Notes
Chapter – 9
Algebraic Expressions and Identities

- Expressions are formed from **variables** and **constants.**
- **Constant:** A symbol having a fixed numerical value. Example: 2, 23, 2.1, etc.
- **Variable:** A symbol which takes various numerical values. Example: x, y, z, etc.
- **Algebric Expression:** A combination of constants and variables connected by the sign +, -, × and ÷ is called algebraic expression.
- Terms are added to form **expressions**. Terms themselves are formed as product of factors.
- Expressions that contain exactly one, two and three terms are called monomials, binomials and **trinomials** respectively. In general, any expression containing one or more terms with non-zero coefficients (and with variables having non- negative exponents) is called a polynomial.
- **Like** terms are formed from the same variables and the powers of these variables are the same, too. Coefficients of like terms need not be the same.
- While adding (or subtracting) polynomials, first look for like terms and add (or subtract) them; then handle the unlike terms.
- There are number of situations in which we need to multiply algebraic expressions: for example, in finding area of a rectangle, the sides of which are given as expressions.
- **Monomial:** An expression containing only one term. Example: -3, 4x, 3xy, etc.
- **Binomial:** An expression containing two terms. Example: 2x-3, 4x+3y, xy-4, etc.
- **Trinomial:** An expression containing three terms. Example: $2x^2 + 3xy + 9$, 3x+2y+5z, etc.
- **Polynomial:** In general, any expression containing one or more terms with non-zero coefficients (and with variables having non-negative exponents). A polynomial may contain any number of terms, one or more than one.
- A monomial multiplied by a monomial always gives a monomial.
- While multiplying a polynomial by a monomial, we multiply every term in the polynomial by the monomial.
- In carrying out the multiplication of a polynomial by a binomial (or trinomial), we multiply term by term, i.e., every term of the polynomial is multiplied by every term in the binomial (or trinomial). Note that in such multiplication, we may get terms in the product which are like and have to be combined.
- An **identity** is an equality, which is true for all values of the variables in the equality. On the other hand, an equation is true only for certain values of its variables. An equation is not an identity.
- The following are the standard identities:
  $(a + b)^2 = a^2 + 2ab + b^2$ (I)
  $(a – b)^2 = a^2 – 2ab + b^2$ (II)
  $(a + b)(a – b) = a^2 – b^2$ (III)
  Another useful identity is (x + a) (x + b) = x2 + (a + b) x + ab (IV)
- The above four identities are useful in carrying out squares and products of algebraic expressions. They also allow easy alternative methods to calculate products of numbers and so on.
- **Coefficients**: In the term of an expression any of the factors with the sign of the term is called the coefficient of the product of the other factors.
- **Terms**: Various parts of an algebraic expression which are separated by + and – signs. Example: The expression 4x + 5 has two terms 4x and 5.

(i) **Constant Term**: A term of expression having no lateral factor.
(ii) **Like term**: The term having the same literal factors. Example 2xy and -4xy are like terms.
(iii) **Unlike term**: The terms having different literal factors. Example: $4x^2$ and 3xy are unlike terms.

- **Factors:** Each term in an algebraic expression is a product of one or more number (s) and/or literals. These number (s) and/or literal (s) are known as the factor of that term. A constant factor is called numerical factor, while a variable factor is known as a literal factor. The term 4x is the product of its factors 4 and x.

CHAPTER - 9

Algebraic Expressions and Identities (Ex. 9.1)

1. Identify the terms, their coefficients for each of the following expressions

(I) $5xyz^2 - 3zy$

(ii) $1 + x + x^2$

(iii) $4x^2y^2 - 4x^2y^2z^2 + z^2$

(iv) $3 - pq + qr - rp$

(v) $\frac{x}{2} + \frac{y}{2} - xy$

(vi) $0.3a - 0.6ab + 0.5b$

Ans. (i) Terms: $5xyz^2$ and $-3zy$

Coefficient in $5xyz^2$ is 5 and in $-3zy$ is -3.

(ii) Terms: $1, x$ and x^2. Coefficient of x and of x^2 is1.

(iii) Terms: $4x^2y^2, -4x^2y^2z^2$ and z^2.

Coefficient in $4x^2y^2$ is 4, coefficient of $-4x^2y^2z^2$ is -4 and coefficient of z^2 is 1.

(iv) Terms: $3, -pq, qr$ and $-rp$

Coefficient of $-pq$ is -1, coefficient of qr is 1 and coefficient of $-rp$ is -1.

(v) Terms: $\frac{x}{2}, \frac{y}{2}$ and $-xy$

Coefficient of $\frac{x}{2}$ is $\frac{1}{2}$, coefficient of $\frac{y}{2}$ is $\frac{1}{2}$ and coefficient of $-xy$ is -1.

(vi) Terms: $0.3a, -0.6ab$ and $0.5b$

Coefficient of $0.3a$ is 0.3, coefficient of $-0.6ab$ is -0.6 and coefficient of $0.5b$ is 0.5.

2. Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories:

$x+y, 1000, x+x^2+x^3+x^4, 7+y+5x, 2y-3y^2, 2y-3y^2+4y^3, 5x-4y+3xy, 4z-15z^2, ab+bc+cd+da, pqr, p^2q+pq^2, 2p+2q$

Ans. (i) Since $x+y$ contains two terms. Therefore it is binomial.

(ii) Since 1000 contains one terms. Therefore it is monomial.

(iii) Since $x+x^2+x^3+x^4$ contains four terms. Therefore it is a polynomial and it does not fit in above three categories.

(iv) Since $7+y+5x$ contains three terms. Therefore it is trinomial.

(v) Since $2y-3y^2$ contains two terms. Therefore it is binomial.

(vi) Since $2y-3y^2+4y^3$ contains three terms. Therefore it is trinomial.

(vii) Since $5x-4y+3xy$ contains three terms. Therefore it is trinomial.

