Chapter -1

Real Numbers -

- Introduction.
- Euclid’s Division Lemma.
- The Fundamental Theorem of Arithmetic.
- Revisiting Irrational Numbers.
- Revisiting Rational Numbers and Their Decimal Expensions.

Real Numbers

- Real numbers consist of all the rational and irrational numbers.
- The real number system has many subsets:
  - Natural Numbers
  - Whole Numbers
  - Integers
  - Rational Number
  - Irrational Numbers

DESCIPTION OF PARTS OF REAL NUMBERS

Natural Numbers

- ***Natural numbers*** are the set of counting numbers.

  {1, 2, 3,...}

Whole Numbers

- ***Whole numbers*** are the set of numbers that include 0 plus the set of natural numbers.

  {0, 1, 2, 3, 4, 5,...}

Integers

- ***Integers*** are the set of whole numbers and their opposites.

  {...,-3, -2, -1, 0, 1, 2, 3,...}

Rational Numbers

- ***Rational numbers*** are any numbers that can be expressed in the form of, where *a* and *b* are integers, and b ≠ 0.
- They can always be expressed by using terminating decimals or repeating decimals.
- Examples: 36.8, 0.125, 4.5

Irrational Numbers

- ***Irrational numbers*** are any numbers that cannot be expressed in the form of, where *a* and *b* are integers, and b ≠ 0.
- They are expressed as ***non-terminating, non-repeating decimals***; decimals that go on forever without repeating a pattern.
- Examples: 0.34334333433334…, 45.86745893…, π (pi), $\sqrt{2}$

EUCLID'S DIVISION ALGORITHMN

Given positive integers a and b, there exist a unique integers q and r satisfying :

$$a = bq + r \; ; \; 0 < r < b$$

QUESTIONS

- **Use Euclid's division algorithm to find the HCF of:**

(i) 135 and 225

Answer:

135 and 225

Since 225 > 135, we apply the division lemma to 225 and 135 to obtain

$225 = 135 \times 1 + 90$

Since remainder $90 \neq 0$, we apply the division lemma to 135 and 90 to obtain

$135 = 90 \times 1 + 45$

We consider the new divisor 90 and new remainder 45, and apply the division lemma to obtain

$90 = 2 \times 45 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 45,

Therefore, the HCF of 135 and 225 is 45.

(ii) 196 and 38220

Answer

196 and 38220

Since 38220 > 196, we apply the division lemma to 38220 and 196 to obtain

$38220 = 196 \times 195 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

(iii) 867 and 255

Answer

867 and 255

Since 867 > 255, we apply the division lemma to 867 and 255 to obtain

$867 = 255 \times 3 + 102$

Since remainder $102 \neq 0$, we apply the division lemma to 255 and 102 to obtain

$255 = 102 \times 2 + 51$

We consider the new divisor 102 and new remainder 51, and apply the division lemma to obtain

$102 = 51 \times 2 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 51, Therefore, HCF of 867 and 255 is 51.

- An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer

HCF (616, 32) will give the maximum number of columns in which they can march.

We can use Euclid's algorithm to find the HCF.

$616 = 32 \times 19 + 8$

$32 = 8 \times 4 + 0$

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

FUNDAMENTAL THEOREM OF ARITHEMATIC

In number theory, the **fundamental theorem of arithmetic**, also called the **unique factorization theorem** or the **unique-prime-factorization theorem**, states that every integer greater than 1 either is prime itself or is the product of prime numbers, and that this product is unique, up to the order of the factors.

Find the total number of divisors of 225.

1. Eight
2. Nine
3. Eleven
4. Fifteen

Find the sum of all divisors of 144.

1. 401
2. 403
3. 405
4. 411

- Express each number as product of its prime factors:

(i) 140

Answer: $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$

(ii) 156

Answer: $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

(iii) 3825

Answer: $3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$

(iv) 5005

Answer: $5005 = 5 \times 7 \times 11 \times 13$

(v) 7429

Answer: $7429 = 17 \times 19 \times 23$

- Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91

Answer

(i) 26 and 91

$26 = 2 \times 13$

$91 = 7 \times 13$

$HCF = 13$

$LCM = 2 \times 7 \times 13 = 182$

Product of the two numbers $= 26 \times 91 = 2366$

$HCF \times LCM = 13 \times 182 = 2366$

Hence, product of two numbers = HCF × LCM

(ii) 510 and 92

Answer

(ii) 510 and 92

$510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

$HCF = 2$

$LCM = 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

Product of the two numbers $= 510 \times 92 = 46920$

$HCF \times LCM = 2 \times 23460$

$= 46920$

Hence, product of two numbers = HCF × LCM

- Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21

Answer

(i) 12,15 and 21

$12 = 2^2 \times 3$

$15 = 3 \times 5$

$21 = 3 \times 7$

HCF = 3

$\text{LCM} = 2^2 \times 3 \times 5 \times 7 = 420$

(ii) 17, 23 and 29

Answer

(ii) 17,23 and 29

$17 = 1 \times 17$

$23 = 1 \times 23$

$29 = 1 \times 29$

$HCF = 1$

$LCM = 17 \times 23 \times 29 = 11339$

(iii) 8, 9 and 25

Answer

(iii) 8,9 and 25

$8 = 2 \times 2 \times 2$

$9 = 3 \times 3$

$25 = 5 \times 5$

$HCF = 1$

$LCM = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

- Given that HCF (306, 657) = 9, find LCM (306, 657).

