Notes
Chapter – 12
Exponents and Powers

- **Numbers with exponents obey the following laws of exponents.**
  (a) $a^m \times a^n = a^{m+n}$
  (b) $a^m \div a^n = a^{m-n}$
  (c) $\left(a^m\right)^n = a^{mn}$
  (d) $a^m \times b^m = (ab)^m$
  (e) $a^0 = 1$
  (f) $\dfrac{a^m}{b^m} = \left(\dfrac{a}{b}\right)^m$

- Very small numbers can be expressed in standard form using negative exponents.
- Use of Exponents to Express Small Number in Standard form:
  (i) Very large and very small numbers can be expressed in standard form.
  (ii) Standard form is also called scientific notation form.
  (iii) A number written as $m \times 10^n$ is said to be in standard form if m is a decimal number such that $1 \leq m < 10$ and n is either a positive or a negative integer.
- Examples: $150,000,000,000 = 1.5 \times 10^{11}$.
- Exponential notation is a powerful way to express repeated multiplication of the same number. For any non-zero rational number 'a' and a natural number n, the product a x a x a x .... x a(n times) = a^n.
  It is known as the nth power of 'a' and is read as '**a' raised to the power n'**. The rational number a is called the base and n is called exponent.

CHAPTER - 12
Exponents and Powers

Ex. 12.1

1. Evaluate:

(i) 3^{-2} **(ii)** $(-4)^{-2}$ **(iii)** $\left(\dfrac{1}{2}\right)^{-5}$

Ans. (i) $3^{-2} = \dfrac{1}{3^2}$

$$\left[\because \ a^{-m} = \dfrac{1}{a^m}\right]$$

$= \dfrac{1}{9}$

(ii) $(-4)^{-2} = \dfrac{1}{(-4)^2}$

$$\left[\because \ a^{-m} = \dfrac{1}{a^m}\right]$$

$= \dfrac{1}{16}$

(iii) $\left(\dfrac{1}{2}\right)^{-5} = \left(\dfrac{2}{1}\right)^{5}$

$$\left[\because \ (a^{-m})^5 = \dfrac{1}{a^m}\right]$$

$= = 32$

2. Simplify and express the result in power notation with positive exponent:

(i) $\left(-4\right)^{5} \div \left(-4\right)^{8}$

(ii) $\left(\dfrac{1}{2^{3}}\right)^{2}$

(iii) $\left(-3\right)^{4} \times \left(\dfrac{5}{3}\right)^{4}$

(iv) $\left(3^{-7} \div 3^{-10}\right) \times 3^{-5}$

(v) $2^{-3} \times \left(-7\right)^{-3}$

Ans. (i) $\left(-4\right)^{5} \div \left(-4\right)^{8} = \left(-4\right)^{5-8}$ $\left[\because \ a^{m} \div a^{n} = a^{m-n}\right]$

$$= \left(-4\right)^{-3} = \dfrac{1}{\left(-4\right)^{3}} \qquad \left[\because \ a^{-m} = \dfrac{1}{a^{m}}\right]$$

(ii) $\left(\dfrac{1}{2^{3}}\right)^{2} = \dfrac{1^{2}}{\left(2^{3}\right)^{2}}$

$$\left[\because \ \left(\dfrac{a}{b}\right)^{m} = \dfrac{a^{m}}{a^{n}}\right]$$

$$= \qquad \left[\because \ \left(a^{m}\right)^{n} = a^{m \times n}\right]$$

(iii) $\left(-3\right)^{4} \times \left(\dfrac{5}{3}\right)^{4} = \left(-3\right)^{4} \times \dfrac{5^{4}}{3^{4}}$ $\left[\because \ \left(\dfrac{a}{b}\right)^{m} = \dfrac{a^{m}}{a^{n}}\right]$

$$\left[\because \ \left(ab\right)^{m} = a^{m}b^{m}\right]$$

$$= 3^{4-4} \times 5^4 \quad \left[\because \ a^m \div a^n = a^{m-n}\right]$$

$$= 3^0 \times 5^4 = 5^4 \left[\because \ a^0 = 1\right]$$

(iv) $\left(3^{-7} \div 3^{-10}\right) \times 3^{-5} = 3^{-7-(-10)} \times 3^{-5} \quad \left[\because \ a^m \div a^n = a^{m-n}\right]$

$$= 3^{-7+10} \times 3^{-5} = 3^3 \times 3^{-5} = 3^{3+(-5)} \quad \left[\because \ a^m \times a^n = a^{m+n}\right]$$

$$= 3^{-2} = \frac{1}{3^2} \quad \left[\because \ a^{-m} = \frac{1}{a^m}\right]$$