(viii) Since $4z-15z^2$ contains two terms. Therefore it is binomial.

(ix) Since $ab+bc+cd+da$ contains four terms. Therefore it is a polynomial and it does not fit in above three categories.

(x) Since pqr contains one terms. Therefore it is monomial.

(xi) Since p^2q+pq^2 contains two terms. Therefore it is binomial.

(xii) Since $2p+2q$ contains two terms. Therefore it is binomial.

3. Add the following:

(i) $ab-bc, bc-ca, ca-ab$

(ii) $a-b+ab, b-c+bc, c-a+ac$

(iii) $2p^2q^2-3pq+4, 5+7pq-3p^2q^2$

(iv) $l^2+m^2, m^2+n^2, n^2+l^2+2lm+2mn+2nl$

Ans. (i) $ab-bc, bc-ca, ca-ab$

$$\begin{array}{l} ab-bc \\ \quad +bc-ca \\ -ab \quad +ca \\ \boxed{0+0+0} \end{array}$$

(ii) $a-b+ab, b-c+bc, c-a+ac$

$$\begin{array}{l} a-b-ab \\ \quad +b \quad -c+bc \\ -a \quad +c \quad +ac \\ \boxed{0+0+ab+0+bc+ac} \end{array}$$

Hence the sum if 0.

Hence the sum is $ab+bc+ac$.

(iii) $2p^2q^2-3pq+4,5+7pq-3p^2q^2$

$$\begin{array}{l} 2p^2q^2-3pq+4 \\ -3p^2q^2+7pq+5 \end{array}$$

$$\boxed{-p^2q^2+4pq+9}$$

(iv) $l^2+m^2, m^2+n^2, n^2+l^2, 2lm+2mn+2nl$

$$\begin{array}{l} l^2+m^2 \\ +\quad m^2+n^2 \\ +l^2\quad +n^2 \\ +\quad\quad\quad 2lm+2mn+2nl \end{array}$$

$$\boxed{2l^2+2m^2+2n^2+2lm+2mn+2nl}$$

Hence the sum is

$2\left(l^2+m^2+n^2+lm+mn+nl\right)$.

4. (a) Subtract $4a-7ab+3b+12$ **from** $12a-9ab+5b-3.$

(b) Subtract $3xy+5yz-7zx$ **from** $5xy-2yz-2zx+10xyz.$

(c) Subtract $4p^2q-3pq+5pq^2-8p+7q-10$ **from**

$18-3p-11q+5pq-2pq^2+5p^2q.$

Ans. (a)

$$\begin{array}{l} 12a-9ab+5b-3 \\ \quad 4a-7ab+3b+12 \\ (-)\quad (+)\quad (-)(-) \end{array}$$

$$\boxed{8a-2ab+2b-15}$$

$$\begin{array}{l} 5xy-2yz-2zx+10xyz \\ 3xy+5yz-7zx \\ (-)\quad (-)\quad (+) \end{array}$$

$$\boxed{2xy-7yz+5zx+10xyz}$$

(b)

(c)

$$5p^2q - 2pq^2 + 5pq - 11q - 3p + 18$$
$$4p^2q + 5pq^2 - 3pq + 7q - 8p - 10$$
$$(-) \quad (-) \quad (+) \quad (-) \quad (+) \quad (+)$$
$$\boxed{p^2q - 7pq^2 + 8pq - 18q + 5p + 28}$$

Ex. 9.2

1. Find the product of the following pairs of monomials:

(i) $4, 7p$

(ii) $-4p, 7p$

(iii) $-4p, 7pq$

(iv) $4p^3, -3p$

(iv) $4p, 0$

Ans.

(i) $4 \times 7p = 4 \times 7 \times p = 28p$

(ii) $-4p \times 7p = (-4 \times 7) \times (p \times p)$

$= -28p^2$

(iii) $-4p \times 7pq = (-4 \times 7)(p \times pq)$

$= -28p^2q$

(iv) $4p^3 \times -3p = (4 \times -3)(p^3 \times p)$

$= -12p^4$

(v) $4p \times 0 = (4 \times 0)(p) = 0$

2. Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively:

$(p, q);\ (10m, 5n);\ (20x^2, 5y^2);\ (4x, 3x^2);\ (3mn, 4np)$

Ans.

(i) Area of rectangle

$= length \times breadth$

$= p \times q = pq$ sq. units

(ii) Area of rectangle

$= length \times breadth$

$= 10m \times 5n = (10 \times 5)(m \times n)$

$= 50mn$ sq. units

(iii) Area of rectangle = $length \times breadth$

$= 20x^2 \times 5y^2 = (20 \times 5)(x^2 \times y^2)$

$= 100x^2y^2$ sq. units

(iv) Area of rectangle = $length \times breadth$

$= 4x \times 3x^2 = (4 \times 3)(x \times x^2)$

$= 12x^3$ sq. units

(v) Area of rectangle = $length \times breadth$

= $3mn \times 4np = (3 \times 4)(mn \times np)$

= $12mn^2p$ sq. units

3. Complete the table of products:

(i)

First monomial $\longrightarrow$ **Second monomial** $\downarrow$	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	.....	.....	.....	.....	.....
$-5y$	.....	.....	$-15x^2y$	.....	.....	.....
$3x^2$	.....	.....	.....	.....	.....	.....
$-4xy$	.....	.....	.....	.....	.....	.....
$7x^2y$	.....	.....	.....	.....	.....	.....
$-9x^2y^2$	.....	.....	.....	.....	.....	.....

