पुUना International School
Shree Swaminarayan Gurukul, Zundal

Grade – 9
MATHS
Specimen copy
Year 21-22

INDEX

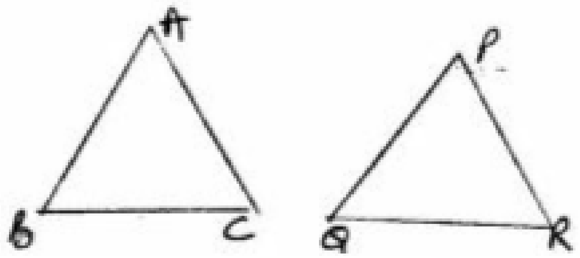
Notes

CHAPTER – 7

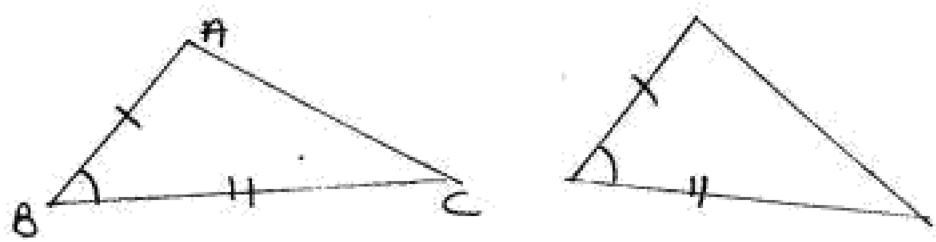
TRIANGLES

1. **Congruence of Triangles**
2. **Criteria for Congruence of Triangles**
3. **Some Properties of a Triangle**
4. **Inequalities in a Triangle**

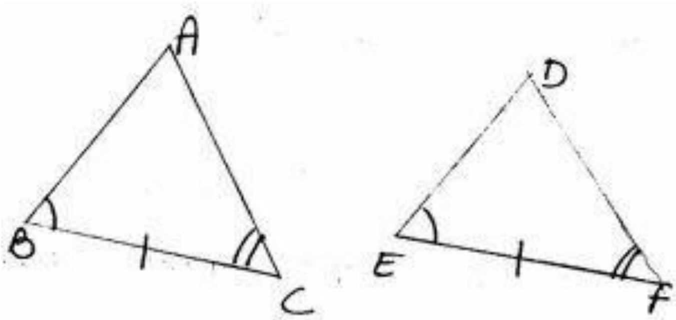
- **Triangle** - A closed figure formed by three intersecting lines is called a triangle. A triangle has three sides, three angles and three vertices.
- **Congruent figures** - Congruent means equal in all respects or figures whose shapes and sizes both are same. For example, two circles of the same radii are congruent. Also two squares of the same sides are congruent.
- **Congruent Triangles** - Two triangles are congruent if and only if one of them can be made to superimpose on the other, so as to cover it completely
- If two triangles ABC and PQR are congruent under the correspondence $A \leftrightarrow P$ $B \leftrightarrow Q$, $and\ C \leftrightarrow R$ then symbolically, it is expressed as $\Delta ABC \cong \Delta PQR$



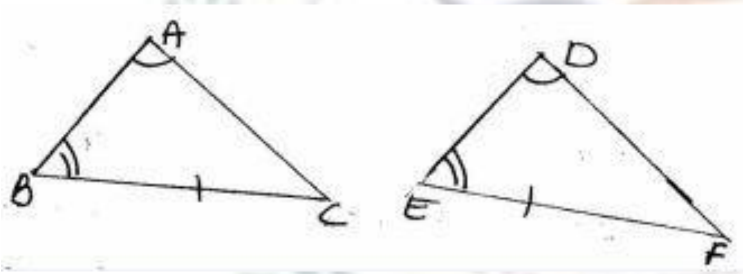
- In congruent triangles, corresponding parts are equal and we write 'CPCT' for corresponding parts of congruent triangles.
- **SAS congruency rule -** Two triangles are congruent if two sides and the included angle between two sides of one triangle are equal to the two sides and the included angle between two sides of the other triangle. For example ΔABC and ΔPQR as shown in the figure satisfy SAS congruence criterion.



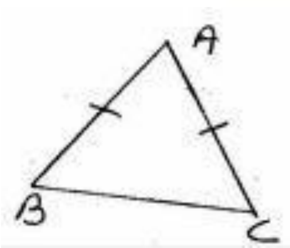
- **ASA Congruence Rule** - Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle. For examples ΔABC and ΔDEF shown below satisfy ASA congruence criterion.



- **AAS Congruence Rule** - Two triangle are congruent if any two pairs of angles and one pair of corresponding sides are equal. For example ΔABC and ΔDEF shown below satisfy AAS congruence criterion.



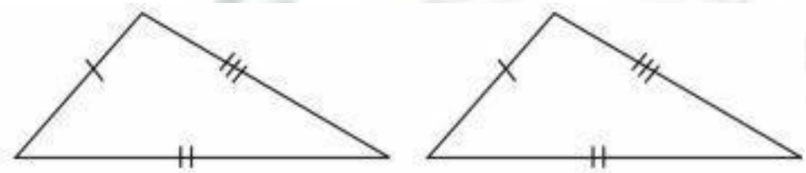
- **AAS criteri**on for congruence of triangles is a particular case of ASA criterion
- . **Isosceles Triangle** - A triangle in which two sides are equal is called an isosceles triangle. For example ΔABC shown below is an isosceles triangle with AB=AC.



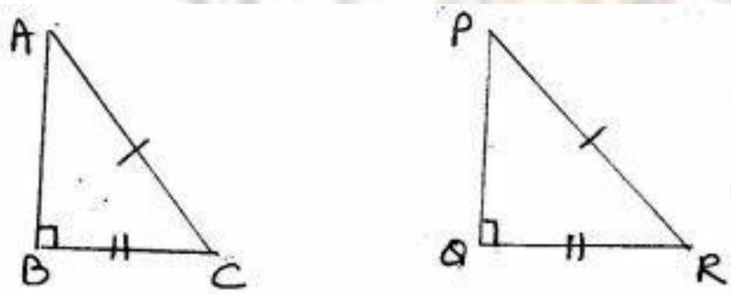
- **Scalene Triangle** - A triangle, no two of whose sides are equal, is called scalene triangle.
- **Equilateral Triangle** - A triangle whose all sides are equal, is called an equilateral triangle.
- **Right angled triangle** - A triangle with one right angle is called a right angled

triangle.

- The sum of all the angles of a triangle is 180^o.
- If a side of a triangle is produced, the exterior angle so formed is equal to the sum of two interior opposite angles.
- Angle opposite to equal sides of a triangle are equal.
- Sides opposite to equal angles of a triangle are equal.
- Each angle of an equilateral triangle is 60^o.
- If the altitude from one vertex of a triangle bisects the base, then the triangle is isosceles triangle.
- **(i) congruence Rule** - If three sides of one triangle are equal to the three sides of another triangle then the two triangles are congruent for example $\Delta ABC\ and\ \Delta DEF$ as shown in the figure satisfy SSS congruence criterion.



- **RHS Congruence Rule** - If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle then the two triangle are congruent. For example $\Delta ABC\ and\ \Delta PQR$ shown below satisfy RHS congruence criterion.

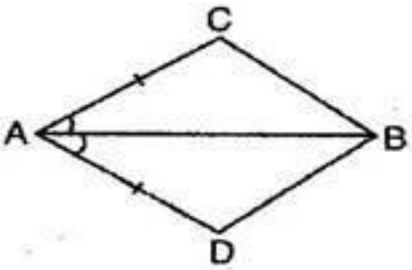


RHS stands for Right angle - Hypotenuse side.

- A point equidistant from two given points lies on the perpendicular bisector of the line segment joining the two points and vice-versa.
- A point equidistant from two intersecting lines lies on the bisectors of the angles formed by the two lines.
- In a triangle, angle opposite to the longer side is larger (greater)
- In a triangle, side opposite to the larger (greater) angle is longer.
- Sum of any two sides of a triangle is greater than the third side.

Ex. 7.1

4. In quadrilateral ABCD (See figure). AC = AD and AB bisects ∠A. Show that ΔABC ≅ ΔABD. What can you say about BC and BD?



Ans. Given: In quadrilateral ABCD, AC = AD and AB bisects ∠A.