(v) $2^{-3} \times (-7)^{-3} = \frac{1}{2^3} \times \frac{1}{(-7)^3} \quad \left[\because \ a^{-m} = \frac{1}{a^m}\right]$

$$= \frac{1}{\{2 \times (-7)\}^3} = \frac{1}{(-14)^3} \quad \left[\because \ (ab)^m = a^m b^m\right]$$

3. Find the value of:

(i) $\left(3^0 + 4^{-1}\right) \times 2^2$

(ii) $\left(2^{-1} \times 4^{-1}\right) \div 2^{-2}$

(iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

(iv) $\left(3^{-1} + 4^{-1} + 5^{-1}\right)^0$

(v) $\left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2$

Ans.

$$\left(1 + \frac{1}{4}\right) \times 2^2 \quad \left[\because \ a^{-m} = \frac{1}{a^m}\right]$$

(i) $\left(3^0 + 4^{-1}\right) \times 2^2 =$

$= \left(\dfrac{4+1}{4}\right) \times 2^2 = \dfrac{5}{4} \times 2^2 = \dfrac{5}{2^2} \times 2^2 = 5 \times 2^{2-2} \quad \left[\because \ a^m \div a^n = a^{m-n}\right]$

$= 5 \times 2^0 = 5 \times 1 = 5 \quad \left[\because \ a^0 = 1\right]$

(ii) $\left(2^{-1} \times 4^{-1}\right) \div 2^{-2} = \left(\dfrac{1}{2^1} \times \dfrac{1}{4^1}\right) \div 2^{-2} \quad \left[\because \ a^{-m} = \dfrac{1}{a^m}\right]$

$= \left(\dfrac{1}{2} \times \dfrac{1}{2^2}\right) \div 2^{-2} = \dfrac{1}{2^3} \div 2^{-2} \quad \left[\because \ a^m \times a^n = a^{m+n}\right]$

$= 2^{-3} \div 2^{-2} = 2^{-3-(-2)} = 2^{-3+2} = 2^{-1} \quad \left[\because \ a^m \div a^n = a^{m-n}\right]$

$= \dfrac{1}{2} \quad \left[\because \ a^{-m} = \dfrac{1}{a^m}\right]$

(iii) $\left(\dfrac{1}{2}\right)^{-2} + \left(\dfrac{1}{3}\right)^{-2} + \left(\dfrac{1}{4}\right)^{-2}$

$= \left(2^{-1}\right)^{-2} + \left(3^{-1}\right)^{-2} + \left(4^{-1}\right)^{-2}$

$\left[\because \ a^{-m} = \dfrac{1}{a^m}\right]$

$= 2^{-1 \times (-2)} + 3^{-1 \times (-2)} + 4^{-1 \times (-2)} \quad \left[\because \ \left(a^m\right)^n = a^{m \times n}\right]$

$= 2^2 + 3^2 + 4^2 = 4 + 9 + 16 = 29$

(iv) $\left(3^{-1} + 4^{-1} + 5^{-1}\right)^0 = \left(\dfrac{1}{3} + \dfrac{1}{4} + \dfrac{1}{5}\right)^0 \quad \left[\because \ a^{-m} = \dfrac{1}{a^m}\right]$

$\left(\dfrac{20+15+12}{60}\right)^0 = \left(\dfrac{47}{60}\right)^0 = 1$

=

$$\left[\because \quad a^{0} = 1\right]$$

(v) $\left\{\left(\dfrac{-2}{3}\right)^{-2}\right\}^{2} = \left(\dfrac{-2}{3}\right)^{-2\times 2}$ $\left[\because \quad \left(a^{m}\right)^{n} = a^{m\times n}\right]$

$= \left(\dfrac{-2}{3}\right)^{-4} = \left(\dfrac{-3}{2}\right)^{4}$ $\left[\because \quad a^{-m} = \dfrac{1}{a^{m}}\right]$