Ans. (i)s

First monomial $\longrightarrow$ **Second monomial** $\downarrow$	$2x$	$-5y$	$3x^2$	$-4xy$	$7x^2y$	$-9x^2y^2$
$2x$	$4x^2$	$-10xy$	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
$-4xy$	$8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27x^4y^2$	$36x^3y^3$	$-63x^4y^3$	$81x^4y^4$

4. Obtain the volume of rectangular boxes with the following length, breadth and height respectively:

(i) $5a, 3a^2 7a^4$

(ii) $2p, 4q, 8r$

(iii) $xy, 2x^2y, 2xy^2$

(iv) $a, 2b, 3c$

Ans. (i) Volume of rectangular box

$= length \times breadth \times height$

$= 5a \times 3a^2 \times 7a^4 = (5 \times 3 \times 7)(a \times a^2 \times a^4)$

$= 105a^7$ cubic units

(ii) Volume of rectangularbox

$= length \times breadth \times height$

$= 2p \times 4q \times 8r = (2 \times 4 \times 8)(p \times q \times r)$

$= 64pqr$ cubic units

(iii) Volume of rectangularbox

$= length \times breadth \times height$

$= xy \times 2x^2y \times 2xy^2$

$= (1 \times 2 \times 2)(x \times x^2 \times x \times y \times y \times y^2)$

$= 4x^4y^4$ cubic units

(iv) Volume of rectangularbox

$= length \times breadth \times height$

$= a \times 2b \times 3c = (1 \times 2 \times 3)(a \times b \times c)$

$= 6abc$ cubic units

5. Obtain the product of:

(i) xy, yz, zx

(ii) $a, -a^2, a^3$

(iii) $2, 4y, 8y^2, 16y^3$

(iv) $a, 2b, 3c, 6abc$

(v) $m, -mn, mnp$

Ans.

(i) $xy \times yz \times zx = x \times x \times y \times y \times z \times z$

$= x^2 y^2 z^2$

(ii) $a \times (-a^2) \times a^3 = (-1)(a \times a^2 \times a^3)$

$= -a^6$

(iii) $2 \times 4y \times 8y^2 \times 16y^3$

$= (2 \times 4 \times 8 \times 16)(y \times y^2 \times y^3)$

$= 1024y^6$

(iv) $a \times 2b \times 3c \times 6abc$

$= (1 \times 2 \times 3 \times 6)(a \times b \times c \times abc)$

$= 36a^2b^2c^2$

(v) $m \times -mn \times mnp$ =(-1) (m x m x m x n x n x p)

= $-m^3n^2p$

Ex. 9.3

1. Carry out the multiplication of the expressions in each of the following pairs:

(i) $4p, q+r$ (ii) $ab, a-b$ (iii) $a+b, 7a^2b^2$ (iv)

$a^2-9, 4a$

(v) $pq+qr+rp, 0$ Ans.

(i) $4p \times (q+r) = 4p \times q + 4p \times r$

$= 4pq + 4pr$

(ii) $ab \times (a-b) = ab \times a - ab \times b$

$= a^2b - ab^2$

(iii) $(a+b) \times 7a^2b^2 = a \times 7a^2b^2 + b \times 7a^2b^2 = 7a^3b^2 + 7a^2b^3$

(iv) $(a^2-9) \times 4a = a^2 \times 4a - 4a \times 9 = 4a^3 - 36a$

(v) $(pq+qr+rp) \times 0 = pq \times 0 + qr \times 0 + rp \times 0$

= 0 + 0 + 0 = 0

2. Complete the table:

	First expression	Second expression	Product

(i)	a	$b+c+d$	.....
(ii)	$x+y-5$	$5xy$	.....
(iii)	p	$6p^2-7p+5$	.....
(iv)	$4p^2q^2$	p^2-q^2	.....
(v)	$a+b+c$	abc	.....

ANS:

	First expression	Second expression	Product
(i)	a	$b+c+d$	$a(b+c+d)$ $= a\times b+a\times c+a\times d$ $= ab+ac+ad$
(ii)	$x+y-5$	$5xy$	$5xy(x+y-5)$ $= 5xy\times x+5xy\times y-5xy\times 5$ $= 5x^2y+5xy^2-25xy$
(iii)	p	$6p^2-7p+5$	$p(6p^2-7p+5)$ $= p\times 6p^2-p\times 7p+p\times 5$ $= 6p^3-7p^2+5p$
(iv)	$4p^2q^2$	p^2-q^2	$4p^2q^2(p^2-q^2)$ $= 4p^2q^2\times p^2-4p^2q^2\times q^2$ $= 4p^4q^2-4p^2q^4$
(v)	$a+b+c$	abc	$abc(a+b+c)$ $= abc\times a+abc\times b+abc\times c$ $= a^2bc+ab^2c+abc^2$

3. Find the product:

(i) $\left(a^2\right)\times\left(2a^{22}\right)\times\left(4a^{26}\right)$

(ii) $\left(\frac{2}{3}xy\right)\times\left(\frac{-9}{10}x^2y^2\right)$

(iii) $\left(\frac{-10}{3}pq^3\right)\times\left(\frac{6}{5}p^3q\right)$

(iv) $x\times x^2\times x^3\times x^4$ Ans.

(i) $\left(a^2\right)\times\left(2a^{22}\right)\times\left(4a^{26}\right)$

$=\left(2\times4\right)\left(a^2\times a^{22}\times a^{26}\right)$

$=8\times a^{2+22+26}=8a^{50}$

(ii) $\left(\frac{2}{3}xy\right)\times\left(\frac{-9}{10}x^2y^2\right)$

$=\left(\frac{2}{3}\times\frac{-9}{10}\right)\left(x\times x^2\times y\times y^2\right)$

$=\frac{-3}{5}x^3y^3$

(iii) $\left(\frac{-10}{3}pq^3\right)\left(\frac{6}{5}p^3q\right)$

$=\left(\frac{-10}{3}\times\frac{6}{5}\right)\left(p\times p^3\times q^3\times q\right)$

$=-4p^4q^4$

(iv) $x\times x^2\times x^3\times x^4=x^{1+2+3+4}=x^{10}$

4. (a) Simplify: $3x\left(4x-5\right)+3$ **and find values for**

(i) $x=3$

(ii) $x=\frac{1}{2}$.

