expression.

- **Variable:** Symbols which are used to represent or replace numbers. They are denoted as x, y, z, a, b, c…… and can take different numerical values. We generally use small letters to represent variables.
- **Constant:** A symbol having a fixed numerical value. Example: 2, -10, etc.
- Expressions are made up of **terms**. Terms are **added** to make an expression. For example, the addition of the terms $4xy$ and 7 gives the expression $4xy + 7$.
- A term is a **product of factors**. The term $4xy$ in the expression $4xy + 7$ is a product of factors x, y and 4. Factors containing variables are said to be **algebraic factors**.
- The **coefficient** is the numerical factor in the term. Sometimes anyone factor in a term is called the coefficient of the remaining part of the term.
- Any expression with one or more terms is called a **polynomial**.
- Specifically a one term expression is called a **monomial**.
- A two-term expression is called a **binomial.**
- A three-term expression is called a **trinomial**.
- Terms which have the same algebraic factors are **like**terms. Terms which have different algebraic factors are **unlike** terms. Thus, terms $4xy$ and $– 3xy$ are like terms; but terms $4xy$ and $– 3x$ are not like terms.
- The **sum** (or **difference**) **of two like terms** is a **like** term with coefficient equal to the **sum** (or **difference**) **of the coefficients** of the two like terms. Thus, $8xy – 3xy = (8 – 3)xy$, i.e., $5xy$.
- When we **add** two algebraic expressions, the like terms are added as given above; the **unlike** terms are **left as they are**. Thus, the sum of $4x^2 + 5x$ and $2x + 3$ is $4x^2 + 7x + 3$; the like terms $5x$ and $2x$ add to $7x$; the unlike terms $4x^2$ and 3 are left as they are.
- In situations such as solving an equation and using a formula, we have to **find the value of an expression**. The value of the expression depends on the value of the variable from which the expression is formed. Thus, the value of $7x – 3$ for $x = 5$ is 32, since $7(5) – 3 = 35 – 3 = 32$.
- **Rules and formulas** in mathematics are **written** in a concise and general form using algebraic expressions: Thus, the area of rectangle = lb, where l is the length and b is the breadth of the rectangle.
- The general (*nth*) term of a number pattern (or a sequence) is an expression in n.
- Thus, the *nth* term of the number pattern 11, 21, 31, 41, . . . is $(10n + 1)$

Ex: 12.1

1(1). Get the algebraic subtraction of z from y, using variables, constants and arithmetic operations.

Sol. $y - z$

1(2). Get the algebraic expression for one-half of the sum of numbers x and y, using variables, constants and arithmetic operations.

Sol. Sum of numbers x and y $= x + y$
One half of the sum of numbers x and y $= 1 / 2\ (x+y) = x/2 + y / 2$

1(3). Get the algebraic expression for the number z multiplied by itself, using variables, constants and arithmetic operations.

Sol. $Z \times z = z^2$

1(4). Get the algebraic expressions for One-fourth of the product of numbers p and q, using variables, constants and arithmetic operations.

Sol. Products of two numbers p and q $= p \times q = pq$
So one - fourth of the above quantity is $1 / 4\ pq$

1(5). Get the algebraic expression numbers x and y both squared and added, using variables, constants and arithmetic operations.

Sol. Square of number x $= x^2$
Square of number y $= y^2$
Addition of squares of x and y $= x^2 + y^2$

1(6). Get the algebraic expression: number 5 added to three times the product of numbers m and n, using variables, constants and arithmetic operations.

Sol. Product of m and n $= m \times n = mn$
Three times of product of m and n $= 3 \times mn = 3mn$
Five added to three times of product of m and n$= 5 + 3mn$

1(7). Get the algebraic product of numbers y and z subtracted from 10, using variables, constants and arithmetic operations.

Sol. Product of number y and z $= xy$
Product of number y and z subtracted from 10 $= 10 - xy$

1(8). Get the algebraic expression: Sum of numbers a and b subtracted from their product, using variables, constants and arithmetic operations.

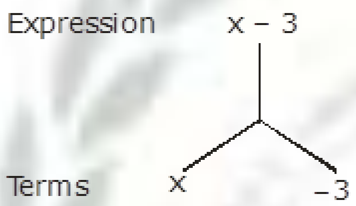
Sol. Sum of numbers a and b $=a+b$

Products of numbers $=ab$

Subtraction of sum from product $=ab-(a+b) \ =ab-a-b$

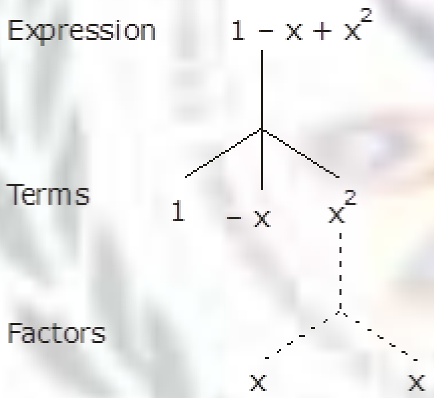
2(1). Identify the term and their factors in $x - 3$. Show the terms and factors by tree diagrams.

Sol.



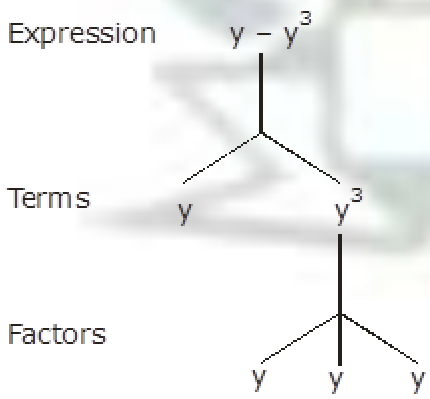
2(2). Identify the term and their factors in the expression $1 + x + x^2$. Show the terms and factors by tree diagrams.

Sol.



