CHAPTER 1

Number Systems(Ex. 1.1)

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Ans. Consider the definition of a rational number.

A rational number is the one that can be written in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Zero can be written as $\frac{0}{1}$, $\frac{0}{2}$, $\frac{0}{3}$, $\frac{0}{4}$, $\frac{0}{5}$

So, we arrive at the conclusion that 0 can be written in the form of $\frac{p}{q}$, where q is any integer.

Therefore, zero is a rational number.

2. Find six rational numbers between 3 and 4.

Ans. We know that there are infinite rational numbers between any two numbers.

A rational number is the one that can be written in the form of $\frac{p}{q}$, where p and q are

We need to rewrite the numbers 3.1, 3.2, 3.3, 3.4, 3.5 and 3.6 in $\frac{p}{q}$ form to get the rational numbers between 3 and 4.

So, after converting, we get $\frac{31}{10}$, $\frac{32}{10}$, $\frac{33}{10}$, $\frac{34}{10}$, $\frac{35}{10}$ and $\frac{36}{10}$.

We can further convert the rational numbers $\frac{32}{10}$, $\frac{34}{10}$, $\frac{35}{10}$ and $\frac{36}{10}$ into lowest fractions.

On converting the fractions into lowest fractions, we get $\frac{16}{5}$, $\frac{17}{5}$, $\frac{7}{2}$ and $\frac{18}{5}$.

Therefore, six rational numbers between 3 and 4 are $\frac{31}{10}$, $\frac{16}{5}$, $\frac{33}{10}$, $\frac{17}{5}$, $\frac{7}{2}$ and $\frac{18}{5}$.

3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Ans. We know that there are infinite rational numbers between any two numbers.

A rational number is the one that can be written in the form of $\frac{p}{q}$, where p and q are Integers and $q \neq 0$.

We know that the numbers $\frac{3}{5}$ and $\frac{4}{5}$ can also be written as 0.6 and 0.8.

We can conclude that the numbers 0.61, 0.62, 0.63, 0.64 and 0.65 all lie between 0.6 and 0.8

We need to rewrite the numbers 0.61, 0.62, 0.63, 0.64 and 0.65 in $\frac{p}{q}$ form to get the rational numbers between 3 and 4.

So, after converting, we get $\frac{61}{100}$, $\frac{62}{100}$, $\frac{63}{100}$, $\frac{64}{100}$ and $\frac{65}{100}$.

We can further convert the rational numbers $\frac{62}{100}$, $\frac{64}{100}$ and $\frac{65}{100}$ into lowest fractions.

On converting the fractions, we get $\frac{31}{50}$, $\frac{16}{25}$ and $\frac{13}{20}$.

Therefore, six rational numbers between 3 and 4 are $\frac{61}{100}$, $\frac{31}{50}$, $\frac{63}{100}$, $\frac{16}{25}$ and $\frac{13}{50}$.

4. State whether the following statements are true or false. Give reasons for your answers.

- (i) Every natural number is a whole number.
- (ii) Every integer is a whole number.
- (iii) Every rational number is a whole number.
- Ans. (i) Consider the whole numbers and natural numbers separately.

We know that whole number series is 0, 1, 2, 3, 4, 5.....

We know that natural number series is 1, 2, 3, 4, 5......

So, we can conclude that every number of the natural number series lie in the whole number series.

Therefore, we conclude that, yes every natural number is a whole number.

- (ii) Consider the integers and whole numbers separately.
- We know that integers are those numbers that can be written in the form of $\frac{p}{q}$, where q

Now, considering the series of integers, we have $\dots -4$, -3, -2, -1, 0, 1, 2, 3, 4.....

We know that whole number series is 0, 1, 2, 3, 4, 5.....

We can conclude that all the numbers of whole number series lie in the series of integers.

But every number of series of integers does not appear in the whole number series.

Therefore, we conclude that every integer is not a whole number.