(b) Simplify: $a\left(a^2+a+1\right)+5$ and find its value for

(i) $a = 0$

(ii) $a = \frac{1}{2}$

(iii) $a = -1$.

Ans. (a) $3x(4x-5)+3$

$= 3x \times 4x - 3x \times 5 + 3$

$= 12x^2 - 15x + 3$

(i) For $x = 3$, $12x^2 + 15x + 3$

$= 12(3)^2 - 15 \times 3 + 3$

$= 12 \times 9 - 45 + 3$ = 108 – 45 + 3 = 66

(ii) For $x = \frac{1}{2}$, $12x^2 - 15x + 3$

$= 12\left(\frac{1}{2}\right)^2 - 15 \times \frac{1}{2} + 3$

$= 12 \times \frac{1}{4} - \frac{15}{2} + 3$

$= 6 - \frac{15}{2} = \frac{12-15}{2} = \frac{-3}{2}$

(b) $a(a^2+a+1)+5$

$= a \times a^2 + a \times a + a \times 1 + 5$

$= a^3 + a^2 + a + 5$

(i) For $a = 0$, $a^3 + a^2 + a + 5$

$= (0)^3 + (0)^2 + (0) + 5$

$= 0 + 0 + 0 + 5 = 5$

(ii) For $a = 1$, $a^3 + a^2 + a + 5$

$= (1)^3 + (1)^2 + (1) + 5$

$= 1 + 1 + 1 + 5 = 8$

(iii) For $a = -1$, $a^3 + a^2 + a + 5$

$= (-1)^3 + (-1)^2 + (-1) + 5$

$= -1 + 1 - 1 + 5 = -2 + 6 = 4$

5. (a) Add: $p(p-q), q(q-r)$ **and** $r(r-p)$.

(b) Add: $2x(z-x-y)$ **and** $2y(z-y-zx)$.

(c) Subtract: $3l(l-4m+5n)$ **from** $4l(10n-3m+2l)$.

(d) Subtract: $3a(a+b+c)-2b(a-b+c)$ **from** $4c(-a+b+c)$.

Ans. (a) $p(p-q)+q(q-r)+r(r-p)$

=

$= p^2 + q^2 + r^2 - pq - qr - rp$

(b) $2x(z-x-y)+2y(z-y-x)$

$= 2xz-2x^2-2xy+2yz-2y^2-2xy$

$= 2xz-2xy-2xy+2yz-2x^2-2y^2$

$= -2x^2-2y^2-4xy+2yz+2zx$

(c) $4l(10n-3m+2l)-3l(l-4m+5n)$

$= 40ln-12lm+8l^2-3l^2+12lm-15ln$

$= 8l^2-3l^2-12lm+12lm+40ln-15ln$

$= 5l^2+25ln$

(d) $4c(-a+b+c)-[3a(a+b+c)-2b(a-b+c)]$

$= -4ac+4bc+4c^2-[3a^2+3ab+3ac-2ab+2b^2-2bc]$

$= -4ac+4bc+4c^2-[3a^2+2b^2+3ab-2bc+3ac-2ab]$

$= -4ac+4bc+4c^2-[3a^2+2b^2+ab-2bc+3ac]$

$= -4ac+4bc+4c^2-3a^2-2b^2-ab-3ac+2bc$

$= -3a^2-2b^2+4c^2-ab+4bc+2bc-4ac-3ac$

$= -3a^2-2b^2+4c^2-ab+6bc-7ac$

Ex. 9.4

1. Multiply the binomials:

(i) (2x+5) and (4x-3)

(ii) $(y-8)$ and $(3y-4)$

(iii) $(2.5l-0.5m)$ and $(2.5l+0.5m)$

(iv) $(a+3b)$ and $(x+5)$

(v) $(2pq+3q^2)$ and $(3pq-2q^2)$

(vi) $\left(\frac{3}{4}a^2+3b^2\right)$ and $4\left(a^2-\frac{2}{3}b^2\right)$

Ans.

(i) $(2x+5)\times(4x-3)$

$= 2x(4x-3)+5(4x-3)$

$= 2x\times 4x-2x\times 3+5\times 4x-5\times 3$

$= 8x^2-6x+20x-15$

$= 8x^2+14x-15$

(ii) $(y-8)\times(3y-4) = y(3y-4)-8(3y-4)$

$= y\times 3y-y\times 4-8\times 3y-8\times -4$

$= 3y^2 - 4y - 24y + 32$

$= 3y^2 - 28\ y + 32$

(iii) $(2.5l-0.5m)\times(2.5l+0.5m)$

$= 2.5l\times(2.5l+0.5m)-0.5m\times(2.5l+0.5m)$

= 2.5l x 2.5l + 2.5l x 0.5m - 0.5m x 2.5l - 0.5m x0.5m

$= 6.25l^2+1.25lm-1.25lm-0.25m^2$

$= 6.25l^2-0.25m^2$

(iv) $(a+3b)\times(x+5)=a(x+5)+3b(x+5)$

$= a\times x+a\times 5+3b\times x+3b\times 5$

$= ax+5a+3bx+15b$

(v) $(2pq+3q^2)(3pq-2q^2)$

$= 2pq\times(3pq-2q^2)+3q^2(3pq-2q^2)$

$= 2pq\times 3pq-2pq\times 2q^2+3q^2\times 3pq-3q^2\times 2q^2$

$= 6p^2q^2-4pq^3+9pq^3-6q^4$

$= 6p^2q^2+5pq^3-6q^4$

(vi) $\left(\frac{3}{4}a^2+3b^2\right)\times 4\left(a^2-\frac{2}{3}b^2\right)$

$=\left(\frac{3}{4}a^2+3b^2\right)\times\left(4a^2-\frac{8}{3}b^2\right)$

$=\frac{3}{4}a^2\times\left(4a^2-\frac{8}{3}b^2\right)+3b^2\times\left(4a^2-\frac{8}{3}b^2\right)$

