पु॰ना International School

Shree Swaminarayan Gurukul, Zundal

Class – IX

Sub – Maths

Specimen Copy

Year 2020-21

Class 9 Maths Chapters:

Real Numbers

Euclid's Division Lemma

- An algorithm is a series of well defined steps which gives a procedure for solving a type of problem.
- A lemma is a proven statement used for proving another statement.
- Euclid's division algorithm is a technique to compute the Highest Common Factor (HCF) of two given positive integers.
- To obtain the HCF of two positive integers, say c and d, with c > d, follow the steps below:
  Step 1: Apply Euclid's division lemma, to c and d. So, we find whole numbers, q and r such that $c = dq + r$, $0 \leq r < d$.
  Step 2: If $r = 0$, d is the HCF of c and d. If $r \neq 0$, apply the division lemma to d and r.
  Step 3: Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

The Fundamental Theorem of Arithmetic

- Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

Rational and Irrational Numbers

- A number 's' is called rational if it can be written in the form p, q
  Where p and q are integers and $q \neq 0$.
- A number 's' is called irrational if it cannot be written in the form p, q
  Where p and q are integers and $q \neq 0$.

Irrationality of Square Roots of 2, 3 and 5

- Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.
- $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ are irrational

Decimal Expansions of Rational Numbers

- Let x be a rational number whose decimal expansion terminates. Then we can express x in the form pq, where p and q are coprime, and the prime factorization of q is of the form 2^n5^m, where n, m are non-negative integers.
- Let x = pq be a rational number, such that the prime factorization of q is of the form 2^n5^m, where n, m are non-negative integers. Then x has a decimal expansion which terminates.
- Let x = pq be a rational number, such that the prime factorization of q is not of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which is non-terminating repeating (recurring).

Polynomials

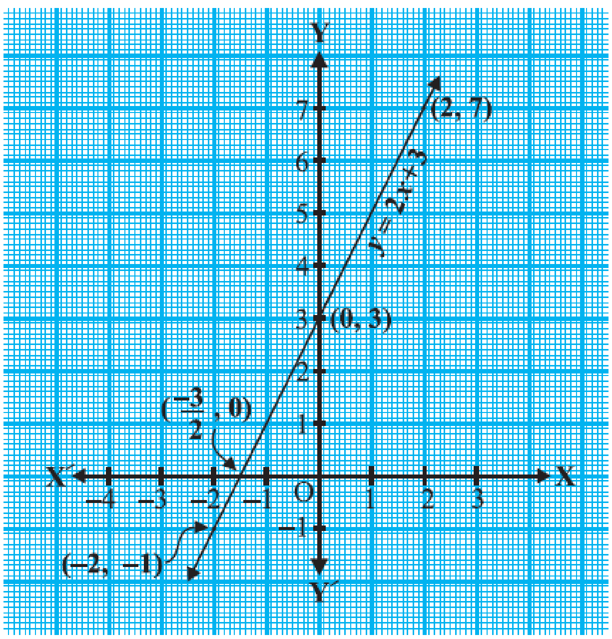
- If p(x) is a polynomial in x, the highest power of x in p(x) is called the degree of the polynomial p(x).

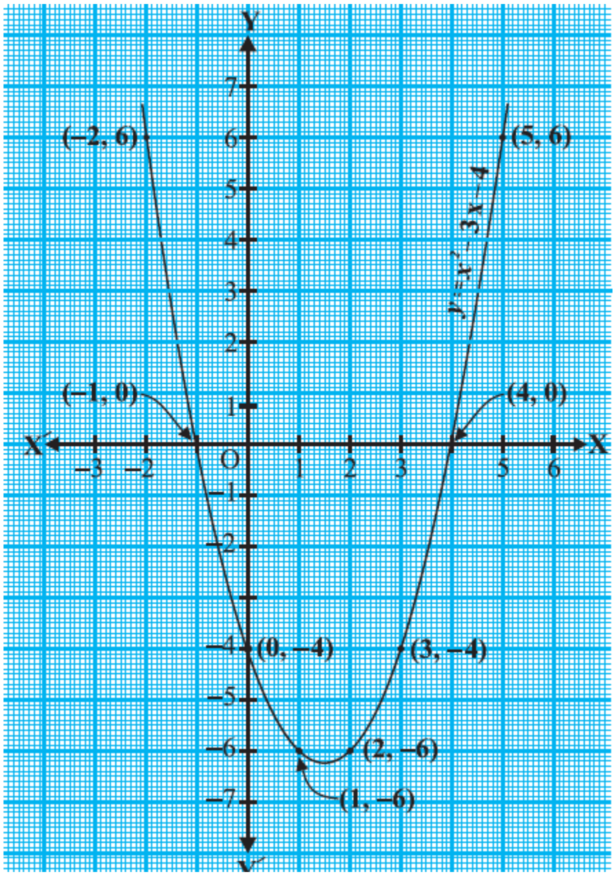
Types of Polynomials

- A polynomial of degree 1 is called a linear polynomial.
- A polynomial of degree 2 is called a quadratic polynomial.
- A polynomial of degree 3 is called a cubic polynomial.

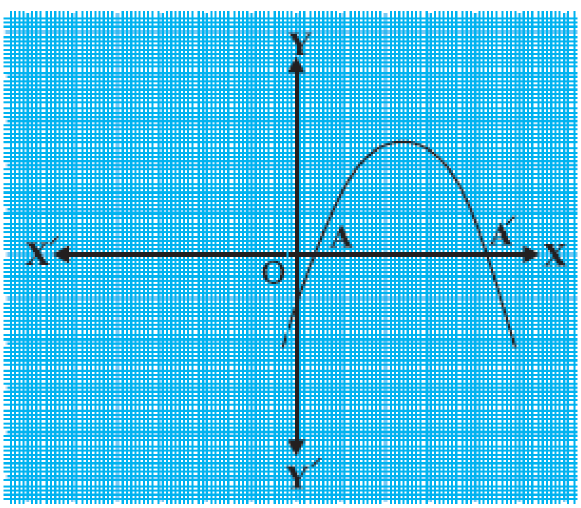
Zeroes of a Polynomial

- If p(x) is a polynomial in x, and if k is any real number, then the value obtained by replacing x by k in p(x), is called the value of p(x) at x = k, and is denoted by p(k).
- A real number k is said to be a zero of a polynomial p(x), if p(k) = 0.
- Geometrical Meaning of Zeroes of Polynomials

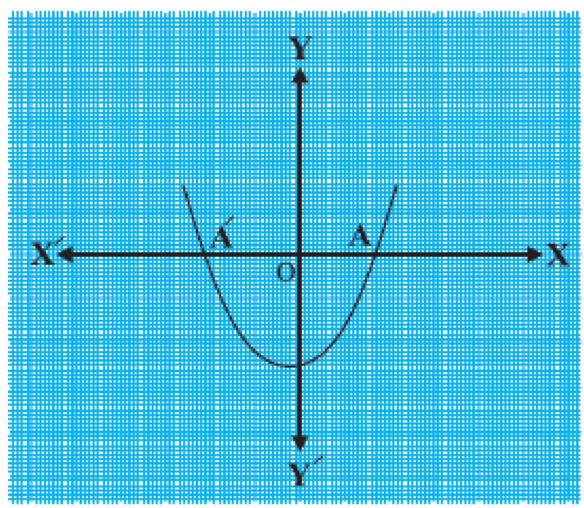




- The equation $ax^2 + bx + c$ can have three cases for the graphs
- Case (i): Here, the graph cuts x-axis at two distinct points A and A′.

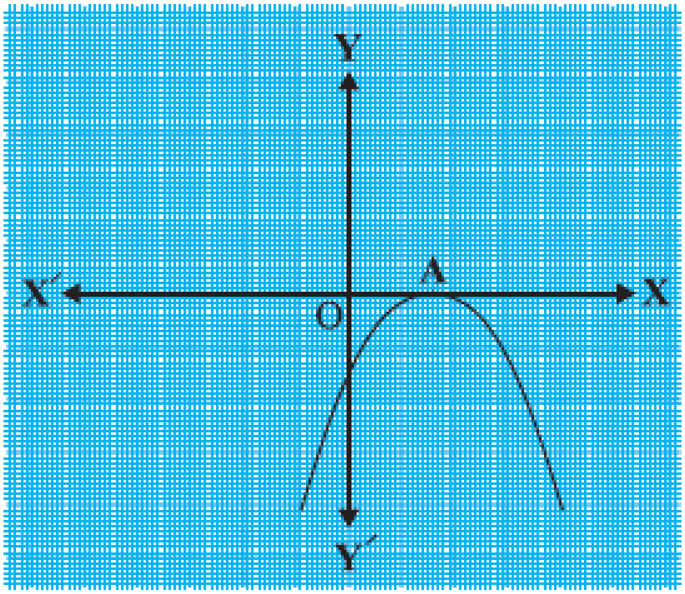


(i)

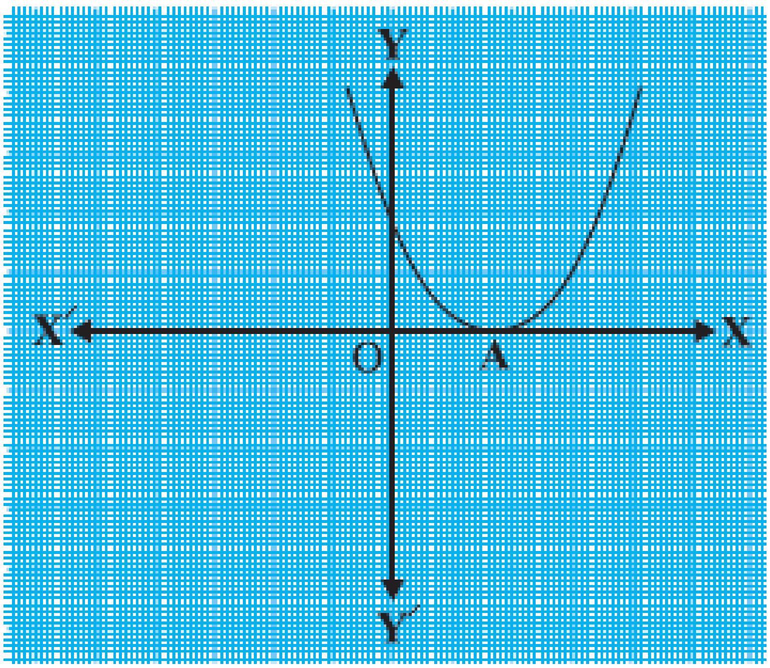


(ii)

Case (ii): Here, the graph cuts the x-axis at exactly one point

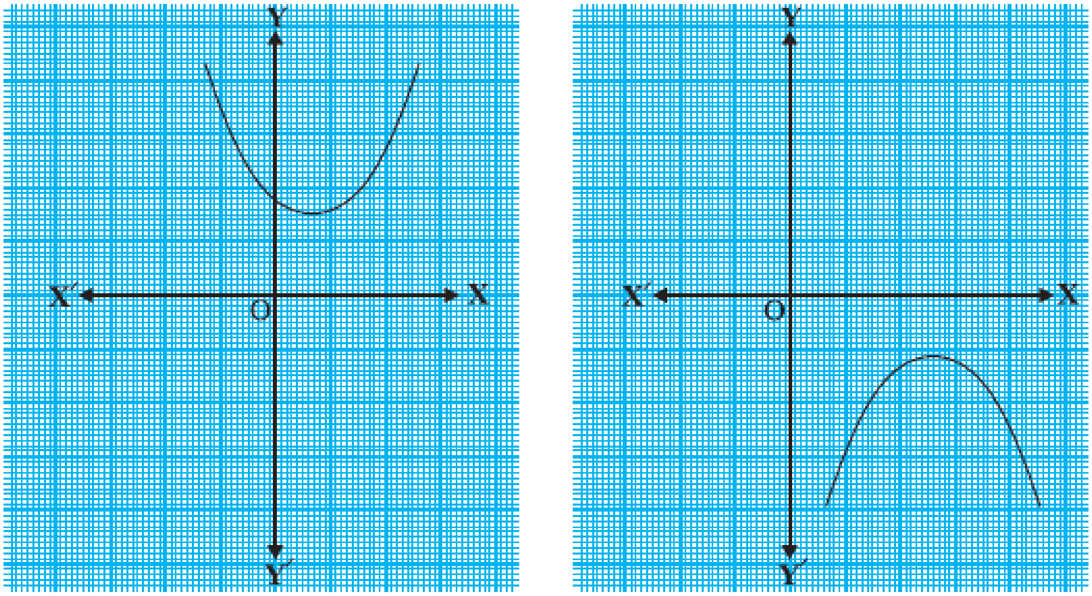


(i)



(ii)

Case (iii): Here, the graph is either completely above the x-axis or completely below the x-axis.