2(3). Identify the term and their factors in the expression $y - y^3$. Show the terms and factors by tree diagrams.

Sol.



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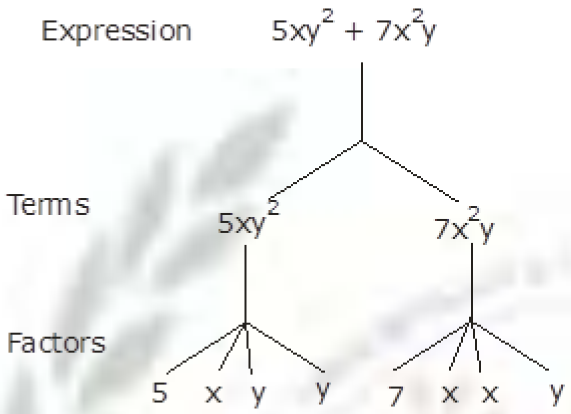
2(4). Identify the terms and their factors in the expression $5xy^2 + 7x^2y$. Show the terms and factors by tree diagrams.

Sol.



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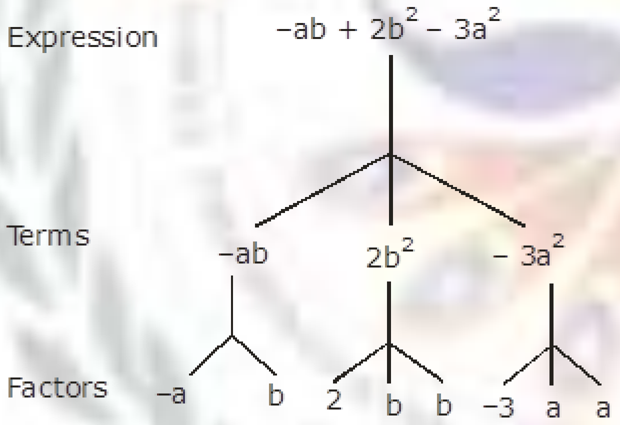
2(5). Identify the term and their factors in the expression: (- ab + $2b^2$ - $3a^2$). Show the terms and factors by tree diagrams.

Sol.



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2(6). Identify terms and factors in the expressions given below:

a. $-4x + 5$
b. $-4x + 5y$
c. $5y + 3y^2$
d. $xy + 2x^2 y^2$
e. $pq + q$
f. $1.2\ ab - 2.4\ b + 3.6\ a$
g. $34x+14$
h. $0.1\ p^2 + 0.2\ q^2$

Sol. The required information is provided below in the table:

S.No	Expression	Terms	Factors

a	$-4x+5$	$-4x$ 5	-4, x 5
b	$-4x+5y$	$-4x$ 5y	-4, x 5, y
c	$5y+3y^2$	5y $3y^2$	5, y 3, y, y
d	$xy+2x^2y^2$	xy $2x^2y^2$	x, y 2, x, x, y, y
e	pq + q	p q q	p, q q
f	1.2 a b - 2.4 b + 3.6 a	1.2 ab -2.4 b 3.6 a	1.2, a, b -2.4, b 3.6, a
g	34x+14	34x 14	34,x 14
h	0.1p2+0.2q2	0.1p2 0.2q2	0.1, p, p 0.2, q, q

3(1). Identify the numerical coefficients of term other than constants in $5 - 3t^2$

Sol. Term other than constant = $-3t^2$
∴ Required Coefficient = -3

3(2). Identify the numerical coefficients of terms other than constants in $1 + t + t^2 + t^3$

Sol. Terms other than constant are: t, t^2, t^3
∴ Required Coefficients = 1, 1, 1

3(3). Identify the numerical coefficients of terms other than constants in x + 2xy + 3y.

Sol. Terms other than constants are: x, 2xy, 3y
∴ Required Coefficients = 1, 2, 3

3(4). Identify the numerical coefficients of terms other than constants in 100m + 1000n

Sol. Terms other than constants are: 100m, 1000n
∴ Required Coefficients = 100, 1000

3(5). Identify the numerical coefficients of terms other than constants in $-p^2q^2 + 7pq$

Sol. Terms other than constants are: $-p^2q^2$, 7pq
∴ Required Coefficients = -1, 7

3(6). Identify the numerical coefficients of terms other than constants in $1.2a + 0.8b$

Sol. Terms other than constants are: $1.2a$, $0.8b$
$\therefore$ Required Coefficients = 1.2, 0.8

3(7). Identify the numerical coefficient of term $3.14r^2$

Sol. Required Coefficient = 3.14

3(8). Identify the numerical coefficients of terms other than constants in $2(l + b)$

Sol. Terms other than constants are: $2l$, $2b$
$\therefore$ Required Coefficients = 2, 2

3(9). Identify the numerical coefficients of terms other than constants in $0.1y + 0.01y^2$

Sol. Terms other than constants are: $0.1y$, $0.01y^2$
$\therefore$ Required Coefficients = 0.1, 0.01

4(1). Identify the term which contains x in the expression $y^2x + y$ also write the coefficient of x.

Sol. Term = y^2x.
Coefficient = y^2.

4(2). In the expression $13y^2 - 8yx$, Identify the term which contains x and write the coefficient of x.

Sol. Required Term = $8yx$
Required Coefficient = $-8y$

4(3). Identify the term which contains x in $x + y + 2$, and write the coefficient of x.

Sol. Required Term = x
Required Coefficient = 1

4(4). Identify the terms which contain x, in the expression $5 + z + zx$, and write the coefficient of x.

Sol. The required term = xz
and The required Coefficient = z

4(5). Identify terms which contain x in 1 + x + xy, and write the coefficients of x in those terms.

Sol. Terms containing x are: xy
∴ Required Coefficients are: 1 and y

4(6). Identify the term which contains x in $12xy^2 + 25$, and write the coefficient of x.