$= \frac{3}{4}a^2\times 4a^2-\frac{3}{4}a^2\times\frac{8}{3}b^2+3b^2\times 4a^2-3b^2\times\frac{8}{3}b^2$

$= 3a^4-2a^2b^2+12a^2b^2-8b^4$

$= 3a^4+10a^2b^2-8b^4$

2. Find the product:

(i) $(5-2x)(3+x)$

(ii) $(x+7y)(7x-y)$

(iii) $\left(a^2+b\right)\left(a+b^2\right)$

(iv) $\left(p^2-q^2\right)(2p+q)$ Ans. (i)

$(5-2x)(3+x)$

$=5\times(3+x)-2x(3+x)$

$= 5\times3+5\times x-2x\times3-2x\times x$

$= 15+5x-6x-2x^2 = 15-x-2x^2$

(ii) $(x+7y)(7x-y)$

$=x(7x-y)+7y\times(7x-y)$

$= x\times7x-x\times y+7y\times7x-7y\times y$

$= 7x^2-xy+49xy-7y^2$

$= 7x^2+48xy-7y^2$

(iii) $\left(a^2+b\right)\left(a+b^2\right)$

$=a^2\times\left(a+b^2\right)+b\times\left(a+b^2\right)$

$= a^2\times a+a^2\times b^2+b\times a+b\times b^2$

$= a^3+a^2b^2+ab+b^3$

(iv) $\left(p^2-q^2\right)(2p+q)$

$=p^2\times(2p+q)-q^2(2p+q)$

=

$p^2 \times 2p + p^2 \times q - q^2 \times 2p - q^2 \times q$

= $2p^3 + p^2q - 2pq^2 - q^3$

3. Simplify:

(i) $(x^2 - 5)(x+5) + 25$

(ii) $(a^2 + 5)(b^2 + 3) + 5$

(iii) $(t + s^2)(t^2 - s)$

(iv) $(a+b)(c-d) + (a-b)(c+d) + 2(ac+bd)$

(v) $(x+y)(2x+y) + (x+2y)(x-y)$

(vi) $(x+y)(x^2 - xy + y^2)$

(vii) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$

(viii) $(a+b+c)(a+b-c)$

Ans. (i) $(x^2 - 5)(x+5) + 25$

$= x^2(x+5) - 5(x+5) + 25$

= $x^2 \times x + x^2 \times 5 - 5 \times x - 5 \times 5 + 25$

= $x^3 + 5x^2 - 5x - 25 + 25$

= $x^3 + 5x^2 - 5x$

(ii) $(a^2 + 5)(b^3 + 3) + 5$

$= a^2(b^3 + 3) + 5(b^3 + 3) + 5$

$= \quad a^2 \times b^3 + a^2 \times 3 + 5 \times b^3 + 5 \times 3 + 5$

$= a^2b^3 + 3a^2 + 5b^3 + 15 + 5$

$= \quad a^2b^3 + 3a^2 + 5b^3 + 20$

(iii) $\left(t + s^2\right)\left(t^2 - s\right) = t\left(t^2 - s\right) + s^2\left(t^2 - s\right)$

$= \quad t \times t^2 - t \times s + s^2 \times t^2 - s^2 \times s$

$= \quad t^3 - st + s^2t^2 - s^3$

(iv) $\left(a + b\right)\left(c - d\right) + \left(a - b\right)\left(c + d\right) + 2\left(ac + bd\right)$

$= a\left(c - d\right) + b\left(c - d\right) + a\left(c + d\right) - b\left(c + d\right) + 2ac + 2bd$

$= ac - ad + bc - bd + ac + ad - bc - bd + 2ac + 2bd$

$= ac + ac - ad + ad + bc - bc - bd - bd + 2ac + 2bd$

$= 2ac - 2bd + 2ac + 2bd$

$= \quad 4ac$

(v) $\left(x + y\right)\left(2x + y\right) + \left(x + 2y\right)\left(x - y\right)$

$= x\left(2x + y\right) + y\left(2x + y\right) + x\left(x - y\right) + 2y\left(x - y\right)$

$= \quad 2x^2 + xy + 2xy + y^2 + x^2 - xy + 2xy - 2y^2$

$= 2x^2 + x^2 + xy + 2xy - xy + 2xy + y^2 - 2y^2$

$= 3x^2 + 4xy - y^2$

(vi) $(x+y)(x^2 - xy + y^2)$

$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$

$= x^3 - x^2y + x^2y + xy^2 - xy^2 + y^3$

$= x^3 + y^3$

(vii) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$

$= 1.5x(1.5x + 4y + 3) - 4y(1.5x + 4y + 3) - 4.5x + 12y$

$= 2.25x^2 + 6.0xy + 4.5x - 6.0xy - 16y^2 - 12y - 4.5x + 12y$

$= 2.25x^2 + 6.0xy - 6.0xy + 4.5x - 4.5x - 16y^2 - 12y + 12y$

$= 2.25x^2 - 16y^2$

(viii) $(a+b+c)(a+b-c)$

$= a(a+b-c) + b(a+b-c) + c(a+b-c)$

$= a^2 + ab - ac + ab + b^2 - bc + ac + bc - c^2$

$= a^2 + ab + ab - ac + ac - bc + bc + b^2 - c^2$

$= a^2 + b^2 - c^2 + 2ab$

Ex. 9.5

1. Use a suitable identity to get each of the following products:

(i) $(x+3)(x+3)$

(ii) $(2y+5)(2y+5)$

(iii) $(2a-7)(2a-7)$

(iv) $\left(3a-\frac{1}{2}\right)\left(3a-\frac{1}{2}\right)$

(v) $(1.1m-0.4)(1.1m+0.4)$

(vi) $(a^2+b^2)(-a^2+b^2)$

(vii) $(6x-7)(6x+7)$

(viii) $(-a+c)(-a+c)$

(ix) $\left(\frac{x}{2}+\frac{3y}{4}\right)\left(\frac{x}{2}+\frac{3y}{4}\right)$