$= \dfrac{81}{16}$

4. Evaluate:

(i) $\dfrac{8^{-1}\times 5^{3}}{2^{-4}}$ **(ii)** $\left(5^{-1}\times 2^{-1}\right)\times 6^{-1}$

Ans. (i)

$\dfrac{8^{-1}\times 5^{3}}{2^{-4}} = \dfrac{\left(2^{3}\right)^{-1}\times 5^{3}}{2^{-4}} = \dfrac{2^{-3}\times 5^{3}}{2^{-4}}$ $\left[\because \quad \left(a^{m}\right)^{n} = a^{m\times n}\right]$

$= 2^{-3-(-4)}\times 5^{3} = 2^{-3+4}\times 5^{3}$ $\left[\because \quad a^{m}\div a^{n} = a^{m-n}\right]$

$= 2\times 125 = 250$

(ii)

$\left(5^{-1}\times 2^{-1}\right)\times 6^{-1} = \left(\dfrac{1}{5}\times \dfrac{1}{2}\right)\times \dfrac{1}{6}$ $\left[\because \quad a^{-m} = \dfrac{1}{a^{m}}\right]$

$= \dfrac{1}{10}\times \dfrac{1}{6} = \dfrac{1}{60}$

5. Find the value of m for which $5^{m}\div 5^{-3} = 5^{5}$. Ans.

$5^{m}\div 5^{-3} = 5^{5}$

$$\Rightarrow \quad 5^{m-(-3)} = 5^5$$

$$\left[\because \quad a^m \div a^n = a^{m-n}\right]$$

$$\Rightarrow \quad 5^{m+3} = 5^5$$

Comparing exponents both sides, we get

$$\Rightarrow \quad m+3 = 5$$

$$\Rightarrow \quad m = 5-3$$

$$\Rightarrow \quad m = 2$$

I. Evaluate:

(i) $\left\{\left(\dfrac{1}{3}\right)^{-1} - \left(\dfrac{1}{4}\right)^{-1}\right\}^{-1}$ **(ii)** $\left(\dfrac{5}{8}\right)^{-7} \times \left(\dfrac{8}{5}\right)^{-4}$

Ans.

(i) $\left\{\left(\dfrac{1}{3}\right)^{-1} - \left(\dfrac{1}{4}\right)^{-1}\right\} = \left\{\left(\dfrac{3}{1}\right)^{1} - \left(\dfrac{4}{1}\right)^{1}\right\} \quad \left[\because \quad a^{-m} = \dfrac{1}{a^m}\right]$

$= \{3-4\} = -1$

(ii) $\left(\dfrac{5}{8}\right)^{-7} \times \left(\dfrac{8}{5}\right)^{-4} = \dfrac{5^{-7}}{8^{-7}} \times \dfrac{8^{-4}}{5^{-4}} \quad \left[\because \quad \left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}\right]$

$= 5^{-7-(-4)} \times 8^{-4-(-7)} \quad \left[\because \quad a^m \div a^n = a^{m-n}\right]$

$= 5^{-7+4} \times 8^{-4+7} = 5^{-3} \times 8^3 = \dfrac{8^3}{5^3} \quad \left[\because \quad a^{-m} = \dfrac{1}{a^m}\right]$

$= \dfrac{512}{125}$

6. Simplify:

(i) $\dfrac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$ $(t \neq 0)$

(ii) $\dfrac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

Ans. (i) $\dfrac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$

$= \dfrac{5^2 \times t^{-4}}{5^{-3} \times 5 \times 2 \times t^{-8}}$

$= \dfrac{5^{2-(-3)-1} \times t^{-4-(-8)}}{2}$

$=$ $\qquad\qquad\qquad =$

$\left[\because \dfrac{5^{2+3-1} \times t^{-4+8}}{a^m \div d^n = a^{m-5^4} \times t^4}{2} \right] \times t^4 \qquad \dfrac{625}{2} t^4$

(ii) $\dfrac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

$= \dfrac{3^{-5} \times (2 \times 5)^{-5} \times 5^3}{5^{-7} \times (2 \times 3)^{-5}}$