Sol. Required Term = $12xy^2$
Required Coefficient = $12y^2$

4(7). Identify term which contains x in the expression: $7x + xy^2$ also find the coefficient of x.

Sol. Term = 7x.
Coefficients = 7.

4(8). Identify the term which contains y^2 in $8 - xy^2$ and write the coefficient of y^2.

Sol. Required Term = $-xy^2$
and Required Coefficient = -x

4(9). Identify the term which contains x in $5y^2 + 7x$ and write the coefficient of x.

Sol. Required Term = 7x
and Required Coefficient = 7

4(10). Identify the terms which contain x in the expression: $2xy - 15xy^2 + 7y^2$ and write the coefficient of x.

Sol. Required Terms are: 2xy, $-15xy^2$
Hence, Required Coefficients = 2y, $-15y^2$

5(1). Classify into monomials, binomials and trinomials: 4y - 7z

Sol. 4y–7z.
As the expression contains two terms expression is Binomial.

5(2). Classify whether the expression given is monomial, binomial or trinomial: y^2

Sol. y^2.
As the expression contains only one term expression is Monomial.

5(3). Classify into monomials, binomials and trinomials: $x+y-xy$

Sol. $x+y-xy$
As the expression contains three terms expression is Trinomial.

5(4). Classify into monomials, binomials and trinomials: 100

Sol. 100 is a constant polynomial.
As the expression contains only one term, it is a Monomial.

5(5). Classify into monomials, binomials and trinomials: $ab - a - b$

Sol. $ab - a - b$
As the expression contains three terms, it is a Trinomial.

5(6). Classify into monomials, binomials and trinomials: $5 - 3t$

Sol. $5 - 3t$
As the expression contains two terms, it is a Binomial.

5(7). Classify into monomials, binomials and trinomials: $4p^2q - 4pq^2$

Sol. $4p^2q - 4pq^2$.
As the expression contains two terms, it is a Binomial.

5(8). Classify into monomials, binomials and trinomials: 7mn

Sol. 7mn
As the expression contains only one term, it is a Monomial.

5(9). Classify into monomials, binomials and trinomials: $z^2 - 3z + 8$

Sol. $z^2 - 3z + 8$
As the expression contains three terms, it is a Trinomial.

5(10). Classify into monomials, binomials and trinomials: $a^2 + b^2$

Sol. $a^2 + b^2$
As the expression contains two terms, it is a Binomial.

5(11). Classify into monomials, binomials and trinomials: $z^2 + z$

Sol. $z^2 + z$

As the expression contains two terms, it is a Binomial.

5(12). Classify into monomials, binomials and trinomials: $1 + x + x^2$

Sol. $1 + x + x^2$

As the expression contains three terms, it is a Trinomial.

6(1). State whether the pair of expressions given is of like or unlike terms: 1, 100

Sol. 1, 100

As each expression contains only a single term, so the expressions are simply of like terms.

6(2). State whether the pair of expressions given are of like or unlike terms: -7x, 5x2

Sol. -7x, 5x2

As each expression contains only a single term, so the expressions are simply of like terms.

6(3). State whether the pair of expressions is of like or unlike terms: -29x, -29y

Sol. 29x, -29y

As both terms don't have the same algebraic factor (As -29x has x and -29 and -29y has y and -29), the terms are unlike.

6(4). State whether the pair of terms is like or unlike: 14xy, 42yx

Sol. 14xy, 42yx

As both term have same algebraic factors as x and y, the pair of terms is like.

6(5). State whether the pair of terms is like or unlike: $4m^2p$, $4mp^2$

Sol. $4m^2p$, $4mp^2$

As term $4m^2p$ has algebraic factors m, m and p but term $4mp^2$ has algebraic factors m, p and p. So, both terms are unlike.

6(6). State whether the pair of terms is like or unlike: $12xz$, $12x^2z^2$

Sol. $12xz$, $12x^2z^2$

As term $12xz$ has algebraic factors x and z but term $12x^2y^2$ has algebraic factors x, x, y and y.

So, both terms are unlike.

7(1). Identify like terms among the following: $-xy^2$, $-4yx^2$, $8x^2$, $2xy^2$, $7y$, $-11x^2$, $-100x$, $-11yx$, $20x2y$, $-6x^2$, y, $2xy$, $3x$

Sol. Here, the given terms can be tabulated as:

Expression	Variable Factors
$-xy^2$	x, y, y
$-4yx^2$	y, x, x
$8x^2$	x, x
$2xy^2$	x, y, y
$7y$	Y
$-11x^2$	x,x
$-100x$	X
$-11yx$	y, x
$20x^2y$	x, x, y
$-6x^2$	x, x
y	Y
2xy	x, y
3x	X

So, From the above table we conclude that sets of like terms are:

i. $-xy^2$, $2xy^2$: As both have common variable factors as x, y and y
ii. $-4yx^2$, $20x^2y$: As both have common variable factors as x, x and y
iii. $8x^2$, $-11x^2$, $-6x^2$: As both have common variable factors as x and x
iv. -11yx, 2xy : As both have common variable factors as x and y
v. -100x, 3x : As both have common variable factor as x
vi. 7y, y : As both have common variable factor as y

7(2). Identify like terms in the following: $10pq$, $7p$, $8q$, $-p^2q^2$, $-7qp$, $-100q$, -23, $12q^2p^2$, $-5p^2$, 41, $2405p$, $78qp$, $13p^2q$, qp^2, $701p^2$

Sol. The given terms and their factors are shown in the following table:

Expression	Variable Factors
10pq	p, q
7p	P
8q	Q
$-p^2q^2$	p, p, q, q
$-7qp$	q, p
$-100q$	Q
-23	Constant
$12q^2p^2$	q, q, p, p
$-5p^2$	p, p

41	Constant
2405p	P
78qp	q, p
$13p^2q$	p, q, p
qp^2	q, p, p
$701p^2$	p, p

So, From the above table we conclude that sets of like terms are

i. 10pq, –7qp, 78qp : As both have common variable factors as p and q
ii. 7p, 2405p: As both have common variable factor as p
iii. 8q, – 100q : As both have common variable factor as q
iv. $-p^2q^2$, $12q^2p^2$: As both have common variable factors as p, q, q and q
v. –23, 41 : As both terms are constant and don't have any variable factor.
vi. $-5p^2$, $701p^2$:As both have common variable factors as p and p.
vii. $13p^2q$, qp^2 : As both have common variable factors as p, p and q.