(x) $(7a-9b)(7a-9b)$

Ans. (i) $(x+3)(x+3)=(x+3)^2$

$=(x)^2+2\times x\times 3+(3)^2$

[Using identity $(a+b)^2=a^2+2ab+b^2$

$=x^2+6x+9$

(ii) $(2y+5)(2y+5)=(2y+5)^2$

$=(2y)^2+2\times 2y\times 5+(5)^2$

[Using identity $(a+b)^2=a^2+2ab+b^2$]

$=4y^2+20y+25$

(iii) $(2a-7)(2a-7)=(2a-7)^2$

$= (2a)^2 - 2\times 2a\times 7 + (7)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= 4a^2 - 28a + 49$

(iv) $\left(3a-\frac{1}{2}\right)\left(3a-\frac{1}{2}\right)=\left(3a-\frac{1}{2}\right)^2$

$= (3a)^2 - 2\times 3a\times \frac{1}{2} + \left(\frac{1}{2}\right)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$

$= 9a^2 - 3a + \frac{1}{4}$

(v) $(1.1m-0.4)(1.1m+0.4)=(1.1m)^2-(0.4)^2$

[Using identity $(a-b)(a+b)=a^2-b^2$]

$= 1.21m^2 - 0.16$

(vi) $(a^2+b^2)(-a^2+b^2)=(b^2+a^2)(b^2-a^2)$

$= (b^2)^2 - (a^2)^2$

[Using identity $(a-b)(a+b)=a^2-b^2$

$= b^4 - a^4$

(vii) $(6x-7)(6x+7)=(6x)^2-(7)^2$

[Using identity $(a-b)(a+b)=a^2-b^2$]

$= 36x^2 - 49$

(viii) $(-a+c)(-a+c)$

$(c-a)(c-a) = (c-a)^2$

$= (c)^2 - 2 \times c \times a + (a)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= c^2 - 2ca + a^2$

(ix) $\left(\frac{x}{2} + \frac{3y}{4}\right)\left(\frac{x}{2} + \frac{3y}{4}\right) = \left(\frac{x}{2} + \frac{3y}{4}\right)^2$

$= \left(\frac{x}{2}\right)^2 + 2 \times \frac{x}{2} \times \frac{3y}{4} + \left(\frac{3y}{4}\right)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

$= \frac{x^2}{4} + \frac{3}{4}xy + \frac{9}{16}y^2$

(x) $(7a-9b)(7a-9b) = (7a-9b)^2$

$= (7a)^2 - 2 \times 7a \times 9b + (9b)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

$= 49a^2 - 126ab + 81b^2$

2. Use the identity $(x+a)(x+b) = x^2 + (a+b)x + ab$ to find the following products:

(i) $(x+3)(x+7)$

(ii) $(4x+5)(4x+1)$

(iii) $(4x-5)(4x-1)$

(iv) $(4x+5)(4x-1)$

(v) $(2x+5y)(2x+3y)$

(vi) $(2a^2+9)(2a^2+5)$

(vii) $(xyz-4)(xyz-2)$

Ans. (i) $(x+3)(x+7)$

$=(x)^2+(3+7)x+3\times 7$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$=x^2+10x+21$

(ii) $(4x+5)(4x+1)$

$=(4x)^2+(5+1)4x+5\times 1$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$=16x^2+6\times 4x+5=16x^2+24x+5$

(iii) $(4x-5)(4x-1)$

$=(4x)^2+(-5-1)4x+(-5)\times(-1)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$=16x^2+(-6)\times 4x+5=16x^2-24x+5$

(iv) $(4x+5)(4x-1)$

= $(4x)^2+\{5+(-1)\}(4x)+(5)(-1)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

= $16x^2+(5-1)\times 4x-5$

= $16x^2+4\times 4x-5$

= $16x^2+16x-5$

(v) $(2x+5y)(2x+3y)$

$=(2x)^2+(5y+3y)\times 2x+5y\times 3y$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

= $4x^2+8y\times 2x+15y^2$

= $4x^2+16xy+15y^2$

$(2a^2+9)(2a^2+5)$

(vi)

$=(2a^2)^2+(9+5)\times 2a^2+9\times 5$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$

= $4a^4+14\times 2a^2+45$

= $4a^4+28a^2+45$

(vii) $(xyz-4)(xyz-2)$

$=(xyz)^2+(-4-2)\times xyz+(-4)\times(-2)$

[Using identity $(x+a)(x+b)=x^2+(a+b)x+ab$]

$= x^2y^2z^2-6xyz+8$

3. Find the following squares by using identities:

(i) $(b-7)^2$

(ii) $(xy+3z)^2$

(iii) $(6x^2-5y)^2$

(iv) $\left(\frac{2}{3}m+\frac{3}{2}n\right)^2$

(v) $(0.4p-0.5q)^2$

(vi) $(2xy+5y)^2$

Ans. (i) $(b-7)^2=(b)^2-2\times b\times 7+(7)^2$

[Using identity $(a-b)^2=a^2-2ab+b^2$

$= b^2-14b+49$

(ii) $(xy+3z)^2=(xy)^2+2\times xy\times 3z+(3z)^2$

[Using identity $(a+b)^2=a^2+2ab+b^2$

$=x^2y^2+6xyz+9z^2$

(iii) $(6x^2-5y)^2$

$=(6x^2)^2-2\times 6x^2\times 5y+(5y)^2$

[Using identity $(a-b)^2=a^2-2ab+b^2$

$=36x^4-60x^2y+25y^2$

(iv) $\left(\frac{2}{3}m+\frac{3}{2}n\right)^2$

$=\left(\frac{2}{3}m\right)^2+2\times\frac{2}{3}m\times\frac{3}{2}n+\left(\frac{3}{2}n\right)^2$