(iii) Consider the rational numbers and whole numbers separately.

We know that rational numbers are the numbers that can be written in the form $\frac{p}{q}$, when $q \neq 0$.

We know that whole number series is 0, 1, 2, 3, 4, 5.....

We know that every number of whole number series can be written in the form of $\frac{p}{q}$ as

$$q \neq 0 \frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}, \dots$$

We conclude that every number of the whole number series is a rational number. But, every rational number does not appear in the whole number series.

Therefore, we conclude that every rational number is not a whole number.

CHAPTER 1

Number Systems(Ex. 1.2)

1. State whether the following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form \sqrt{m} , where m is a natural number.

(iii) Every real number is an irrational number.

Ans. (i) Consider the irrational numbers and the real numbers separately.

We know that irrational numbers are the numbers that cannot be converted in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

We know that a real number is the collection of rational numbers and irrational numbers.

Therefore, we conclude that, yes every irrational number is a real number.

(ii)False, Consider a number line. We know that on a number line, we can represent negative as well as positive numbers.

We know that we cannot get a negative number after taking square root of any number.

Therefore, we conclude that not every number point on the number line is of the form \sqrt{m} , where m is a natural number.

(iii)False, Consider the irrational numbers and the real numbers separately.

We know that irrational numbers are the numbers that cannot be converted in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

We know that a real number is the collection of rational numbers and irrational numbers.

So, we can conclude that every irrational number is a real number. But every real number is not an irrational number.

square root of a number that is a rational number.

Ans. We know that square root of every positive integer will not yield an integer. We know that $\sqrt{4}$ is 2, which is an integer. But, $\sqrt{7}$ or $\sqrt{10}$ will give an irrational number.

Therefore, we conclude that square root of every positive integer is not an irrational number.

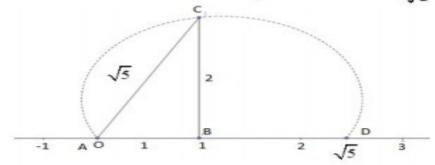
3. Show how $\sqrt{5}$ can be represented on the number line.

Ans. According to the Pythagoras theorem, we can conclude that

$$(\sqrt{5})^2 = (2)^2 + (1)^2$$

We need to draw a line segment AB of 1 unit on the number line. Then draw a straight line segment BC of 2 units. Then join the points C and A, to form a line segment BC.

Then draw the arc ACD, to get the number $\sqrt{5}$ on the number line.



CHAPTER 1

Number Systems(Ex. 1.3)

1. Write the following in decimal form and say what kind of decimal expansion each has:

(i)
$$\frac{36}{100}$$
 (ii) $\frac{1}{11}$

(ii)
$$\frac{1}{11}$$

(iii)
$$4\frac{1}{8}$$

(iv) $\frac{3}{13}$
(v) $\frac{2}{11}$

(iv)
$$\frac{3}{13}$$

(v)
$$\frac{2}{11}$$

(vi)
$$\frac{329}{400}$$

Ans. (i)
$$\frac{36}{100}$$

On dividing 36 by 100, we get

Therefore, we conclude that $\frac{36}{100} = 0.36$, which is a terminating decimal.

(ii)
$$\frac{1}{11}$$

On dividing 1 by 11, we get

We can observe that while dividing 1 by 11, we got the remainder as 1, which will continue to be 1.

Therefore, we conclude that $\frac{1}{11} = 0.0909...$ or $\frac{1}{11} = 0.\overline{09}$, which is a non-terminating repeating decimal.

(iii)
$$4\frac{1}{8} = \frac{33}{8}$$

On dividing 33 by 8, we get

We can observe that while dividing 33 by 8, we got the remainder as 0.

Therefore, we conclude that $4\frac{1}{8} = \frac{33}{8} = 4.125$, which is a terminating decimal.

(iv)
$$\frac{3}{13}$$

On dividing 3 by 13, we get

We can observe that while dividing 3 by 13 we got the remainder as 3, which will continue to be 3 after carrying out 6 continuous divisions.