EX: 12.2

1(1). Simplify combining like term: 21b – 32 + 7b – 20 b

Sol. =21b – 32 + 7b – 20 b
= 21b + 7b– 20b – 32 [rearranging terms]
= 8b – 32
= 8(b - 4)

1(2). Simplify combining like term: $-z^2 + 13z^2 - 5z + 7z^3 - 15z$

Sol. $=-z^2 + 13z^2 - 5z + 7z^3 - 15z$ [rearranging terms]
$= 13z^2 - z^2 - 5z - 15z + 7z^3$
$= 12z^2 - 20z + 7z^3$
$= 7z^3 + 12z^2 - 20z$

1(3). Simplify combining like term: p – (p –q) – q – (q – p)

Sol. = p – (p – q) – q – (q – p)
= p – p + q – q – q + p ... [rearranging terms]
= p + p – p + q – q – q
= p – q

1(4). Simplify combining like term: 3a – 2b –ab –(a –b+ ab) + 3ab + b – a

Sol. = 3a – 2b –ab –(a –b+ ab) + 3ab + b – a
= 3a – 2b – ab – a + b – ab + 3ab + b – a
= 3a – a – a + b + b – 2b + 3ab – ab – ab [rearranging terms]
= a + ab

1(5). Simplify combining like term : $5x^2y - 5x^2 + 3yx^2 - 3y^2 + x^2 - y^2 + 8xy^2 - 3y^2$

Sol. $= 5x^2y - 5x^2 + 3yx^2 - 3y^2 + x^2 - y^2 + 8xy^2 - 3y^2$
$= 5x^2y + 3yx^2 + x^2 - 5x^2 - y^2 - 3y^2 - 3y^2 + 8xy^2$ [rearranging terms]
$= 8x^2y - 4x^2 - 7y^2 + 8xy^2$

1(6). Simplify combining like term: $(3y^2 + 5y - 4) - (8y - y^2 - 4)$

Sol. $= (3y^2 + 5y - 4) - (8y - y^2 - 4)$
$= 3y^2 + 5y - 4 - 8y + y^2 + 4$
$= 3y^2 + y^2 + 5y - 8y - 4 + 4$ [rearranging terms]
$= 4y^2 - 3y$

2(1). Add: $3mn, -5mn, 8mn, -4mn$

Sol. The required expression is:
$3mn + (-5mn) + 8mn + (-4mn)$
$= 3mn - 5mn + 8mn - 4mn$
$= 11mn - 9mn$
$= 2mn$

2(2). Add: $t - 8tz, 3tz - z, z - t$

Sol. The required expression is:
$t - 8tz, 3tz - z, z - t$
$= t - 8tz + 3tz - z + z - t$
$= t - t - z + z - 8tz + 3tz$
$= -5tz$

2(3). Add: $-7mn + 5, 12mn + 2, 9mn - 8, -2mn - 3$

Sol. Here, we have
$-7mn + 5, 12mn + 2, 9mn - 8, -2mn - 3$
$= (-7mn + 5) + (12mn + 2) + (9mn - 8) + (-2mn - 3)$
$= -7mn + 12mn + 9mn - 2mn + 5 + 2 - 8 - 3$
$= -9mn + 21mn + 7 - 11$
$= 12mn - 4$

2(4). Add : $a + b - 3, b - a + 3, a - b + 3$

Sol. The required expression is:
$a + b - 3, b - a + 3, a - b + 3$

$= (a + b - 3) + (b - a + 3) + (a - b + 3)$
$= a - a + a + b + b - b - 3 + 3 + 3$
$= a + b + 3$

2(5). Add: $14x + 10y - 12xy - 13 + (18 - 7x - 10y + 8xy) + 4xy$

Sol. Here, we have
$14x + 10y - 12xy - 13 + (18 - 7x - 10y + 8xy) + 4xy$
$= 14x + 10y - 12xy - 13 + 18 - 7x - 10y + 8xy + 4xy$
$= 14x - 7x + 10y - 10y - 12xy + 8xy + 4xy - 13 + 18$
$= 7x + 5$

2(6). Add : $5m - 7n$, $3n - 4m + 2$, $2m - 3mn -5$

Sol. Thre required expression is:
$(5m - 7n) + (3n - 4m + 2) + (2m - 3mn -5)$
$= 5m - 7n + 3n - 4m + 2 + 2m - 3mn -5$
$= 3m - 4n - 3mn – 3$

2(7). Add: $4x^2y + (-3xy^2) + (-5xy^2) + 5x^2y$

Sol. Here, we have:
$4x^2y + (-3xy^2) + (-5xy^2) + 5x^2y$
$= 4x^2y - 3xy^2 - 5xy^2 +5x^2y$
$= 4x^2y + 5x^2y - 3xy^2 - 5xy^2$
$= 9x^2y - 8xy^2$

2(8). Add : $(3p^2q^2 - 4pq + 5)$, $(-10p^2q^2)$, $(15 + 9pq + 7p^2q^2)$

Sol. Here, we have
$(3p^2q^2 - 4pq + 5) + (-10p^2q^2) + (15 + 9pq + 7p^2q^2)$
$= 3p^2q^2 - 4pq + 5 - 10p^2q^2 + 15 + 9pq + 7p^2q^2$
$= 3p^2q^2 - 10p^2 q^2 + 7p^2q^2 - 4pq + 9pq + 5 + 15$
$= 5pq + 20$