Answer

$\text{HCF}(306, 657) = 9$

We know that, $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$

$\therefore \text{LCM} \times \text{HCF} = 306 \times 657$

$$\text{LCM} = \frac{306 \times 657}{\text{HCF}} = \frac{306 \times 657}{9}$$

$\text{LCM} = 22338$

- Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Answer

Numbers are of two types - prime and composite. Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1) = 13 \times (77 + 1)$

$= 13 \times 78$

$= 13 \times 13 \times 6$

The given expression has 6 and 13 as its factors. Therefore, it is a composite number.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$= 5 \times (1008 + 1)$

$= 5 \times 1009$

1009 cannot be factorised further. Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

- There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Answer

It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.

$18 = 2 \times 3 \times 3$

And, $12 = 2 \times 2 \times 3$

LCM of 12 and 18 $= 2 \times 2 \times 3 \times 3 = 36$

Therefore, Ravi and Sonia will meet together at the starting point after 36 minutes.

Examples:-

Example 1

Use Euclid’s algorithm to find the HCF of 4052 and 12576.

Since 12576 > 4052,

We divide 12576 by 4052

$$
\begin{array}{r|l|l}
4052 & 12576 & 3 \\
 & (-)\ 12156 & \\ \hline
 & \quad 420 &
\end{array}
$$

Since remainder is not 0

We divide 4052 by 420

420) 4052 (9

(-) 3780

272

Since remainder is not zero 0

We divide 420 by 272

272) 420 (1

(-) 272

148

Since remainder is not zero 0

We divide 272 by 148

$$\begin{array}{rcl} 148 \,\big) & 272 & \big(\, 1 \\ (-) & \underline{148} & \\ & \underline{124} & \end{array}$$

Since remainder is not zero

We divide 148 by 124

$$\begin{array}{rcl} 124 \,\big) & 148 & \big(\, 1 \\ (-) & \underline{124} & \\ & \underline{24} & \end{array}$$

Since remainder is not zero

We divide 124 by 24

$$
\begin{array}{r|l|l}
24 & 124 & 5 \\
(-) & 120 & \\
\hline
 & 4 &
\end{array}
$$

Since remainder is not zero

We divide 24 by 4

$$
\begin{array}{r|l|l}
4 & 24 & 6 \\
(-) & 24 & \\
\hline
 & 0 &
\end{array}
$$

Since remainder is 0

HCF of 12576 and 4052 is 4

Example 4

A sweet seller has 420 kaju barfis and 130 badam barfis. She wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. What is the maximum number of barfis that can be placed in each stack for this purpose?

Maximum number of Barfi's that can be placed

= HCF of 420 and 130

Using Euclid's Division Algorithm

Since 420 > 130

We divide 420 by 130

$$\begin{array}{r|l} 130 & 420 \;(3 \\ & \underline{(-)\,390} \\ & \quad \underline{30} \end{array}$$

Since the remainder is not 0,

We divide 130 by 30

$$
\begin{array}{r|l|l}
30 & 130 & 4 \\
 & (-)\,120 & \\
\hline
 & 10 & \\
\hline
\end{array}
$$

Since the remainder is not 0,

We divide 30 by 10

$$
\begin{array}{r|l|l}
10 & 30 & 3 \\
 & (-)\,30 & \\
\hline
 & 0 & \\
\hline
\end{array}
$$

Since remainder is 0

HCF of 420 & 130 is 10

Therefore,

Maximum number of Barfi’s that can be placed = HCF of 420 and 130

= 10

Example 5

Consider the numbers 4^n , where n is a natural number. Check whether there is any value of n for which 4^n ends with the digit zero.

Let us take the example of a number which ends with the digit 0

So, $10 = 2 \times 5$

$100 = 2 \times 2 \times 5 \times 5$

Here we note that numbers ending with 0 has both 2 and 5 as their prime factors

Whereas $4^n = (2 \times 2)^n$

Does not have 5 as a prime factor.

So, it does not end with zero.

Therefore, 4^n cannot end with zero for any natural number n

Example 6

Find the LCM and HCF of 6 and 20 by the prime factorisation method.

$$\begin{array}{c|l} 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad\qquad \begin{array}{c|l} 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$6 = 2 \times 3$

$6 = 2^1 \times 3^1$

$20 = 2 \times 2 \times 5$

$20 = 2^2 \times 5$

H.C.F = Product of smallest power of each common prime factor

$= 2^1$

$= 2$

L.C.M = Product of greatest power of each prime factor

$= 2^2 \times 3^1 \times 5$

$= 4 \times 3 \times 5$

$= 60$

Example 7

Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM

$$\begin{array}{r|l} 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 404 \\ \hline 2 & 202 \\ \hline 101 & 101 \\ \hline & 1 \end{array}$$

$$96 = \boxed{2 \times 2} \times 2 \times 2 \times 2 \times 3$$

$$404 = \boxed{2 \times 2} \times 101$$

$$\text{H.C.F} = 2 \times 2$$

$$= 4$$

We know that

H.C.F × L.C.M = Product of numbers

$4 \times L.C.M = 96 \times 404$

$$L.C.M = \frac{96 \times 404}{4}$$

L.C.M = 9696

Example 8 (Method 2)

Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.

$$\begin{array}{r|l} 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 120 \\ \hline 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$$6 = \boxed{2} \times 3$$

$$72 = \boxed{2} \times 2 \times 2 \times \boxed{3} \times 3$$

$$120 = \boxed{2} \times 2 \times 2 \times \boxed{3} \times 5$$

$$\text{H.C.F} = 2 \times 3$$

$$= 6$$

Finding LCM

2	6	72	120
2	3	36	60
2	3	18	30
3	3	9	15
3	1	3	5
5	1	1	5
	1	1	1

$$\text{L.C.M} = 2 \times 2 \times 2 \times 3 \times 3 \times 5$$
$$= 360$$