To prove: ΔABC ≅ ΔABD

Proof: In ΔABC and ΔABD,

AC = AD [Given]

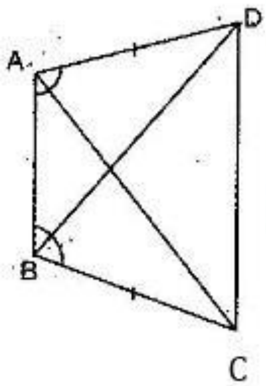
∠BAC = ∠BAD [∵ AB bisects ∠A]

AB = AB [Common]

∴ ΔABC ≅ ΔABD [By SAS congruency]

Thus **BC = BD** [By C.P.C.T.]

(ii) ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA. (See figure). Prove that:



(i) ΔABD ≅ ΔBAC

(ii) BD=AC

(iii) $\angle$ ABD = $\angle$ BAC

Ans. (i) In ΔABC and ΔBAD,

BC = AD [Given]

$\angle$DAB = $\angle$CBA [Given]

AB = AB [Common]

$\therefore$ ΔABC $\cong \Delta$ ABD [By SAS congruency]

Thus AC = BD [By C.P.C.T.]

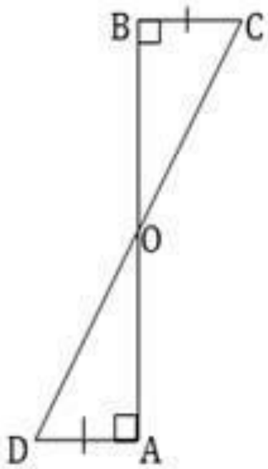
(ii) Since ΔABC $\cong \Delta$ ABD

$\therefore$ **AC = BD** [By C.P.C.T.]

(ii) Since ΔABC $\cong \Delta$ ABD

$\therefore$ $\angle$**ABD =** $\angle$**BAC** [By C.P.C.T.]

(iv) AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB (See figure)



Ans. In ΔBOC and ΔAOD,

$\angle$OBC = $\angle$OAD = 90° [Given]

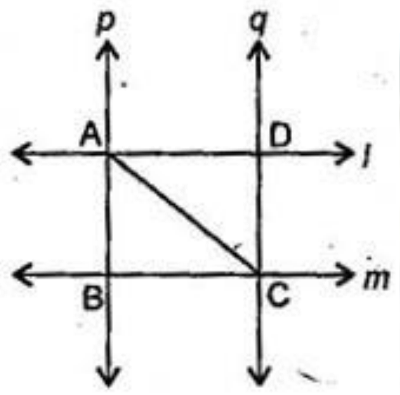
∠BOC = ∠AOD [Vertically Opposite angles]

BC = AD [Given]

∴ ΔBOC ≅ ΔAOD [By AAS congruency]

⇒ **OB = OA** [By C.P.C.T., Also, OC = OD again by C.P.C.T.]

4. l and m are two parallel lines intersected by another pair of parallel lines p and q (See figure). Show that ΔABC ≅ ΔCDA.



Ans. AC being a transversal. [Given]

Therefore ∠DAC = ∠ACB [Alternate angles]

Now p ∥ q [Given]

And AC being a transversal. [Given]

Therefore ∠BAC = ∠ACD [Alternate angles]

Now In ΔABC and ΔADC,

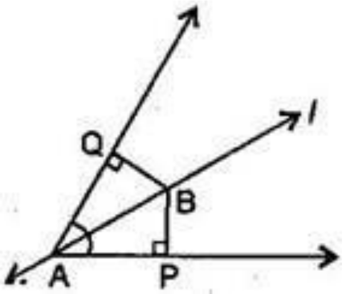
∠ACB = ∠DAC [Proved above]

∠BAC = ∠ACD [Proved above]

AC = AC [Common]

∴ Δ**ABC** ≅ Δ**CDA** [By ASA congruency]

5. Line l is the bisector of the angle A and B is any point on BP and BQ are perpendiculars from B to the arms of ∠A. Show that:



(i) △APB ≅ △AQB

(ii) BP = BQ or B is equidistant from the arms of ∠A (See figure). Ans. Given:

Line l bisects ∠A.

∴ ∠BAP = ∠BAQ

(i) In △ABP and △ABQ,

∠BAP = ∠BAQ [Given]

∠BPA = ∠BQA = 90°

[Given] AB = AB [Common]

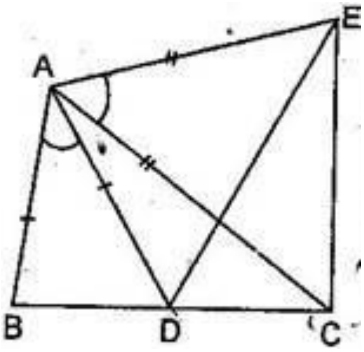
∴ △APB ≅ △AQB [By AAS congruency]

(ii) Since △APB ≅ △AQB

∴ BP = BQ [By C.P.C.T.]

⇒ B is equidistant from the arms of ∠A.

6. In figure, AC = AE, AB = AD and ∠BAD = ∠EAC. Show that BC = DE.



Ans. Given that ∠BAD = ∠EAC

Adding ∠DAC on both sides, we get

∠BAD + ∠DAC = ∠EAC + ∠DAC

⇒ ∠BAC = ∠EAD ……….(i)

Now in ΔABC and ΔADE,

AB = AD [Given]

AC = AE [Given]

∠BAC = ∠DAE [From eq. (i)]

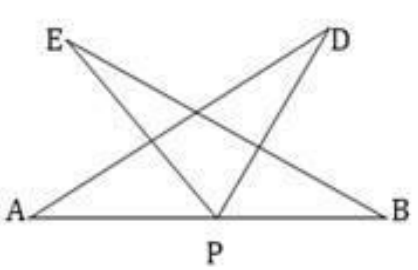
∴ ΔABC ≅ ΔADE [By SAS congruency]

⇒ BC = DE [By C.P.C.T.]

7. AB is a line segment and P is the mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB. Show that:

(i) ΔDAP ≅ ΔEBP

(ii) AD = BE (See figure)



Ans. Given that ∠EPA = ∠DPB

Adding ∠EPD on both sides, we get

∠EPA + ∠EPD = ∠DPB + ∠EPD

⇒ ∠APD = ∠BPE ……….(i)

Now in ΔAPD and ΔBPE,

$\angle$PAD = $\angle$PBE [$\because$ $\angle$BAD = $\angle$ABE (given),

$\therefore$ $\angle$PAD = $\angle$PBE]

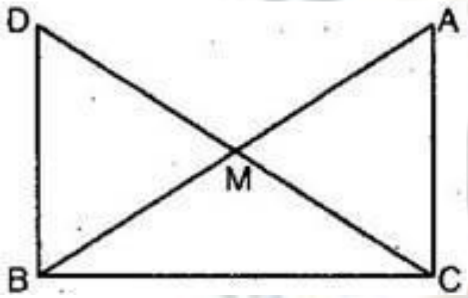
AP = PB [P is the mid-point of AB]

$\angle$APD = $\angle$BPE [From eq. (i)]

$\therefore$ ΔDAP $\cong$ ΔEBP [By ASA congruency]

$\Rightarrow$ AD = BE [By C.P.C.T.]

8. In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. (See figure)



Show that:

(i) ΔAMC $\cong$ ΔBMD

(ii) $\angle$DBC is a right angle.

(iii) ΔDBC $\cong$ ΔACB

(iv) CM = $\frac{1}{2}$ AB

Ans. (i) In ΔAMC and ΔBMD,
AM = BM [M is the mid-point of AB]

$\angle$AMC = $\angle$BMD [Vertically opposite angles]
CM = DM [Given]

$\therefore$ ΔAMC $\cong$ ΔBMD [By SAS congruency]
$\therefore$ $\angle$ACM = $\angle$BDM ……….(i)

$\angle$ CAM = $\angle$ DBM and AC = BD [By C.P.C.T.]

(ii) For two lines AC and DB and transversal DC, we have,

$\angle$ ACD = $\angle$ BDC [Alternate angles]

$\therefore$ AC $\parallel$ DB

Now for parallel lines AC and DB and for transversal BC. $\angle D$
$BC + \angle ACB = 180^0$[cointerior angles].....(ii)

But Δ ABC is a right angled triangle, right angled at C. $\therefore$
$\angle$ ACB = 90^0 ....(iii)

Therefore $\angle$ DBC = 90^0 [Using eq. (ii) and (iii)]

$\Rightarrow$ $\angle$ DBC is a right angle.

