पु॰ना International School

Shree Swaminarayan Gurukul, Zundal

Grade VIII

MATHEMATICS

Specimen Copy

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INDEX

CHAPTER NO: 1

CHAPTER NAME : RATIONAL NUMBERS

KEY POINTS TO REMEMBER:

(1) Natural Numbers:-Counting numbers starting from 1 are known as Natural numbers , and denoted by N. i.e. N = {1,2,3,4,5,………}

(2)Whole Numbers: All natural numbers together with 0 are called whole Numbers and denoted by W.

i.e. W = { 0 , 1, 2, 3, 4, 5, ……….}

(3)Integers :All natural numbers and negative of natural numbers including 0 are called Integers.

i.e. ………….−5,−4,−3,−2, −1 , 0, 1,2,3,4,5,……..etc. are all integers.

(4) Rational Numbers:The numbers of the form $\frac{a}{b}$, where a and b are integers and b≠0 , are called rational numbers.

(5) Properties of Rational numbers:

(i) Closure

(ii) Commutativity

(iii)Associativity

(iv) Distributivity of multiplication over addition

(v) The role of zero

(vi) The role of 1

(vii) Reciprocal

(6) Representation of Rational numbers on the Number line

CHAPTER NO. – 1

CHAPTER NAME – RATIONAL NUMBERS

KEY POINTS TO REMEMBER –

Rational Number: A number is called rational if we can write the number in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$
i.e., $1 = \frac{1}{1}$, $2 = \frac{2}{1}$, $0 = \frac{0}{1}$ and $\frac{5}{8}$, $\frac{-3}{14}$, $\frac{-7}{15}$ are all rational numbers.

Between two rational numbers x and y, there exists a rational number $\frac{x+y}{2}$

$\frac{-x}{y}$ is called the additive inverse of $\frac{x}{y}$ and vice-versa.

$\frac{y}{x}$ is called the multiplicative inverse or reciprocal of $\frac{x}{y}$.

Rational number 0 is the additive identity for all rational numbers because a number does not change when 0 is added to it.

Rational number 1 is the multiplicative identity for all rational numbers because on multiplying a rational number with 1, its value does not change.

Rational numbers can be represented on a number line.

- **Properties on Rational Numbers**

(i) Closure Property

Rational numbers are closed under :

- **Addition**

eg. $\frac{3}{5} + \frac{(-4)}{9} = \frac{27-20}{45} = \frac{7}{45}$

which is a rational number.

- **Subtraction**

  $\frac{5}{8} - \frac{3}{7} = \frac{35-24}{56} = \frac{11}{56}$

  $\frac{2}{5} - \frac{3}{4} = \frac{8-15}{20} = \frac{-7}{20}$

  are rational numbers.

- **Multiplication:**

  $\frac{-5}{7} \times \frac{2}{9} = \frac{-10}{63}$

$$\frac{2}{3}\times\frac{5}{11} = \frac{10}{33}$$

are rational numbers.

Rational numbers are closed under addition subtraction and multiplication.

- **Division**:eq$\frac{-3}{5}\div \frac{2}{3}=\frac{-9}{10}$, which is also a rational number. For any rational number a, a ÷ 0 is not defined. So, rational number are not closed under division.

However, if we exclude zero then the rational numbers are closed under division.

(ii) Commutativity:

Addition: Two rational numbers can be added in any order, i.e., commutativity holds for rational numbers under addition, i.e., for any two rational number a and b, a + b = b + a.

$$\frac{-3}{4}+\frac{5}{11} = \frac{-13}{44}$$

$$\frac{5}{11}+\left(\frac{-3}{4}\right) = -\frac{13}{44}$$

- Subtraction:

$$\frac{2}{5}-\frac{5}{6} = \frac{12-25}{30} = \frac{-13}{30}$$

$$\frac{5}{6}-\frac{2}{5} = \frac{25-12}{20} = \frac{13}{30}$$

Hence, subtraction is not associative for rational numbers.

- **(iii) Multiplication**: Multiplication is commutative for rational numbers.
- In general, a × b = b × a, for any two rational numbers a and b.

$$\frac{-3}{4}\times\frac{5}{6} = \frac{5}{6}\times\left(\frac{-3}{4}\right) = \frac{-15}{24}$$

eg. $$\frac{-2}{3}\times\left(\frac{2}{5}\times\frac{6}{7}\right) = \frac{-2}{3}\times\frac{12}{35} = \frac{-24}{105} = \frac{-8}{35}$$

and $$\left(\frac{-2}{3}\ 2\right)\ 6 \quad -4\ \ 6 \quad -24 \quad -8$$

$$\frac{-3}{7}\div\frac{2}{5} = \frac{-3}{7}\times\frac{5}{2} = \frac{-15}{14}$$

$$\frac{2}{5}\div\left(\frac{-3}{7}\right) = \frac{2}{5}\times\frac{7}{-3} = \frac{14}{-15}$$

$$\frac{-3}{7}\div\frac{2}{5} \neq \frac{2}{5}\div\left(\frac{-3}{7}\right)$$

Hence, division is not commutative for rational numbers.

iii) Associativity:

Addition:

$$eg.\ \frac{-2}{5}+\left[\frac{3}{4}+\left(\frac{-7}{8}\right)\right] = \frac{-2}{5}+\left(\frac{-1}{8}\right) = \frac{-21}{40} = \left[\frac{-2}{5}+\frac{3}{4}\right]+\left(\frac{-7}{8}\right) = \frac{7}{20}-\frac{7}{8} = \frac{-21}{40}$$

So, addition is associative for rational numbers, i.e., for any three rational numbers a, b and c, a + (b + c) = (a + b) + c.

Subtraction:

$$eg.\quad \frac{-3}{4}-\left[-\frac{5}{6}-\frac{2}{3}\right] = \frac{-3}{4}-\left(\frac{-9}{6}\right) = \left(-\frac{9}{6}\right) = \frac{9}{12} = \frac{3}{4}$$

and

$$\left[\frac{-3}{4}-\left(\frac{-5}{6}\right)\right]-\frac{2}{3} = \frac{1}{12}-\frac{2}{3} = \frac{-7}{12}$$

i.e., $\frac{3}{4} \neq \frac{-7}{12}$.

Hence, subtraction is not associative for rational numbers.

Multiplication:

So, multiplication is associative for rational number, i.e., for any three rational numbers a, b and c, a × (b × c) = (a × b) × c.

$$eg.\quad \frac{-2}{3}\times\left(\frac{2}{5}\times\frac{6}{7}\right) = \frac{-2}{3}\times\frac{12}{35} = \frac{-24}{105} = \frac{-8}{35}$$

and

$$\left(\frac{-2}{3}\times\frac{2}{5}\right)\times\frac{6}{7} = \frac{-4}{15}\times\frac{6}{7} = \frac{-24}{105} = \frac{-8}{35}$$

Division:

eg. $\frac{2}{3} \div \left[\frac{-1}{2} \div \frac{2}{5}\right] = \left[\frac{2}{3} \div \left(\frac{-1}{2}\right)\right] \div \frac{2}{5}$

We have, LHS $\frac{2}{3} \div \left[-\frac{1}{2} \div \frac{2}{5}\right] = \left[\frac{2}{3} \div \frac{1}{2} \times \frac{5}{2}\right]$

$= \frac{2}{3} \div \left(\frac{-5}{4}\right) = \frac{2}{3} \times \frac{4}{-5} = \frac{8}{-15}$

RHS $\left[\frac{2}{3} \div \left(\frac{-1}{2}\right)\right] \div \frac{2}{5} = \left[\frac{2}{3} \times \left(\frac{-2}{1}\right)\right] \div \frac{2}{5} = \left(\frac{-4}{3}\right) \div \frac{2}{5}$

$= \frac{-4}{3} \times \frac{5}{2} = \frac{-20}{6} = \frac{-10}{3}$

But LHS $\neq$ RHS

Hence, the division is not associative for rational numbers.

Distributivity of multiplication over addition for rational number:

For all rational numbers a, b and c, a(b + c) = ab + ac

eg. Let $\frac{-2}{5}, \frac{3}{8}$ and $\frac{-3}{4}$ are any three rational numbers, then

$\frac{-2}{5} \times \left\{\frac{3}{8} + \left(\frac{-3}{4}\right)\right\} = \frac{-2}{5} \times \left\{\frac{3+(-6)}{8}\right\}$

$= \frac{-2}{5} \times \left(\frac{-3}{8}\right) = \frac{6}{40} = \frac{3}{20}$

Also, $\frac{-2}{5} \times \frac{3}{8} = \frac{-6}{40} = \frac{-3}{20}$

and $\frac{-2}{5} \times \frac{-3}{4} = \frac{6}{20}$

Therefore, $\left(\frac{-2}{5} \times \frac{3}{8}\right) + \left(\frac{-2}{5} \times \frac{-3}{4}\right) = \frac{-3}{20} + \frac{6}{20} = \frac{3}{20}$

Thus, $\frac{2}{5} \times \left\{\frac{3}{8} + \left(\frac{-3}{4}\right)\right\} = \left(\frac{-2}{5} \times \frac{3}{8}\right) + \left\{\frac{-2}{5} \times \left(\frac{-3}{4}\right)\right\}$

Distributivity of multiplication over subtraction for rational number: For any three rational numbers a, b and c, a (b – c) = ab – ac

eg.Let $\frac{1}{2}$, $\frac{-2}{5}$ an $\frac{-3}{10}$ are any three rational numbers, then

$\frac{1}{2} \times \left\{\frac{-2}{5} - \left(\frac{-3}{10}\right)\right\} = \frac{1}{2} \times \left\{\frac{-4-(-3)}{10}\right\} = \frac{1}{2} \times \left(\frac{-4+3}{10}\right)$

$= \frac{1}{2} \times \left(\frac{-1}{10}\right) = \frac{-1}{20}$

Also, $\frac{1}{2} \times \left(\frac{-2}{5}\right) = \frac{-2}{10} = -\frac{1}{5}$

and $\frac{1}{2} \times \left(\frac{-3}{10}\right) = \frac{-3}{10}$

Therefore, $\left(\frac{1}{2} \times \frac{-2}{5}\right) - \left(\frac{1}{2} \times \frac{-3}{10}\right) = \frac{-1}{5} - \left(\frac{-3}{20}\right)$

$= \frac{-1}{5} + \frac{3}{20}$

$= \frac{-4+3}{20} = \frac{-1}{20}$

Thus, $\frac{1}{2} \times \left\{\frac{-2}{5} - \left(\frac{-3}{10}\right)\right\} = \left(\frac{1}{2} \times \frac{-2}{5}\right) - \left(\frac{1}{2} \times \frac{-3}{10}\right)$

The numbers 1, 2, 3, 4, ………. are called natural numbers.