2(9). Add : $(ab - 4a) + (4b - ab) + (4a - 4b)$

Sol. The required expression is:
$(ab - 4a) + (4b - ab) + (4a - 4b)$
$= ab - 4a + 4b - ab + 4a - 4b$
$= ab - ab - 4a + 4a + 4b - 4b$
$= 0$

2(10). Add: $(x^2 - y^2 - 1)$, $(y^2 - 1 - x^2)$, $(1 - x^2 - y^2)$

Sol. Here, we have

$(x^2 - y^2 - 1) + (y^2 - 1 - x^2) + (1 - x^2 - y^2)$

$= x^2 - y^2 - 1 + y^2 - 1 - x^2 + 1 - x^2 - y^2$

$= x^2 - x^2 - x^2 - y^2 + y^2 - y^2 - 1 - 1$

$= -x^2 - y^2 – 1$

3(1). Subtract: $–5y^2$ from y^2

Sol. $–5y^2$ from y^2

$y^2 – (–5\ y^2)$

$= y^2 + 5\ y^2 = 6y^2$

3(2). Subtract: 6xy from – 12 xy

Sol. 6xy from – 12 xy

$–12xy – 6xy = –18\ xy$

3(3). Subtract: (a – b) from (a + b)

Sol. (a – b) from (a + b)

$(a + b) – (a – b) = a + b – a + b$

$= a – a + b + b$ ... [rearranging terms]

$= 2b$

3(4). Subtract: a(b – 5) from b (5 – a)

Sol. a(b – 5) from b (5 – a)

$b(5–a) – a (b – 5) = 5b –ab –ab + 5a$

$= 5a + 5b – ab –ab$ ... [rearranging terms]

$= 5a + 5b – 2ab$

3(5). Subtract: $–m^2 + 5\ mn$ from $4\ m^2 – 3mn + 8$

Sol. $= 4m^2 –3mn + 8 – (– m^2 + 5mn)$

$= 4m^2 - 3\ mn + 8 + m^2 - 5mn$

$= 4\ m^2 + m^2 – 3mn – 5mn + 8$ [rearranging terms]

$= 5\ m^2 – 8mn + 8$

3(6). Subtract: $–x^2 + 10\ x – 5$ from $5x – 10$

Sol. $-x^2 + 10x - 5$ from $5x - 10$

$5x - 10 - (-x^2 + 10x - 5)$

$= 5x - 10 + x^2 - 10x + 5$

$= x^2 + 5x - 10x - 10 + 5$ ... [rearranging terms]

$= x^2 - 5x - 5$

3(7). Subtract: $5a^2 - 7ab + 5b^2$ from $3ab - 2a^2 - 2b^2$

Sol. $3ab - 2a^2 - 2b^2 - (5a^2 - 7ab + 5b^2)$

$= 3ab - 2a^2 - 2b^2 - 5a^2 + 7ab - 5b^2$

$= -2a^2 - 5a^2 - 2b^2 - 5b^2 + 3ab + 7ab$ ... [rearranging terms]

$= -7a^2 - 7b^2 + 10ab$

3(8). Subtract: $4pq - 5q^2 - 3p^2$ from $5p^2 + 3q^2 - pq$

Sol. $4pq - 5q^2 - 3p^2$ from $5p^2 + 3q^2 - pq$

$5p^2 + 3q^2 - pq - (4pq - 5q^2 - 3p^2)$

$= 5p^2 + 3p^2 + 3q^2 + 5q^2 - pq - 4pq$ [rearranging terms]

$= 8p^2 + 8q^2 - 5pq$

4(1). What should be added to $x^2 + xy + y^2$ to obtain $2x^2 + 4xy$

Sol. Required expression

$= 2x^2 + 4xy - (x^2 + xy + y^2)$

$= 2x^2 + 4xy - x^2 - xy - y^2$ ... [rearranging terms]

$= 2x^2 - x^2 + 4xy - xy - y^2$

$= x^2 + 3xy - y^2$

4(2). What should be subtracted from $2a + 8b + 10$ to get $-3a + 7b + 16$

Sol. Required expression

$= (2a + 8b + 10) - (-3a + 7b + 16)$

$= 2a + 8b + 10 + 3a - 7b - 16$

$= 2a + 3a + 8b - 7b + 10 - 16$ ... [rearranging terms]

$= 5a + b - 6$

5. What should be taken away from $3x^2 - 4y^2 + 5xy + 20$ to obtain $-x^2 - y^2 + 6xy + 20$

Sol. Required expression

$= (3x^2 - 4y^2 + 5xy + 20) - (-x^2 - y^2 + 6xy + 20)$

$= 3x^2 - 4y^2 + 5xy + 20 + x^2 + y^2 - 6xy - 20$

$= 3x^2 + x^2 + y^2 - 4y^2 + 5xy - 6xy + 20 - 20$ ... [rearranging terms]

$= 4x^2 - 3y^2 - xy$

6(1). From the sum of $3x - y + 11$ and $- y - 11$ subtract $3x - y - 11$

Sol. Resulting expression = $\{(3x - y + 11) + (- y - 11)\} - (3x - y - 11)$
$= 3x - y + 11 - y - 11 - 3x + y + 11$
$= 3x - 3x + y - y - y + 11 + 11 - 11$ ... [rearranging terms]
$= -y + 11$
Coefficient of x and y in the result are 0 and –1 respectively.