(iii) Now in Δ DBC and Δ ABC,

DB = AC [Proved in part (i)]

$\angle$ DBC = $\angle$ ACB = 90^0 [Proved in part (ii)]

BC = BC [Common]

$\therefore$ Δ DBC $\cong \Delta$ ACB [By SAS congruency]

(iv) Since Δ DBC $\cong \Delta$ ACB [Proved above]

$\therefore$ DC=AB

$\Rightarrow$ DM+CM=AB

$\Rightarrow$ CM+CM=AB[$\because$ DM=CM]

$\Rightarrow$ 2CM = AB

$\Rightarrow$ CM= $\frac{1}{2}$AB

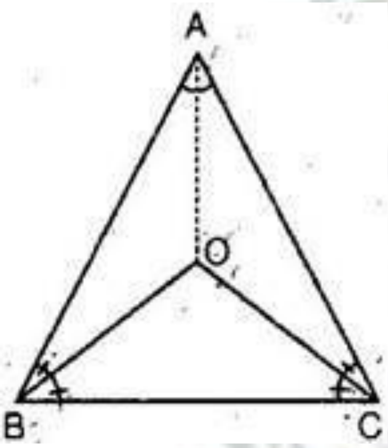
Ex. 7.2

5. In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Join A to O. Show that:

(iii) OB=OC

(iv) AO bisects ∠A.

Ans. (i) ABC is an isosceles triangle in which AB = AC.



∴ ∠C = ∠B [Angles opposite to equal sides]

⇒ ∠OCA + ∠OCB = ∠OBA + ∠OBC

∵ OB bisects ∠B and OC bisects ∠C

∴ ∠OBA = ∠OBC and ∠OCA = ∠OCB

⇒ ∠OCB + ∠OCB = ∠OBC + ∠OBC

⇒ 2∠OCB=2∠OBC

⇒ ∠OCB = ∠OBC

Now in ΔOBC,

$\angle$OCB = $\angle$OBC [Proved above]

$\therefore$ OB = OC [Sides opposite to equal angles]

(iv) In ΔAOB and ΔAOC,

AB = AC [Given]

OA = OA [Common]

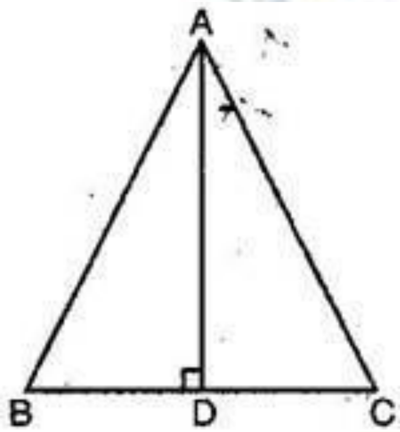
OB = OC [Prove above]

$\therefore$ ΔAOB $\cong$ ΔAOC [By SSS congruency]

$\Rightarrow$ $\angle$OAB = $\angle$OAC [By C.P.C.T.]

Hence AO bisects $\angle$A.

(iii) In ΔABC, AD is the perpendicular bisector of BC (See figure). Show that ΔABC is an isosceles triangle in which AB = AC.



Ans. In ΔADB and ΔADC,

BD = CD [AD bisects BC]

$\angle$ADB = $\angle$ADC = 90° [AD $\perp$ BC]

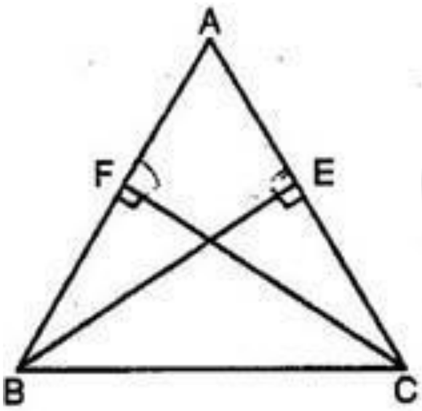
AD = AD [Common]

$\therefore$ ΔABD $\cong$ ΔACD [By SAS congruency]

$\Rightarrow$ AB = AC [By C.P.C.T.]

Therefore, ABC is an isosceles triangle with AB = AC. Hence, proved.

(iii) ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (See the given figure). Show that these altitudes are equal.



Ans. In Δ ABE and Δ ACF,

$\angle$A= $\angle$A [Common]

$\angle$AEB = $\angle$AFC = 90° [Given]

AB = AC [Given]

$\therefore$ ΔABE $\cong$ Δ ACF [By AAS congruency]

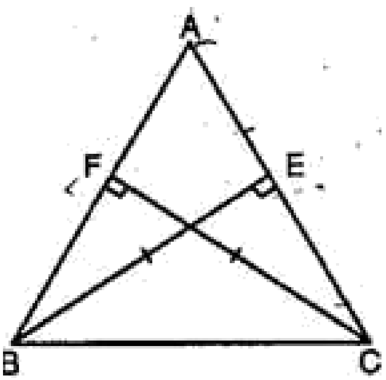
$\Rightarrow$ BE = CF [By C.P.C.T.]

$\Rightarrow$ **Altitudes are equal.**

(v) ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (See figure). Show that:

(iii) ΔABE $\cong$ Δ ACF

(iv) AB = AC or ΔABC is an isosceles triangle.



Ans. (i) In ΔABE and ΔACF,

$\angle$A= $\angle$A [Common]

$\angle$AEB = $\angle$AFC = 90° [Given]

BE = CF [Given]

$\therefore$ ΔABE $\cong$ ΔACF [By AAS congruency]

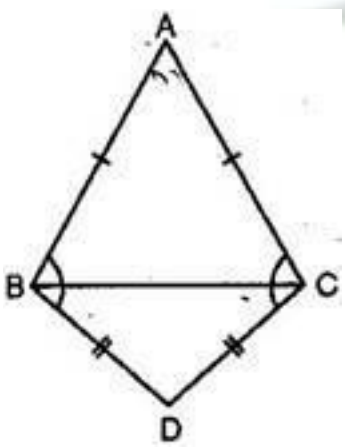
(iii) Since ΔABE $\cong$ Δ

ACF $\Rightarrow$ BE = CF [By

C.P.C.T.]

$\Rightarrow$ ABC is an isosceles triangle.

8. ABC and DBC are two isosceles triangles on the same base BC (See figure). Show that $\angle$ABD = $\angle$ACD.



Ans. In isosceles triangle ABC,

AB = AC [Given]

∠ACB = ∠ABC …….(i) [Angles opposite to equal sides]

Also in Isosceles triangle BCD.

BD=DC

∴ ∠BCD = ∠CBD ……….(ii) [Angles opposite to equal sides]

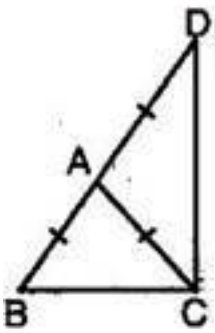
Adding eq. (i) and (ii),

∠ACB + ∠BCD = ∠ABC + ∠CBD

⇒ ∠ACD = ∠ABD

Or ∠ABD = ∠ACD

(iii) Δ ABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB. Show that ∠BCD is a right angle (See figure).



Ans. In isosceles triangle ABC,

AB = AC [Given]

∠ACB = ∠ABC …….(i) [Angles opposite to equal sides]

Now AD = AB [By construction]

But AB = AC [Given]

∴ AD=AB=AC

$\Rightarrow$ AD=AC

Now in triangle ADC,

AD=AC

$\Rightarrow \angle$ADC = $\angle$ACD ………(ii) [Angles opposite to equal sides]

In triangle BCD,

$\Rightarrow \angle ABC + \angle BCD + \angle CDA = 180^0$ [Angle sum property]

$\Rightarrow \angle ACB + \angle BCD + \angle CDA = 180^0$ [Because $\angle$ACB = $\angle$ABC, see (i)]

$\Rightarrow \angle ACB + \angle ACB + \angle ACD + \angle CDA = 180^0$ [Because $\angle BCD = \angle ACB + \angle ACD$]

$\Rightarrow 2\angle ACB + \angle ACD + \angle CDA = 180^0$

$\Rightarrow 2\angle ACB + \angle ACD + \angle ACD = 180^0$ [Because $\angle$ADC = $\angle$ACD, see (ii)]

$\Rightarrow 2\angle ACB + 2\angle ACD = 180^0$

$\Rightarrow 2(\angle ACB + \angle ACD) = 180^0$ [Taking out 2 common]

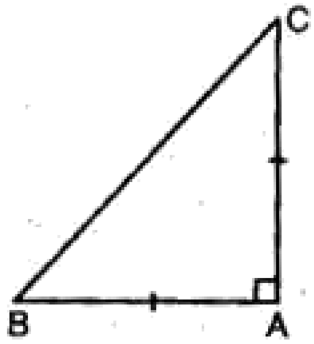
$\Rightarrow 2\angle BCD = 180^0$ [Because, $\angle ACD + \angle ACB = \angle BCD$]

$\Rightarrow \angle$BCD = 90°

Hence $\angle$BCD is a right angle.