If we add 0 to the collection of natural numbers, what we get is called the collection of whole numbers.

Thus, 0, 1, 2, 3, 4, are whole numbers.

Natural numbers are also known as positive integers. If we put a negative sign before each positive integer, we get negative integers. Thus, -1, -2, -3, -4, ………. are negative integers. A number of the form pq, where p and q are integers and $q \neq 0$ is called a rational number. All the above types of numbers are needed to solve various types of simple algebraic equations.

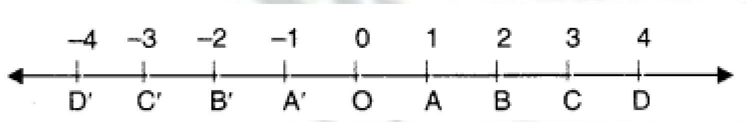
Properties of Rational Numbers

The list of properties of rational numbers can be given as follows:

- Closure
- Commutativity
- Associativity
- The role of zero (0)
- The role of 1
- Negative of a number
- Reciprocal
- Distributivity of multiplication over addition for rational numbers.

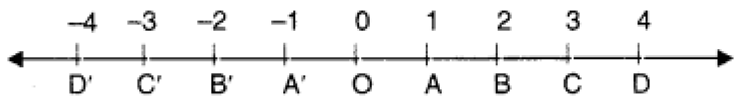
Distributivity of Multiplication Over Addition for Rational Numbers

For all rational numbers a, b and c,
$a(b + c) = ab + ac$
$a(b – c) = ab – ac$.



- We draw a line.
- We mark a point O on it and name it 0. Mark a point to the right of 0. Name it 1. The distance between these two points is called unit distance.
- Mark a point to the right of 1 at unit distance and name it 2.
- Proceeding in this manner, we can mark points 3, 4, 5,
- Similarly we can mark – 1, – 2, – 3, – 4, – 5, ……… to the left of 0. This line is called the number line.
- This line extends indefinitely on both sides.

Representation of Rational Numbers on the Number Line



- We draw a line.
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- Mark a point to the right of 1 at unit distance and name it 2.
- Proceeding in this manner, we can mark points 3, 4, 5,
- Similarly we can mark – 1, – 2, – 3, – 4, – 5, ……… to the left of 0. This line is called the number line.
- This line extends indefinitely on both sides.

The positive rational numbers are represented by points on the number line to the right of O whereas the negative rational numbers are represented by points on the number line to the left of O.

Any rational number can be represented on this line. The denominator of the rational number indicates the number of equal parts into which the first unit has been divided whereas the numerator indicates as to how many of these parts are to be taken into consideration.

Rational Numbers Between Two Rational Numbers

We can find infinitely many rational numbers between any two given rational numbers. We can take the help of the idea of the mean for this purpose.

CHAPTER 1

CHAPTER NAME : RATIONAL NUMBERS

EXERCISE 1.1

Using appropriate properties find:

$(i)\ -\frac{2}{3}\times\frac{3}{5}+\frac{5}{2}-\frac{3}{5}\times\frac{1}{6}$

$(ii)\ \frac{2}{5}\times\left(\frac{-3}{7}\right)-\frac{1}{6}\times\frac{3}{2}+\frac{1}{14}\times\frac{2}{5}$

Solution:

$= -\frac{2}{3} \times \frac{3}{5} + \frac{5}{2} - \frac{3}{5} \times \frac{1}{6}$

$= -\frac{2}{3} \times \frac{3}{5} - \frac{3}{5} \times \frac{1}{6} + \frac{5}{2}$ (By regrouping)

$= \frac{3}{5} \times \left(-\frac{2}{3} - \frac{1}{6}\right)$

(using distributive property)

$= \frac{3}{5} \times \left(\frac{-2\times2}{3\times2} - \frac{1\times1}{6\times1}\right) + \frac{5}{2}$

$= \frac{3}{5} \times \left(-\frac{4}{6} - \frac{1}{6}\right) + \frac{5}{2}$

$= \frac{3}{5} \times \left(\frac{-4-1}{6}\right) + \frac{5}{2}$

$= \frac{3}{5} \times \left(\frac{-5}{6}\right) + \frac{5}{2}$

$= \frac{3}{\cancel{5}} \times \left(\frac{\cancel{-5}}{6}\right) + \frac{5}{2}$

$= -\frac{3}{6} + \frac{5}{2} =$

$-\frac{1}{2} + \frac{5}{2}$

$= \frac{-1+5}{2} = \frac{4}{2} = 2$

Q.**1. Using appropriate properties find:**

Q.1.(b) $\frac{2}{5} \times \left(-\frac{3}{7}\right) - \frac{1}{6} \times \frac{3}{2} + \frac{1}{14} \times \frac{2}{5} +$

$= \frac{2}{5} \times \left(-\frac{3}{7}\right) + \frac{1}{14} \times \frac{2}{5} - \frac{1}{6} \times \frac{3}{2}$ (By regrouping)

(by associativity)

$= [\frac{2}{5} \times (-\frac{3}{7}) + \frac{1}{14} \times \frac{2}{5}] - \frac{1}{6} \times \frac{3}{2}$

$= \frac{2}{5} \times [-\frac{3}{7} + \frac{1}{14}] - \frac{1}{\cancel{6}_{2}} \times \frac{\cancel{3}^{1}}{2}$ [by distributivity]

$= \frac{2}{5} \times [-\frac{3 \times 2}{7 \times 2} + \frac{1}{14}] - \frac{1}{4}$

$= \frac{2}{5} \times [-\frac{6}{14} + \frac{1}{14}] - \frac{1}{4}$

$= \frac{2}{5} \times [\frac{-6+1}{14}] - \frac{1}{4}$

$= \frac{2}{\cancel{5}} \times [-\frac{\cancel{5}}{14}] - \frac{1}{4}$

$= -\frac{2}{14} - \frac{1}{4}$

$= \frac{-\cancel{2}^{1}}{\cancel{14}_{7}} - \frac{1}{4}$

$= -\frac{1}{7} - \frac{1}{4}$

L.C.M. of 4 and 7 = 2×2×7=28

2	4	7
2	2	7
7	1	7
	1	1

$= \frac{-4-7}{28}$

$= \frac{-11}{28}$

Q.2. Write the additive inverse of each of the following:

(a) $\frac{2}{8}$

The additive inverse of a rational number $\frac{a}{b}$ is $(\frac{-a}{b})$

(i)The additive inverse of $\frac{2}{8}$ is $\frac{-2}{8}$ because $\frac{2}{8} + (\frac{-2}{8}) = 0$

(ii) The additive inverse of $\frac{-5}{9} = \frac{5}{9}$

(iii) The additive inverse of $\frac{-6}{-5} = \frac{-6}{5}$

(iv) The additive inverse of $\frac{2}{-9}$ is $\frac{2}{9}$

(v) The additive inverse of $\frac{19}{-6} = \frac{19}{6}$

Q.3. Verify that –(−x) = x for

(i) $x = \frac{11}{15}$

$-(-x) = -(-\frac{11}{15}) = \frac{11}{15}$

$-(-x) = -(-(-\frac{13}{17})) = -(\frac{13}{17}) = \frac{-13}{17}$

Q.4. Find the multiplicative inverse of the following :

(a) The multiplicative inverse of -13 is $-\frac{1}{13}$

(b) The multiplicative inverse of $\frac{-13}{19} = \frac{1}{\frac{-13}{19}} = \frac{-19}{13} = -\frac{19}{13}$

(c) The multiplicative inverse of $\frac{1}{5}$ is 5.

(d) The multiplicative inverse of $\frac{-5}{8} \times \frac{-3}{7}$

$\frac{(-5)\times(-3)}{8\times 7} = \frac{15}{56}$

Multiplicative inverse of $\frac{15}{56} = \frac{1}{\frac{15}{56}} = \frac{56}{15}$

(e) The multiplicative inverse of $-1 \times \frac{-2}{5}$ = ??

First of all $-1 \times \frac{-2}{5} = \frac{2}{5}$

Now Multiplicative inverse is $\frac{1}{\frac{2}{5}} = \frac{5}{2}$

(f) The multiplicative inverse of -1 is $\frac{1}{-1} = -1$

Q.5. Name the property under multiplication used in each of the following

(i) $\frac{-4}{5} \times 1 = 1 \times \frac{-4}{5} = \frac{-4}{5}$

Multiplicative identity of 1. Or 1 is the multiplicative identity for rationals.

(ii) $-\frac{13}{17} \times \frac{-2}{7} = \frac{-2}{7} \times \frac{-13}{17}$

Commutativity

(iii) $\frac{-19}{29} \times \frac{29}{-19} = 1$

Multiplicative inverse.

Q.6. Multiply $\frac{6}{13}$ by the reciprocal of $\frac{-7}{16}$

Reciprocal of $\frac{-7}{16} = \frac{-16}{7}$

$\frac{6}{13} \times$ [Reciprocal of $\frac{-7}{16}$] $= \frac{6}{13} \times [\frac{-16}{7}] = \frac{6 \times (-16)}{13 \times 7} = \frac{-96}{91}$

Q.7. Tell what property allows you to compute $\frac{1}{3} \times (6 \times \frac{4}{3})$ as $(\frac{1}{3} \times 6) \times \frac{4}{3}$

By using associativity

a × (b × c) = (a × b) × c

Q.8. Is $\frac{8}{9}$ the multiplicative inverse of $-1\frac{1}{8}$? Why or why not?

No; $\frac{8}{9}$ is not the multiplicative inverse of $-1\frac{1}{8}$

Because $-1\frac{1}{8} = \frac{-9}{8}$ and $\frac{8}{9} \times \frac{-9}{8} = -1$ [which is not equal to 1]

$\therefore$ $\frac{8}{9}$ is not the multiplicative inverse of $\frac{-9}{8}$

$\because$ The product of $\frac{-9}{8}$ and its multiplicative inverse must be equal to 1.

Q.9. Is 0.3 the multiplicative inverse of $3\frac{1}{3}$? Why or why not ?

$\mathbf{0.3} \times \mathbf{3\frac{1}{3}} = \frac{\mathbf{3}}{\mathbf{10}} \times \frac{\mathbf{10}}{\mathbf{3}} = 1$

Yes, 0.3 is multiplicative inverse of $\frac{1}{3}$.

Q.10. Write:

(i) The rational number that does not have a reciprocal

Ans 0 (zero) is a rational number that does not have a reciprocal

(ii) The rational number that are equal to their reciprocals.

Ans. 1 and -1 are rational number that are equal to their reciprocals.