6(2). From the sum of $4 + 3x$ and $5 - 4x + 2x^2$, subtract the sum of $3x^2 - 5x$ and $-x^2 + 2x + 5$

Sol. As per the given condition in the question, we have
$[(4 + 3x) + (5 - 4x + 2x^2)] - [(3x^2 - 5x) + (-x^2 + 2x + 5)]$
$= [4 + 3x + 5 - 4x + 2x^2] - [3x^2 - 5x - x^2 + 2x + 5]$
$= 4 + 3x + 5 - 4x + 2x^2 - 3x^2 + 5x + x^2 - 2x - 5$
$= 2x^2 - 3x^2 + x^2 + 3x - 4x - 2x + 5x + 4 + 5 - 5$
$= 2x + 4$

Ex: 12.3

1(1). If m = 2, find the value of $m - 2$

Sol. $m - 2$
$m - 2 = 2 - 2 = 0$

1(2). If m = 2, find the value of $3m - 5$

Sol. $3m - 5$
$3m - 5 = 3 \times 2 - 5 = 6 - 5 = 1$

1(3). If m = 2, find the value of $9 - 5m$

Sol. $9 - 5m = 9 - 5 \times 2 = 9 - 10 = -1$

1(4). If m = 2, find the value of $3m^2 - 2m - 7$

Sol. $3m^2 - 2m - 7 = 3(2)^2 - 2(2) - 7 = 3(4) - 4 - 7 = 12-4-7 = 1$

1(5). If m = 2, find the value of 5m2-4

Sol. 5m2 - 4 = 5×22 - 4 = 5 - 4 = 1

2(1). If p = –2, find the value of $4p + 7$

Sol. $4p + 7$

$4p + 7 = 4(-2) + 7 = -8 + 7 = -1$

2(2). If $p = -2$, find the value of $-3p^2 + 4p + 7$

Sol. $-3p^2 + 4p + 7$

$-3p^2 + 4p + 7 = -3(-2)^2 + 4(-2) + 7$

$= -12 - 8 + 7 = -20 + 7 = -13$

2(3). If $p = -2$, find the value of $-2p^3 - 3p^2 + 4p + 7$

Sol. $= -2p^3 - 3p^2 + 4p + 7$

$= -2p^3 - 3p^2 + 4p + 7$

$= -2(-2)^3 - 3(-2)^2 + 4(-2) + 7$

$= -2(-8) - 3(4) - 8 + 7 = 16 - 12 - 8 + 7$

$= 16 + 7 - 12 - 8 = 23 - 20 = 3$

3(1). Find the value of the expression $2x - 7$, when $x = -1$

Sol. $2x - 7$

$2x - 7 = 2(-1) - 7 = -2 - 7 = -9$

3(2). Find the value of the expression: $-x + 2$, when $x = -1$

Sol. The given expression is: $-x + 2$

Put $x = -1$, we get

$-x + 2 = -(-1) + 2$

$= 1 + 2 = 3$

3(3). Find the value of the expression $x^2 + 2x + 1$, when $x = -1$

Sol. The given expression is: $x^2 + 2x + 1$

Put $x = -1$, we get

$x^2 + 2x + 1 = (-1)^2 + 2(-1) + 1$

$= 1 - 2 + 1$

$= 2 - 2 = 0$

3(4). Find the value of the expression $2x^2 - x - 2$, when $x = -1$

Sol. The given expression is: $2x^2 - x - 2$

Put $x = -1$, we get

$2x^2 - x - 2 = 2(-1)^2 - (-1) - 2$
$= 2(1) + 1 - 2$
$= 2 + 1 - 2$
$= 3 - 2 = 1$

4(1). If a = 2, b = –2 find the value of $a^2 + b^2$

Sol. $a^2 + b^2$
$a^2 + b^2 = (2)^2 + (-2)^2 = 4 + 4 = 8$

4(2). If a = 2, b = –2 find the value of $a^2 + ab + b^2$

Sol. $= a^2 + ab + b^2$
$= a^2 + ab + b^2$
$= (2)^2 + (2)(-2) + (-2)^2$
$= 4 - 4 + 4 = 4 + 4 - 4$
$= 8 - 4$
$= 4$

4(3). If a = 2, b = –2 find the value of $a^2 - b^2$

Sol. $a^2 - b^2$
$a^2 - b^2 = (2)^2 - (-2)^2 = 4 - 4 = 0$

5(1). When a = 0, b = – 1, find the value of: 2a + 2b

Sol. The given expression is: 2a + 2b
Put a = 0 and b = -1, we get
$2a + 2b = 2(0) + 2(-1)$
$= 0 - 2 = -2$

5(2). When a = 0, b = – 1, find the value of: $2a^2 + b^2 + 1$

Sol. The given expression is: $2a^2 + b^2 + 1$
Put a = 0 and b = -1, we get
$2a^2 + b^2 + 1 = 2(0)^2 + (-1)^2 + 1$
$= 2(0) + 1 + 1$
$= 0 + 1 + 1 = 2$

5(3). When a = 0, b = – 1, find the value of: $2a^2b + 2ab^2 + ab$

Sol. The given expression is: $2a^2b + 2ab^2 + ab$
Put a = 0 and b = -1, we get
$2a^2b + 2ab^2 + ab = 2(0)^2(-1) + 2(0)(-1)^2 + 0(-1)$
$= 2(0)(-1) + 2(0)(1) + 0$
$= 0 + 0 + 0 = 0$

5(4). When a = 0, b = –1, find the value of the given expression: $a^2 + ab + 2$

Sol. $=a^2 + ab + 2$
$=a^2 + ab + 2$
$= (0)^2 + (0)(–1) + 2$
$= 0 + 0 + 2$
$= 2$

6(1). Simplify the expression x + 7 + 4 (x – 5) and find the value if x is equal to 2.

Sol. $= x + 7 + 4 (x – 5)$
$= x + 7 + 4x – 20$
$= x + 4x + 7 – 20$ [rearranging terms]
$= 5x – 13$
$= 5(2) – 13$
$= 10 – 13$
$= –3$

6(2). Simplify the expression 3 (x + 2) + 5x – 7 and find the value If x is equal to 2.