9. ABC is a right angled triangle in which $\angle$A = 90° and AB = AC. Find $\angle$B and $\angle$C. Ans. ABC

is a right triangle in which,



$\angle A = 90^\circ$ And AB = AC

In ΔABC,

AB=AC

$\Rightarrow \angle C = \angle B$ ……….(i)

We know that, in ΔABC,

$\angle A + \angle B + \angle C = 180^\circ$ [Angle sum property]

$\Rightarrow 90^\circ + \angle B + \angle B =$

[$\angle A = 90^\circ$ (given) and $\angle B = \angle C$ (from eq. (i)]

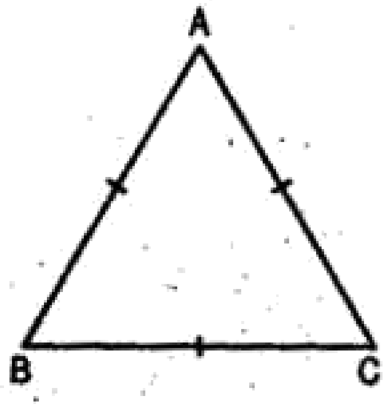
$\Rightarrow 2\angle B = 90^\circ$

$\Rightarrow \angle B = 45^\circ$

Also $\angle C = 45^\circ$ [$\angle B = \angle C$]

(iv) Show that the angles of an equilateral triangle are 60° each.

Ans. Let ABC be an equilateral triangle.



$\therefore$ AB=BC=AC

$\Rightarrow$ AB=BC

$\Rightarrow \angle C = \angle A$ ……….(i)

Similarly, AB = AC

$\Rightarrow \angle C = \angle B$ ……….(ii)

From eq. (i) and (ii),

$\angle A = \angle B = \angle C$ ……….(iii)

Now in Δ ABC

$\angle A + \angle B + \angle C = 180^\circ$ ……….(iv)

$\Rightarrow \angle A + \angle A + \angle A = 180^\circ$

$\Rightarrow 3\angle A = 180^\circ$

$\Rightarrow \angle A = 60^\circ$

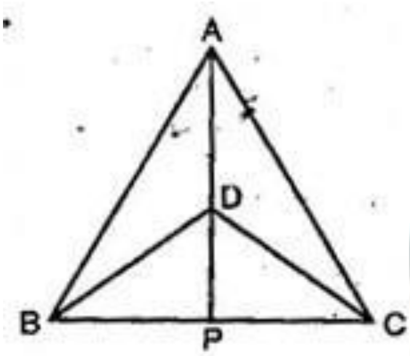
Since $\angle A = \angle B = \angle C$ [From eq. (iii)]

$\therefore \angle A = \angle B = \angle C =$

Hence each angle of equilateral triangle is 60°.

Ex. 7.3

6. ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (See figure). If AD is extended to intersect BC at P, show that:



(v) ΔABD $\cong \Delta$ ACD

(vi) ΔABP $\cong \Delta$ ACP

(vii) AP bisects $\angle$A as well as $\angle$D.

(viii)AP is the perpendicular bisector of BC. Ans. (i)

ΔABC is an isosceles triangle.

$\therefore$ AB=AC

ΔDBC is an isosceles triangle.

$\therefore$ BD=CD

Now in ΔABD and ΔACD,

AB = AC [Given]

BD = CD [Given] AD

= AD [Common]

$\therefore \Delta$ABD $\cong \Delta$ ACD [By SSS congruency]

$\Rightarrow \angle$ BAD = $\angle$ CAD [By C.P.C.T.] ……….(i)

(v) Now in ΔABP and ΔACP,

AB = AC [Given]

$\angle$ BAD = $\angle$ CAD [From eq. (i)]

AP = AP [Common]

$\therefore \Delta$ABP $\cong \Delta$ ACP [By SAS congruency]

Also, BP = CP [By C.P.C.T.]............(ii)

(iv) Since ΔABP $\cong \Delta$ ACP [From part (ii)]

$\Rightarrow \angle$ BAP = $\angle$ CAP [By C.P.C.T.]

$\Rightarrow$ AP bisects $\angle$ A.

In ΔBDP and ΔC

DP, BD = CD [Given]

DP = DP [Commmon] BP

= CP [From eqn (ii)]

Therefore, $\Delta BDP \cong \Delta CDP$ [By SSS Conruency]

=> $\angle BDP = \angle CDP$ [By C.P.C.T.]..............(iii)

and $\angle BPD = \angle CPD$ [By C.P.C.T.] ....(iv)

Hence, AP bisects $\angle D$ from (iii)

(iv) Since=> $\angle BPD = \angle CPD$[By eqn (iv)]

Now $\angle$ BPD + $\angle$ CPD = 180° [Linear pair]

$\Rightarrow \angle BPD + \angle BPD = 180^\circ$ [Using eq. (iii)]

$\Rightarrow 2\angle BPD = 180^\circ$

$\Rightarrow \angle BPD = 90^\circ$

$\Rightarrow AP \perp BC$ ……….(v)

From eq. (iv) and (v), we have AP = BP and AP $\perp$ BC. So, collectively AP is perpendicular bisector of BC.

(vi) AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that:

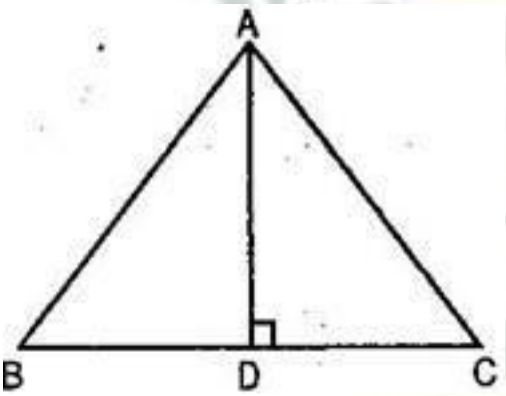
(i) AD bisects BC. (ii)

AD bisects $\angle$A.

Ans. In ΔABD and ΔACD,

AB = AC [Given]

$\angle ADB = \angle ADC = 90^\circ$ [AD $\perp$ BC]



AD = AD [Common]

$\therefore \Delta ABD \cong \Delta ACD$ [RHS rule of congruency]

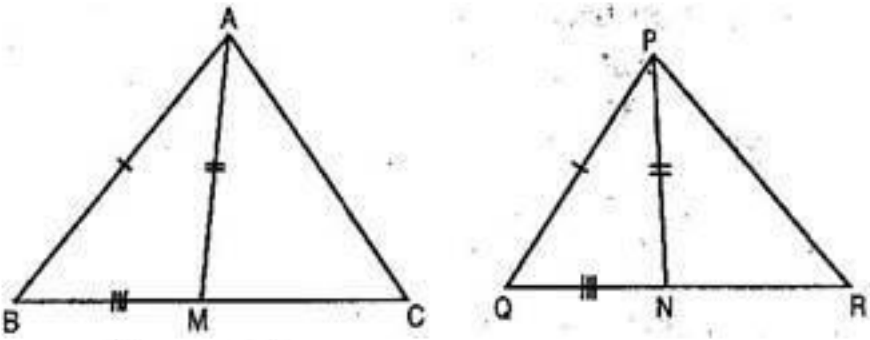
$\Rightarrow$ BD = DC [By C.P.C.T.]

$\Rightarrow$ AD bisects BC

Also $\angle BAD = \angle CAD$ [By C.P.C.T.]

$\Rightarrow$ AD bisects $\angle$A. Hence proved.