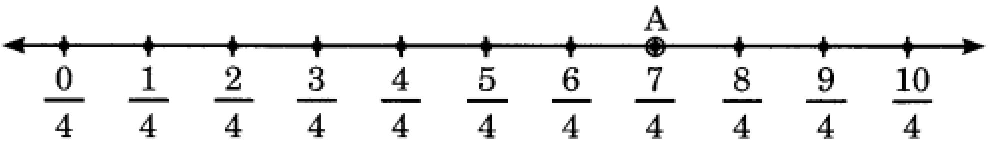
(iii) The rational number that is equal to its negative.

Ans. 0 is the rational number equal to its negative.

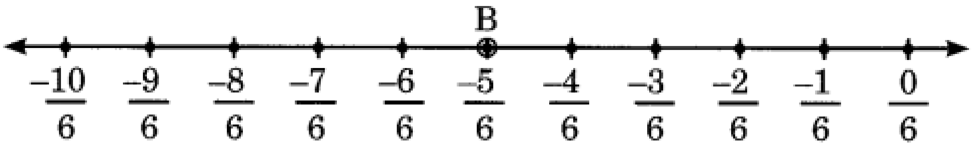
Textual Exercise 1.2 (Textbkpg 20)

Q.1. Represent these numbers on the number line. (i)$\frac{7}{4}$ (ii) $\frac{-5}{6}$

(i) $\frac{7}{4}$



(ii) $(-5)/6$

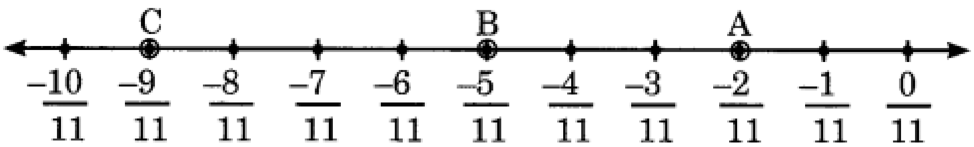


Here, point B represents $(-5)/6$ on the number line.

Q.2. Represent $\frac{-2}{11}, \frac{-5}{11}, \frac{-9}{11}$on a number line.

Solution:

We have $\frac{-2}{11}, \frac{-5}{11}$and $\frac{-9}{11}$



Here, point A represents $\frac{-2}{11}$, point B represents $\frac{-5}{11}$, point C represents $\frac{-9}{11}$

Q.3. Write five rational numbers which are smaller than 2.

Solution:

Required five rational numbers smaller than 2 are

$1, 0, \frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$

Or

$-1, -\frac{1}{2}, 0, \frac{1}{2}, 1$

Q.4. Find ten rational numbers between $-\frac{2}{5}$ and $\frac{1}{2}$

Solution :

First of all given 2 rational numbers have different denominators.

So to convert them into rationals having same denominator find L.C.M.

L.C.M. of 2 and 5 = 10

$-\frac{2}{5} = -\frac{2}{5} \times \frac{2}{2} = -\frac{4}{10}$

$\frac{1}{2} = \frac{1}{2} \times \frac{5}{5} = \frac{5}{10}$

Again , $-\frac{4}{10} = -\frac{4}{10} \times \frac{2}{2} = -\frac{8}{20}$

$\frac{5}{10} = \frac{5}{10} \times \frac{2}{2} = \frac{10}{20}$

$\therefore$ Ten rational numbers between $-\frac{2}{5}$ and $\frac{1}{2}$ are :

$-\frac{7}{20}, -\frac{6}{20}, -\frac{5}{20}, -\frac{4}{20}, -\frac{3}{20}, -\frac{2}{20}, -\frac{1}{20}, \frac{0}{20}, \frac{1}{20}, \frac{2}{20}$

Q.5. Find five rationals between

(i) $\frac{2}{3}$ and $\frac{4}{5}$

Solution :Converting $\frac{2}{3}$ and $\frac{4}{5}$ into same denominators such that the difference between the numerators is more than 5.

L.C.M OF 3 AND 5 is 15 (15×4=60)

1	3	5
	3	5

$\frac{2}{3} = \frac{2\times20}{3\times20} = \frac{40}{60}$

$\frac{4}{5} = \frac{4\times12}{5\times12} = \frac{48}{60}$

Now five rationals between $\frac{40}{60}\left(=\frac{2}{3}\right)$ and $\frac{48}{60}$ $(=\frac{4}{5})$ are

$\frac{41}{60}$; $\frac{42}{60}$; $\frac{43}{60}$; $\frac{44}{60}$; $\frac{45}{60}$; $\frac{46}{60}$

Q5(ii) $-\frac{3}{2}$ and $\frac{5}{3}$

Solution :Converting $-\frac{3}{2}$ and $\frac{5}{3}$ with the same denominators we have

L.C.M. of 2 and 3 = 6

1	2	3
	2	3

$$-\frac{3}{2} = -\frac{3\times3}{2\times3} = -\frac{9}{6}$$

$$\frac{5}{3} = \frac{5\times2}{3\times2} = \frac{10}{6}$$

Now ; five rationals between $-\frac{9}{6}$ $(=-\frac{3}{2})$and $\frac{10}{6}$ $(= 5/3)$are

$-\frac{2}{2}$; $-\frac{1}{2}$; $\frac{0}{2}$; $\frac{1}{2}$; $\frac{2}{2}$; $\frac{3}{2}$

Q.5.(iii) $\frac{1}{4}$ and $\frac{1}{2}$

Solution : Converting with same denominators we get

L.C.M of 2 and 4 = 4

2	2	4
2	1	2
	1	1

$\frac{1}{4} = \frac{1}{4}$; $\frac{1}{2} = \frac{1\times2}{2\times2} = \frac{2}{4}$

$\frac{1}{4} = \frac{1\times8}{4\times8} = \frac{8}{32}$; $\frac{1}{2} = \frac{2}{4} = \frac{2\times8}{4\times8} = \frac{16}{32}$

So , now five rationals between $\frac{1}{4}$ and $\frac{1}{2}$ are

$\frac{9}{32}$, $\frac{10}{32}$, $\frac{11}{32}$, $\frac{12}{32}$, $\frac{13}{32}$

Q.6. Write five rationals numbers greater than −2.

Solution : greater $>(-2)$ means $>(-\frac{4}{2})$

So $> \frac{-4}{2}$ are $-\frac{3}{2}$, $-\frac{2}{2}$, $-\frac{1}{2}$, $\frac{0}{2}$, $\frac{1}{2}$

Q.7. Find ten rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$

Solution: Converting given rationals $\frac{3}{5}$ and $\frac{3}{4}$ into rationals with same denominator

And their numerators with difference of more than 10

$\frac{3}{5} = \frac{3\times20}{5\times20} = \frac{60}{100}$ and $\frac{3}{4} = \frac{3\times25}{4\times25} = \frac{75}{100}$

$\therefore$ Ten rational numbers between $\frac{3}{5}$ and $\frac{3}{4}$ are as follows:

$\frac{61}{100}$; $\frac{62}{100}$; $\frac{63}{100}$; $\frac{64}{100}$; $\frac{65}{100}$; $\frac{66}{100}$; $\frac{67}{100}$; $\frac{68}{100}$; $\frac{69}{100}$; $\frac{70}{100}$.

Q. Fill in the blanks:

(1) $\left(\frac{-3}{17}\right) + \left(\frac{-12}{5}\right) = \left(\frac{-12}{5}\right) + (\ \dots\dots\dots\ )$

(2) $\left(\frac{-8}{13} + \frac{3}{7}\right) + \left(\frac{-13}{4}\right) = (\dots\dots) + [\frac{3}{7} + (\frac{-13}{4})]$

(3) $\frac{-16}{7} + \dots\dots\dots = \dots\dots\dots + \frac{-16}{7} = \frac{-16}{7}$

(4) Zero has _____reciprocal.

(5) The numbers ______and ____are their own reciprocals.

(6) The reciprocal of −5 is ______.
(7) Reciprocal of $\frac{1}{x}$, where $x \neq 0$ is ______.
(8) The product of two rational numbers is always a ________.
(9) The reciprocal of a positive rational number is _________.

Solution:

(1) $\left(\frac{-3}{17}\right)$
(2) $(-\frac{8}{13})$
(3) 0 , 0
(4) no
(5) 1 , −1
(6) $-\frac{1}{5}$

(7) x
(8) rational number
(9) positive

Q. Multiple choice questions:

1. An integer can be:
a) Only Positive
b) Only Negative
c) Both positive and negative
d) None of the above

Answer: (c)
Explanation: An integer can be both positive and negative as well as zero.

2. A rational number can be represented in the form of:
a) p/q
b) pq
c) p+q
d) p-q

Answer: (a)
Explanation: A rational number can be represented in the form p/q where p and q are integers and q is not equal to zero.

3. *The value of* ½ *x* ⅗ *is equal to*:
a) ½
b) 3/10
c) ⅗
d) ⅖

Answer: (*b*)
Explanation: ½ *x* ⅗ $= \frac{1 \times 3}{2 \times 5} = \frac{3}{10}$

3. The value of $\frac{1}{2} \times \frac{3}{5}$ is equal to:

a) $\frac{1}{2}$

b) $\mathbf{\frac{3}{10}}$

c) $\mathbf{\frac{3}{5}}$

d)

Higher Order Thinking Skills (HOTS)

Q.1.Rajni had a certain amount of money in her purse. She spent ₹ $10\frac{1}{4}$ in the school canteen, bought a gift worth ₹ $25\frac{3}{4}$ and gave ₹ $16\frac{1}{2}$ to her friend. How much she have to begin with?

Solution:

Amount given to school canteen = ₹ $10\frac{1}{4}$

Amount given to buy gift = ₹ $25\frac{3}{4}$

Amount given to her friend = ₹ $16\frac{1}{2}$

To begin with Rajni had

$$= ₹10\frac{1}{4} + ₹25\frac{3}{4} + ₹16\frac{1}{2}$$

$$= ₹\left(\frac{41}{4} + \frac{103}{4} + \frac{33}{2}\right)$$

$$= ₹\left(\frac{41+103+66}{4}\right) = ₹\frac{210}{4}$$

$$= ₹52\frac{2}{4} = ₹52\frac{1}{2}$$

Q.2.One-third of a group of people are men. If the number of women is 200 more than the men, find the total number of people.

Solution:

Number of men in the group = $\frac{1}{3}$ of the group

Number of women = $1 - \frac{1}{3} = \frac{2}{3}$

Difference between the number of men and women = $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

If difference is $\frac{1}{3}$, then total number of people = 1

If difference is 200, then total number of people

= $200 \div \frac{1}{3} = 200 \times 3 = 600$

Hence, the total number of people = 600

Rational Numbers **Extra Questions**

Very Short Answer Type

Q.1. Pick up the rational numbers from the following numbers.