(i) (ii)

Relationship between Zeroes and Coefficients of a Polynomial

If α and β are the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, $a \neq 0$, then you know that $x - \alpha$ and $x - \beta$ are the factors of p(x).

$A + \beta = - b/a$

$\alpha\beta = c/a$

Division Algorithm for Polynomials

- If p(x) and g(x) are any two polynomials with $g(x) \neq 0$, then we can find polynomials q(x) and r(x) such that

  $p(x) = g(x) \times q(x) + r(x)$,

  where $r(x) = 0$ or degree of r(x) < degree of g(x).

  This result is known as the Division Algorithm for polynomials.

- Consider the cubic polynomial $x^3 - 3x^2 - x + 3$.

  If one of its zeroes is 1, then $x - 1$ is a factor of $x^3 - 3x^2 - x + 3$.

  So, you can divide $x^3 - 3x^2 - x + 3$ by $x - 1$,

  Next, you could get the factors of $x^2 - 2x - 3$, by splitting the middle term, as: $(x + 1)(x - 3)$. This would give you:

  $x^3 - 3x^2 - x + 3 = (x - 1)(x^2 - 2x - 3)$

  $= (x - 1)(x + 1)(x - 3)$

  So, all the three zeroes of the cubic polynomial are now known to you as 1, − 1, 3.

Pair of Linear Equations in Two Variables

Linear Equation

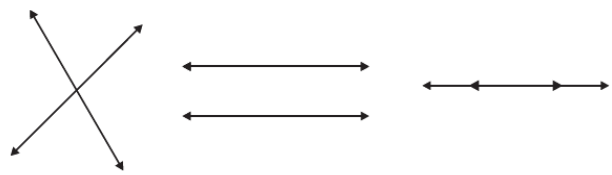
- An equation which can be put in the form ax + by + c = 0, where a, b and c are real numbers and both a and b are nonzero is called a linear equation in two variables.

Solution of an Equation

- Each solution (x, y) of a linear equation in two variables.
  ax + by + c = 0, corresponds to a point on the line representing the equation, and vice-versa.

Pair of Linear Equations in Two Variables

- The general form for a pair of linear equations in two variables x and y is $a_1x + b_1y + c_1 = 0$
  And $a_2x + b_2y + c_2 = 0$
- Geometrically they look like the following:

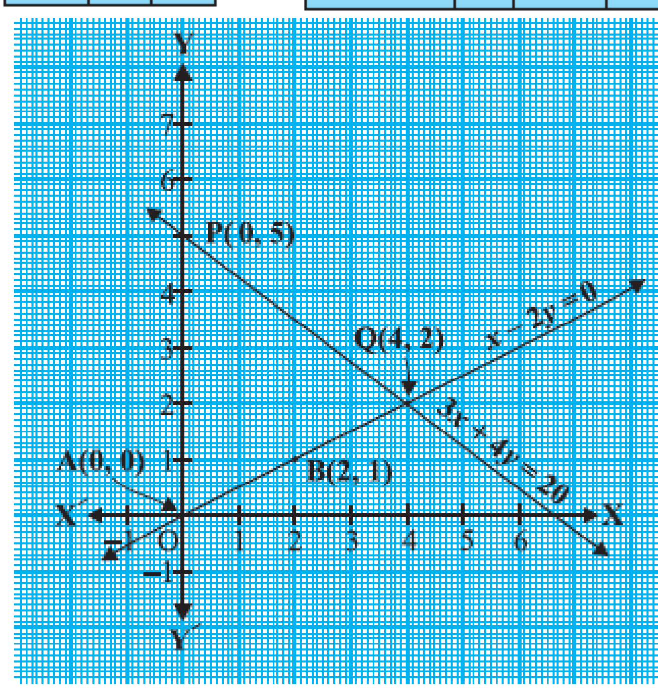


Graphical Method of Solutions

- $x - 2y = 0$
  $3x + 4y = 20$

x	0	2
$y = \frac{x}{2}$	0	1

x	0	$\frac{20}{3}$	4
$y = \frac{20-3x}{4}$	5	0	2



The solution is (4, 2), the point of intersection.

- To summarize the behavior of lines representing a pair of linear equations in two variables:
- The lines may intersect in a single point. In this case, the pair of equations has a unique solution (consistent pair of equations).
- The lines may be parallel. In this case, the equations have no solution (inconsistent pair of equations).
- The lines may be coincident. In this case, the equations have infinitely many solutions [dependent (consistent) pair of equations]

Substitution Method

The following are the steps:

- Find the value of one variable, say y in terms of the other variable, i.e., x from either equation, whichever is convenient.
- Substitute this value of y in the other equation, and reduce it to an equation in one variable, i.e., in terms of x, which can be solved. Sometimes, you can get statements with no variable. If this statement is true, you can conclude that the pair of linear equations has infinitely many solutions. If the statement is false, then the pair of linear equations is inconsistent.
- Substitute the value of x (or y) obtained in Step 2 in the equation used in Step 1 to obtain the value of the other variable.

Elimination Method

- Steps in the elimination method:
- First multiply both the equations by some suitable non-zero constants to make the coefficients of one variable (either x or y) numerically equal.
- Then add or subtract one equation from the other so that one variable gets eliminated. If you get an equation in one variable, go to Step 3.
  If in Step 2, we obtain a true statement involving no variable, then the original pair of equations has infinitely many solutions.
  If in Step 2, we obtain a false statement involving no variable, then the original pair of equations has no solution, i.e., it is inconsistent.
- Solve the equation in one variable (x or y) so obtained to get its value.
- Substitute this value of x (or y) in either of the original equations to get the value of the other variable.

Cross Multiplication Method

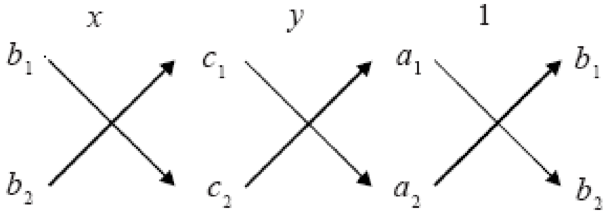
Steps:

Write the given equations in the form

$a_1x + b_1y + c_1 = 0$

And $a_2x + b_2y + c_2 = 0$

Taking the help of the diagram



Write Equations as

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Find x and y, provided $a_1b_2 - a_2b_1 \neq 0$

Quadratic Equations

A quadratic equation in the variable x is an equation of the form $ax^2 + bx + c = 0$, where a , b, c are real numbers, a ≠ 0.

Roots of a Quadratic Equation:

- A real number α is called a root of the quadratic equation
  $ax^2 + bx + c = 0$, $a \neq 0$ if
  $a\alpha^2 + b\alpha + c = 0$.
- ***x*** **=**α is a solution of the quadratic equation, or α satisfies the quadratic equation.
- The zeroes of the quadratic polynomial $\boldsymbol{ax^2 + bx + c}$ and the roots of the quadratic. Equation $\boldsymbol{ax^2 + bx + c = 0}$ are the same.

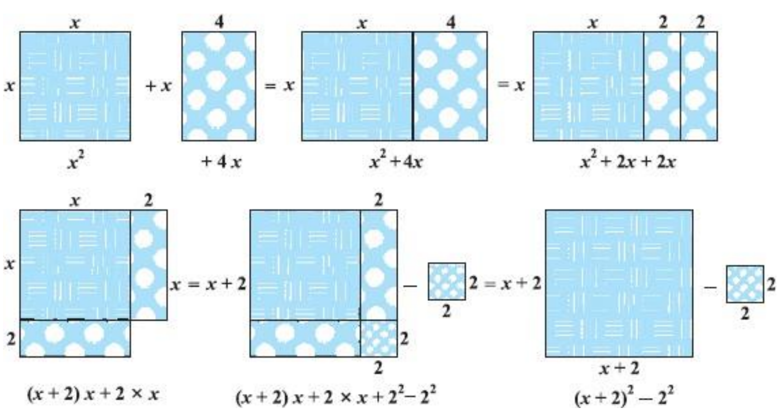
Solution of Quadratic Equation by Factorisation:

- To factorise quadratic polynomials the middle term is split.
- By factorizing the equation into linear factors and equating each factor to zero the roots are determined.

Quadratic Equations - Method of Squares

Solution of Quadratic Equation by method of Squares

- We can convert any quadratic equation to the form
  ( )
  $x^2 + 4x$ is being converted to
  $(x + 2)^2$ 4. = $(x + 2)^2$ 2^2



The process is as follows:

(−) −

=

( )

( )

( ) ( )

( ) ( )

( )

So, ( ) ( )

So, can be written as ( ) by this process of completing the square. This is known as the **method of completing the square.**

Solution of Quadratic Equation by using Formula.

The formula is as follows:

The roots of √ √

If

Thus, if then the roots of the quadratic equation are given by

√

This formula for finding the roots of a quadratic equation is known as the Quadratic formula.

Nature of Roots

We know that roots of the equation are

√ √

Where is known as discriminant.

Nature of roots based on the discriminant value

1. If then the roots are real and equal.
2. If then the roots are real and distinct (unequal)
3. If then the roots are imaginary (not real)

Arithmetic Progressions

- Look at the list of numbers 1, 3, 5, 7………
- Each of the numbers in the list is called a term.
- An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.
- This fixed number is called the common difference of the AP. It can be positive, negative or zero
- The general form of an AP is: a , $a + d$, $a + 2d$, $a + 3d$, . . .
- An AP with finite number of terms is a finite AP. That means the AP has a last term.
- An AP which does not have finite number of terms is an infinite AP. That means the AP does not have a last term. n^{th} term of an AP:

Let a_1, a_2, a_3,…… Be an AP whose first term a_1 is a and the common difference is d.

Then,

The **Second** term $a_2 = a + d = a + \mathbf{(2 – 1)}\, d$

The **third** term $a_3 = a_2 + d = (a + d) + d = a + 2d = a + \mathbf{(3 – 1)}\, d$

The **fourth** term $a_4 = a_3 + d = (a + 2d) + d = a + 3d = a + \mathbf{(4 – 1)}\, d$

……………

……………

Looking at the pattern, we can say that the n^{th} term a_n = a + (n 1) d.