(v) Two sides AB and BC and median AM of the triangle ABC are respectively equal to side PQ and QR and median PN of ΔPQR (See figure). Show that:



(iv) ΔABM $\cong \Delta$ PQN

(v) ΔABC $\cong \Delta$ PQR

Ans. AM is the median of ΔABC.

$\therefore$ BM = MC = $\frac{1}{2}$ BC ……….(i)

PN is the median of ΔPQR.

$\therefore$ QN = NR = $\frac{1}{2}$ QR ……….(ii)

Now BC = QR [Given] $\Rightarrow \frac{1}{2}$ BC = $\frac{1}{2}$ QR

$\therefore$ BM = QN ……….(iii)

9. Now in ΔABM and ΔPQN,

AB = PQ [Given]

AM = PN [Given]

BM = QN [From eq. (iii)]

$\therefore \Delta$ABM $\cong \Delta$ PQN [By SSS congruency]

$\Rightarrow \angle$B = $\angle$Q [By C.P.C.T.] ……….(iv)

10. In ΔABC and ΔPQR,

AB = PQ [Given]

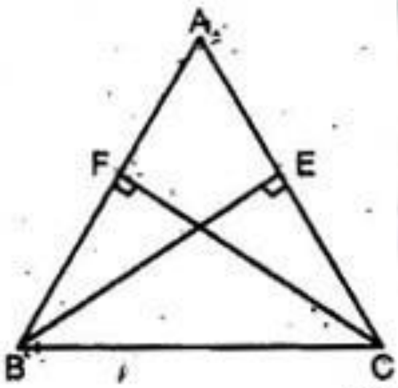
$\angle$B = $\angle$Q [Prove above]

BC = QR [Given]

$\therefore$ ΔABC $\cong$ Δ PQR [By SAS congruency]

(iv) BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Ans. In ΔBEC and ΔCFB,



$\angle$BEC = $\angle$CFB [Each 90°]

BC = BC [Common]

BE = CF [Given]

$\therefore$ ΔBEC $\cong$ Δ CFB [RHS congruency]

$\Rightarrow$ EC = FB [By C.P.C.T.] …..(i)

Now In ΔAEB and ΔAFC

$\angle$AEB = $\angle$AFC [Each 90°]

$\angle$A = $\angle$A [Common]

BE = CF [Given]

$\therefore$ ΔAEB $\cong$ Δ AFC [AAS congruency]

$\Rightarrow$ AE = AF [By C.P.C.T.] …………(ii)

Adding eq. (i) and (ii), we get,

EC+AE=FB+AF

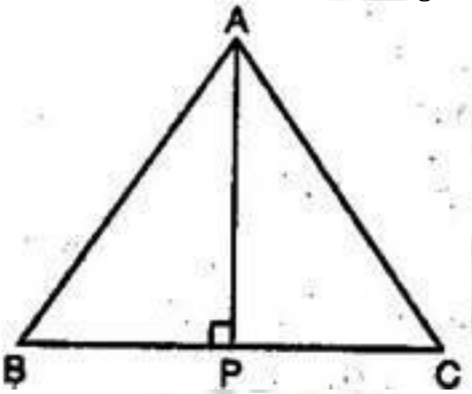
⇒ AB=AC

⇒ ABC is an isosceles triangle.

Hence proved.

10. ABC is an isosceles triangles with AB = AC. Draw AP $\perp$ BC and show that $\angle$B = $\angle$C. Ans.

Given: ABC is an isosceles triangle in which AB = AC



To prove: $\angle$B = $\angle$C

Construction: Draw AP $\perp$ BC

Proof: In ΔABP and ΔACP

$\angle$APB = $\angle$APC = 90° [By construction]

AB = AC [Given]

AP = AP [Common]

$\therefore$ ΔABP $\cong$ ΔACP [RHS congruency]

⇒ $\angle$ B = $\angle$C [By C.P.C.T.]

Hence proved.

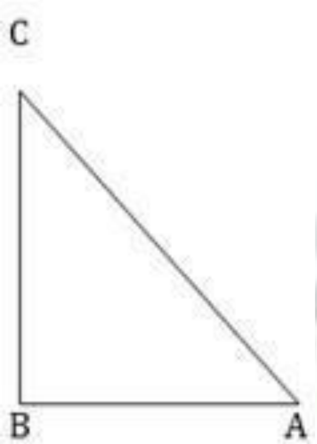
Ex. 7.4

1. Show that in a right angles triangle, the hypotenuse is the longest side.

Ans. Given: Let ABC be a right angled triangle, right angled at B.

To prove: Hypotenuse AC is the longest side.

Proof: In right angled triangle ABC,



$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle A + 90^\circ + \angle C = 180^\circ$ [$\because \angle B = 90^\circ$]

$\Rightarrow \angle A + \angle C = 180^\circ - 90^\circ$

=> $+ \ C \quad 90$

And $\angle B = 90^\circ$

$\Rightarrow \angle B > \angle C$ and $\angle B > \angle A$

Since the greater angle has a longer side opposite to it.

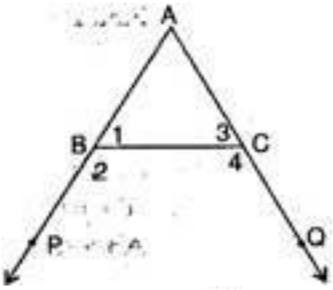
$\Rightarrow$ AC > AB and AC > BC

Therefore $\angle$B being the greatest angle has the longest opposite side AC, i.e. hypotenuse.

Hence, proved.

2. In figure, sides AB and AC of ΔABC are extended to points P and Q respectively. Also

$\angle$PBC < $\angle$QCB. Show that AC > AB.



Ans. Given: In ΔABC, $\angle$PBC < $\angle$QCB

To prove: AC > AB

Proof: In the given figure,

$\angle$4 > $\angle$2 [Given]

Now $\angle$1 + $\angle$2 = $180°$ [Linear pair]

=> $\angle 1 = 180^0 - \angle 2$

And, $\angle$3 + $\angle$4 = $180°$

=> $\angle 3 = 180^0 - \angle 4$

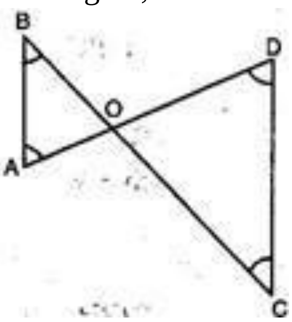
Because, $\angle$4 is greater than $\angle$2, therefore when we will subtract it from $180°$ we will get a value which would be lesser than the quantity obtained on deducting $\angle$2 from $180°$.

$\therefore \angle 1 > \angle 3$

$\Rightarrow$ AC > AB [Side opposite to greater angle is longer]

Hence, proved.

3. In figure, $\angle$B < $\angle$A and $\angle$C < $\angle$D. Show that AD < BC.



Ans. In ΔAOB,

$\angle$A > $\angle$B [Given]

$\Rightarrow$ OB > OA ……….(i) [Side opposite to greater angle is longer]

Similarly, In ΔCOD,

$\angle$D > $\angle$C [Given]

$\Rightarrow$ OC > OD ……….(ii) [Side opposite to greater angle is longer]

Adding eq. (i) and (ii),

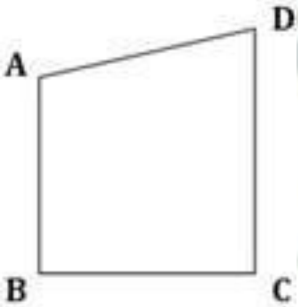
OB+OC>OA+OD

=>BC>AD

$\Rightarrow$ AD<BC

Hence, proved.

7. AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (See figure). Show that $\angle$A > $\angle$C and $\angle$B > $\angle$D.

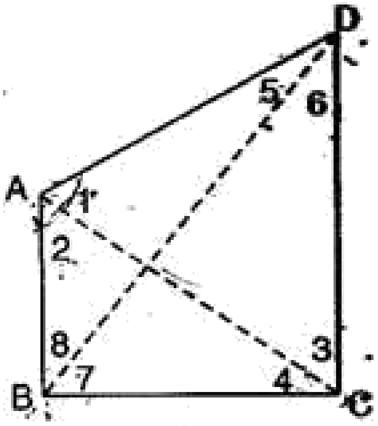


Ans. Given: ABCD is a quadrilateral with AB as smallest and CD as longest side.