Therefore, we conclude that $\frac{3}{13} = 0.230769...$ or $\frac{3}{13} = 0.\overline{230769}$, which is a non-terminating repeating decimal.

(v)
$$\frac{2}{11}$$

On dividing 2 by 11, we get

We can observe that while dividing 2 by 11, first we got the remainder as 2 and then 9, which will continue to be 2 and 9 alternately.

Therefore, we conclude that $\frac{2}{11}=0.1818...$ or $\frac{2}{11}=0.\overline{18}$, which is a non-terminating repeating decimal.

(vi)
$$\frac{329}{400}$$

On dividing 329 by 400, we get

We can observe that while dividing 329 by 400, we got the remainder as 0.

Therefore, we conclude that $\frac{329}{400} = 0.8225$, which is a terminating decimal.

2. You know that $\frac{1}{7} = 0.142857...$ Can you predict what the decimal expansions of $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$ are, without actually doing the long division? If so, how?

[Hint: Study the remainders while finding the value of $\frac{1}{7}$ carefully.]

Ans. We are given that $\frac{1}{7} = 0.\overline{142857}$ or $\frac{1}{7} = 0.142857...$

We need to find the values of $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$ and $\frac{6}{7}$, without performing long division.

We know that, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$ and $\frac{6}{7}$ can be rewritten as $2 \times \frac{1}{7}$, $3 \times \frac{1}{7}$, $4 \times \frac{1}{7}$, $5 \times \frac{1}{7}$ and $6 \times \frac{1}{7}$.

On substituting value of $\frac{1}{7}$ as 0.142857...., we get

$$2 \times \frac{1}{7} = 2 \times 0.142857.... = 0.285714...$$

$$3 \times \frac{1}{7} = 3 \times 0.142857.... = 0.428571$$

$$4 \times \frac{1}{7} = 4 \times 0.142857... = 0.571428$$

$$5 \times \frac{1}{7} = 5 \times 0.142857.... = 0.714285$$

$$6 \times \frac{1}{7} = 6 \times 0.142857.... = 0.857142...$$

Therefore, we conclude that, we can predict the values of $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$ and $\frac{6}{7}$, without performing long division, to get

$$\frac{2}{7} = 0.\overline{285714}, \frac{3}{7} = 0.\overline{428571}, \frac{4}{7} = 0.\overline{571428}, \frac{5}{7} = 0.\overline{714285}, \text{ and } \frac{6}{7} = 0.\overline{857142}$$

3. Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

- (i) 0.6
- (ii) 0.47
- (iii) 0.001

Ans. Solution:

(i) Let
$$x = 0.\overline{6} \Rightarrow x = 0.6666...(a)$$

We need to multiply both sides by 10 to get

$$10x = 6.6666....(b)$$

We need to subtract (a) from (b), to get

$$10x = 6.6666...$$

$$-x = 0.6666....$$

$$9x = 6$$

We can also write 9x = 6 as $x = \frac{6}{9}$ or $x = \frac{2}{3}$.

Therefore, on converting $0.\overline{6}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{2}{3}$.

(ii) Let
$$x = 0.47 \Rightarrow x = 0.47777......(a)$$

We need to multiply both sides by 10 to get

$$10x = 4.7777....(b)$$

We need to subtract (a) from (b), to get

$$10x = 4.7777...$$

 $-x = 0.4777...$
 $9x = 4.3$

We can also write 9x = 4.3 as $x = \frac{4.3}{9}$ or $x = \frac{43}{90}$.

Therefore, on converting $0.4\overline{7}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{43}{90}$.

(iii) Let
$$x = 0.\overline{001} \Rightarrow x = 0.001001.....(a)$$

We need to multiply both sides by 1000 to get

$$1000x = 1.001001....$$
(b)

We need to subtract (a) from (b), to get

$$1000x = 1.001001....$$
$$-x = 0.001001....$$
$$999x = 1$$

We can also write 999x = 1 as $x = \frac{1}{999}$.