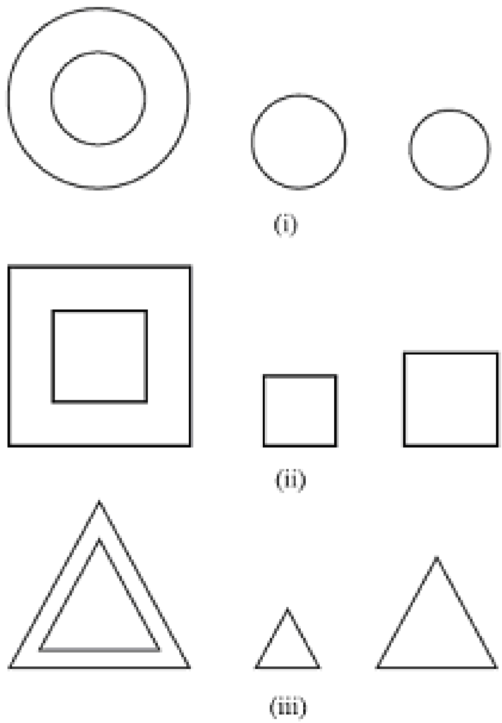
So, **the n^{th} term a_n of the AP with first term a and common difference d is given by $a_n = a + (n\ 1)\, d$.**

Sum on n terms in an AP:

The sum of the first n terms of an AP is given by S = ( )

Triangles

Similar Figures



The above are similar figures.

- All congruent figures are similar but similar figures need not be congruent.
- Two regular polygons of the same number of sides are similar, if
  - (i) their corresponding angles are equal and
  - (ii) their corresponding sides are in the same ratio (or proportion).

Similarity of Triangles:

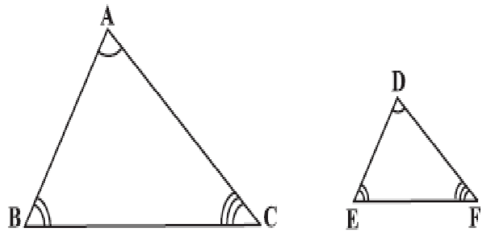
- Two triangles are similar, if
  - a) their corresponding angles are equal and
  - b) their corresponding sides are in the same ratio (or proportion).

- Criterion of similarity:

  In ΔABC and ΔDEF, if

  (i) if ∠A = ∠D, ∠B = ∠E, ∠C = ∠F and

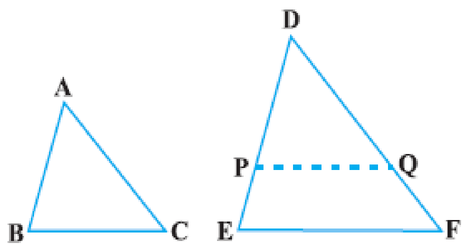
  (ii) $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$, then the two triangles are similar.



Theorems in similarity (SSS, AA, SAS, BPT)
AA criterion of similarity or AAA criterion:
If in two triangles, the corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.

This criterion is referred to as the AAA (Angle-Angle-Angle) criterion of similarity of two triangles.

This theorem can be proved by taking two triangles ABC and DEF such that ∠A = ∠D, ∠B = ∠E, ∠C = ∠F.



Cut DP = AB and DQ = AC and join PQ.
So, ΔABC ≅ ΔDPQ
This gives ∠B = ∠P = ∠E and PQ || EF
Therefore, $\frac{DP}{PE} = \frac{DQ}{QF}$

i.e. $\frac{AB}{DE} = \frac{AC}{DF}$

Similarly, $\frac{AB}{DE} = \frac{BC}{EF}$ and so

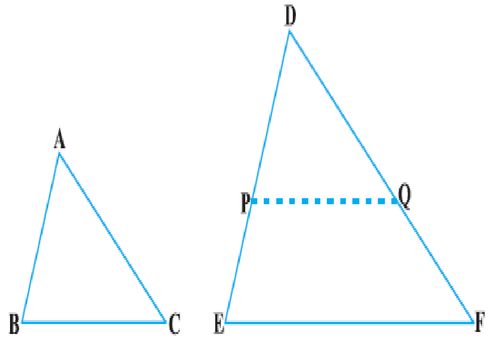
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

SSS criterion of similarity:
If in two triangles, the sides of one triangle are proportional to (i.e. in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

This criterion is referred to as the SSS (Side-Side-Side) similarity criterion for two triangles.

This theorem can be proved by taking two triangles ABC and DEF such that

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} (< 1)$$



Cut DP = AB and DQ = AC and join PQ.

It can be seen that $\frac{DP}{PE} = \frac{DQ}{QF}$ and

PQ || EF

So, ∠P = ∠E and ∠Q = ∠F

Therefore, $\frac{DP}{DE} = \frac{DQ}{DF} = \frac{PQ}{EF}$

So, $\frac{DP}{DE} = \frac{DQ}{DF} = \frac{BC}{EF}$

So, BC = PQ

SAS criterion of similarity:

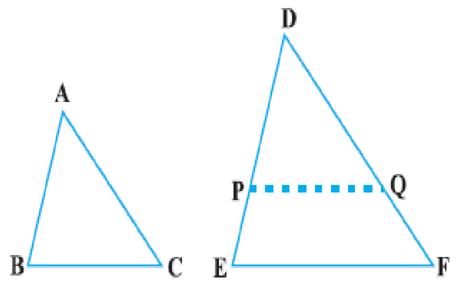
If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

This criterion is referred to as the SAS (Side-Angle-Side) similarity criterion for two triangles.

As before, this theorem can be proved by taking two triangles ABC and DEF such that

$\frac{AB}{DE} = \frac{AC}{DF} (< 1)$ and ∠A = ∠D.

Cut DP = AB, DQ = AC and join PQ.



Now, PQ || EF and $\Delta ABC \cong \Delta DPQ$

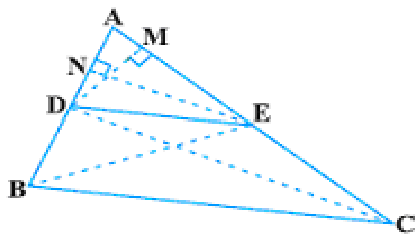
So, $\angle A = \angle D$, $\angle B = \angle P$ and $\angle C = \angle Q$ Therefore,

$\Delta ABC \sim \Delta DEF$

Basic Proportionality theorem:

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Proof: We are given a triangle ABC in which a line parallel to side BC intersects the other two sides AB and AC at D and E respectively.



We need to prove that $\frac{AD}{DB} = \frac{AE}{EC}$

Let us join BE and CD and then draw

DM $\perp$ AC and EN $\perp$ AB

Now, area of $\Delta ADE = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times AD \times EN$

Recall from Class IX, that area of ΔADE is denoted as ar(ADE).

So, ar(ADE) $= \frac{1}{2} \times AD \times EN$

Similarly, ar(BDE) $= \frac{1}{2} \times DB \times EN$

ar(ADE) $= \frac{1}{2} \times AE \times DM$ and ar(DEC) = $\frac{1}{2} \times EC \times DM$

Therefore, $\frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN}$

$= \frac{AD}{DB}$... (1)

and $\frac{ar(ADE)}{ar(DEC)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM}$

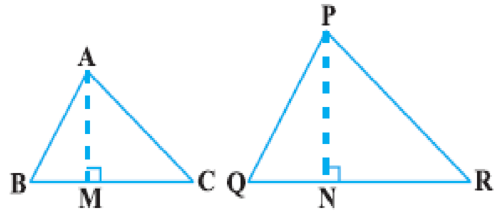
$= \frac{AE}{EC}$...(2)

Note that ΔBDE and ΔDEC are on the same base DE and between the same parallels BC and DE.
So, ar(BDE) = ar(DEC)...(3)
Therefore, from (1), (2) and (3), we have:

$\frac{AD}{DB} = \frac{AE}{EC}$

Areas of Similar Triangles:
The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Proof: We are given two triangles ABC and PQR such that ΔABC ~ ΔPQR



We need to prove that

$$\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$

For finding the areas of the two triangles, we draw altitudes AM and PN of the triangles.

Now, ar(ABC) = $\frac{1}{2} \times BC \times AM$

and ar(PQR) = $\frac{1}{2} \times QR \times PN$

So, $\frac{ar(ABC)}{ar(PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN}$ … (1)

Now, in ΔABM and ΔPQN,

$\angle B = \angle Q$ (As ΔABC ~ ΔPQR) and

$\angle M = \angle N$ (Each is 90°)

So, ΔABM ~ ΔPQN (AA similarity criterion)

Therefore, $\frac{AM}{PN} = \frac{AB}{PQ}$ … (2)

Also, ΔABC ~ ΔPQR (Given)

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ … (3)

Therefore, $\frac{ar(ABC)}{ar(PQR)} = \frac{AB}{PQ} \times \frac{AM}{PN}$

[From (1) & (3)]

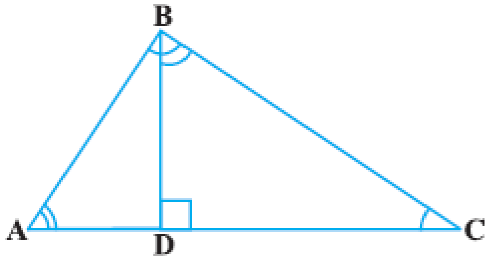
$= \frac{AB}{PQ} \times \frac{AB}{PQ}$ [From (2)]

$= \left(\frac{AB}{PQ}\right)^2$

Now using (3) we get:

$\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$

Proof of Pythagoras Theorem Using Similarity



You may note that in ΔADB and ΔABC

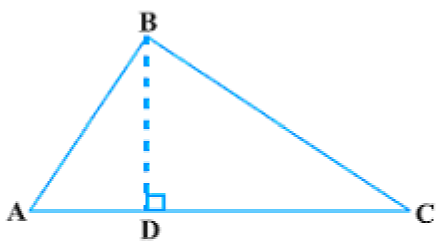
∠A = ∠A

and $\angle ADB = \angle ABC$ So,
$\Delta ADB \sim \Delta ABC$
Similarly, $\Delta BDC \sim \Delta ABC$
From (1) and (2), triangles on both sides of the perpendicular BD are similar to the whole triangle ABC.
Also, since $\Delta ADB \sim \Delta ABC$ and $\Delta BDC \sim \Delta ABC$
So, $\Delta BDC \sim \Delta BDC$

If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and to each other.

Let us now apply this theorem in proving the Pythagoras Theorem:
In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Proof: We are given a right triangle ABC right angled at B.

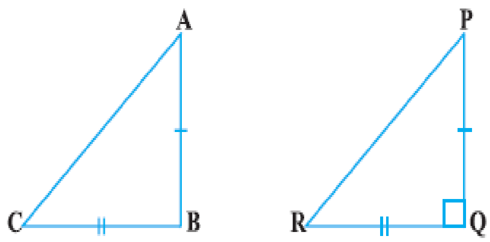


We need to prove that $AC^2 = AB^2 + BC^2$
Let us draw $BD \perp AC$.
Now, $\Delta ADB \sim \Delta ABC$
So, $\frac{AD}{AB} = \frac{AB}{AC}$ (Sides are proportional)

or $AD \cdot AC = AB^2$ … (1)
Also, $\Delta BDC \sim \Delta ABC$
So, $\frac{CD}{BC} = \frac{BC}{AC}$
or $CD \cdot AC = BC^2$ … (2)
Adding (1) and (2),
$AD \cdot AC + CD \cdot AC = AB^2 + BC^2$ Or
$AC (AD + CD) = AB^2 + BC^2$ Or
$AC \cdot AC = AB^2 + BC^2$
Or $AC^2 = AB^2 + BC^2$

Converse of Pythagoras Theorem:
In a right triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite the first side is a right angle.