Sol. $= 3 (x + 2) + 5x – 7$
$= 3 (2 + 2) + 5(2) - 7$
$= 3(4) + 10 - 7$
$= 12 + 10 - 7$
$= 22 - 7$
$= 15$

6(3). Simplify 6x + 5(x – 2) and find the value if x is equal to 2.

Sol. The given expression is: $6x + 5(x - 2)$
Opening the brackets, we get
$6x + 5x - 10 = 11x - 10$
Put x = 2, we get
$11x - 10 = 11(2) - 10$
$= 22 - 10 = 12$

6(4). Simplify the expression 4(2x – 1) + 3x + 11 and find the value if x is equal to 2.

Sol. $= 4(2x - 1) + 3x + 11$
$= 8x - 4 + 3x + 11$
$= 8x + 3x + 11 - 4$ [rearranging terms]
$= 11x + 7$
$= 11\ (2) + 7$
$= 22 + 7$
$= 29$

7(1). Simplify $3x - 5 - x + 9$ and find the value if $x = 3$.

Sol. The given expression is:
$3x - 5 - x + 9$
$= 3x - x - 5 + 9$
$= 2x - 5 + 9$
$= 2x + 4$
Put $x = 3$, we get
$2x + 4 = 2(3) + 4$
$= 6 + 4$
$= 10$

7(2). Simplify the expression $2 - 8x + 4x + 4$ and find its value if $x = 3$.

Sol. $=2 - 8x + 4x + 4$
$= 4x - 8x + 2 + 4$ [rearranging terms]
$= -4x + 6 = - 4(3) + 6$
$= -12 + 6$
$= -6$

7(3). Simplify the expression $3a + 5 - 8a + 1$ and find its value, if $a = -1$

Sol. $3a + 5 - 8a + 1$
$= 3(-1) + 5 - 8(-1) + 1$
$= -3 + 5 + 8+ 1$
$= -3 + 14$
$= 1$

7(4). Simplify the expression $10 - 3b - 4 - 5b$ and find their value if $b = -2$.

Sol. $=10 - 3b - 4 - 5b$
$=10 - 3b - 4 - 5b$
$= -3b - 5b + 10 - 4$ [rearranging terms]
$= -8b + 6$
$= - 8(-2) + 6$
$= 16 + 6$
$= 22$

7(5). Simplify the expression 2a – 2b – 4 – 5 + a and find their value if a = –1, b = –2.

Sol. $=2a - 2b - 4 - 5 + a$
$= 2a - 2b - 4 - 5 + a$
$= 2a + a - 2b - 4 - 5$ [rearranging terms]
$= 3a - 2b - 9$
$= 3(-1) - 2(-2) - 9$
$= -3 + 4 - 9$
$= -12 + 4$
$= -8$

8(1). If z = 10, find the value of $z^3 - 3(z - 10)$

Sol. $=z^3 - 3(z - 10)$
$= z^3 - 3z + 30$
$= (10)^3 - 3(10) + 30$
$= 1000 - 30 + 30$
$= 1000 + 30 - 30$
$= 1030 - 30$
$=1000$

8(2). If p= –10, find the value of $p^2 - 2p - 100$

Sol. $=p^2 - 2p - 100$
$= (-10)^2 - 2(-10) - 100$
$= 100 + 20 - 100$
$= 120 - 100$
$=20$

9. What should be the value of a if the value of $2x^2 + x - a$ equals 5, when x = 0?

Sol. $2x^2 + x - a$
$= 2(0)^2 + (0) - a$ ... (when x = 0)
$= -a$
According to the question,
$-a = 5$
$a = -5$

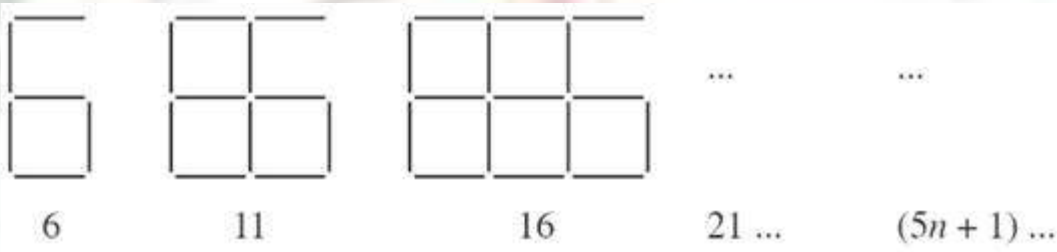
10. Simplify the expression $2(a^2 + ab) + 3 - ab$ and find its value when a = 5 and b = –3

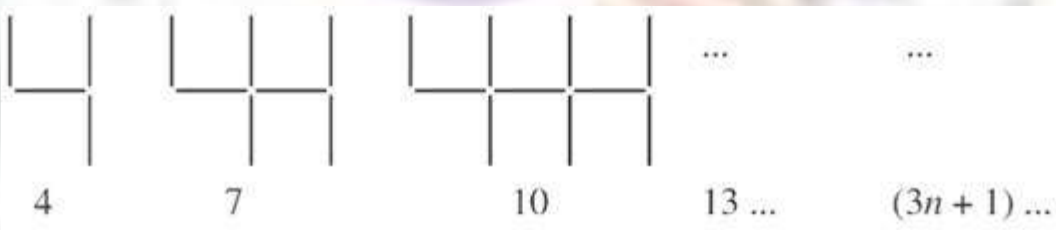
Sol. $= 2a^2 + 2ab + 3 - ab$
$= 2a^2 + 2ab - ab + 3$ ... [rearranging terms]
$= 2a^2 + ab + 3$