Therefore, on converting $0.\overline{001}$ in the $\frac{p}{q}$ form, we get the answer as $\frac{1}{999}$.

4. Express 0.99999... in the form $\frac{p}{q}$. Are you surprised by your answer? Discuss why the answer makes sense with your teacher and classmates.

Ans. Let
$$x = 0.999999.....(a)$$

We need to multiply both sides by 10 to get

$$10x = 9.9999.....(b)$$

We need to subtract (a) from (b), to get

$$10x = 9.999999...$$

 $-x = 0.999999...$

We can also write
$$9x = 9$$
 as $x = \frac{9}{9}$ or $x = 1$.

Therefore, on converting 0.99999... in the $\frac{p}{q}$ form, we get the answer as 1.

Yes, at a glance we are surprised at our answer.

But the answer makes sense when we observe that 0.9999...... goes on forever. SO there is not gap between 1 and 0.9999...... and hence they are equal.

5. What can the maximum number of digits be in the recurring block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Ans. We need to find the number of digits in the recurring block of $\frac{1}{17}$.

Let us perform the long division to get the recurring block of $\frac{1}{17}$.

We need to divide 1 by 17, to get

We can observe that while dividing 1 by 17 we got the remainder as 1, which will continue to be 1 after carrying out 16 continuous divisions.

Therefore, we conclude that

 $\frac{1}{17}=0.0588235294117647..... \text{ or } \frac{1}{17}=0.\overline{0588235294117647} \text{ , which is a non-terminating decimal and recurring decimal.}$

6. Look at several examples of rational numbers in the form $\frac{p}{q}$ ($q \neq 0$), where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Ans. Solution:

Let us consider the examples of the form $\frac{p}{q}$ that are terminating decimals.

$$\frac{5}{2} = 2.5$$

$$\frac{5}{4} = 1.25$$

$$\frac{2}{5} = 0.4$$

$$\frac{2}{10} = 0.2$$

$$\frac{5}{16} = 0.3125$$

We can observe that the denominators of the above rational numbers have powers of 2, 5 or both.

Therefore, we can conclude that the property, which q must satisfy in $\frac{p}{q}$, so that the rational

number $\frac{p}{q}$ is a terminating decimal is that q must have powers of 2, 5 or both.

is not equal to

7. Write three numbers whose decimal expansions are non-terminating non-recurring.

Ans. The three numbers that have their expansions as non-terminating on recurring decimal are given below.

- 0.04004000400004....
- 0.07007000700007....
- 0.013001300013000013....
- 8. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Ans. Let us convert $\frac{5}{7}$ and $\frac{9}{11}$ into decimal form, to get

$$\frac{5}{7} = 0.714285...$$
 and $\frac{9}{11} = 0.818181...$

Three irrational numbers that lie between 0.714285.... and 0.818181.... are:

- 0.73073007300073....
- 0.74074007400074....
- 0.76076007600076....
- 9. Classify the following numbers as rational or irrational:
- (i) $\sqrt{23}$

(ii)
$$\sqrt{225}$$

(iii) 0.3796

(iv) 7.478478...

(v) 1.101001000100001...

Ans. (i)
$$\sqrt{23}$$

We know that on finding the square root of 23, we will not get an integer.

Therefore, we conclude that $\sqrt{23}$ is an irrational number.

(ii)
$$\sqrt{225}$$

We know that on finding the square root of 225, we get 15, which is an integer.

Therefore, we conclude that $\sqrt{225}$ is a rational number.

(iii) 0.3796

We know that 0.3796 can be converted into $\frac{p}{q}$.

While, converting 0.3796 into $\frac{p}{q}$ form, we get

$$0.3796 = \frac{3796}{10000}$$
.

The rational number $\frac{3796}{10000}$ can be converted into lowest fractions, to get $\frac{949}{2500}$.