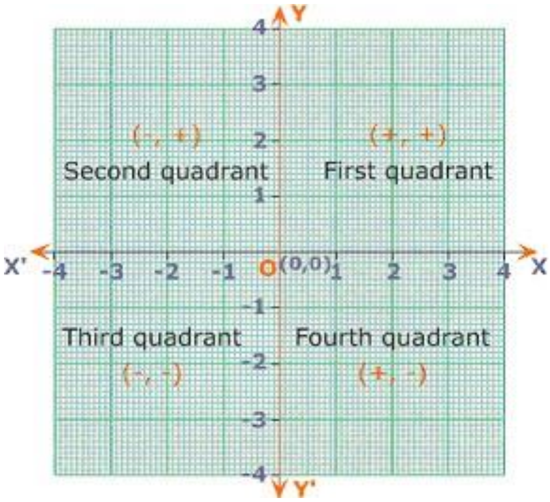
Proof: Here we are given a triangle ABC in which $AC^2 = AB^2 + BC^2$
We need to prove that $\angle B = 90°$
To start with, we construct a ΔPQR right angled at Q such that PQ = AB and QR = BC



Now, from ΔPQR, we have:
$PR^2 = PQ^2 + QR^2$ (Pythagoras Theorem, as $\angle Q = 90°$)
Or $PR^2 = AB^2 + BC^2$ (By construction). (1)
But $AC^2 = AB^2 + BC^2$ (Given)… (2)
So, AC = PR [From (1) and (2)] … (3)
Now, in ΔABC and ΔPQR,
AB = PQ (By construction)
BC = QR (By construction)
AC = PR [Proved in (3) above]
So, ΔABC ≅ ΔPQR (SSS congruence) Therefore, $\angle B = \angle Q$ (CPCT)
But $\angle Q = 90°$ (By construction) So, $\angle Q = 90°$

Co−ordinate Geometry

Important Terms and Concepts



" If a pair of perpendicular lines XOX' and YOY' intersects at O, then these lines are called the **co−ordinate axes**". The axes divide the plane into four quadrants.
The plane containing the axes is called the **Cartesian plane.**

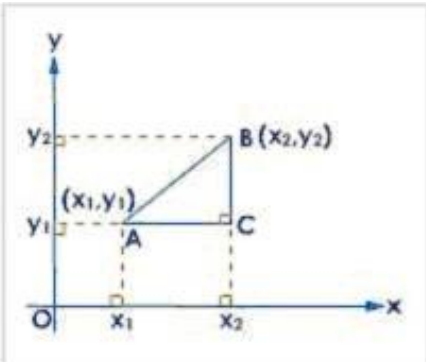
The lines XOX' and YOY' are usually drawn horizontally and vertically as shown in the figure, and are known as x−axis and y−axis respectively.
O, the point of intersection of the axes is called the **origin**.

Values of x are measured from O along the x−axis and are called **abscissae**. Along OX, x has positive values while OX', x has negative values.
Similarly, the values of y are measured from O along the y−axis and are called **ordinate**. Along OY, y has positive values while OY', y has negative values.

The ordered pair containing the abscissa and the ordinate of a point is called the **coordinates of the point**.

Distance Formula:

To find the distance two points A (x_1, y_1) and $B(x_2, y_2)$



From the figure

$AC = x_2 - x_1$

$BC = y_2 - y_1$

∴ In ΔABC,

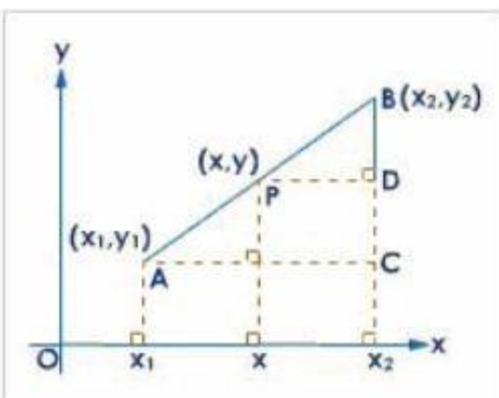
$AB^2 = AC^2 + BC^2$ (By Pythagoras Theorem)

( ) ( )

∴ √( ) ( )

Section Formula:

To find the coordinates of a point which divides the line segment joining two given points in a given ratio (internally)



Let P(x, y) divide the join of $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio m : n

∴ $AC = x - x_1$, $PD = x_2 - x$

∴From similarity property — — —— —

Now, Make x the subject of the formula, $nx - nx_1 = mx_2 - mx$

$mx + nx = mx_2 + nx_1$

$x(m + n) = mx_2 + nx_1$

∴ x =

Similarly, we can show that

y =

Thus coordinate of P are . /

Mid−Point Formula:

If P is the mid−point of AB, then m=n,

∴The ratio becomes 1:1

∴ x =

Similarly, we get

Thus coordinates of point are . /

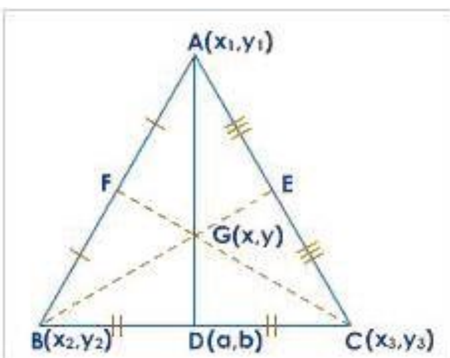
Note:

When the point P divides the line joining AB in the ration m: n externally then

. /

Centroid of a Triangle

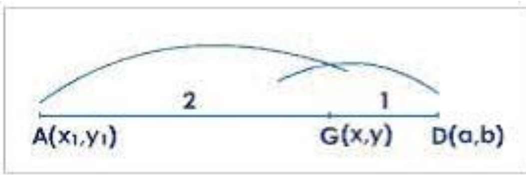
Centroid is the point of intersection of three medians. It is the point of intersection of a median AG: GD is 2: 1



To find the coordinates of the centroid of a triangle.

Let the coordinates of the vertices of △ABC, be ( ) ( ) ( ). Let G(x, y) be the centroid of the △ABC.

By applying the mid−point formula



Again, by applying the section formula

( ) ( )

. /

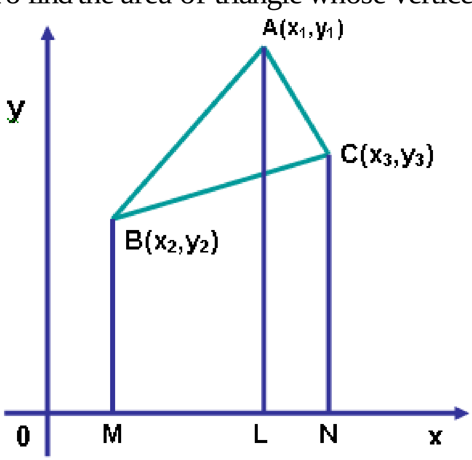
( ) ( )

. /

∴Coordinates of the centroid are . /

Area of the Triangle

To find the area of triangle whose vertices are ( ) ( ) ( )



Let ( ) ( ) ( ) be the vertices of a triangle ABC Area of ΔABC = Area of Trapezium ABML

+ Area of Trapezium ALNC

Area of Trapezium BMNC

_ ( ) ( ) ( ) _

_( ) ( )

_( )( ) (_) ( )

_ ( ) ( )

_ * ( ) ( ) ( )+

Arrow Method:

It is to obtain the formula for the area of the triangle

_ | |

_ , ( ) ( ) ( )-

Note:

1. If the points A, B and C we take in the anticlockwise direction, then the area will be positive. If the points we take in clockwise direction the area will be negative.
   So we always take the absolute value of the area calculated.
   Area of triangle

   _ , ( ) ( ) ( )-

2. If the area of a triangle is zero, then the three points are collinear.

Trigonometry

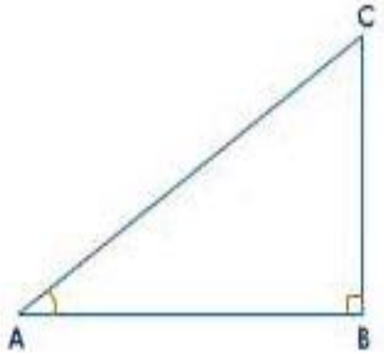
Introduction

Trigonometry is based on the right triangle. It is the study of the relationship between the sides and angles of a triangle. The height of a building, tree, tower, width of a river etc. can be determined with the help of trigonometry.

Trigonometric Ratios:

There are six t-ratios.

Δ ABC is a right-angled triangle, $\angle B = 90^{o}$.



(i) $\text{sine } A = \frac{\text{side opposite to} \angle A}{\text{hypotenuse}}$

$\text{sine } A = \frac{BC}{AC}$

In short, $\sin A = \frac{BC}{AC}\left[\frac{\text{opp.}}{\text{hyp.}}\right]$

(ii) $\text{cosine } A = \frac{\text{side adjacent to } \angle A}{\text{hypotenuse}}$

$\text{cosine } A = \frac{AB}{AC}$ In short, $\cos A = \frac{AB}{AC}\left[\frac{\text{adj.}}{\text{hyp.}}\right]$

(iii) $\text{tangent } A = \frac{\text{side opposite to } \angle A}{\text{side adjacent to } \angle A}$

$\text{tangent } A = \frac{BC}{AB}$ In short, $\tan A = \frac{BC}{AB}\left[\frac{\text{opp.}}{\text{adj.}}\right]$

Reciprocal Ratios:

(iv) cosecant $A = \frac{1}{\text{sine } A}$ In short, $\text{cosec } A = \frac{1}{\sin A}$

(v) secant $A = \frac{1}{\text{cosine } A}$ In short, $\sec A = \frac{1}{\cos A}$

(vi) cotangent $A = \frac{1}{\text{tangent } A}$ In short, $\cot A = \frac{1}{\tan A}$

$\sin C = \frac{AB}{AC}$

$\Rightarrow \text{cosec } C = \frac{AC}{AB}$

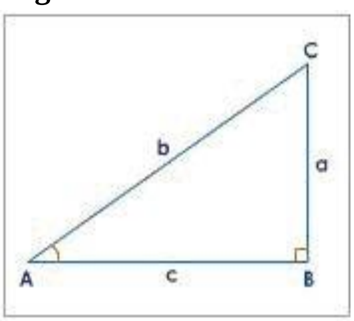
$\cos C = \frac{BC}{AC}$

$\Rightarrow \sec C = \frac{AC}{BC}$

$\tan C = \frac{AB}{BC}$

$\cot C = \frac{BC}{AB}$

Trigonometric Identities:



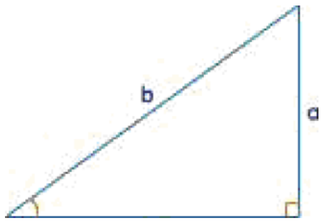
1. $\sin^2 A + \cos^2 A = 1$

2. $1 + \tan^2 A = \sec^2 A$

3. $1 + \cot^2 A = \text{cosec}^2 A$

Proof 1:

$a^2 + c^2 = b^2$ (by Pythagoras Theorem)



by b^2 a^2 c^2 b^2

b^2 ” b^2 b^2

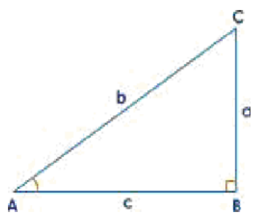
$$\Rightarrow \left(\frac{}{b}\right)^2 + \left(\frac{c}{b}\right)^2 = 1$$

Butsin A= band COSA= $\frac{c}{b}$,

$(\sin A)^2 + (\cos A)^2 = 1$

” $\sin^2 A + ca\ s^2 A = 1$

Proof 2:



a + c* = b^2 (by Pytb ago ras Theo rem)

by c^2 $\frac{a^2}{c'} + \frac{c^2}{c^2} = \frac{b^2}{c^2}$

$\frac{a}{c}^2 + 1 = \frac{b}{c}^2$

But tan A = $\frac{'}{c}$ and sec A = $\frac{b}{c}$

$(\tan A)^2 + 1 = (\sec A)^2$

” $1 + \tan^2 A = \sec^2 A$

Proof 3:

Again $a^2 + c^2 = b^2$(by Pythagoras Theorem)

$\div$ by a^2 $\frac{a^2}{a^2} + \frac{c^2}{a^2} = \frac{b^2}{a^2}$

$\Rightarrow 1 + \left(\frac{c}{a}\right)^2 = \left(\frac{b}{a}\right)^2$

But $\cot A = \frac{c}{a}$ and $\operatorname{cosec} A = \frac{b}{a}$

$\therefore 1 + (\cot A)^2 = (\operatorname{cosec} A)^2$

$\Rightarrow 1 + \cot^2 A = \operatorname{cosec}^2 A$

Trigonometric Ratios of Standard Angles

0^0, 30^0, 45^0, 60^0 and 90^0 are called standard angles.

Trigonometric ratios of 45^0

Let $\angle A = 45^0, \angle C = 45^0, \angle B = 90^0$

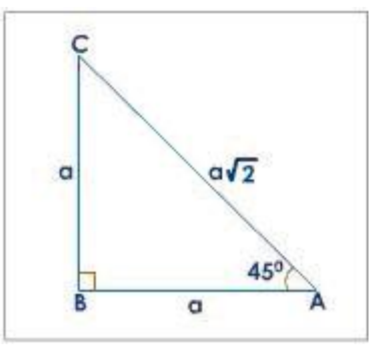
$\therefore \angle A = \angle C$

$\therefore AB = BC = a$

Using Pythagoras Theorem

$AC^2 = a^2 + a^2 = 2a^2$

$AC = a\sqrt{2}$



$\sin 45^0 = \frac{\text{opposite side}}{\text{hypotensue}}$

$= \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\cos 45^0 = \frac{\text{adjacent side}}{\text{hypotensue}}$

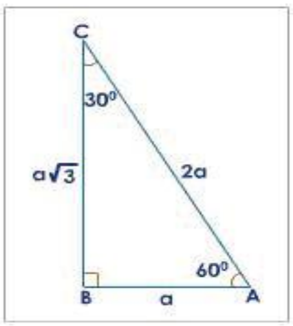
$= \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\tan 45^0 = \frac{\text{oppsotieside}}{\text{adjacent side}}$

$= \frac{a}{a} = 1$

Trigonometric ratios of 30^0 and 60^0

Let $\angle A = 60^0$ and $\angle C = 30^0$



In a $30^0 - 60^0 - 90^0$ triangle, it can be proved that the hypotenuse is double the side opposite to 30^0.

AC = 2AB

Let AB = a

∴ AC = 2a

Using Pythagoras Theorem

$BC^2 = (2a)^2 - a^2$

$= 4a^2 - a^2$

$= 3a^2$

$\therefore Bc = a\sqrt{3}$

$\sin 30^0 = \frac{a}{2a} = \frac{1}{2},$

$\sin 60^0 = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2},$

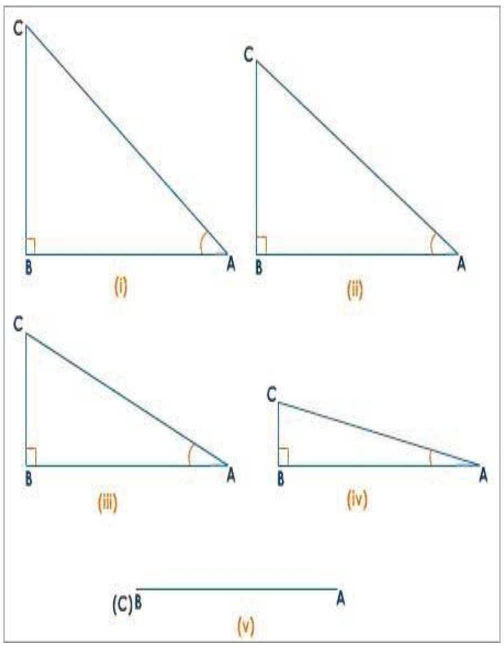
$$\cos 30^0 = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2},$$

$$\cos 60^0 = \frac{a}{2a} = \frac{1}{2},$$

$$\tan 30^0 = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}},$$

$$\tan 60^0 = \frac{a\sqrt{3}}{a} = \sqrt{3}$$

Trigonometric ratios of 0^0



ABC is a right-angled triangle with $\angle B = 90^0$. $\angle A$ is an acute angle and is made smaller and smaller till it becomes zero (fig v). As $\angle A$ gets smaller and smaller, the length of the side BC keeps decreasing. The point 'C' gets closer to the point B and at one time it coincides with 'B', so that $\angle A$ becomes zero.

If $\angle A = 0$, BC = 0

$$\sin A = \frac{BC}{AC} = \frac{0}{AC} = 0$$

$\Rightarrow \sin 0 = 0$

$$\cos A = \frac{AB}{AC} = \frac{AB}{AB} = 1$$

$\Rightarrow \cos 0 = 1$

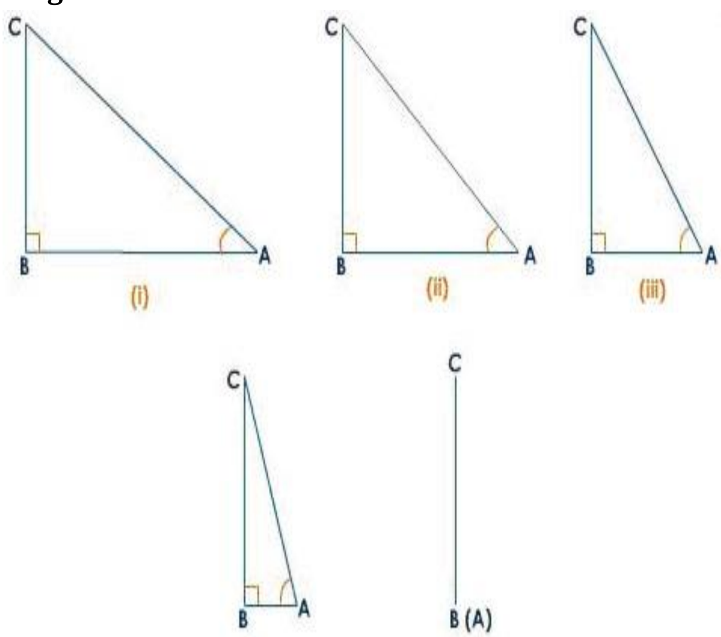
$\tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$

$\cot 0 = \frac{1}{\tan 0} = \frac{1}{0}$ = not defined

$\sec 0 = \frac{1}{\cos 0} = \frac{1}{1} = 1$

and $\text{cosec } 0 = \frac{1}{\sin 0} = \frac{1}{0}$ = not defined

Trigonometric ratios of 90^0



In right-angled triangle ABC, ∠A is made larger and larger till it becomes 90^0 (fig v). As ∠A gets larger and larger, ∠C gets smaller and smaller and the length of the side AB keeps decreasing. The point 'A' gets closer to the point B and at one time it coincides with 'B', so that ∠C becomes zero.

BC = AC

$\text{Sin } 90 = \frac{BC}{AC} = \frac{AC}{AC} = 1$

$\text{Cos } 90 = \frac{AB}{AC} = \frac{0}{BC} = 0$

$\text{Tan } 90 = \frac{\sin 90}{\cos 90} = \frac{1}{0}$, not defined

$\text{Cot } 90 = \frac{1}{\tan 90} = \frac{0}{1} = 0$

$\text{Sec } 90 = \frac{1}{\cos 90} = \frac{1}{0}$ not defined

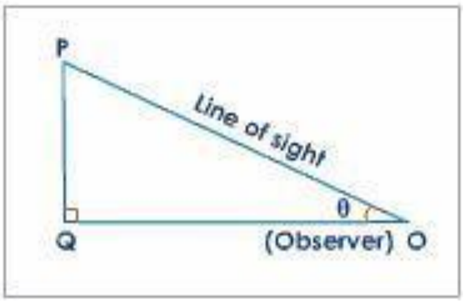
$\text{Cosec } 90 = \frac{1}{\sin 90} = \frac{1}{1} = 1$

Line of Sight

- It is an imaginary line drawn from the eye of the observer to the point of the object viewed by the observer.

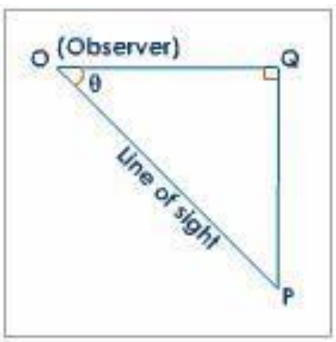
Angle of Elevation

- It is the angle formed by the line of sight with the horizontal when the object is above the horizontal level. It is the case when we look up to see the object.



Angle of Depression

- It is the angle formed by the line of sight with the horizontal when the object is below the horizontal level. It is the case when we look down to see the object.

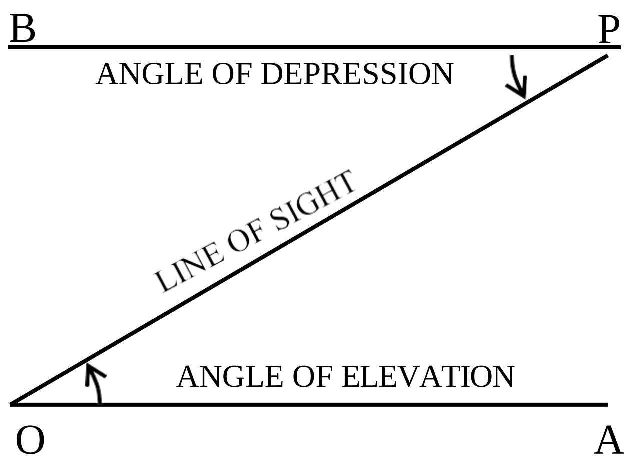


Applications of Trigonometry

Angles of Elevation and Depression

Let O and P be two points such that the point P is at higher level. Let OA and PB be horizontal lines through O and P respectively.

If an observer is at O and the point P is the object under consideration, then the line OP is called the line of sight of the point P and the angle AOP, between the line of sight and the horizontal line OA, is known as the angle of elevation of Point P as seen from O.