$\frac{6}{7}$, $\frac{-1}{2}$, 0 , $\frac{1}{0}$, $\frac{100}{0}$

Solution:

Since rational numbers are in the form of ab where b ≠ 0.
Only $\frac{6}{7}$, $\frac{-1}{2}$ and 0 are the rational numbers.

Q.2. Find the reciprocal of the following rational numbers:

(a) $\frac{-3}{4}$
(b) 0
(c) $\frac{6}{11}$
(d) $\frac{5}{-9}$

Solution:
(a) Reciprocal of −34 is −43
(b) Reciprocal of 0, i.e. 10 is not defined.
(c) Reciprocal of 611 is 116
(d) Reciprocal of 5−9 = −95

$\frac{6}{7}$, $\frac{-1}{2}$, 0, $\frac{1}{0}$, $\frac{100}{0}$

CHAPTER 2 LINEAR EQUATIONS IN ONE VARIABLE

Equation:
An equation is a statement of equality which contains one or more unknown quantity/ quantities (or variable/variables) is called an equation.

Linear Equation in One Variable: An equation is called linear equation if it has only one degree i.e., the highest power of the variable appearing in equation is 1, and the form of linear equation is
$P(x) = ax + b = 0$
e.g., $x + 5 = 0$, $x/2 - 7 = 15$.

Solving an Equation: Solving an equation means determining its root i.e., determining (finding) the value of the variable which satisfies it.

A linear equation may have any rational number, as its solution.

An equation may have a linear expression on both sides of the equation.

Some equations may not be linear in the beginning, but they can be brought to be linear by using usual methods.

The utility of linear equations is in their diverse applications, different problems on numbers, ages, perimeters, the combination of currency notes and so on can be solved using linear equations.
A number which satisfies an equation is called the solution of the equation.

A term may be transposed from one side of the equation to the other side, but its sign will be changed.
An equation essentially contains a sign of equality (=).

The expression we use to form a linear equation is linear only, i.e., the highest power of the variable occurring in the expression is 1 and that too only in one variable.

The expression on the left of the sign of equality is called the Left-Hand Side whereas the expression on the right of the sign of equality is called the Right Hand Side.

The value of the variable for which LHS = RHS is called a solution of the linear equation.

To find the solution of a linear equation in one variable, we assume that the two sides of the equation are balanced.
We are free to perform the same operation (suitable) on both sides of the equation such as we can add to or subtract from both sides of the equation the same quantity (number).

Also, we can multiply or divide both sides of the equation by the same non-zero quantity (number).
Solving Equations which have Linear Expressions on One Side and Numbers on the Other Side
We transpose the numbers from LHS to RHS and simplify. Then, we divide both sides by the coefficient of the variable to get the solution.
Finally, we check the solution by substituting it in LHS and checking whether RHS is obtained or not by doing so.

If LHS = RHS, the solution is said to be valid. Reject if the solution is invalid. Note: The process of carrying out a term from one side to the other side is called transposition. In this process, the sign of the term gets changed.
Note: The process of carrying out a term from one side to the other side is called transposition. In this process, the sign of the term gets changed.

In daily life, there come many simple problems which can be easily solved by first forming a linear equation in one variable according to the given condition of the problem and then solving this linear equation by following usual procedure.

Stepwise procedure to solve a word problem.

Read the given word problem carefully and identify what is given and what is required.
Use letters x, y, z, etc. to represent the unknown quantity.
Transform the statements of the problem into mathematical statements.
Form the linear equation according to the given condition (s) of the problem.
Solve the equation by the usual method.
Check the solution for its validity.
Reject if the solution is invalid.

Solving Equations having the Variable on Both Sides

We transpose the terms in such a manner that the terms containing the variables are on the LHS and constant numbers on RHS.
Then, simplifying both sides and dividing by a suitable number (if required), we can solve the equation.
Finally, check the validity of the solution obtained. Reject if the solution is invalid.

Some More Applications

Problems of multi-varieties occur in our daily life. All these can be solved by forming a suitable linear equation according to the condition given in the problem and then solving this equation by usual method. Finally check the validity of the solution. Reject if the solution is invalid.

Reducing Equations to Simpler Form

We multiply both sides of the equation by the LCM of the denominators of the terms in the expressions occurring in the given equation.

We transpose properly so that all the variable terms come on LHS and constant terms on RHS.
Then, combining like terms on both sides of the equation and dividing both sides by a suitable number (if required), we can find out the required solution.
Finally, we check this solution for its validity. Reject if the solution is invalid.

Equations Reducible to Linear Form

Sometimes the given equation is not linear in form. By cross-multiplication and further simplification, it can be transformed into a linear equation in one variable. Then, it can be solved by the usual method.

Textual Exercise 2.1

Q.1. Solve the following equations:

1. x – 2 = 7

Solution:

Given: x – 2 = 7

⇒ x – 2 + 2 = 7 + 2 (adding 2 on both sides)

⇒ x = 9 (Required solution)

2.Solve the equation: y + 3 = 10.

Given: y + 3 = 10

⇒ y + 3 – 3 = 10 – 3 (subtracting 3 from each side)

⇒ y = 7 (Required solution)

3. Solve the equation: 6 = z + 2

Solution:

We have 6 = z + 2

⇒ 6 – 2 = z + 2 – 2 (subtracting 2 from each side)

⇒ 4 = z

Thus, z = 4 is the required solution.

4. Solve the equations: $\frac{3}{7}$ **+ x =** $\frac{17}{7}$

Solution

We have $\frac{3}{7}+x = \frac{17}{7}$

$$\Rightarrow \quad \frac{3}{7}-\frac{3}{7}+x = \frac{17}{7}-\frac{3}{7}$$

$$\left(\text{subtracting } \frac{3}{7} \text{ from each side}\right)$$

$$\Rightarrow \quad x = \frac{17-3}{7}$$

$$\Rightarrow \quad x = \frac{\cancel{14}^{2}}{\cancel{7}^{1}}$$

$$\Rightarrow \quad x = 2$$

Thus, $x = 2$ is the required solution.

5.Solve the equation 6x = 12.

Solution:

We have 6x = 12

⇒ 6x ÷ 6 = 12 ÷ 6 (dividing each side by 6)

⇒ x = 2

Thus, x = 2 is the required solution.

6.Solve the equation $\frac{t}{5}$ **= 10**

Solution:

Given $\frac{t}{5}$= 10

$\Rightarrow \frac{t}{5} \times 5 = 10 \times 5$ (multiplying both sides by 5)

$\Rightarrow$ t = 50

Thus, t = 50 is the required solution.

7. Solve the equation $\frac{2x}{3} = 18$

Solution:

$\frac{2x}{3} \times 3 = 18 \times 3$ [Multiplying both sides by 3]

$\Rightarrow$2x = 54

$\Rightarrow$ 2x ÷ 2 = 54 ÷ 2 (dividing both sides by 2)

$\Rightarrow$ x = 27

Thus, x = 27 is the required solution.

8. Solve the equation 1.6 = $\frac{y}{1.5}$

Solution:

Given: 1.6 = $\frac{y}{1.5}$

$\Rightarrow 1.6 \times 1.5 = \frac{y}{1.5} \times 1.5$ (multiplying both sides by 1.5)

$\Rightarrow$ 2.40 = y

Thus, y = 2.40 is the required solution.

9.Solve the equation 7x – 9 = 16.

Solution:

We have 7x – 9 = 16

$\Rightarrow$ 7x – 9 + 9 = 16 + 9 (adding 9 to both sides)

$\Rightarrow$ 7x = 25

$\Rightarrow$ 7x ÷ 7 = 25 ÷ 7 (dividing both sides by 7)

$\Rightarrow$ x = 25/7

Thus, x = 25/7 is the required solution.

10. Solve the equation 14y – 8 = 13.

Solution :We have 14y – 8 = 13

$\Rightarrow$ 14y – 8 + 8 = 13 + 8 (adding 8 to both sides)

$\Rightarrow$ 14y = 21

$\Rightarrow$ 14y ÷ 14 = 21 ÷ 14 (dividing both sides by 14)

$\Rightarrow$ y = 21/14

$\Rightarrow y = \frac{3}{2}$

Thus, y $=\frac{3}{2}$ is the required solution.

11. Solve the equation 17 + 6p = 9.

Solution:
We have, 17 + 6p = 9

⇒ 17 – 17 + 6p = 9 – 17 (subtracting 17 from both sides)

⇒ 6p = -8

⇒ 6p ÷ 6 = -8 ÷ 6 (dividing both sides by 6)

⇒ p = $-\frac{8}{6}$

⇒ p = $-\frac{4}{3}$

Thus, p = $-\frac{4}{3}$ the required solution.

12. Solve the equation $\frac{x}{3} + 1 = \frac{7}{15}$

Solution:

⇒ $\frac{x}{3} = \frac{-8}{15}$

⇒ $\frac{x}{3} \times 3 = \frac{-8}{\cancel{15}_5} \times \cancel{3}^1$

(multiplying both sides by 3)

⇒ $x = \frac{-8}{5}$

Thus, $x = \frac{-8}{5}$ is the required solution.

EXERCISE 2.2

Q.1.If you subtract $\frac{1}{2}$ from a number and multiply the result by $\frac{1}{2}$, you get $\frac{1}{8}$.

What is the number?

Solution:

Let the required number be x.

Condition I: $x - \frac{1}{2}$

Condition II: $\frac{1}{2} \times \left(x - \frac{1}{2}\right)$

Condition III: $\frac{1}{2} \times \left(x - \frac{1}{2}\right) = \frac{1}{8}$

$\Rightarrow \frac{1}{2} \times 8 \times \left(x - \frac{1}{2}\right) = \frac{1}{8} \times 8$ (multiplying both sides by the LCM of 2 and 8, i.e. 8)

$\Rightarrow 4 \times \left(x - \frac{1}{2}\right) = 1$

$\Rightarrow 4x - \frac{1}{2} \times 4 = 1$ (solving the bracket)

$\Rightarrow 4x - 2 = 1$

$\Rightarrow 4x - 2 + 2 = 1 + 2$ (adding 2 to both sides)

$\Rightarrow 4x = 3$

$\Rightarrow 4x \div 4 = 3 \div 4$ (dividing both sides by 4)

$\Rightarrow x = \frac{3}{4}$

Thus, the required number is $\frac{3}{4}$.

Q.2. The perimeter of a rectangular swimming pool is 154 m. Its length is 2 m more than twice its breadth. What are the length and the breadth of the pool?

Solution:

Let the breadth of the pool be x m.

Condition I: Length = (2x + 2) m.