We can observe that 0.3796 can be converted into a rational number.

Therefore, we conclude that 0.3796 is a rational number.

(iv) 7.478478....

We know that 7.478478.... is a non-terminating recurring decimal, which can be converted into $\frac{p}{a}$ form.

While, converting 7.478478.... into $\frac{p}{q}$ form, we get

$$x = 7.478478...$$
(a)

$$1000x = 7478.478478....(b)$$

While, subtracting (b) from (a), we get

$$1000x = 7478.478478...$$

$$-x = 7.478478...$$

$$999x = 7471$$

We know that 999x = 7471 can also be written as $x = \frac{7471}{999}$.

Therefore, we conclude that 7.478478.... is a rational number.

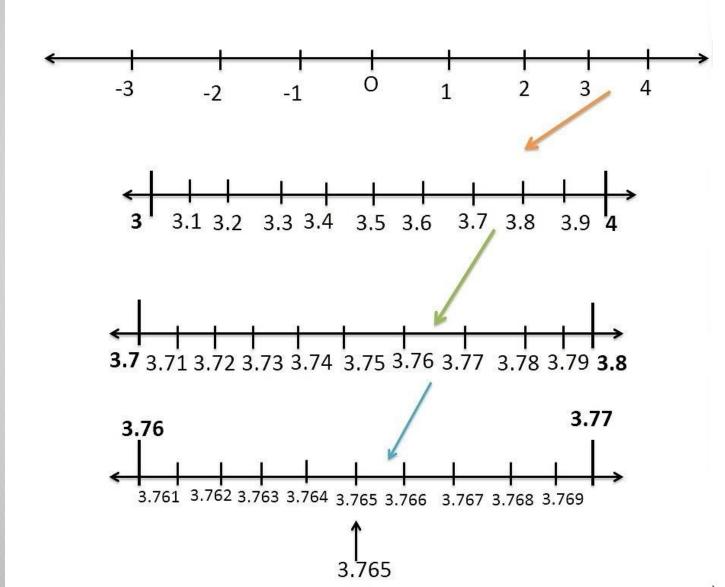
(v) 1.101001000100001....

We can observe that the number 1.101001000100001.... is a non-terminating on recurring decimal.

We know that non-terminating and non-recurring decimals cannot be converted into $\frac{p}{q}$ form.

Therefore, we conclude that 1.101001000100001.... is an irrational number.

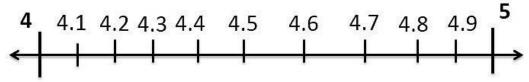
Ex1.4, 1
Visualise 3.765 on the number line using successive magnification.

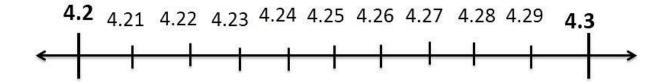


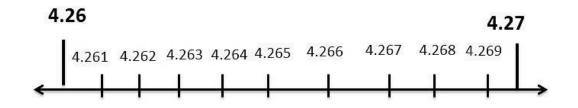
Ex1.4, 2

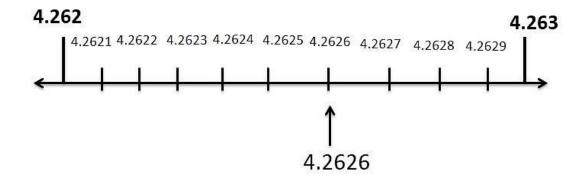
Visualise $4.\overline{26}$ on the number line, up to 4 decimal places.











Classify the following numbers as rational or irrational:

(i)
$$2 - \sqrt{5}$$

2 is rational

 $\sqrt{5}$ = 2.236.... which is non-terminating non-repeating is an irrational number

We know that Rational – Irrational = Irrational

2 - $\sqrt{5}$ is an irrational number

Classify the following numbers as rational or irrational:

(ii)
$$(3 + \sqrt{23}) - \sqrt{23}$$

$$(3 + \sqrt{23}) - \sqrt{23} = 3 + \sqrt{23} - \sqrt{23}$$
$$= 3$$
$$= \frac{3}{1}$$

Since it is of the form $\frac{p}{q}$

So, it is a rational number.