To prove: (i) $\angle$A > $\angle$C (ii) $\angle$B > $\angle$D

Construction: Join AC and BD.

Proof: (i) In ΔABC, AB is the smallest side.



∴ ∠4 < ∠2 ……….(i)

[Angle opposite to smaller side is smaller]

In ADC, DC is the longest side.

∴ ∠3 < ∠1 ……….(ii)

[Angle opposite to smaller side is smaller]

Adding eq. (i) and (ii),

∠4+ ∠3< ∠1+ ∠2

⇒ ∠C < ∠A

=> ∠A> ∠C

(ix) In ΔABD, AB is the smallest side.

∴ ∠5 < ∠8 ……….(iii)

[Angle opposite to smaller side is smaller] In Δ

BDC, DC is the longest side.

∴ ∠6 < ∠7 ……….(iv)

[Angle opposite to smaller side is smaller]

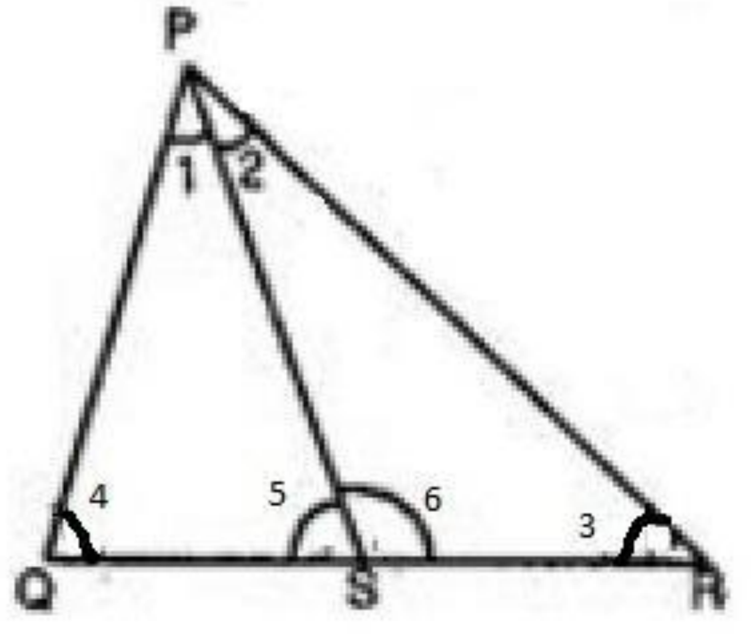
Adding eq. (iii) and (iv),

∠5+ ∠6< ∠7+ ∠8

$\Rightarrow \angle D < \angle B$

=> $\angle B > \angle D$

5. In figure, PR > PQ and PS bisects $\angle$QPR. Prove that $\angle$PSR > $\angle$PSQ.



Ans. In ΔPQR, PR > PQ [Given]

$\therefore \angle 4 > \angle 3$ …..(i) [Angle opposite to longer side is greater]

Again $\angle 1 = \angle 2$ …..(ii) [$\because$ PS is the bisector of $\angle$P]

Now, $\angle 6$ is exterior angle of ΔPQS,

=> $\angle 6 = \angle 4 + \angle 1$ ....(iii)

Again, $\angle 5$ is exterior angle of ΔPSR

=> $\angle 5 = \angle 2 + \angle 3$ .... (iv)

Adding (i) and (ii), we get :-

=> $\angle 4 + \angle 1 > \angle 2 + \angle 3$

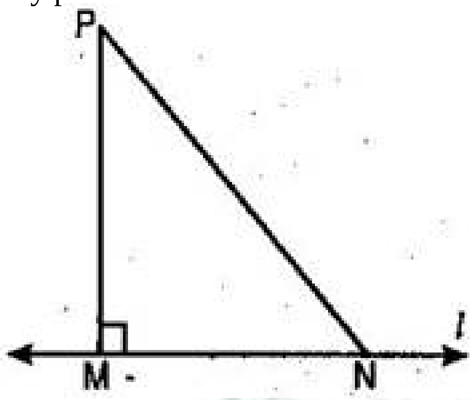
=> $\angle 6 > \angle 5$ [From, (iii) and (iv)]

i.e. $\angle$PSR > $\angle$PSQ

Hence, Proved.

(vi) Show that all the line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Ans. Given: l is a line and P is point not lying on l. PM $\perp l$ N is any point on l other than M.



To prove: PM < PN

Proof: In ΔPMN $\angle$M is the right angle.

$\therefore$ N is an acute angle. (Angle sum property of Δ)

$\therefore$ $\angle$M > $\angle$N

$\therefore$ PN > PM [Side opposite greater angle]

$\Rightarrow$ PM<PN

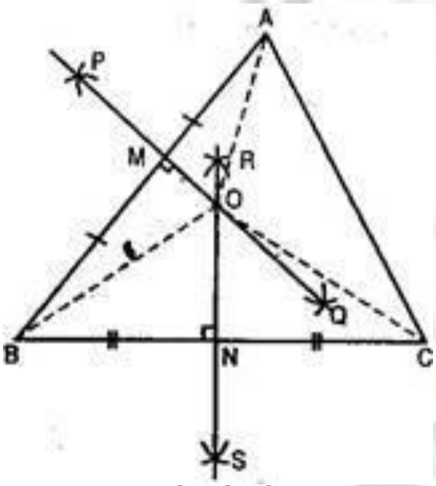
Hence of all line segments drawn from a given point not on it, the perpendicular is the shortest.

Ex. 7.5

8. ABC is a triangle. Locate a point in the interior of ΔABC which is equidistant from all the vertices of ΔABC.

Ans. The point which is equidistant from all the vertices of a triangle is known as the circum-centre of the triangle. This point acts as the centre of a circle which can be drawn by passing through the vertices of the given triangle. And to find out the circum-centre we usually, draw the perpendicular bisectors of any two sides, their point of intersection is the required point which is equidistant from the vertices (being the radius). So we will proceed with drawing a circum-centre.

Let ABC be a triangle.



Draw perpendicular bisectors PQ and RS of sides AB and BC respectively of triangle ABC. Let PQ bisects AB at M and RS bisects BC at point N.

Let PQ and RS intersect at point O.

Join OA, OB and OC.

Now in ΔAOM and Δ BOM,

AM = MB [By construction]

$\angle$AMO = $\angle$BMO = 90° [By construction]

OM = OM [Common]

$\therefore$ ΔAOM $\cong$ Δ BOM [By SAS congruency]

$\Rightarrow$ OA = OB [By C.P.C.T.] …..(i)

Similarly, ΔBON $\cong \Delta$CON

$\Rightarrow$ OB = OC [By C.P.C.T.] …..(ii)

From eq. (i) and (ii),

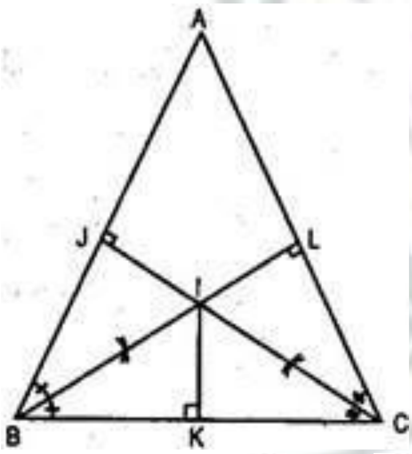
OA=OB=OC

Hence O, the point of intersection of perpendicular bisectors of any two sides of ΔABC equidistant from its vertices.

(x) In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

Ans. The point which is equidistant from all the sides of a triangle is known as its in-centre and is the point of intersection of the angle bisectors. Hence we will proceed with finding the in-centre of the given triangle.

Let ABC be a triangle.



Draw bisectors of $\angle$B and $\angle$C.

Let these angle bisectors intersect each other at point I.
Draw IK $\perp$ BC

Also draw IJ $\perp$ AB and IL $\perp$ AC.

Join AI.

In ΔBIK and ΔBIJ,

$\angle$IKB = $\angle$IJB = 90° [By construction]

$\angle$IBK = $\angle$IBJ

[$\because$ BI is the bisector of $\angle$B (By construction)] BI =

BI [Common]

$\therefore$ ΔBIK $\cong$ Δ BIJ [ASA criteria of congruency]

$\therefore$ IK = IJ [By C.P.C.T.] ……….(i)

Similarly, ΔCIK $\cong$ Δ CIL

$\therefore$ IK = IL [By C.P.C.T.] ……….(ii)

From eq (i) and (ii),

IK=IJ=IL

Hence, I is the point of intersection of angle bisectors of any two angles of ΔABC equidistant from its sides.