Condition II: Perimeter = 154 m.

We know that Perimeter of rectangle = 2 × [length + breadth]

2 × [2x + 2 + x] = 154

⇒ 2 × [3x + 2] = 154

⇒ 6x + 4 = 154 (solving the bracket)

⇒ 6x = 154 – 4 [Transposing 4 from (+) to (-)]

⇒ 6x = 150

⇒ x = 150 ÷ 6 [Transposing 6 from (×) to (÷)]

⇒ x = 25

Thus, the required breadth = 25 m

and the length = 2 × 25 + 2 = 50 + 2 = 52 m.

Q.3. The base of an isosceles triangle is $4/3$ cm. The perimeter of the triangle is $4\ 2/(15\ )$.

What is the length of either of the remaining equal sides?

Solution :

Let the length of each of equal sides of the triangle be x cm.

Perimeter of the triangle = sum of the three sides

$$= \left(x + x + \frac{4}{3}\right) \text{cm} = \left(2x + \frac{4}{3}\right) \text{cm}$$

$$\therefore \quad 2x + \frac{4}{3} = 4\frac{2}{15}$$

$$\Rightarrow \quad 2x + \frac{4}{3} = \frac{62}{15}$$

$$\Rightarrow \quad 2x = \frac{62}{15} - \frac{4}{3}$$

$$\left[\text{Transposing } \frac{4}{3} \text{ from } (+) \text{ to } (-)\right]$$

$$\Rightarrow \quad 2x = \frac{62 - 20}{15}$$

$$\Rightarrow \quad 2x = \frac{42}{15}$$

$$\Rightarrow \quad x = \frac{42}{15} \div 2$$

[transposing 2 from (×) to (÷)]

$$x = \frac{42}{15} \times \frac{1}{2}$$

$$\Rightarrow \quad x = \frac{\cancel{21}^{7}}{\cancel{15}^{5}}$$

$$\therefore \quad x = \frac{7}{5} = 1\frac{2}{5}$$

Thus, the required length of each of equal side $= 1\frac{2}{5}$ cm.

Q.4. Sum of two numbers be 95. If one exceeds the other by 15, find the numbers.

Solution:

Let one number be x

Other number = x + 15

As per the condition of the question, we get

$x + (x + 15) = 95$

$\Rightarrow x + x + 15 = 95$

$\Rightarrow 2x + 15 = 95$

$\Rightarrow 2x = 95 – 15$ [transposing 15 from (+) to (-)]

$\Rightarrow 2x = 80$

$\Rightarrow x = \frac{80}{2}$[transposing 2 from (×) to (÷)]

$\Rightarrow x = 40$

Other number = 95 – 40 = 55

Thus, the required numbers are 40 and 55.

Q.5. Two numbers are in the ratio 5 : 3. If they differ by 18, what are the numbers?

Solution:

Let the two numbers be 5x and 3x.

As per the conditions, we get

$5x – 3x = 18$

$\Rightarrow 2x = 18$

$\Rightarrow x = 18 \div 2$ [Transposing 2 from (×) to (÷)]

$\Rightarrow x = 9$.

Thus, the required numbers are $5 \times 9 = 45$ and $3 \times 9 = 27$

Q.6.Three consecutive integers add up to 51. What are these integers?

Solution

Let the three consecutive integers be x, x + 1 and x + 2.

As per the condition, we get

$x + (x + 1) + (x + 2) = 51$

$\Rightarrow x + x + 1 + x + 2 = 51$

$\Rightarrow 3x + 3 = 51$

$\Rightarrow 3x = 51 - 3$ [transposing 3 to RHS]

$\Rightarrow 3x = 48$

$\Rightarrow x = 48 \div 3$ [transposing 3 to RHS]

$\Rightarrow x = 16$

Thus, the required integers are 16, 16 + 1 = 17 and 16 + 2 = 18, i.e., 16, 17 and 18.

Q.7.The sum of three consecutive multiples of 8 is 888. Find the multiples.

Solution:

Let the three consecutive multiples of 8 be 8x, 8x + 8 and 8x + 16.

As per the conditions, we get

$8x + (8x + 8) + (8x + 16) = 888$

$\Rightarrow 8x + 8x + 8 + 8x + 16 = 888$

$\Rightarrow 24x + 24 = 888$

$\Rightarrow 24x = 888 - 24$ (transposing 24 to RHS)

$\Rightarrow 24x = 864$

$\Rightarrow x = 864 \div 24$ (transposing 24 to RHS)

	36
24	864
	72
	144
	144

$\Rightarrow x = 36$

Thus, the required multiples are

$36 \times 8 = 288$, $36 \times 8 + 8 = 296$ and $36 \times 8 + 16 = 304$,

i.e., 288, 296 and 304.

Q.8. Three consecutive integers are such that when they are taken in increasing order and multiplied by 2, 3, and 4 respectively, they add up to 74. Find these numbers.

Solution :

Let the three consecutive integers be x, x + 1 and x + 2.

As per the condition, we have

$2x + 3(x + 1) + 4(x + 2) = 74$

$\Rightarrow 2x + 3x + 3 + 4x + 8 = 74$

$\Rightarrow 9x + 11 = 74$

$\Rightarrow 9x = 74 - 11$ **(transposing 11 to RHS)**

$\Rightarrow 9x = 63$

$\Rightarrow x = 63 \div 9$

$\Rightarrow x = 7$ **(transposing 7 to RHS)**

Thus, the required numbers are 7, 7 + 1 = 8 and 7 + 2 = 9, i.e., 7, 8 and 9.

Q.9.The ages of Rahul and Haroon are in the ratio 5 : 7. Four years later the sum of their ages will be 56 years. What are their present ages?

Solution :

Let the present ages of Rahul and Haroon he 5x years and 7x years respectively.

4 years later, the age of Rahul will be (5x + 4) years.

4 years later, the age of Haroon will be (7x + 4) years.

As per the conditions, we get

$(5x + 4) + (7x + 4) = 56$

$\Rightarrow 5x + 4 + 7x + 4 = 56$

$\Rightarrow 12x + 8 = 56$

$\Rightarrow 12x = 56 - 8$ **(transposing 8 to RHS)**

$\Rightarrow 12x = 48$

$\Rightarrow x = 48 \div 12 = 4$ **(transposing 12 to RHS)**

Hence, the required age of Rahul = 5 × 4 = 20 years.

and the required age of Haroon = 7 × 4 = 28 years.

Q.10. The number of boys and girls in a class are in the ratio 7 : 5. The number of boys is 8 more than the numbers of girls. What is the total class strength?

Solution:

Let the number of boys be 7x

and the number of girls be 5x

As per the conditions, we get

$7x - 5x = 8$

$\Rightarrow 2x = 8$

$\Rightarrow x = 8 \div 2 = 4$ (transposing 2 to RHS)

the required number of boys = 7 × 4 = 28

and the number of girls = 5 × 4 = 20

Hence, total class strength = 28 + 20 = 48

Q.11. Baichung's father is 26 years younger than Baichung's grandfather and 29 years older than Baichung. The sum of the ages of all the three is 135 years. What is the age of each one of them?

Solution:

Let the age of Baichung be x years.

The age of his father = x + 29 years,

and the age of his grandfather = x + 29 + 26 = (x + 55) years.

As per the conditions, we get

$x + x + 29 + x + 55 = 135$

$\Rightarrow 3x + 84 = 135$

$\Rightarrow 3x = 135 - 84$ (transposing 84 to RHS)

$\Rightarrow 3x = 51$

$\Rightarrow x = 51 \div 3$ (transposing 3 to RHS)

$\Rightarrow x = 17$

Hence Baichung's age = 17 years

Baichung's father's age = 17 + 29 = 46 years, and grand father’s age = 46 + 26 = 72 years.

Q.12. Fifteen years from now Ravi’s age will be four times his present age. What is Ravi’s present age?

Solution:

Let the present age of Ravi be x years.

After 15 years, his age will be = (x + 15) years

As per the conditions, we get

⇒ x + 15 = 4x

⇒ 15 = 4x – x (transposing x to RHS)

⇒ 15 = 3x

⇒ 15 ÷ 3 = x (transposing 3 to LHS)

⇒ x = 5

Hence, the present age of Ravi = 5 years.

Q.13.A rational number is such that when you multiply it by $\frac{5}{2}$ and add $\frac{2}{3}$ to the product, you get $-\frac{7}{12}$. What is the number?

Solution:

$$x \times \frac{5}{2} + \frac{2}{3} = \frac{-7}{12}$$

$$\Rightarrow \quad \frac{5}{2}x + \frac{2}{3} = \frac{-7}{12}$$

$$\Rightarrow \quad \frac{5}{2}x = \frac{-7}{12} - \frac{2}{3}$$

$$\left(\text{transposing } \frac{2}{3} \text{ to RHS}\right)$$

$$\Rightarrow \quad \frac{5}{2}x = \frac{-7-8}{12}$$

$$\Rightarrow \quad \frac{5}{2}x = \frac{-15}{12}$$

$$\therefore \quad x = \frac{-15}{12} \div \frac{5}{2}$$

$$\left(\text{transposing } \frac{5}{2} \text{ to RHS}\right)$$

$$\Rightarrow \quad x = \frac{-\cancel{15}^{\cancel{3}^{1}}}{\cancel{12}_{\cancel{6}_{2}}} \times \frac{\cancel{2}}{\cancel{5}} = \frac{-1}{2}$$

Hence, the required rational number is $-\ 1/2$

Q.14. Lakshmi is a cashier in a bank. She has currency notes of denominations ₹ 100, ₹ 50 and ₹ 10, respectively. The ratio of these notes is 2 : 3 : 5. The total cash with Lakshmi is ₹ 4,00,000. How many notes of each denomination does she have?

Solution :

Let the number of ₹ 100, ₹ 50 and ₹ 10 notes be 2x, 3x and 5x respectively.

Converting all the denominations into rupees, we have

$2x \times 100$, $3x \times 50$ and $5x \times 10$ i.e. 200x, 150x and 50x

As per the conditions, we have

$200x + 150x + 50x = 4,00,000$

$\Rightarrow 400x = 4,00,000$

$\Rightarrow x = 4,00,000 \div 400$ (transposing 400 to RHS)

$\Rightarrow x = 1,000$

Hence, the required number of notes of

₹ 100 notes = $2 \times 1000 = 2000$

₹ 50 notes = $3 \times 1000 = 3000$

and ₹ 10 notes = $5 \times 1000 = 5000$

Q.15.I have a total of ₹ 300 in coins of denomination ₹ 1, ₹ 2 and ₹ 5. The number of ₹ 2 coins is 3 times the number of ₹ 5 coins. The total number of coins is 160. How many coins of each denomination are with me?