Classify the following numbers as rational or irrational:

(iii)
$$\frac{2\sqrt{7}}{7\sqrt{7}}$$

$$\frac{2\sqrt{7}}{7\sqrt{7}} = \frac{2}{7},$$

Since it is of the form $\frac{p}{q}$

So, it is a rational number.

Classify the following numbers as rational or irrational:

(iv)
$$\frac{1}{\sqrt{2}}$$

1 is rational

 $\sqrt{2}$ = 1.4142... which is non-terminating non-repeating is an irrational number

We know that
$$\frac{Rational}{Irrational}$$
 = Irrational

So, $\frac{1}{\sqrt{2}}$ is irrational number

Classify the following numbers as rational or irrational: (v) 2π

2 is rational

 π = 3.1415.... which is non-terminating non-repeating is an irrational number

We know that Rational \times Irrational = Irrational Thus, 2π is an irrational number

Simplify each of the following expressions:

(i)
$$(3 + \sqrt{3})(2 + \sqrt{2})$$

$$(3 + \sqrt{3}) (2 + \sqrt{2})$$

$$= 3(2 + \sqrt{2}) + \sqrt{3}(2 + \sqrt{2})$$

$$= 3 \times 2 + 3 \times \sqrt{2} + \sqrt{3} \times 2 + \sqrt{3} \times \sqrt{2}$$

$$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{3} \times 2$$

$$= 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$$

Simplify each of the following expressions:

(ii)
$$(3 + \sqrt{3})(3 - \sqrt{3})$$

$$(3+\sqrt{3})(3-\sqrt{3})$$

We know that, $(a + b) (a-b) = a^2 - b^2$

Putting
$$a = 3$$
, $b = \sqrt{3}$

$$=(3)^2-(\sqrt{3})^2$$

$$= 9 - 3$$

Simplify each of the following expressions:

(iii)
$$(\sqrt{5} + \sqrt{2})^2$$

$$(\sqrt{5} + \sqrt{2})^2$$

We know that
$$(a + b)^2 = a^2 + b^2 + 2ab$$

where
$$a = \sqrt{5}$$
, $b = \sqrt{2}$

$$= (\sqrt{5})^2 + (\sqrt{2})^2 + 2(\sqrt{5})(\sqrt{2})$$

$$= 5 + 2 + 2\sqrt{5 \times 2}$$

$$= 5 + 2 + 2\sqrt{10}$$

$$= 7 + 2\sqrt{10}$$

Simplify each of the following expressions:

(iv)
$$(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$$

$$(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$$

We know that, $(a + b)(a-b) = a^2 - b^2$

Putting
$$a = \sqrt{5}$$
, $b = \sqrt{2}$

$$= (\sqrt{5})^2 - (\sqrt{2})^2$$

$$= 5 - 2$$

Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, $\pi = \frac{c}{d}$. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

$$\pi = \frac{c}{d}$$
 is a rational number

when c & d both are rational numbers

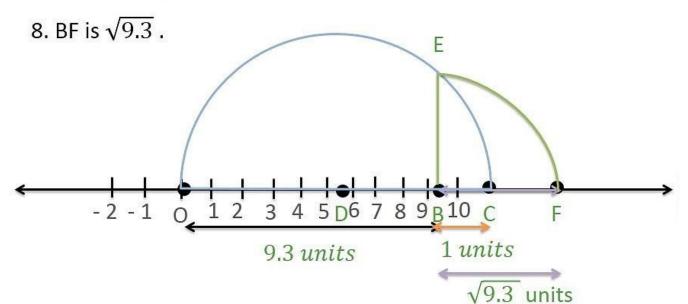
But the c & d we calculate is not exactly a rational number and either c or d or both can be irrational numbers,

Since,
$$\frac{Rational}{Irrational}$$
 = Irrational

So, π is irrational.