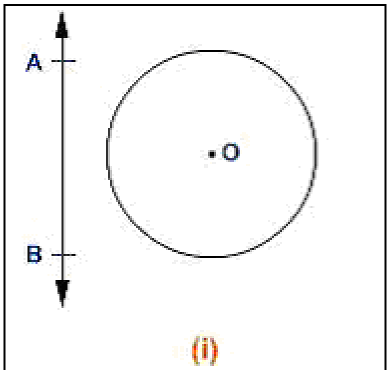
If an observer is at P and the object under consideration is at O, then the angle BPO is known as the angle of depression of O as seen from P.

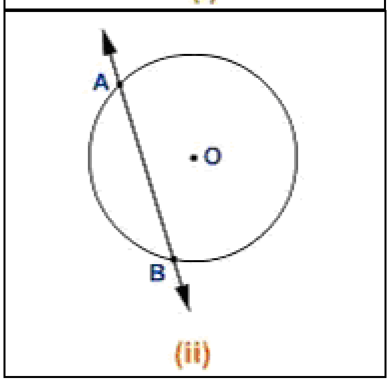
Obviously, the angle of elevation of a point P as seen from a point O is equal to the angle of depression of O as seen from P.

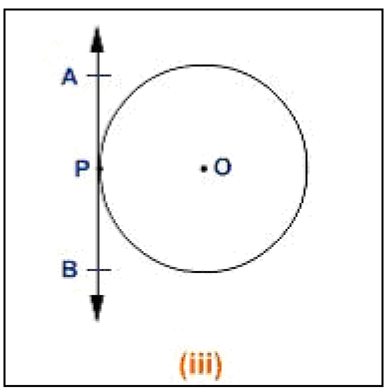
Circles

Tangent to a Circle

A tangent is a line touching a circle at one point

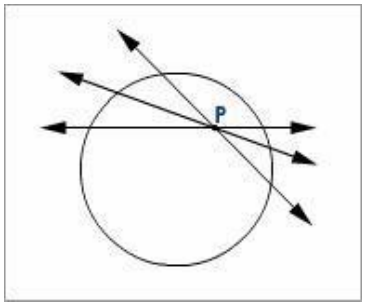




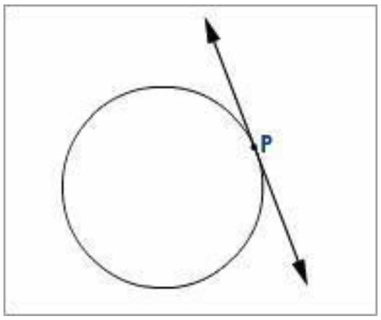


1. **Non-intersecting line** - fig (i): The circle and the line AB have no common point.
2. **Secant** - fig (ii): The line AB intersects the circle at two points A and B. AB is the secant of the circle.
3. **Tangent** - fig (iii): The line AB touches the circle at only one point. P is the point on the line and on the circle. P is called the point of contact. AB is the tangent to the circle at P.

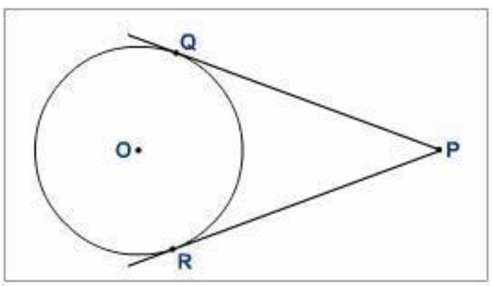
Number of Tangents from a Point on a Circle



From a point inside a circle, no tangents can be drawn to the circle.



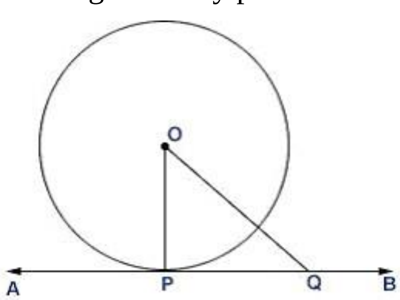
From a point on a circle, only 1 tangent can be drawn to the circle.
In this figure, P is a point on the circle. There is only 1 tangent at P. P is called the point of contact.



From a point outside a circle, exactly 2 tangents can be drawn to the circle. In this figure, P is the external point. PQ and PR are the tangents to the circle at points Q and R respectively. The length of a tangent is the length of the segment of the tangent from the external point to the point of contact. In this figure, PQ and PR are the lengths of the 2 tangents.

Theorem 1:

The tangent at any point of a circle is perpendicular to the radius through the point of contact.



Given:

AB is a tangent to the circle with centre O. P is the point of contact. OP is the radius of the circle.

To prove:

OP ⊥ AB

Proof:

Let Q be any point (other than P) on the tangent AB.

Then Q lies outside the circle.

For any point Q on the tangent other than P.

⇒ OP is the shortest distance between the point O and the line AB.

OP ⊥ AB

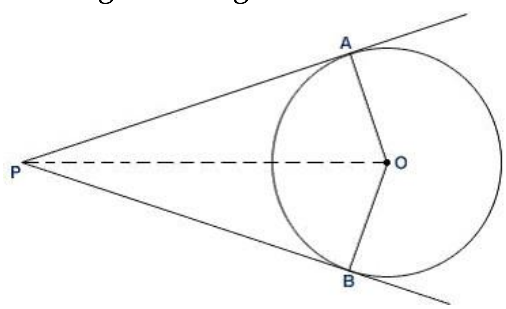
(∵The shortest line segment drawn from a point to a given line, is perpendicular to the line) Thus, the theorem is proved.

From the above theorem,

1. The perpendicular drawn from the centre to the tangent of a circle passes through the point of contact.
2. If OP is a radius of a circle with centre O, a perpendicular drawn on OP at P, is the tangent to the circle at P.

Theorem2:

The lengths of tangents drawn from an external point to a circle are equal.



Given:

P is an external point to a circle with centre O. PA and PB are the tangents from P to the circle. A and B are the points of contact.

To prove:

PA = PB

Construction:

Join OA, OB, OP.

Proof:

In triangle APO and BPO,

Statement	Reason
OA = OB	Radii of the same circle
	The radius is perpendicular to the tangent at the point of the contact.
OP = OP	Common
	By SAS postulate
PA = PB	CPCT(Third side of the triangles)

From the above theorem,

1. (CPCT) This states that the two tangents subtend equal angles at the centre of the circle
2. (CPCT) The tangents are equally inclined to the line joining the point and the centre of the circle.
   Or the centre of the circle lies on the angle bisector of the .

Constructions

Division of a Line Segment

To divide a line segment internally in a given ratio m: n, where both m and n are positive integers.

Steps:

Step 1: Draw a line segment AB of given length using a ruler.

Step 2: Draw any ray AX making an acute angle with AB.

Step 3: Along AX mark off (m + n) points, namely A_1, A_2… A_m, A_{m+1}… A_{m+n}

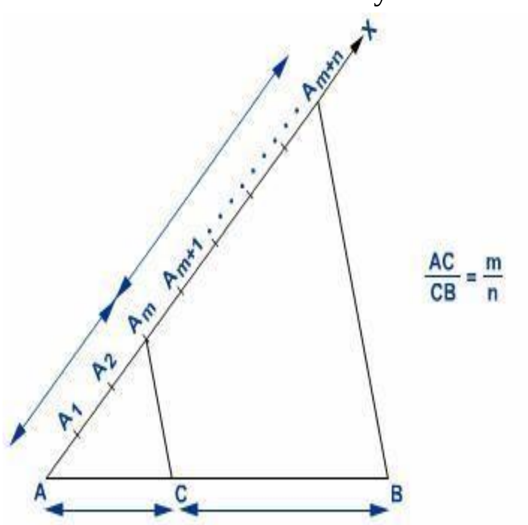
Step 4: Join B to A_{m+n}

Step 5: Through the point A_m draw a line parallel to A_{m+n} B at A_m. Let this line meet AB at 'C' which divides AB internally in the ratio m: n.

Proof: In ΔABA_{m+n}, CA_m is parallel to BA_{m+n}.

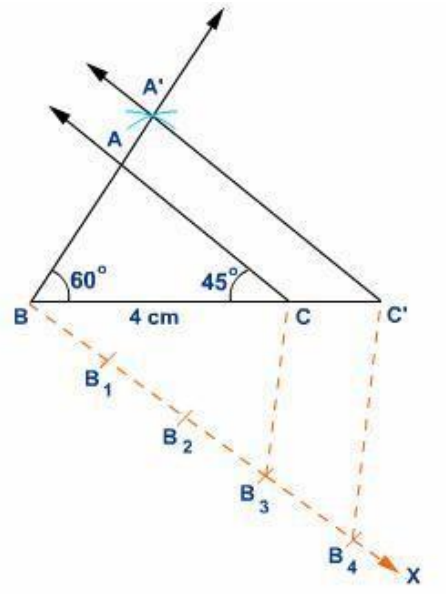
By basic proportionality theorem, we get, ——— — —

Here 'C' divides AB internally in the ratio m: n.



To Construct a Triangle Similar To a Given Triangle as Per the Given Scale Factor

Construct a ΔABC in which BC = 4cm, ∠B = 60° and ∠C = 45°. Also construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of ΔABC.



Steps of construction:

Step 1: Construct a triangle ABC with the given measurement i.e. BC = 4cm, ∠B = 60° and ∠C = 75°

Step 2: Construct an acute angle CBX downwards.

Step 3: On BX, make 4 equal parts and mark them B_1, B_2, B_3, B_4.

Step 4: Join 'C' to B_3 and draw a line through B_4 parallel to B_3C, intersecting the extended line segment BC at C'.

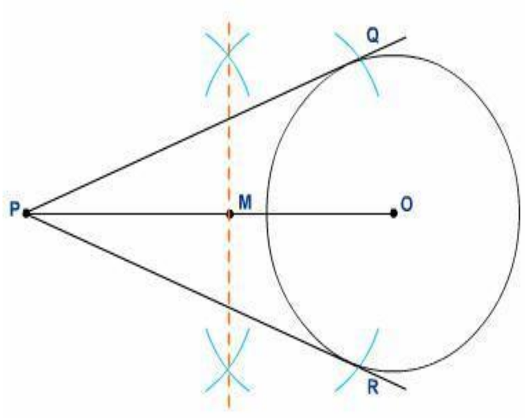
Step 5: In the same way draw C'A' parallel to CA. Thus ΔA'BC' is the required triangle similar to ΔABC whose sides are $\frac{4}{3}$ times the corresponding sides of ΔABC.

Construction of Tangents to a Circle

To construct the tangents to a circle from a point outside it

Given: A circle with centre 'O' and a point 'P' outside it

Required: To construct the tangents to the circle from P.



Steps of construction:

Step 1: Draw a circle with centre 'O'

Step 2: Join OP.

Step 3: Draw the perpendicular bisector OP. It meets OP at 'M'.

Step 4: Taking 'M' as centre and OM as radius draw arcs which cut the circle with centre 'O' at two points. Name them as Q and R.

Step 5: Join PQ and PR

Step 6: PQ and PR are the required tangents to the circle with centre 'O' from an external point 'P'.

Note:

We can prove that the length of PQ and PR are equal.