Solution:

Let the number of ₹ 5 coins be x.

Number of ₹ 2 coins = 3x

Total number of coins = 160

Number of ₹ 1 coin = $160 - (x + 3x) = 160 - 4x$

Converting the number of coins into rupees, we have

x coins of ₹ 5 amount to ₹ 5x

3x coins of ₹ 2 amount to ₹ $3x \times 2$ = ₹ 6x

and (160 – 4x) coins of ₹ 1 amount to ₹ $1 \times (160 - 4x)$ = ₹ (160 – 4x)

As per the conditions, we have

$5x + 6x + 160 - 4x = 300$

⇒ 7x + 160 = 300

⇒ 7x = 300 – 160 (transposing 160 to RHS)

⇒ 7x = 140

⇒ x = 140 ÷ 7 (transposing 7 to RHS)

⇒ x = 20

Thus, number of ₹ 5 coins = 20

Number of ₹ 2 coins = 3 × 20 = 60

and Number of ₹ 1 coins = 160 – 4 × 20 = 160 – 80 = 80

Q.16.The organisers of an essay competition decide that a winner in the competition gets a prize of ₹ 100 and a participant who does not win gets a prize of ₹ 25. The total prize money distributed is ₹ 3,000. Find the number of winners, if the total number of participants is 63.

Solution:

Let the number of winners = x

Number of participants who does not win the prize = (63 – x)

Amount got by winners = ₹ 100 × x = ₹ 100x

Amount got by loosers = ₹ (63 – x) × 25 = ₹ (1575 – 25x)

As per the conditions, we get

100x + 1575 – 25x = 3000

⇒ 75x + 1575 = 3000

⇒ 75x = 3000 – 1575 (transposing 1575 to RHS)

⇒ 75x = 1425

⇒ x = 1425 ÷ 75 (Transposing 75 to RHS)

$$\left[\begin{array}{r} 75\,)\,\overline{1425}\,(\,19 \\ -75 \\ \hline 675 \\ -675 \\ \hline 0 \\ \hline \end{array}\right]$$

⇒ x = 19

Thus, the number of winners = 19

Textual Exercise 2.3

Solve the following equations and check your results.

Q.1.3x = 2x + 18

Solution:

We have 3x = 2x + 18

⇒ 3x – 2x = 18 (Transposing 2x to LHS)

⇒ x = 18

Hence, x = 18 is the required solution.

Check: 3x = 2x + 18

Putting x = 18, we have

LHS = 3 × 18 = 54

RHS = 2 × 18 + 18 = 36 + 18 = 54

LHS = RHS

Q.2. 5t – 3 = 3t – 5

Solution:

We have 5t – 3 = 3t – 5

⇒ 5t – 3t – 3 = -5 (Transposing 3t to LHS)

⇒ 2t = -5 + 3 (Transposing -3 to RHS)

⇒ 2t = -2

⇒ t = -2 ÷ 2

⇒ t = -1

Hence t = -1 is the required solution.

Check: 5t – 3 = 3t – 5

Putting t = -1, we have

LHS = 5t – 3 = 5 × (-1)-3 = -5 – 3 = -8

RHS = 3t – 5 = 3 × (-1) – 5 = -3 – 5 = -8

LHS = RHS

Hence verified.

Q.3. 5x + 9 = 5 + 3x

Solution:

We have 5x + 9 = 5 + 3x

⇒ 5x – 3x + 9 = 5 (Transposing 3x to LHS) => 2x + 9 = 5

⇒ 2x = 5 – 9 (Transposing 9 to RHS)

⇒ 2x = -4

⇒ x = -4 ÷ 2 = -2

Hence x = -2 is the required solution.

Check: 5x + 9 = 5 + 3x

Putting x = -2, we have

LHS = 5 × (-2) + 9 = -10 + 9 = -1

RHS = 5 + 3 × (-2) = 5 – 6 = -1

LHS = RHS

Hence verified.

Q.4. 4z + 3 = 6 + 2z

Solution:

We have 4z + 3 = 6 + 2z

⇒ 4z – 2z + 3 = 6 (Transposing 2z to LHS)

⇒ 2z + 3 = 6

⇒ 2z = 6 – 3 (Transposing 3 to RHS)

⇒ 2z = 3

⇒ z = $\frac{3}{2}$

Hence z = $\frac{3}{2}$ **is the required solution.**

Check: 4z + 3 = 6 + 2z

Putting z = $\frac{3}{2}$ **we have**

LHS = 4z + 3 = 4 × $\frac{3}{2}$**+ 3 = 6 + 3 = 9**

RHS = 6 + 2z = 6 + 2 ×$\frac{3}{2}$**= 6 + 3 = 9**

LHS = RHS

Q.5. $2x - 1 = 14 - x$

Solution:

We have $2x - 1 = 14 - x$

$\Rightarrow 2x + x = 14 + 1$ (Transposing x to LHS and 1 to RHS)

$\Rightarrow 3x = 15$

$\Rightarrow x = 15 \div 3 = 5$

Hence $x = 5$ is the required solution.

Check: $2x - 1 = 14 - x$

Putting $x = 5$

LHS we have $2x - 1 = 2 \times 5 - 1 = 10 - 1 = 9$

RHS = $14 - x = 14 - 5 = 9$

LHS = RHS

Hence verified.

Q.6. $8x + 4 = 3(x - 1) + 7$

Solution:

We have $8x + 4 = 3(x - 1) + 7$

$\Rightarrow 8x + 4 = 3x - 3 + 7$ (Solving the bracket)

$\Rightarrow 8x + 4 = 3x + 4$

$\Rightarrow 8x - 3x = 4 - 4$ [Transposing 3x to LHS and 4 to RHS]

$\Rightarrow 5x = 0$

$\Rightarrow x = 0 \div 5$ [Transposing 5 to RHS]

or $x = 0$

Thus $x = 0$ is the required solution.

Check: $8x + 4 = 3(x - 1) + 7$

Putting $x = 0$, we have

$8 \times 0 + 4 = 3(0 - 1) + 7$

$\Rightarrow 0 + 4 = -3 + 7$

$\Rightarrow 4 = 4$

LHS = RHS

Q.7. $x = \frac{4}{5}(x + 10)$

Solution:

We have $x = \frac{4}{5}\ (x + 10)$

⇒ 5 × x = 4(x + 10) (Transposing 5 to LHS)

⇒ 5x = 4x + 40 (Solving the bracket)

⇒ 5x – 4x = 40 (Transposing 4x to LHS)

⇒ x = 40

Thus x = 40 is the required solution.

Check: $x = \frac{4}{5}\ (x + 10)$

Putting x = 40, we have

40 = 45 (40 + 10)

⇒ 40 = 45 × 50

⇒ 40 = 4 × 10

⇒ 40 = 40

LHS = RHS

Hence verified.$x = \frac{4}{5}\ (x + 10)$

Q.8. $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$

	1	5
3	3	15
	3	
	0	15
–	0	15
	0	0

Solution:

We have $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$

$15(\frac{2x}{3} + 1) = 15(\frac{7x}{15} + 3)$

LCM of 3 and 15 is 15

$\frac{2x}{3} \times 15 + 1 \times 15 = \frac{7x}{15} \times 15 + 3 \times 15$ [Multiplying both sides by 15]

⇒ 2x × 5 + 15 = 7x + 45

⇒ 10x + 15 = 7x + 45

⇒ 10x – 7x = 45 – 15 (Transposing 7x to LHS and 15 to RHS)

⇒ 3x = 30

⇒ x = 30 ÷ 3 = 10 (Transposing 3 to RHS)

Thus the required solution is x = 10

Check: $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$

Putting $x = 10$, we have

$$\frac{2 \times 10}{3} + 1 = \frac{7 \times 10}{15} + 3$$

$$\Rightarrow \quad \frac{20}{3} + 1 = \frac{70}{15} + 3$$

$$\Rightarrow \quad \frac{20+3}{3} = \frac{70+45}{15}$$

$$\Rightarrow \quad \frac{23}{3} = \frac{\cancel{115}^{23}}{\cancel{15}_{3}}$$

$$\Rightarrow \quad \frac{23}{3} = \frac{23}{3}$$

LHS = RHS

Hence verified.

Q.9. Ex 2.3

Q.9. $2y + \frac{5}{3} = \frac{26}{3} - y$

Solution:

We have $2y + \frac{5}{3} = \frac{26}{3} - y$

$\Rightarrow \quad 2y + y = \frac{26}{3} - \frac{5}{3}$

$\left(\text{Transposing } y \text{ to LHS and } \frac{5}{3} \text{ to RHS}\right)$

$\Rightarrow \quad 3y = \frac{26-5}{3}$

$\Rightarrow \quad 3y = \frac{21}{3}$

$\Rightarrow \quad 3y = 7$

$\Rightarrow \quad y = \frac{7}{3}$ (Transposing 3 to RHS)

Thus, $y = \frac{7}{3}$ is the required solution.

Check: $2y + \frac{5}{3} = \frac{26}{3} - y$

Putting $y = \frac{7}{3}$, we have

$2 \times \frac{7}{3} + \frac{5}{3} = \frac{26}{3} - \frac{7}{3}$

$\Rightarrow \quad \frac{14}{3} + \frac{5}{3} = \frac{26}{3} - \frac{7}{3}$

$\Rightarrow \quad \frac{14+5}{3} = \frac{26-7}{3}$

$\Rightarrow \quad \frac{19}{3} = \frac{19}{3}$

LHS = RHS

Hence verified.

Q.10 3m = 5m – $\frac{8}{5}$

Solution:

We have

$$3m = 5m - \frac{8}{5}$$

$$\Rightarrow \quad 3m - 5m = \frac{-8}{5} \quad \text{(Transposing } 5m \text{ to LHS)}$$

$$\Rightarrow \quad -2m = \frac{-8}{5}$$

$$\Rightarrow \quad m = \frac{-8}{5} \div -2$$

(Transposing – 2 to RHS)

$$\Rightarrow \quad m = \frac{-8}{5} \times \frac{1}{-2}$$

$$\Rightarrow \quad m = \frac{4}{5}$$

Thus, $m = \frac{4}{5}$ is the required solution.

Check: $3m = 5\,m - \frac{8}{5}$

Putting $m = \frac{4}{5}$, we have

$$\Rightarrow \quad 3 \times \frac{4}{5} = 5 \times \frac{4}{5} - \frac{8}{5}$$

$$\Rightarrow \quad \frac{12}{5} = \frac{20}{5} - \frac{8}{5}$$

$$\Rightarrow \quad \frac{12}{5} = \frac{20-8}{5}$$

$$\Rightarrow \quad \frac{12}{5} = \frac{12}{5}$$

LHS = RHS

Hence verified.