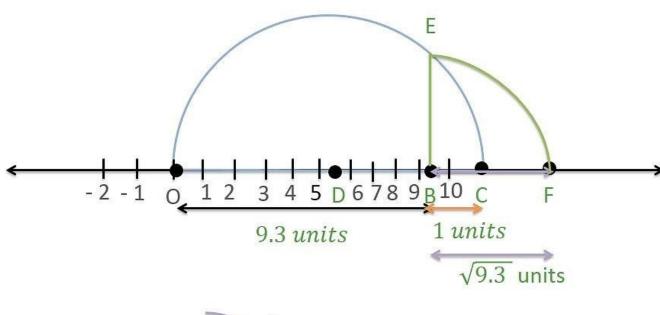
Represent $\sqrt{9.3}$ on the number line.

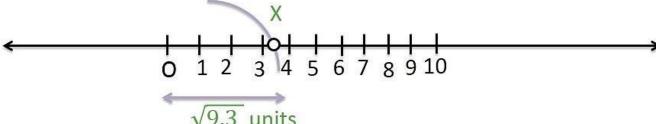
- 1.Mark a line segment OB = 9.3 on number line.
- 2. Mark point C at a distance of 1 unit from B.
- 3. Find the mid-point of OC and mark it point D
- 4. Draw a semi-circle on OC while taking D as its centre.
- 5. Draw a perpendicular to line OC passing through point B.
- 6. Let it intersect the semi-circle at E.
- 7. Taking B as centre and BE as radius, draw an arc intersecting number line at F.



- 8. BF is $\sqrt{9.3}$.
- 9. Draw a new number line of same size as that of previous.
- 10. Take compass, take B as pointed end and F as pencil end and draw an arc on the new number line.
- 11. Where the arc intersects say X.

12. OX =
$$\sqrt{9.3}$$





Rationalize the denominators of the following:

(i)
$$\frac{1}{\sqrt{7}}$$

We need to rationalize i.e. remove root from denominator Hence multiplying and dividing by $\sqrt{7}$

$$\frac{1}{\sqrt{7}}$$

$$= \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$$

$$= \frac{\sqrt{7}}{(\sqrt{7})^2}$$

$$\sqrt{7}$$

Rationalize the denominators of the following:

(ii)
$$\frac{1}{\sqrt{7}-\sqrt{6}}$$

$$\frac{1}{\sqrt{7} - \sqrt{6}}$$

$$= \frac{1}{\sqrt{7} - \sqrt{6}} \times \frac{\sqrt{7} + \sqrt{6}}{\sqrt{7} + \sqrt{6}}$$

$$= \frac{\sqrt{7} + \sqrt{6}}{(\sqrt{7})^2 - (\sqrt{6})^2} \qquad (As (a + b)(a - b) = a^2 - b^2)$$

$$= \frac{\sqrt{7} + \sqrt{6}}{7 - 6}$$

$$= \frac{\sqrt{7} + \sqrt{6}}{7 - 6}$$

$$= \frac{\sqrt{7} + \sqrt{6}}{1}$$

$$= \sqrt{7} + \sqrt{6}$$

Rationalize the denominators of the following:

(iii)
$$\frac{1}{\sqrt{5} + \sqrt{2}}$$

$$\frac{1}{\sqrt{5} + \sqrt{2}}$$

$$=\frac{1}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{\sqrt{5} - \sqrt{2}}{(\sqrt{5})^2 - (\sqrt{2})^2}$$
 (As $(a + b)(a - b) = a^2 - b^2$)

$$=\frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$=\frac{\sqrt{5}-\sqrt{2}}{3}$$