(vii) In a huge park, people are concentrated at three points (See figure).

A: where there are different slides and swings for children.

B: near which a man-made lake is situated.

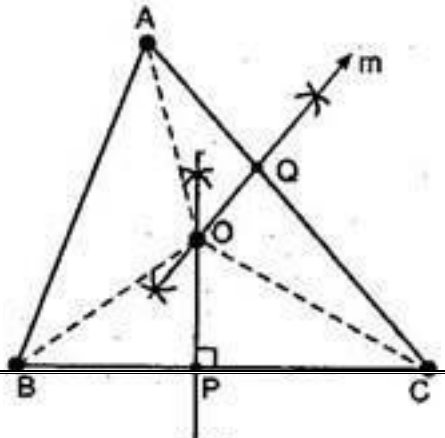
C: which is near to a large parking and exit.

Where should an ice cream parlour be set up so that maximum number of persons can approach it?

Ans. The parlour should be equidistant from A, B and C. So we should find out the circum-centre

of the triangle obtained by joining A, B and C respectively.

For this let us draw perpendicular bisector say l of line joining points B and C also draw perpendicular bisector say m of line joining points A and C.



Let l and m intersect each other at point O. O is the required point.

Proof that O is the required point:

Join OA, OB and OC.

Proof: In ΔBOP and ΔCOP,

OP = OP [Common]

$\angle$OPB = $\angle$OPC = 90°

BP = PC [P is the mid-point of BC]

$\therefore$ ΔBOP $\cong$ Δ COP [By SAS congruency]

$\Rightarrow$ OB = OC [By C.P.C.T.] …..(i)

Similarly, ΔAOQ $\cong$ Δ COQ
$\Rightarrow$ OA = OC [By C.P.C.T.] …..(ii)

From eq. (i) and (ii),

OA=OB=OC

Therefore, O is the required point as it is equidistant from the given points. Thus, ice cream parlour should be set up at point O, the point of intersection of perpendicular bisectors of any two sides out of three formed by joining these points.

(v) Complete the hexagonal rangoli and the star rangolies (See figure) but filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?

Ans. In hexagonal rangoli, Number of equilateral triangles each of side 5 cm are 6.

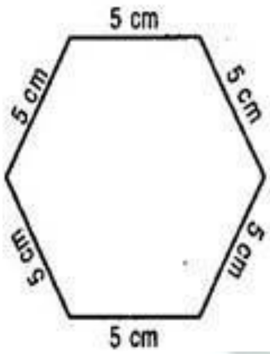
Area of equilateral triangle = $\frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(5)^2 = \frac{\sqrt{3}}{4}\times 25$ sq. cm

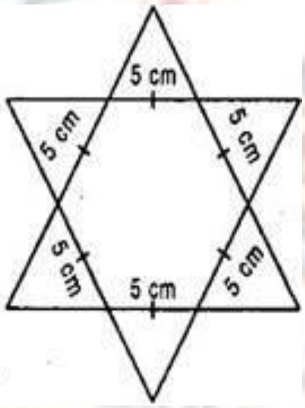
Area of hexagonal rangoli = 6 x Area of an equilateral triangle

= $6\times\frac{\sqrt{3}}{4}\times 25 = 150\times\frac{\sqrt{3}}{4}$ sq. cm ……….(i)

Now area of equilateral triangle of side 1 cm = = $\frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(1)^2 = \frac{\sqrt{3}}{4}$ sq. cm …..(ii)

Number of equilateral triangles each of side 1 cm in hexagonal rangoli

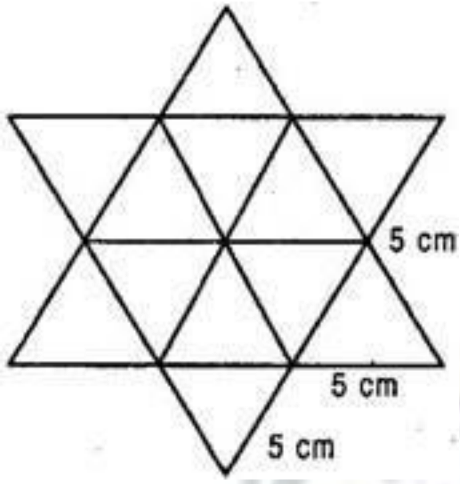




$= 150\times\frac{\sqrt{3}}{4} \div \frac{\sqrt{3}}{4} \quad 150\times\frac{\sqrt{3}}{4}\times\frac{4}{\sqrt{3}}$ = 150 …..(iii)

Now in Star rangoli,

Number of equilateral triangles each of side 5 cm = 12



Therefore, total area of star rangoli = 12 $\times$ Area of an equilateral triangle of side 5 cm

$= 12\times\left(\frac{\sqrt{3}}{4}(5)^2\right)$

$= 12\times\frac{\sqrt{3}}{4}\times 25$

(v)$300\frac{\sqrt{3}}{4}$ sq. cm ……….(iv)

Number of equilateral triangles each of side 1 cm in star rangoli

$= 300\frac{\sqrt{3}}{4} \div \frac{\sqrt{3}}{4}$

$= 300\frac{\sqrt{3}}{4}\times\frac{4}{\sqrt{3}}$

= 300 ……….(v)

From eq. (iii) and (v), we observe that star rangoli has more equilateral triangles each of side 1 cm.

Notes

Chapter 15

PROBABILITY

• Probability – An Experimental Approach

1. Experiment - A procedure which produces some well defined possible outcomes..
2. Random experiment - An experiment which when performed produces one of the several possible outcomes called a random experiment.
3. Trial - When we perform an experiment it is called a trial of the experiment.
4. Event - The set of outcomes of an experiment to which probability is assigned.It is usually denoted by capital letter of English alphabets like A, B, E etc.
5. A collection of two or more possible outcomes (elementary events) of an experiment called a compound event.
6. An event is said to be happen in trial if any one of the elementary events (or outcomes) satisfying its conditions is an outcome.
7. The empirical (or experimental) probability P(E) of an event E is given by

$$P(E) = \frac{Number\ of\ trials\ in\ which\ E\ has\ happend}{Total\ no.\ of\ trial}$$

The probability of an event lies between 0 and 1 (0 and 1 are included)

Impossible event: Event which never happen.

Certain event - event which definitely happen.

Ex. 15.1

1. In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.

Ans. Probability = $\dfrac{\text{Favourable outcomes}}{\text{Total outcomes}}$

Number of times on boundary is not hit = 30 – 6 = 24

$\therefore$ P (did not hit a boundary) = $\dfrac{24}{30} = \dfrac{4}{5}$

2. 1500 families with 2 children were selected randomly and the following data were recorded:

No. of girls in a family	No. of families
2	475
1	814
0	211

Compute the probability of a family, chosen at random, having:

(i) 2 girls (ii) 1 girl (iii) No girl

Also check whether the sum of these probabilities is 1.

Ans. (i) Total number of families = 1500

No. of families having 2 girls = 475

$\therefore$ P (Family having 2 girls) = $\dfrac{475}{1500} = \dfrac{19}{60}$

No of families having 1 girl = 814 $\therefore$ P

(Family having 1 girl) = $\dfrac{814}{1500} = \dfrac{407}{750}$

No. of families having no girl = 211 ∴ P

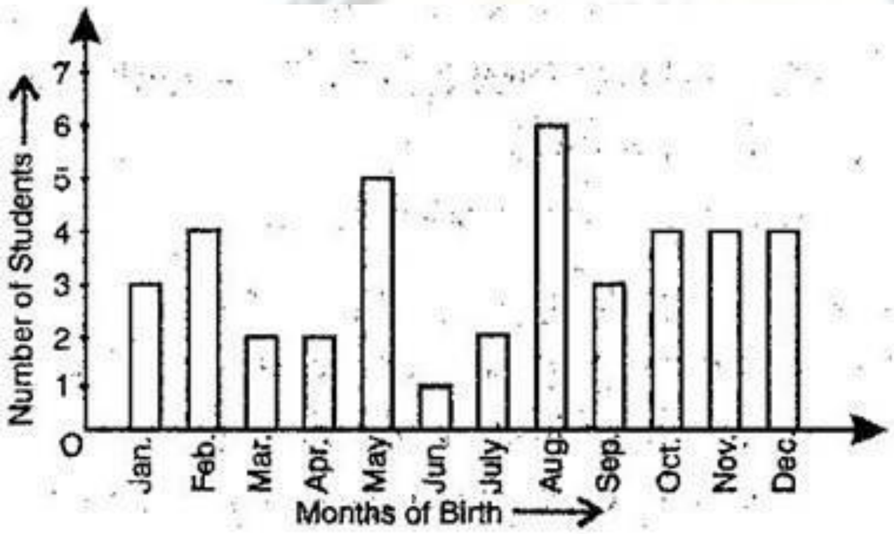
(Family having no girl) = $\frac{211}{1500}$

Checking: Sum of all probabilities = $\frac{19}{60}+\frac{407}{750}+\frac{211}{1500}$

$=\frac{475+814+211}{1500}=\frac{1500}{1500}$ = 1

Yes, the sum of probabilities is 1.