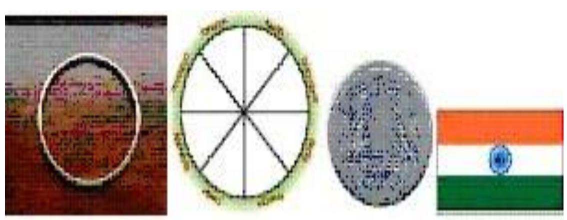
Areas related to Circles

Perimeter and Area of a Circle - A Review Circle

A circle is a path of a point which moves in such a way that its distance from a fixed point is a constant. The path of the point is called the locus of the point. The fixed point is called the centre of the circle. The constant distance is called the radius of the circle.

In our daily life we see a lot of things which are circular in shape.

1. Bangle
2. Wheel
3. Coin
4. Chakra in the middle of the flag
5. Eyeball
6. Clock
7. Papad
8. Ring
9. Dosa



Bangle Wheel Coin Flag

Eye ball Clock Papad Ring

Dosa

Circumference

The perimeter of a circle is called the circumference of the circle. Using a rape or thread we measure the circumference of a wheel or a bangle. We can also measure the diameter. Diameter is the largest chord of a circle. In each case divide the circumference by the diameter. What do you notice? We find that in each case, the ratio of the circumference to the diameter is the same. This ratio is a constant called π (pi) which is a Greek letter.

$\pi = \underline{\qquad\qquad\qquad}$

Form this we get circumference of a circle is the product of π and its diameter.

$C = \pi d$

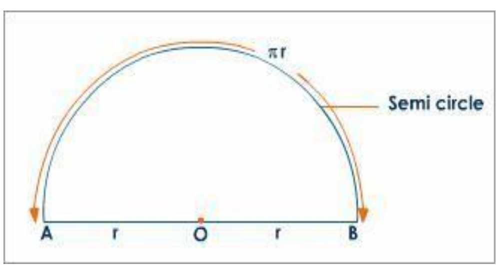
$= 2\pi r$ (since $d = 2r$)

Know about π

π is a Greek letter. It is a ratio of the circumference to diameter of a circle. Many mathematicians have given many values for this ratio. In the chapter Real numbers, we saw that ' π' is an irrational number. Aryabhatta gave the value of π as $\frac{62832}{20000}$ approximately, whereas Ramanujan found the value of π correct to a million places of decimals. However, for practical purposes we use π as $\frac{22}{7}$. But $\frac{22}{7}$ is a rational number. It is only an approximate value.

Semi Circle

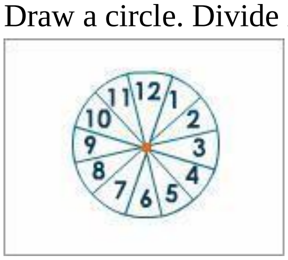
A diameter of a circle divides the circle into two halves called semi circles.

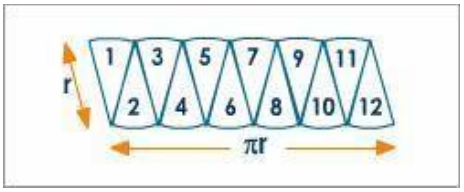


Perimeter of a semicircle is $\pi r + 2r = r(\pi + 2)$ units.

Area of Circle

Draw a circle. Divide it into equal (even) sectors and arrange them in line as shown below.

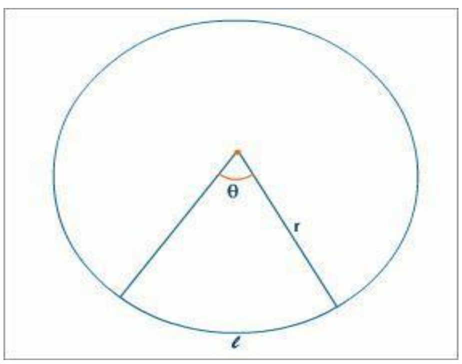




This shape looks like a rectangle of length 'πr' and breath 'r' units
Area of the circle = Area of the rectangle
= length x breadth
= $\pi r \times r$
= πr^2 Sq. units
Area of a circle is πr^2 Sq. units
Area of a semicircle = $\frac{1}{2}\pi r^2$ Sq. units.

Area of a quadrant = $\frac{1}{4}\pi r^2$ Sq. units.

Area of Sector of a Circle



Sector of a circle is a portion of a circular region enclosed between 2 radii and the corresponding arc. θ is the central angle, l is the length of the arc. When the central angle is 360^o, area of the circle is πr^2. When the central angle is θ,
Area of sector = — πr^2

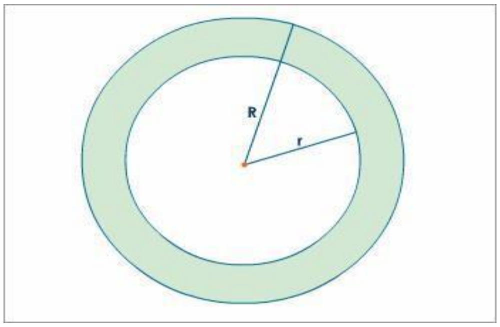
When central angle is 360°,
Length of arc = $2\pi r$
When central angle is θ°,
Length of arc = — $2\pi r$

∴ Length of the arc = — $2\pi r$

Perimeter of a sector = 2r + l

Area of a Ring

Let 'R' and 'r' be the radii of the outer and inner circles respectively.

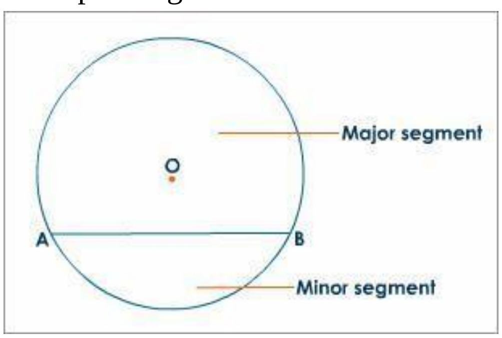


Area of ring = Area of outer circle - Area of inner circle

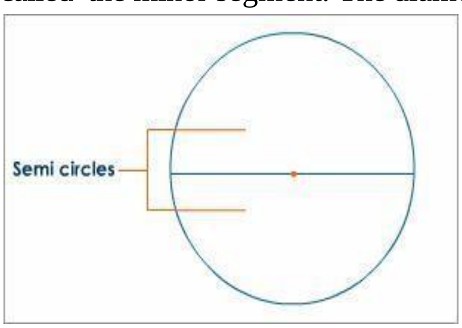
i.e. $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

Segment of a Circle

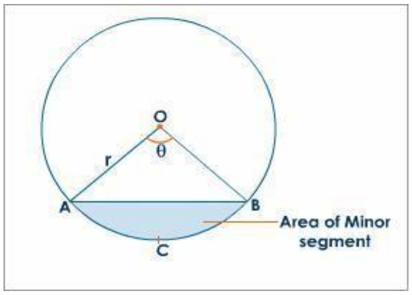
A segment of a circle is a portion of the circular region enclosed between a chord and the corresponding arc. A chord divides a circle into two portions called segments.



The segment which contains the centre of the circle is called the major segment. The other one is called the minor segment. The diameter divides the circle into two equal halves called semicircles.



Area of a Segment

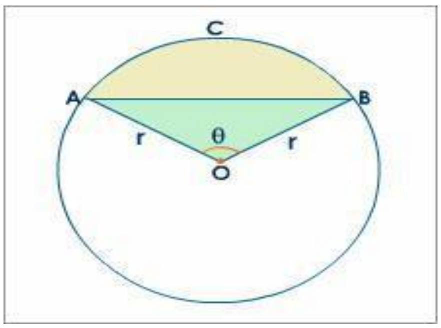


Area of the minor segment = Area of the sector OACB Area of the ΔAOB.

Area of a Major Segment

Area of a major segment = Area of circle - Area of minor segment

Formula for finding the area of a segment of a circle



Draw a circle of radius 'r'. Let the chord AB cut the circle into two segments. We want to find the area of the minor segment (Coloured portion). Join O to A, O to B.

Let $\angle AOB = \theta$. We get a sector; part of it is the segment whose area is to be found.

Area of the sector OACB = area of segment ACB + area of ΔAOB.

Area of segment ACB = Area of sector OACB - Area of ΔAOB.

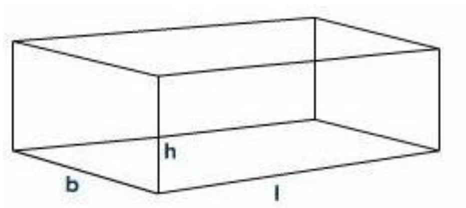
Area of $\Delta AOB = \frac{1}{2} \times r \sin\theta \times r = \frac{1}{2} r^2 \sin\theta$

Area of segment ACB = πr^2 $\frac{1}{2} r^2 \sin\theta$

$[\frac{\pi}{\quad} \quad \frac{}{\quad}]$

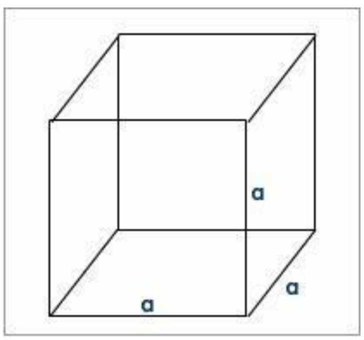
Surface Areas and Volumes

Cuboid



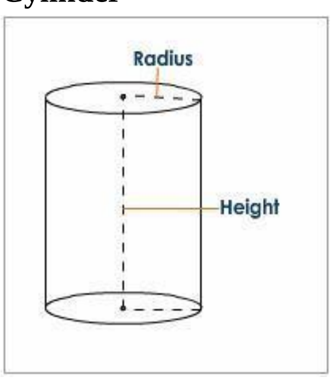
Total surface area = 2 (lb + bh + hl) Sq. units
Lateral surface area = Area of 4 walls = 2h (l + b) Sq. units
Volume = lbh Cu. units

Cube



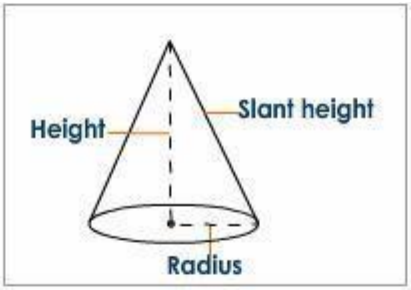
Let each side be 'a' units
Total surface area = $6a^2$Sq. units
Lateral surface area = $4a^2$Sq. units
Volume = a^3Cu. units

Cylinder



C.S.A = Sq. units
T.S.A = ( ) Sq. units
Volume = Cu. units

Cone

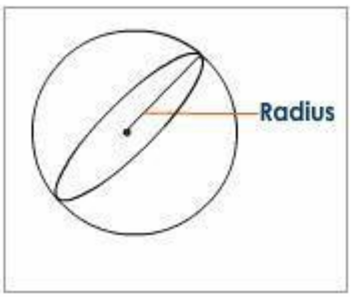


C.S.A = sq. units

T.S.A = ( )sq. units

Volume = – cu. units

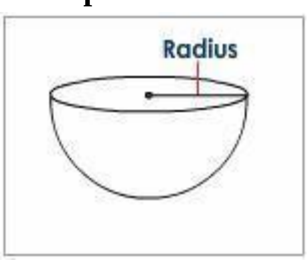
Sphere



Area = Sq. units

Volume = – Cu. units

Hemisphere

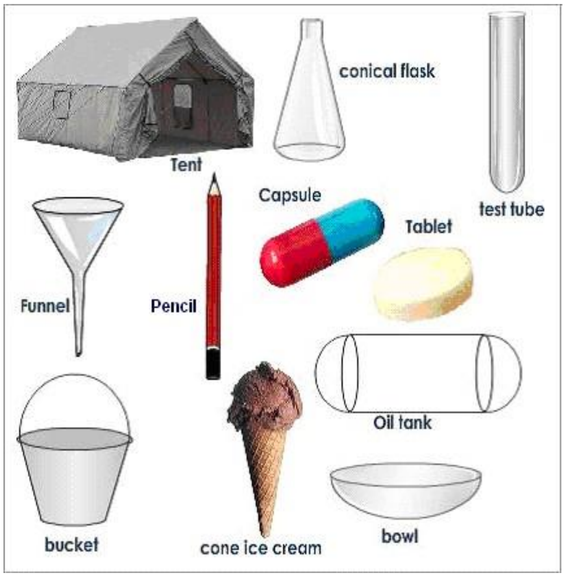


C.S.A = Sq. units

T.S.A = Sq. units

Volume = – Cu. units

Surface Area of a Combination of Solids



Total surface area of such a combined solid is found by adding the curved surface areas of the individual parts.