(i)
$$64^{\frac{1}{2}}$$

$$64^{\frac{1}{2}} = (8 \times 8)^{\frac{1}{2}}$$

$$= (82)^{\frac{1}{2}}$$

$$= 8^{2 \times \frac{1}{2}}$$

$$= 8^{1}$$

$$= 8$$

(ii)
$$32^{\frac{1}{5}}$$

$$32^{\frac{1}{5}} = (2 \times 2 \times 2 \times 2 \times 2)^{\frac{1}{5}}$$

$$= (25)^{\frac{1}{5}}$$

$$= 2^{5 \times \frac{1}{5}}$$

$$= 2^{1}$$

$$= 2$$

(iii)
$$125^{\frac{1}{3}}$$

$$125^{\frac{1}{3}} = (5 \times 5 \times 5)^{\frac{1}{3}}$$

$$= (53)^{\frac{1}{3}}$$

$$= 5^{3 \times \frac{1}{3}}$$

$$= 5^{1}$$

$$= 5$$

(i)
$$9^{\frac{3}{2}}$$

$$9^{\frac{3}{2}} = (3 \times 3)^{\frac{3}{2}}$$

$$= (3^{2})^{\frac{3}{2}}$$

$$= 3^{2 \times \frac{3}{2}}$$

$$= 3^{3}$$

$$= 3 \times 3 \times 3$$

$$= 27$$

(ii)
$$32^{\frac{2}{5}}$$

$$32^{\frac{2}{5}} = (2 \times 2 \times 2 \times 2 \times 2)^{\frac{2}{5}}$$

$$= (25)^{\frac{2}{5}}$$

$$= 2^{5 \times \frac{2}{5}}$$

$$= 2^{2}$$

$$= 4$$

(iii)
$$16^{\frac{3}{4}}$$

$$16^{\frac{3}{4}} = (2 \times 2 \times 2 \times 2)^{\frac{3}{4}}$$

$$= (2^{4})^{\frac{3}{4}}$$

$$= 2^{4 \times \frac{3}{4}}$$

$$= 2^{3}$$

$$= 2 \times 2 \times 2$$

$$= 8$$

(iv)
$$125^{\frac{-1}{3}}$$

$$125^{\frac{-1}{3}} = \frac{1}{125^{\frac{1}{3}}}$$

$$= \frac{1}{(5 \times 5 \times 5)^{\frac{1}{3}}}$$

$$= \frac{1}{(53)^{\frac{1}{3}}}$$

$$= \frac{1}{5^{3 \times \frac{1}{3}}}$$

$$= \frac{1}{5^{1}}$$

$$= \frac{1}{5}$$

(i)
$$2^{\frac{2}{3}}$$
 . $2^{\frac{1}{5}}$

$$2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}} = 2^{\frac{2}{3}} \times 2^{\frac{1}{5}}$$

$$= 2^{\left(\frac{2}{3} + \frac{1}{5}\right)}$$

$$= 2^{\left(\frac{2 \times 5 + 1 \times 3}{3 \times 5}\right)}$$

$$= 2^{\left(\frac{10 + 3}{15}\right)}$$

$$= 2^{\frac{13}{15}}$$

(ii)
$$\left(\frac{1}{3^3}\right)^7$$

$$\left(\frac{1}{3^3}\right)^7 = (3^{-3})^7$$
$$= 3^{-3 \times 7}$$
$$= 3^{-21}$$

(iii)
$$\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$$

$$\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}} = 11^{\left(\frac{1}{2} - \frac{1}{4}\right)}$$

$$= 11^{\left(\frac{1 \times 4 - 1 \times 2}{2 \times 4}\right)}$$

$$= 11^{\left(\frac{4 - 2}{8}\right)}$$

$$= 11^{\left(\frac{2}{8}\right)}$$

$$= 11^{\frac{1}{4}}$$

(iv)
$$7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$$

$$7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}} = (7 \times 8)^{\frac{1}{2}}$$
$$= 56^{\frac{1}{2}}$$