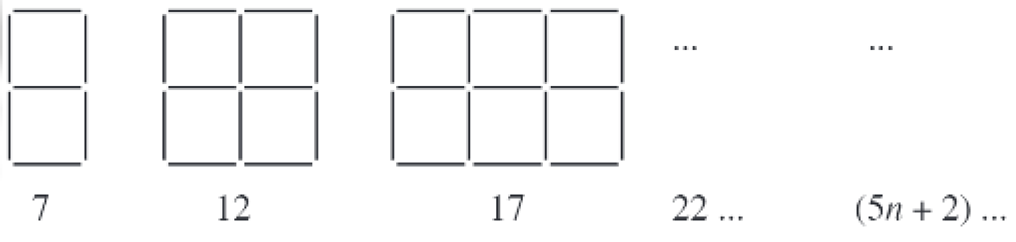
when a = 5, b = -3 then
$= 2(5)^2 + (5)(-3) + 3$
$= 50 - 15 + 3$
$= 50 + 3 - 15$
$= 53 - 15$
$= 38$

EX: 12.4

1. Observe the patterns of digits made from line segments of equal length. You will find such segmented digits on the display of electronic watches or calculators.

i.


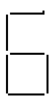
ii.


iii.


If the number of digits formed is taken to be n, the number of segments required to form n digits is given by the algebraic expression appearing on the right of each pattern.

How many segments are required to form 5, 10, 100 digits of the kind 

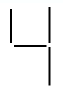
Sol. i. For digit



Expression: $5n + 1$
Where n = No. of digits

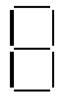
For 5 digits
No. of segments required = 5(5) + 1 = 25 + 1 = 26
For 10 digits
No. of segments required = 5(10) + 1 = 50 + 1 = 51
For 100 digits
No. of segments required = 5(100) + 1 = 500 + 1 = 501

ii. For digit



Expression : 3n + 1
Where n = No. of digits
For 5 digits
No. of segments required = 3(5) + 1 = 15 + 1 = 16
For 10 digits
No. of segments required = 3(10) + 1 = 30 + 1 = 31
For 100 digits
No. of segments required = 3(100) + 1 = 300 + 1 = 301

iii. For digit



Expression : 5n + 2
Where n = No. of digits
For 5 digits
No. of segments required = 5(5) + 2 = 25 + 2 = 27
For 10 digits
No. of segments required = 5(10) + 2 = 50 + 2 = 52
For 100 digits
No. of segments required = 5(100) + 2 = 500 + 2 = 502

2(1). Use the given algebraic expression to complete the table of number patterns.

S.No. Expression	Terms									
	1st	2nd	3rd	4th	5th	..	10th	..	100th	..
2n - 1	1	3	5	7	9	-	19	-	-	..

Sol. Given expression = 2n - 1
For n = 100, we get
2n - 1 = 2(100) - 1
= 200 - 1
= 199
Therefore, 100^{th} term = 199
So, the required table is

S.No. Expression	Terms									
	1st	2nd	3rd	4th	5th	..	10th	..	100th	..
2n - 1	1	3	5	7	9	-	19	-	199	..

2(2). Use the given algebraic expression to complete the table of number patterns.

S.No. Expression	Terms									
	1st	2nd	3rd	4th	5th	..	10th	..	100th	..
3n + 2	5	8	11	14	-	-	-	-	-	..

Sol. Here, the given expression is = 3n + 2
5^{th} term (n = 5) = 3(5) + 2
= 15 + 2 = 17
10^{th} terms (n =10) = 3(10) + 2
= 30 + 2 = 32
100^{th} term (n = 100) = 3(100) + 2
= 300 + 2 = 302
So, the complete table is as follows:

S.No. Expression	Terms									
	1st	2nd	3rd	4th	5th	..	10th	..	100th	..
3n + 2	5	8	11	14	17	-	32	-	302	..

2(3). Use the given algebraic expression to complete the table of number patterns.

S.No. Expression	Terms									
	1st	2nd	3rd	4th	5th	..	10th	..	100th	..
4n + 1	5	9	13	17	-	-	-	-	-	..

Sol. The given expression = 4n + 1
Therefore, we have
5^{th} term (n = 5) = 4(5) + 1
= 20 + 1
= 21
The 10^{th} term (n =10) = 4(10) + 1
= 40 + 1
= 41
The 100^{th} term (n = 100) = 4(100) + 1
= 400 + 1
= 401
So, the complete table is:

S.No. Expression	Terms									
	1st	2nd	3rd	4th	5th	..	10th	..	100th	..
4n + 1	5	9	13	17	21	-	41	-	401	..

2(4). Use the given algebraic expression to complete the table of number patterns.

S.No. Expression	Terms									
	1st	2nd	3rd	4th	5th	..	10th	..	100th	..
7n + 20	27	34	41	48	-	-	-	-	-	..

Sol. The given expression = 7n + 20

Clearly,

The 5^{th} term (n = 5) = 7(5) + 20

= 35 + 20

= 55

The 10^{th} term (n =10) = 7(10) + 20

= 70 + 20

= 90

The 100^{th} term (n = 100) = 7(100) + 20

= 700 + 20

= 720

So, the complete table is:

S.No. Expression	Terms								
	1st	2nd	3rd	4th	5th	..	10th	..	100th	..
7n + 20	27	34	41	48	55	-	90	-	720	..

2(5). Use the given algebraic expression to complete the table of number patterns.

S.No. Expression	Terms									
	1st	2nd	3rd	4th	5th	..	10th	..	100th	..
$n^2 + 1$	2	5	10	17	-	-	-	-	10001	..

Sol. The given expression = $n^2 + 1$

Clearly, we have

The 5^{th} term (n = 5) = $(5)^2 + 1$

= 25 + 1

= 26

The 10^{th} term (n =10) = $(10)^2 + 1$

= 100 + 1

= 101

So, the complete Table is:

S.No. Expression	Terms									
	1st	2nd	3rd	4th	5th	..	10th	..	100th	..
$n^2 + 1$	2	5	10	17	26	-	101	-	10001	..