TEXTUAL EXERCISE 2.4

Q.1Amina thinks of a number and subtracts $\frac{5}{2}$from it. She multiplies the result by 8. The result now obtained is 3 times the same number she thought of. What is the number?

Solution:
Let the required number be x.
Condition I: x – 52
Condition II: 8 × (x – 52)
Condition III: 8 × (x – 52) = 3x
⇒ 8x – 52 × 8 = 3x (Solving the bracket)
⇒ 8x – 20 = 3x
⇒ 8x – 3x = 20 (Transposing 3x to LHS and 20 to RHS)
⇒ 5x = 20
⇒ x = 20 ÷ 5 = 4 (Transposing 5 to RHS)
Thus, x = 4 is the required number.

Q.2.A positive number is 5 times another number. If 21 is added to both the numbers, then one of the new numbers becomes twice the other number. What are the numbers?

Solution:
Let the positive number be x.
Other number = 5x
Condition I: x + 21 and 5x + 21
Condition II: 5x + 21 = 2 (x + 21)
⇒ 5x + 21 = 2x + 42 (Solving the bracket)
⇒ 5x – 2x = 42 – 21 (Transposing 2x to LHS and 21 to RHS)
⇒ 3x = 21
⇒ x = 21 ÷ 3 = 7 (Transposing 3 to RHS)
Thus, the required numbers are 7 and 7 × 5 = 35.

Q.3. Sum of the digits of a two digit number is 9. When we interchange the digits, it is found that the resulting new number is greater than the original number by 27. What is the two-digit number?

Solution:
Let unit place digit be x.
Ten's place digit = 9 – x
Original number = x + 10(9 – x)
Condition I: 10x + (9 – x) (Interchanging the digits)
Condition II: New number = original number + 27
⇒ 10x + (9 – x) = x + 10(9 – x) + 27
⇒ 10x + 9 – x = x + 90 – 10x + 27 (solving the brackets)
⇒ 9x + 9 = -9x + 117 (Transposing 9x to LHS and 9 to RHS)
⇒ 9x + 9x = 117 – 9
⇒ 18x = 108
⇒ x = 108 ÷ 18 (Transposing 18 to RHS)
⇒ x = 6
Unit place digit = 6
Ten's place digit = 9 – 6 = 3
Thus, the required number = 6 + 3 × 10 = 6 + 30 = 36

Q.4. One of the two digits of a two digit number is three times the other digit. If you interchange the digits of this two-digit number and add the resulting number to the original number, you get 88. What is the original number?

Solution:

Let unit place digit be x.

Ten's place digit = 3x

Original number = x + 3x × 10 = x + 30x = 31x

Condition I: 10x + 3x = 13x (interchanging the digits)

Condition II: New number + original number = 88

13x + 31x = 88

⇒ 44x = 88

⇒ x = 88 ÷ 44 (Transposing 44 to RHS)

⇒ x = 2

Thus, the original number = 31x = 31 × 2 = 62

Hence the required number = 62

Q.5. Shobo's mother's present age is six times Shobo's present age. Shobo's age five years from now will be one third of his mother's present age. What are their present ages?

Solution:

Let Shobo's present age be x years.

Shobo's mother's age = 6x years.

After 5 years Shobo's age will be (x + 5) years.

As per the condition, we have

$x + 5 = \frac{1}{3} \times 6x$

⇒ x + 5 = 2x

⇒ 5 = 2x – x (Transposing x to RHS)

⇒ 5 = x

Hence Shobo's present age = 5 years

and Shobo's mother's present age 6x = 6 × 5 = 30 years.

Q.6. There is a narrow rectangular plot, reserved for a school, in Mahuli village. The length and breadth of the plot are in the ratio 11 : 4. At the rate of ₹ 100 per metre, it will cost the village panchayat ₹ 75000 to fence the plot. What are the dimensions of the plot?

Solution:
Let the length and breadth of the plot be 11x m and 4x m respectively.
Fencing all around = perimeter of the rectangular plot
Perimeter of the plot = 75000100 = 750 m
2(l + b) = 750
⇒ 2(11x + 4x) = 750
⇒ 2(15x) = 750
⇒ 30x = 750
⇒ x = 750 ÷ 30 = 25
length = 11 × 25 m = 275 m
and breadth = 4 × 25 m = 100 m

Q.7. Hasan buys two kinds of cloth materials for school uniforms, shirt material that costs him ₹ 50 per metre and trousers material that costs him ₹ 90 per metre. For every 3 metres of the shirt material, he buys 2 metres of the trouser material. He sells the materials at 12% and 10% profit respectively. His total sale is ₹ 36,600. How much trouser material did he buy?
Solution:
Ratio of shirt material bought to the trouser material bought = 3 : 2
Let the shirt material bought = 3x m
and trouser material bought = 2x m
Cost of shirt material = 50 × 3x = ₹ 150x
Cost of trouser material = 90 × 2x = ₹ 180x

$$= ₹\left(150x + 150x \times \frac{12}{100}\right)$$
$$= ₹(150x + 18x)$$
$$= ₹\ 168x$$

Selling price of trouser material

$$= ₹\left(180x + 180x \times \frac{10}{100}\right)$$
$$= ₹\ (180x + 18x)$$
$$= ₹\ 198x$$

As per the conditions, we have
168x + 198x = 36,600
⇒ 366x = 36,600
⇒ x = 36600 ÷ 366 = 100
Length of trouser material bought = 2 × 100 = 200 m.

Q.8. Half of a herd of deer are grazing in the field and three-fourths of the remaining are playing nearby. The rest 9 are drinking water from the pond. Find the number of deer in the herd.

Solution:
Let the number of deer be x.
As per the condition, we have

$\frac{x}{2}$ are grazing in the field.

Remaining numbers $= x - \frac{x}{2} = \frac{x}{2}$

$\therefore \frac{x}{2} \times \frac{3}{4} = \frac{3x}{8}$ are playing nearby.

Rest of the deer = 9

$\therefore \quad \frac{x}{2} + \frac{3x}{8} + 9 = x$

LCM of 2 and 8 = 8

$\therefore \quad \frac{x}{2} \times 8 + \frac{3x}{8} \times 8 + 9 \times 8 = x \times 8$

(Multiplying both sides by 8)

⇒ 4x + 3x + 72 = 8x
⇒ 7x + 72 = 8x
⇒ 72 = 8x – 7x (Transposing 7x to RHS)
⇒ x = 12
Hence, the required number of deer = 72.

Q.9. A grandfather is ten times older than his granddaughter. He is also 54 years older than her. Find their present ages.

Solution:
Let the present age of granddaughter = x years.
the present age of grandfather = 10x years.
As per the conditions, we have
10x – x = 54
⇒ 9x = 54
⇒ x = 54 ÷ 9 = 6 [Transposing 9 to RHS]
Hence, the present age of the granddaughter = 6 years
and the present age of grandfather = 6 × 10 = 60 years.

Q.10. Aman's age is three times his son's age. Ten years ago he was five times his son's age. Find their present ages.

Solution:
Let the present age of the son be x years.
Present age of Aman = 3x years
10 years ago, the son's age was = (x – 10) years
10 years ago, the father's age was = (3x – 10) years
As per the conditions, we have
5(x – 10) = 3x – 10

$\Rightarrow 5x - 50 = 3x - 10$
$\Rightarrow 5x - 3x = 50 - 10$(Transposing 3x to LHS and 50 to RHS)
$\Rightarrow 2x = 40$
$\Rightarrow x = 40 \div 2 = 20$
Hence, the son's age = 20 years.
and the age of Aman = 20 × 3 = 60 years.

EXERCISE 2.5

Q.1. $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$

$$\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$$

Solution:

We have $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$

LCM of 2, 5, 3 and 4 = 60

$$\frac{x}{2} \times 60 - \frac{1}{5} \times 60 = \frac{x}{3} \times 60 + \frac{1}{4} \times 60$$

[Multiplying both sides by 60]

$\Rightarrow 30x - 12 = 20x + 15$
$\Rightarrow 30x - 20x = 15 + 12$ (Transposing 20x to LHS and 12 to RHS)
$\Rightarrow 10x = 27$
$\Rightarrow x = \frac{27}{10}$

Q.2. $\frac{n}{2} - \frac{3n}{4} + \frac{5n}{6} = 21$

$$\frac{n}{2} - \frac{3n}{4} + \frac{5n}{6} = 21$$

Solution:
LCM of 2, 4 and 6 = 12

$$\frac{n}{2} \times 12 - \frac{3n}{4} \times 12 + \frac{5n}{6} \times 12 = 21 \times 12$$

$\Rightarrow \frac{n}{2} \times 12 - \frac{3n}{4} \times 12 + \frac{5n}{6} \times 12 = 21 \times 12$
(Multiplying both sides by 12)
$\Rightarrow 6n - 9n + 10n = 252$
$\Rightarrow 7n = 252$
$\Rightarrow n = 252 \div 7$
$\Rightarrow n = 36$

Q.3. $x + 7 - \frac{8x}{3} = \frac{17}{6} - \frac{5x}{2}$

$$x + 7 - \frac{8x}{3} = \frac{17}{6} - \frac{5x}{2}$$

We have $x + 7 - \frac{8x}{3} = \frac{17}{6} - \frac{5x}{2}$

LCM of 3, 6 and 2 = 6

$\therefore 6 \times x + 6 \times 7 - 6 \times \frac{8x}{3}$

$= 6 \times \frac{17}{6} - 6 \times \frac{5x}{2}$

[Multiplying both sides by 6]

$\Rightarrow 6x + 42 - 16x = 17 - 15x$

⇒ -10x + 42 = 17 – 15x
⇒ -10x + 15x = 17 – 42 [Transposing 15x to LHS and 42 to RHS]
⇒ 5x = -25
⇒ x = -25 ÷ 5 [Transposing 5 to RHS]
⇒ x = -5

Q.4. $\frac{x-5}{3} = \frac{x-3}{5}$

$$\frac{x-5}{3} = \frac{x-3}{5}$$

We have $\frac{x-5}{3} = \frac{x-3}{5}$

LCM of 3 and 5 is 15

$\therefore \quad \frac{x-5}{3} \times 15 = \frac{x-3}{5} \times 15$

(Multiplying both sides by 15)

⇒ (x – 5) × 5 = (x – 3) × 3
⇒ 5x – 25 = 3x – 9 (Solving the brackets)
⇒ 5x – 3x = 25 – 9 (Transposing 3x to LHS and 25 to RHS)
⇒ 2x = 16
⇒ x = 16 ÷ 2 = 8 (Transposing 2 to RHS)
⇒ x = 8

Q.5. $\frac{3t-2}{4} - \frac{2t+3}{3} = \frac{2}{3} - t$

$$\frac{3t-2}{4} - \frac{2t+3}{3} = \frac{2}{3} - t$$

L.C.M of 4 and 3 = 12

$\therefore \frac{3t-2}{4} \times 12 - \frac{2t+3}{3} \times 12 = \frac{2}{3} \times 12 - t \times 12$

(Multiplying both sides by 12)

We have $\frac{3t-2}{4} - \frac{2t+3}{3} = \frac{2}{3} - t$

LCM of 4 and 3 = 12

$$\therefore \frac{3t-2}{4} \times 12 - \frac{2t+3}{3} \times 12 = \frac{2}{3} \times 12 - t \times 12$$

(Multiplying both sides by 12)

⇒ (3t – 2) × 3 – (2t + 3) × 4 = 2 × 4 – 12t
⇒ 9t – 6 – 8t – 12 = 8 – 12t (Solving the brackets)
⇒ t – 18 = 8 – 12t
⇒ t + 12t = 8 + 18 (Transposing 12t to LHS and 18 to RHS)
⇒ 13t = 26
⇒ t = 2 (Transposing 13 to RHS)
Hence t = 2 is the required solution.