[Using identity $(a+b)^2=a^2+2ab+b^2$]

$=\frac{4}{9}m^2+2mn+\frac{9}{4}n^2$

(v) $(0.4p-0.5q)^2$

$=(0.4p)^2-2\times 0.4p\times 0.5q+(0.5q)^2$

[Using identity $(a-b)^2=a^2-2ab+b^2$

$=0.16p^2-0.40pq+0.25q^2$

(vi) $(2xy+5y)^2$

$=(2xy)^2+2\times 2xy\times 5y+(5y)^2$

[Using identity $(a+b)^2=a^2+2ab+b^2$]

$=4x^2y^2+20xy^2+25y^2$

4. Simplify:

(i) $(a^2-b^2)^2$

(ii) $(2x+5)^2-(2x-5)^2$

(iii) $(7m-8n)^2+(7m+8n)^2$

(iv) $(4m+5n)^2+(5m+4n)^2$

(v) $(2.5p-1.5q)^2-(1.5p-2.5q)^2$

(vi) $(ab+bc)^2-2ab^2c$

(vii) $(m^2-n^2m)^2+2m^3n^2$

Ans. (i) $(a^2-b^2)^2$

$=(a^2)^2-2\times a^2\times b^2+(b^2)^2$

[Using identity $(a-b)^2=a^2-2ab+b^2$

$=a^4-2a^2b^2+b^4$

(ii)

$(2x+5)^2-(2x-5)^2$

= {(2x+5)+(2x-5)}{(2x+5)-(2x-5)}

[Using identity $(a^2-b^2) = (a + b)(a - b)$]

= {4x}{2x+5-2x+5}

= (4 x) (10)

= 40 x

(iii) $(7m-8n)^2+(7m+8n)^2$

$=(7m)^2-2\times 7m\times 8n+(8n)^2+\left[(7m)^2+2\times 7m\times 8n+(8n)^2\right]$

[Using identities $(a+b)^2=a^2+2ab+b^2$ and $(a-b)^2=a^2-2ab+b^2$]

$=49m^2-112mn+64n^2+\left[49m^2+112mn+64n^2\right]$

$=49m^2-112mn+64n^2+49m^2+112mn+64n^2$

$=98m^2+128n^2$

(iv) $(4m+5n)^2+(5m+4n)^2$

$=(4m)^2+2\times 4m\times 5n+(5n)^2+(5m)^2+2\times 5m\times 4n+(4n)^2$

[Using identity $(a+b)^2=a^2+2ab+b^2$]

$=16m^2+40mn+25n^2+25m^2+40mn+16n^2$

$=16m^2+25m^2+40mn+40mn+25n^2+16n^2$

$=41m^2+80mn+41n^2$

(v)

$$(2.5p-1.5q)^2-(1.5p-2.5q)^2$$

$$=(2.5p)^2-2\times 2.5p\times 1.5q+(1.5q)^2-\left[(1.5p)^2-2\times 1.5p\times 2.5q+(2.5q)^2\right]$$ [Using identity $(a-b)^2=a^2-2ab+b^2$]

$$=6.25p^2-7.50pq+2.25q^2-\left[2.25p^2-7.50pq+6.25q^2\right]$$

$$=6.25p^2-7.50pq+2.25q^2-2.25p^2+7.50pq-6.25q^2$$

$$=4p^2-4q^2$$

(vi) $(ab+bc)^2-2ab^2c=(ab)^2+2\times ab\times bc+(bc)^2-2ab^2c$

[Using identity $(a+b)^2=a^2+2ab+b^2$]

$$=a^2b^2+2ab^2c+b^2c^2-2ab^2c$$

$$=a^2b^2+b^2c^2$$

(vii) $(m^2-n^2m)^2+2m^3n^2$

$$=(m^2)^2-2\times m^2\times n^2m+(n^2m)^2+2m^3n^2$$

[Using identity $(a-b)^2=a^2-2ab+b^2$]

$$=m^4-2m^3n^2+n^4m^2+2m^3n^2$$

$$=m^4+n^4m^2$$

5. Show that:

(i) $(3x+7)^2-84x=(3x-7)^2$

(ii) $(9p-5q)^2+180pq=(9p+5q)^2$

$$\left(\frac{4}{3}m-\frac{3}{4}n\right)^2+2mn=\frac{16}{9}m^2+\frac{9}{16}n^2$$

(iii)

(iv) $(4pq+3q)^2-(4pq-3q)^2=48pq^2$

(v) $(a-b)(a+b)+(b-c)(b+c)+(c-a)(c+a)=0$

Ans. (i) L.H.S. = $(3x+7)^2-84x$

$=(3x)^2+2\times 3x\times 7+(7)^2-84x$

[Using identity $(a+b)^2=a^2+2ab+b^2$]

$=9x^2+42x+49-84x$

$=9x^2-42x+49$

$=(3x-7)^2$ [$\because (a-b)^2=a^2-2ab+b^2$]

= R.H.S.

(ii) L.H.S. = $(9p-5q)^2+180pq$

$=(9p)^2-2\times 9p\times 5q+(5q)^2+180pq$

[Using identity $(a-b)^2=a^2-2ab+b^2$]

$=81p^2-90pq+25q^2+180pq$

$=81p^2+90pq+25q^2$

$= (9p+5q)^2 \because (a+b)^2 = a^2+2ab+b^2$ [ ]

(iii) L.H.S. $= \left(\frac{4}{3}m-\frac{3}{4}n\right)^2+2mn$

$=\left(\frac{4}{3}m\right)^2-2\times\frac{4}{3}m\times\frac{3}{4}n+\left(\frac{3}{4}n\right)^2+2mn$

[Using identity $(a-b)^2 = a^2-2ab+b^2$]

$= \frac{16}{9}m^2-2mn+\frac{9}{16}n^2+2mn$

$= \frac{16}{9}m^2+\frac{9}{16}n^2$

= R.H.S.