Volume of a Combination of Solids

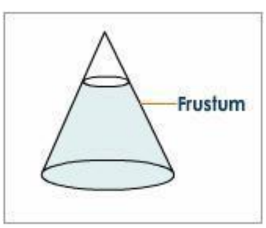
Volume of the new solid formed by the combination of solids is the sum of the volumes of the individual solids.

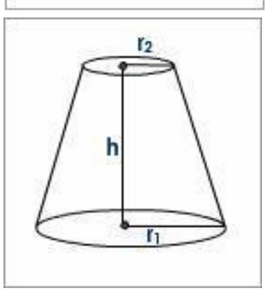
Conversion of solid from one Shape to another

When one solid is converted into other solid, then their volumes are the same.

Frustum of a Cone

Frustum of a cone is a solid obtained from a cone. It is a part of a cone. When we cut the cone by a plane parallel to its base and remove the top portion of the cone, the portion left over is called frustum of a cone.

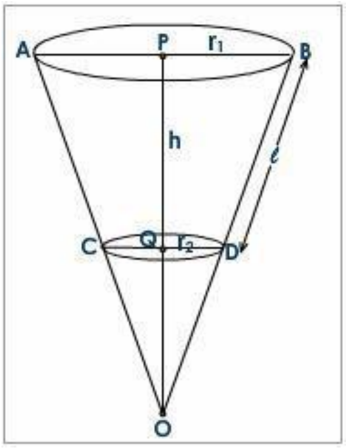




' r_1' is the radius of the bigger circular portion. 'r_2' is the radius of the smaller circular portion. 'h' is the perpendicular distance between two centres. It is height of the frustum.

1. Volume of a frustum = [–]
2. Curved surface area of a frustum
   ( )
   Where √ ( )
3. Total surface area of a frustum = ( )

Derivation of a formula to find the volume and surface area of a frustum of a cone



Let 'h' be the height, 'l' the slant height and r_1 and r_2 be the radii of the bases of the frustum of a cone and $r_1 > r_2$. Now complete the conical portion OCD. Volume of frustum of the right circular cone is the difference in the volumes of the two right circular cones OAB and OCD.

Let the height of the cone OAB be 'h_1' and its slant height be 'l_1' OP = h_1, OA = OB = l_1

The height of the cone OCD = h_1- h

ΔOQD ~ ΔOPB (AA similarity)

…… (1)

Height of the cone OCD =

= …… (2)

Volume of the frustum of cone = volume of the cone OAB Volume of the cone OCD

= – – ( )

=[ ] —— ——

From (1) and (2)

= — [ ]— — {( ) (———)}

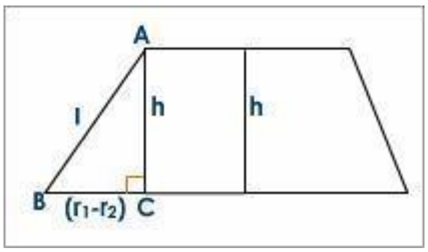
Volume = — [ ]

In the same way we can find the C.S.A and T.S.A of a frustum.

C.S.A = ( ) √ ( )

T.S.A = ( )

Relations among l, h, r_1 and r_2



In a right angled triangle ABC,

$AB^2 = AC^2 + BC^2$

$l^2 = h^2 + (r_1 - r_2)^2$

Statistics

Mean of Grouped Data
Mean is that value of central tendency which is the average of the observations.
There are three methods to find mean for a frequency distribution.

(i) Direct method

$$M = \frac{\sum fx}{\sum f}$$

where x is the mid-interval
f is the frequency
M is the mean

(ii) (ii) Assumed Mean method

$$M = A + \frac{\sum fd}{\sum f}$$

where A = assumed mean
d = x - A

(iii) (iii) Step-deviation method

$$M = A + i\frac{\sum ft}{\sum t}$$

where i = class size

$$t = \frac{d}{i}$$

Mode of Grouped Data
Mode is that value among the observations which has the maximum frequency.
In a grouped frequency distribution, we locate the modal class and find the mode using the following formula.

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

l - lower limit of the modal class
h - size of the class interval
f_1 - frequency of the modal class
f_0 - frequency of the class preceding the modal class
f_2 - frequency of the class succeeding the modal class

Median of Grouped Data
Median is a measure of central tendency which gives the value of the middle-most observation in the data.
In a grouped frequency distribution, we locate the median class and find the median using the following formula.

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c}{f} \right) \times h$$

l - Lower limit of the median class
c - Cumulative frequency preceding the median class frequency
h - Width of the class interval
N = Sum of the frequencies

Working rule

Step 1: Prepare the table containing less than the cumulative frequency with the help of the given frequencies.

Step 2: Find out the cumulative frequency to which $\frac{N}{2}$ belongs. Class interval of this cumulative frequency is the median class interval.

Step 3: Find out the frequency f and lower limit l of this median class.
Step 4: Find the width 'h' of the median class interval.
Step 5: Find the cumulative frequency c of the class preceding the median class.
Step 6: Apply the formula

Median = $l + \left(\frac{\frac{N}{2} - c}{f} \right) \times h$, to find the median.

Graphs in Statistics

Graphical Representation of Cumulative Frequency Distribution

Cumulative frequency is obtained by adding the frequency of a class interval and the frequencies of the preceding intervals up to that class interval.

Ogive (Cumulative Frequency Curve)

There are two ways of constructing an Ogive or cumulative frequency curve. (Ogive is pronounced as O-jive). The curve is usually of 'S' shape.

To Plot an Ogive:

(i) We plot the points with coordinates having abscissae as actual limits and ordinates as the cumulative frequencies
(ii) Join the plotted points by a smooth curve.
(iii) An Ogive is connected to a point on the X-axis representing the actual lower limit of the first class.

Probability

A Theoretical Approach

Here, we try to predict what will happen without actually performing the experiment. We assume that the outcomes of an experiment are equally like ly. We find that the experimental probability of an event approaches its theoretical probability if the number of trials of an experiment is very large.

Random Experiment

When an experiment is repeated several times the result may not be the same. These repetitions are called trials.

The theoretical probability (classical probability) of an event 'E' written as P(E) is defined as, P (E)

Compound Event

An event connected to a random experiment is a compound event if it is obtained by combining two or more elementary events connected to the random experiment.

Occurrence of an event

An event corresponding to a random experiment is said to occur if any one of the elementary events corresponding to the event is the outcome.

Impossible events

The event which never occurs is an impossible event. So the probability of an impossible event is always zero.

Sure event

The event which certainly occurs is a sure event.

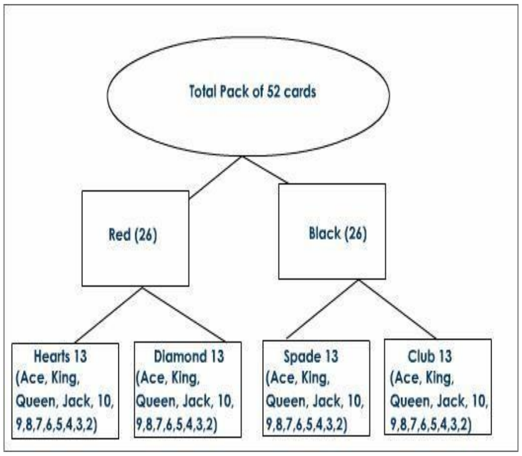
In general, it is true that for an event E,

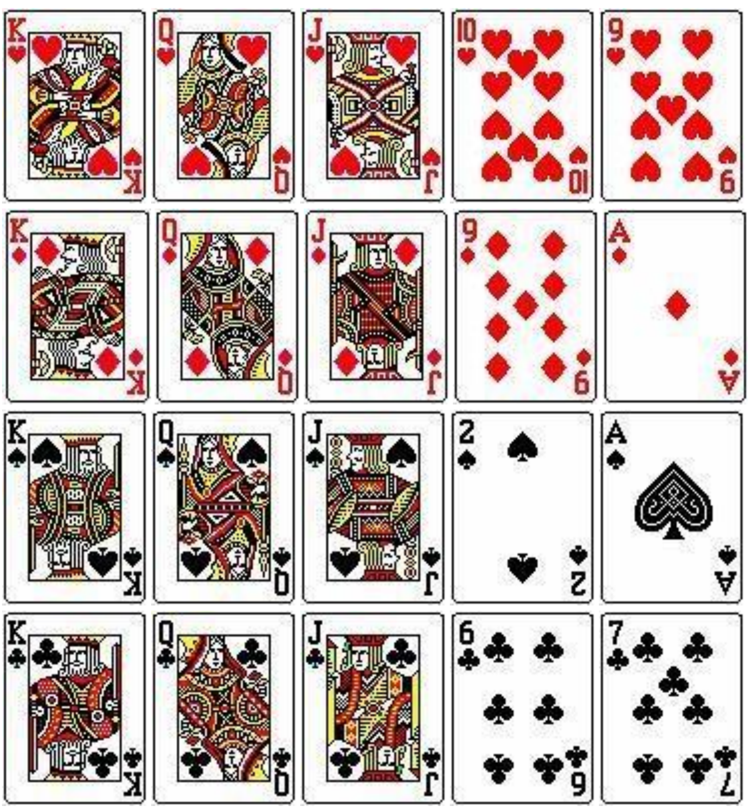
$(\overline{\ }) \; (\;)$

Here the event $\overline{E}$ is representing "not E". This is called the compound of the event 'E'. So 'E' and $\overline{E}$ are complementary events.

Cards: A pack of cards consists of four suits.
They are
Spades ♠,
Hearts ♥,
Diamond ♦and
Clubs ♣

Each suit consists of 13 cards, nine cards numbered 2, 3, 4 ….10, an ace a jack, a queen, and a king. Spades and Clubs are black in colour. Hearts and Diamonds are red in colour. So there are 26 black cards and 26 red cards. King, queen and jack are called face cards. There are totally 12 (4 x 3) face cards in a pack of 52 cards. I.e. in each suit we have 3 face cards.





Coins: A coin has two sides namely head and tail. In the experiment of tossing a coin once, there are 2 possible outcomes - 1 head, 1 tail.

P (Head) = $\frac{1}{2}$ = P (Tail)

Die: A die is a well balanced cube with six faces numbered from 1 to 6. Dice is the plural form. There are six equally likely outcomes 1, 2, 3, 4, 5, 6 in a single throw.