(vi) In a particular section of Class IX, 40 students were asked about the months of their birth and the following graph was prepared for the data so obtained:



Find the probability that a student of the class was born in August.

Ans. From the bar graph, we observe,

Total no. of students of Class IX = 40

No. of students of Class IX born in August = 6

∴ P (A student born in August) = $\frac{6}{40}=\frac{3}{20}=0.15$

(vii) Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes:

Outcomes	Frequency
3 heads	23
2 heads	72
1 head	77
No head	28

If the three coins are simultaneously tossed again, compute the probability of 2 heads coming up.

Ans. No. of 2 heads = 72

Total number of outcomes = 23 + 72 + 77 + 28 = 200

$\therefore$ P (2 heads) = $\frac{72}{200} = \frac{9}{25}$

(vi) An organization selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a family. The information gathered is listed in the table below:

Monthly income (in Rs.)	Vehicles per family			
	0	1	2	Above 2
Less than 7000	10	160	25	0
7000 – 10000	0	305	27	2
10000 – 13000	1	535	29	1
13000 – 16000	2	469	59	25
16000 or more	1	579	82	88

Suppose a family is chosen. Find the probability that the family chosen is:

(vi) earning Rs. 10000 – 13000 per month and owning exactly 2 vehicles.

(vii) earning Rs. 16000 or more per month and owning exactly 1 vehicle.

(viii) earning less than Rs. 7000 per month and does not own any vehicle.

(ix) earning Rs. 13000 – 16000 per month and owning more than 2 vehicles.

(v) not more than 1 vehicle.

Ans. (i) P (earning Rs. 10000 – 13000 per month and owning exactly 2 vehicles) = $\frac{29}{2400}$

11. P (earning Rs. 16000 or more per month and owning exactly 1 vehicles) = $\frac{579}{2400}$

12. P (earning Rs. 7000 per month and does not own any vehicles) = $\frac{10}{2400} = \frac{1}{240}$

13. P (earning Rs. 13000 – 16000 per month and owning more than 2 vehicles) = $\frac{25}{2400} = \frac{1}{96}$

14. Number of families owning not more than 1 vehicle = 10 + 160 + 0 + 305 + 1 + 532 + 2 + 469 + 1 579 = 2062

Therefore, P (owning not more than 1 vehicle) = $\frac{2062}{2400} = \frac{1031}{1200}$

(vii) A teacher analyses the performance of two sections of students in a mathematics test of 100 marks given in the following table:

Marks	No. of students
0 – 20	7
20 – 30	10
30 – 40	10
40 – 50	20
50 – 60	20
60 – 70	15
70 and above	8
Total	90

11. Find the probability that a student obtained less than 20% in the mathematics test.

12. Find the probability that a student obtained 60 or above.

Ans. (i) No. of students obtaining marks less than 20 out of 100, i.e. 20% = 7 Total

students in the class = 90

$\therefore$ P (A student obtained less than 20%) = $\frac{7}{90}$

(v) No. of students obtaining marks 60 or above = 15 + 8 = 23 $\therefore$ P

(A student obtained marks 60 or above) = $\frac{23}{90}$

(iv) To know the opinion of the students about the subject statistics, a survey of 200 students was conducted. The data is recorded in the following table:

Opinion	No. of students
likes	135
dislikes	65

Find the probability that a student chosen at random:

(i) likes statistics (ii) dislikes it.

Ans. Total no. of students on which the survey about the subject of statistics was conducted = 200

(v) No. of students who like statistics = 135 $\therefore$ P

(a student likes statistics) = $\frac{135}{200} = \frac{27}{40}$

(vi) No. of students who do not like statistics = 65 $\therefore$ P

(a student does not like statistics) = $\frac{65}{200} = \frac{13}{40}$

8. Refer Q.2, Exercise 14.2. What is the empirical probability than an engineer lives:

(i) less than 7 km from her place of work?

(ii) more than or equal to 7 km from her place of work?

(iii) within $\frac{1}{2}$ km from her place of work?

Ans. Total number of engineers = 40

(i) No. of engineers living less than 7 km from her place of work = 9 ∴ P

(Engineer living less than 7 km from her place of work) = $\frac{9}{40}$

(ii) No. of engineers living more than or equal to 7 km from her place of work = 40 - 9 = 31 ∴ P

(Engineer living more than or equal to 7 km from her place of work) = $\frac{31}{40}$

(iii) No. of engineers living within $\frac{1}{2}$ km from her place of work = 0

∴ P (Engineer living within $\frac{1}{2}$ km from her place of work) = $\frac{0}{40}$ = 0

9. Activity: Note the frequency of two wheelers, three wheelers and four wheelers going past during a time interval, in front of your school gate. Find the probability that any one vehicle out of the total vehicles you have observed is a two wheeler.

Ans. Let you noted the frequency of types of wheelers after school time (i.e. 3 pm to 3.30 pm) for half an hour.

Let the following table shows the frequency of wheelers.

Type of wheelers	Frequency of wheelers
Two wheelers	125
Three wheelers	45
Four wheelers	30

Probability that a two wheelers passes after this interval = $\frac{125}{200} = \frac{5}{8}$

10. Activity: Ask all the students in your class room to write a 3-digit number. Choose any student from the room at random. What is the probability that the number written by him is divisible by 3, if the sum of its digits is divisible by 3.

Ans. Let number of students in your class is 24.

Let 3-digit number written by each of them is as follows:

837, 172, 643, 371, 124, 512, 432, 948, 311, 252, 999, 557, 784, 928, 867, 798, 665, 245, 107, 463, 267, 523, 944, 314

Numbers divisible by 3 are = 837, 432, 948, 252, 999, 867, 798 and 267 Number

of 3-digit numbers divisible by 3 = 8

$\therefore$ P (3-digit numbers divisible by 3) = $\frac{8}{24} = \frac{1}{3}$

11. Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of four (in kg): 4.97, 5.05, 5.08, 5.03, 5.00, 5.06, 5.08, 4.98, 5.04, 5.07, 5.00

Find the probability that any of these bags chosen at random contains more than 5 kg of flour.

Ans. Number of bags containing more than 5 kg of wheat flour = 7

Total number of wheat flour bags = 11

$\therefore$ P (a bag containing more than 5 kg of wheat flour) = $\frac{7}{11}$

12. In Q.5, Exercise 14.2, you were asked to prepare a frequency distribution table, regarding the concentration of Sulphur dioxide in the air in parts per million of a certain city for 30 days. Using this table, find the probability of the concentration of

Sulphur dioxide in the interval 0.12 – 0.16 on any of these days.

Ans. From the frequency distribution table we observe that:

No. of days during which the concentration of Sulphur dioxide lies in interval 0.12 – 0.16 = 2 Total no. of days during which concentration of Sulphur dioxide recorded = 30

∴ P (day when concentration of Sulphur dioxide (in ppm) lies in 0.12 – 0.16) = $\frac{2}{30} = \frac{1}{15}$

13. In Q.1, Exercise 14.1 you were asked to prepare a frequency distribution table regarding the blood groups of 30 students of a class. Use this table to determine the probability that a student of this class selected at random has blood group AB.

Ans. From the frequency distribution table we observe that:

Number of students having blood group AB = 3

Total number of students whose blood group were recorded = 30

∴ P (a student having blood group AB) = $\frac{3}{30} = \frac{1}{10}$

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