(iv) L.H.S. $= (4pq+3q)^2-(4pq-3q)^2$

$= (4pq)^2+2\times4pq\times3q+(3q)^2-\left[(4pq)^2-2\times4pq\times3q+(3q)^2\right]$ [Using identities $(a+b)^2 = a^2+2ab+b^2$ and $(a-b)^2 = a^2-2ab+b^2$]

$= 16p^2q^2+24pq^2+9q^2-\left[16p^2q^2-24pq^2+9q^2\right] =$

$16p^2q^2+24pq^2+9q^2-16p^2q^2+24pq^2-9q^2 = 48pq^2$

= R.H.S.

(v) L.H.S.$= (a-b)(a+b)+(b-c)(b+c)+(c-a)(c+a) = a^2-b^2+b^2-c^2+c^2-a^2$

[Using identity $(a-b)(a+b)=a^2-b^2$]

= 0

= R.H.S.

6. Using identities, evaluate:

(i) 71^2

(ii) 99^2

(iii) 102^2

(iv) 998^2

(v) 5.2^2

(VI) 297×303

(VII) 78×82

(VIII) 8.9^2

(ix) $10.5 \times \quad 9.5$

Ans. (i) $71^2 = (70+1)^2$

$= (70)^2 + 2 \times 70 \times 1 + (1)^2$

[Using identity $(a+b)^2 = a^2 - 2ab + b^2$]

= 4900 + 140 + 1 = 5041

(ii) $99^2 = (100-1)^2$

$= (100)^2 - 2 \times 100 \times 1 + (1)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$]

= 10000 – 200 + 1 = 9801

(iii) $102^2 = (100+2)^2$

$= (100)^2 + 2\times100\times2 + (2)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

= 10000 + 400 + 4 = 10404

(iv) $998^2 = (1000-2)^2$

$= (1000)^2 - 2\times1000\times2 + (2)^2$

[Using identity $(a-b)^2 = a^2 - 2ab + b^2$

= 1000000 – 4000 + 4 = 996004

(v) $5.2^2 = (5+0.2)^2$

$= (5)^2 + 2\times5\times0.2 + (0.2)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

= 25 + 2.0 + 0.04 = 27.04

(vi) 297×303

$= (300 - 3)\times(300 + 3)$

$= (300)^2 - (3)^2$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

= 90000 – 9 = 89991

(vii) $78\times82 = (80 - 2)\times(80 + 2)$

$= (80)^2 - (2)^2$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

= 6400 – 4 = 6396

(viii) $8.9^2 = (8+0.9)^2$

$= (8)^2 + 2\times8\times0.9 + (0.9)^2$

[Using identity $(a+b)^2 = a^2 + 2ab + b^2$]

= 64 + 14.4 + 0.81 = 79.21

(ix) $10.5 \text{ x } 9.5 = (10 + 0.5)\times(10 - 0.5)$

$= (10)^2 - (0.5)^2$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

= 100 – 0.25 = 99.75

7. Using $a^2 - b^2 = (a+b)(a-b)$, find

(i) $51^2 - 49^2$

(ii) $(1.02)^2 - (0.98)^2$

(iii) $153^2 - 147^2$

(iv) $12.1^2 - 7.9^2$

Ans. (i) $51^2 - 49^2 = (51+49)(51-49)$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

$= 100 \times 2 = 200$

(ii) $(1.02)^2 - (0.98)^2$

$= (1.02+0.98)(1.02-0.98)$

[Using identity $(a-b)(a+b) = a^2 - b^2$

$= 2.00 \times 0.04 = 0.08$

(iii) $153^2 - 147^2 = (153+147)(153-147)$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

$= 300 \times 6 = 1800$

(iv) $12.1^2 - 7.9^2 = (12.1+7.9)(12.1-7.9)$

[Using identity $(a-b)(a+b) = a^2 - b^2$]

$= 20.0 \times 4.2 = 84.0 = 84$

8. Using $(x+a)(x+b) = x^2 + (a+b)x + ab$, find

(i) 103×104

(ii) 5.1×5.2

(iii) 103×98

(iv) 9.7×9.8

Ans. (i) $103 \times 104 = (100 + 3) \times (100 + 4)$

$= (100)^2 + (3+4) \times 100 + 3 \times 4$

[Using identity $(x+a)(x+b) = x^2 + (a+b)x + ab$]

$= 10000 + 7 \times 100 + 12$

= 10000 + 700 + 12 = 10712

(ii) $5.1 \times 5.2 = (5 + 0.1) \times (5 + 0.2)$

$= (5)^2 + (0.1 + 0.2) \times 5 + 0.1 \times 0.2$

[Using identity $(x+a)(x+b) = x^2 + (a+b)x + ab$]

$= 25 + 0.3 \times 5 + 0.02$

= 25 + 1.5 + 0.02 = 26.52

(iii) $103 \times 98 = (100 + 3) \times (100 - 2)$

$= (100)^2 + (3-2) \times 100 + 3 \times (-2)$

[Using identity $(x+a)(x+b) = x^2 + (a+b)x + ab$]

$= 10000 + (3-2) \times 100 - 6$

= 10000 + 100 – 6 = 10094

(iv) $9.7 \times 9.8 = (10 - 0.3) \times (10 - 0.2)$

$= (10)^2 + \{(-0.3) + (-0.2)\} \times 10 + (-0.3) \times (-0.2)$ [Using identity $(x+a)(x+b) = x^2 + (a+b)x + ab$]

$100 + \{-0.3 - 0.2\} \times 10 + 0.06$

=

= $100 - 0.5 \times 10 + 0.06$

= 100 – 5 + 0.06 = 95.06