Q.6. $m - \frac{m-1}{2} = 1 - \frac{m-2}{3}$

$$m - \frac{m-1}{2} = 1 - \frac{m-2}{3}$$

We have $m - \frac{m-1}{2} = 1 - \frac{m-2}{3}$

LCM of 2 and 3 = 6

$$\therefore m \times 6 - \frac{m-1}{2} \times 6 = 1 \times 6 - \frac{m-2}{3} \times 6$$

(Multiplying both sides by 6)

⇒ 6m – (m – 1) × 3 = 6 – (m – 2) × 2
⇒ 6m – 3m + 3 = 6 – 2m + 4 (Solving the brackets)
⇒ 3m + 3 = 10 – 2m
⇒ 3m + 2m = 10 – 3 (Transposing 2m to LHS and 3 to RHS)
⇒ 5m = 7
⇒ m = $\frac{7}{5}$ (Transposing 5 to RHS)

Simplify and solve the following linear equations.

Q.7. 3(t – 3) = 5(21 + 1)

Solution:

We have

3(t – 3) = 5(2t + 1)
⇒ 3t – 9 = 10t + 5 (Solving the brackets)
⇒ 3t – 10t = 9 + 5 (Transposing 10t to LHS and 9 to RHS)
⇒ -7t = 14
⇒ t = -2(Transposing -7 to RHS)
Hence, t = -2 is the required solution.

Q.8. 15(y – 4) – 2(y – 9) + 5(y + 6) = 0

Solution:
We have 15(y – 4) – 2(y – 9) + 5(y + 6) = 0
⇒ 15y – 60 – 2y + 18 + 5y + 30 = 0 (Solving the brackets)
⇒ 8y – 12 = 0
⇒ 8y = 12 (Transposing 12 to RHS)
⇒ y = 23
Hence, y = 23 is the required solution.

Q.9. 3(5z – 7) – 2(9z – 11) = 4(8z – 13) – 17

Solution:
We have
3(5z – 7) – 2(9z – 11) = 4(8z – 13) – 17
⇒ 15z – 21 – 18z + 22 = 32z – 52 – 17 (Solving the bracket)
⇒ -3z + 1 = 32z – 69
⇒ -3z – 32z = – 69 – 1 (Transposing 322 to LHS and 1 to RHS)
⇒ -35z = -70
⇒ z = 2
Hence, z = 2 is the required solution.

Q.10 0.25(4f – 3) = 0.05(10f – 9)

Solution:
We have
0.25(4f – 3) = 0.05(10f – 9)
⇒ 0.25 × 4f – 3 × 0.25 = 0.05 × 10f – 9 × 0.05 (Solving the brackets)
⇒ 1.00f – 0.75 = 0.5f – 0.45
⇒ f – 0.5f = -0.45 + 0.75 (Transposing 0.5 to LHS and 0.75 to RHS)
⇒ 0.5f = 0.30
⇒ f = 0.6

Exercise 2.6

Solve the following equations.

Q.1. $\frac{8x-3}{3x} = 2$

Solution:
We have $\frac{8x-3}{3x} = 2$
$\frac{8x-3}{3x} = \frac{2}{1}$

⇒ $8x - 3 = 2 \times 3x$ (Cross-multiplication)
⇒ $8x - 3 = 6x$
⇒ $8x - 6x = 3$ (Transposing 6x to LHS and 3 to RHS)
⇒ $2x = 3$
⇒ $x = \frac{3}{2}$

Q.2. $\frac{9x}{7-6x} = 15$

Solution:

we have $\frac{9x}{7-6x} = 15$

⇒ = 151
⇒ $9x = 15(7 - 6x)$ (Cross-multiplication)
⇒ $9x = 105 - 90x$ (Solving the bracket)
⇒ $9x + 90x = 105$ (Transposing 90x to LHS)
⇒ $99x = 105$
⇒ $x = \frac{105}{99}$
⇒ $x = \frac{35}{33}$

Q.3. $\frac{z}{z+15} = \frac{4}{9}$

Solution:

We have $\frac{z}{z+15} = \frac{4}{9}$

⇒ $9z = 4(z + 15)$ (Cross-multiplication)
⇒ $9z = 4z + 60$ (Solving the bracket)
⇒ $9z - 42 = 60$
⇒ $5z = 60$
⇒ $z = 12$

Q.4. $\frac{3y+4}{2-6y} = \frac{-2}{5}$

We have $\frac{3y+4}{2-6y} = \frac{-2}{5}$
⇒ $5(3y + 4) = -2(2 - 6y)$ (Cross-multiplication)
⇒ $15y + 20 = -4 + 12y$ (Solving the bracket)
⇒ $15y - 12y = -4 - 20$ (Transposing 12y to LHS and 20 to RHS)
⇒ $3y = -24$ (Transposing 3 to RHS) -24
⇒ $y = -8$

Q.5. $\frac{7y+4}{y+2} = \frac{-4}{3}$

Solution:

we have $\frac{7y+4}{y+2} = \frac{-4}{3}$

⇒ 3(7y + 4) = -4 (y + 2) (Cross-multiplication)
⇒ 21y + 12 = -4y – 8 [Solving the bracket]
⇒ 21y + 4y = -12 – 8 [Transposing 4y to LHS and 12 to RHS]
⇒ 25y = -20 [Transposing 25 to RHS]
⇒ y = $\frac{-20}{25}$

⇒y = $\frac{-4}{5}$

Q.6. The ages of Hari and Harry are in the ratio 5 : 7. Four years from now the ratio of their ages will be 3 : 4. Find their present ages.

Solution:
Let the present ages of Hari and Harry be 5x years and 7x years respectively.
After 4 years Hari's age will be (5x + 4) years and Harry's age will be (7x + 4) years.
As per the conditions, we have
5x+47x+4=34
⇒ 4(5x + 4) = 3(7x + 4) (Cross-multiplication)
⇒ 20x + 16 = 21x + 12 (Solving the bracket)
⇒ 20x – 21x = 12 – 16 (Transposing 21x to LHS and 16 to RHS)
⇒ -x = -4
⇒ x = 4
Hence the present ages of Hari and Harry are 5 × 4 = 20years and 7 × 4 = 28years respectively.

Q.7. The denominator of a rational number is greater than its numerator by 8. If the numerator is increased by 17 and the denominator is decreased by 1, the number obtained is $\frac{3}{2}$. Find the rational number.

Solution:
Let the numerator of the rational number be x.
Denominator = (x + 8)
As per the conditions, we have

$$\frac{x+17}{x+8-1} = \frac{3}{2} \Rightarrow \frac{x+17}{x+7} = \frac{3}{2}$$

⇒ 2(x + 17) = 3(x + 7) (Cross-multiplication)
⇒ 2x + 34 = 3x + 21 (Solving the bracket)
⇒ 2x – 3x = 21 – 34 (Transposing 3x to LHS and 34 to RHS)
⇒ -x = -13

⇒ x = 13
Thus, numerator = 13
and denominator = 13 + 8 = 21

Extra Questions Very Short Answer Type

Question 1.
Q.1. Identify the algebraic linear equations from the given expressions.

(a) $x^2 + x = 2$
(b) $3x + 5 = 11$
(c) $5 + 7 = 12$
(d) $x + y^2 = 3$
Solution:
(a) $x^2 + x = 2$ is not a linear equation.
(b) $3x + 5 = 11$ is a linear equation.
(c) $5 + 7 = 12$ is not a linear equation as it does not contain variable.
(d) $x + y^2 = 3$ is not a linear equation.

Linear Equations in One Variable)

General Instructions: All questions are compulsory.

Q.1 to Q.2 carries one mark each.

Q.3 to Q.7 carries two marks each.

Q.8 and Q.9 carries three marks each.

Q.10 to Q.12 carries four marks each.

1.
   a. Two numbers are in the ratio 6:5. If the sum of the numbers is 110, find the numbers.
   b. Hari's father gave him Rs 70. Now, he has Rs 130. How much money did Hari have in the beginning?
2.
   i. What is the length of the stick, if 1 / 6 of the length of a stick is 5cm.

ii. Five times Raju' pocket money is Rs 80. What is his pocket money?

3. State whether the following statements are True or False:

   a. Sum of two numbers is 95. If one exceeds the other by 15 then the larger number is 40.
   b. x = 4 is the root of 12x + 8 = 56.
   c. The rational number is −1 / 2, when you multiply it by 2 / 5 and 2 / 3 is added to the product, you get 7 / 15.

   d. M = - 5/ 7, for $m - \frac{m-1}{2} = 1 - \frac{m-2}{3}$

4. Fill in the blanks:-

   a. The other name of a solution of an equation is _____.
   b. A polynomial of degree 1 is called _____ polynomial.
   c. The literal symbol which takes on various numerical values is called a _____.
   d. A combination of constants and variables, connect by basic operations is called an _____.Solve the following equation :5t – 3 = 3t – 5

5. Solve the following equation : 37+x=17737+x=177.
6. Solve the following linear equation :x−53=x−35x−53=x−35
7. Solve the following equation : x3+1=715x3+1=715
8. The numerator of a fraction is 2 less than the denominator. If 1 is added to its denominator, it becomes 1212. Find fraction.
9. If you subtract 1212 from a number and multiply the result 1212, you get 1818. What is the number ?
10. Half of a herd of deer are grazing in the field and three fourths of the remaining are playing nearby. The rest 9 are drinking water from the pond. Find the number of deer in the herd.