$= \dfrac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}}$

$$\left[\because \ (ab)^m = a^m b^m \right]$$

=

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5+3}}{5^{-7} \times 2^{-5} \times 3^{-5}} = \frac{3^{-5-(-5)} \times 2^{-5-(-5)} \times 5^{-2}}{5^{-7} \times 2^{-5} \times 3^{-5}} \left[\because a^m \div a^n = a^{m-n} \quad a^m \times a^n = a^{m+n} \right]$$

$$= 3^{-5+5} \times 2^{-5+5} \times 5^{-2+7} \qquad = 3^0 \times 2^0 \times 5^5$$

$$= 1 \times 1 \times 3125 \left[\because a^0 = 1 \right]$$

$$= 3125$$

Exponents and Powers

Ex. 12.2

1. Express the following numbers in standard form:

(i) 0.0000000000085

(ii) 0.00000000000942

(iii) 6020000000000000

(iv) 0.00000000837

(v) 31860000000

Ans. (i) 0.0000000000085

$= 0.0000000000085 \quad \times \dfrac{10^{12}}{10^{12}} \quad = 8.5 \times 10^{-12}$

(ii) 0.00000000000942

$= 0.00000000000942 \quad \times \dfrac{10^{12}}{10^{12}} = 9.42 \times 10^{-12}$

(iii) 6020000000000000

$= 6020000000000000 \quad \times \dfrac{10^{15}}{10^{15}} = 6.02 \times 10^{15}$

(iv) 0.00000000837

$= 0.00000000837 \quad \times \dfrac{10^{9}}{10^{9}} = 8.37 \times 10^{-9}$

(v) 31860000000

$= 31860000000 \quad \times \dfrac{10^{10}}{10^{10}} = 3.186 \times 10^{10}$

2. Express the following numbers in usual form:

(i) 3.02×10^{-5}

(ii) 4.5×10^{4}

(iii) 3×10^{-8}

(iv) 1.0001×10^{9}

(v) 5.8×10^{12}

(vi) 3.61492×10^{6}

Ans. (i) 3.02 X1/10⁵ = 302/100 X 1 / 100000 = 302/10000000 = 0.0000302.

(ii) $4.5 \times 10^{4} =$ 4.5 ×10000 = 45000

(iii) $3 \times 10^{-8} = \dfrac{3}{10^{8}}$ = 0.00000003

(iv) 1.0001×10^9 = 1000100000

(v) 5.8×10^{12} = 5.8 x 1000000000000

= 5800000000000

(vi) 3.61492×10^6 = 3.61492 X   1000000

= 3614920

3. Express the number appearing in the following statements in standard form:

(i) 1 micron is equal to $\dfrac{1}{1000000}$ **m.**

(ii) Charge of an electron is 0.000,000,000,000,000,000,16 coulomb.

(iii) Size of a bacteria is 0.0000005 m.

(iv) Size of a plant cell is 0.00001275 m.

(v) Thickness if a thick paper is 0.07 mm.

Ans. (i) 1 micron

$= \dfrac{1}{1000000} = \dfrac{1}{10^6} = 1 \times 10^{-6} \text{ m}$

(ii) Charge of an electron is 0.00000000000000000016 coulombs.

$= 0.00000000000000000016 \times \dfrac{10^{19}}{10^{19}}$

$= 1.6 \times 10^{-19}$ **coulomb**

(iii)   Size of bacteria = 0.0000005

$\dfrac{5}{10000000} = \dfrac{5}{10^7} = \quad \times 10^{-7} \quad \text{m}$

(iv)   Size of a plant cell is 0.00001275 m

$\times \dfrac{10^5}{10^5}$

=0.00001275 $= 1.275 \times 10^{-5}$ m

(v) Thickness of a thick paper = 0.07 mm

$= \dfrac{7}{100}$ mm $= \dfrac{7}{10^2} = 7 \times 10^{-2}$ mm

4. In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Ans. Thickness of one book = 20 mm Thickness

of 5 books = 20 $\times$ 5 = 100 mm Thickness of one

paper = 0.016 mm Thickness of 5 papers = 0.016

$\times$ 5

= 0.08 mm

Total thickness of a stack = 100 + 0.08

=100.08 mm

= 100.08 $\times \dfrac{10^2}{10^2}$

$= 1.0008 \times 10^2$ **mm**

---THE END ---