PUNA INTERNATIONAL SCHOOL

- **CLASS – 9**
- **SUBJECT - MATHS**
- **CHAPTER - 2**

SAMPLE NOTE-BOOK

SUBJECT: MATHS **STANDARD – 9TH**

CHAPTER – 02

(i) Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$

(ii) $y^2 + \sqrt{2}$

(iii) $3\sqrt{t} + t\sqrt{2}$

(iv) $y + \frac{2}{y}$

(v) $x^{10} + y^3 + t^{50}$

Ans. (i) $4x^2 - 3x + 7$

We can observe that in the polynomial $4x^2 - 3x + 7$, we have *x* as the only variable and the powers of *x* in each term are a whole number.

Therefore, we conclude that $4x^2 - 3x + 7$ is a polynomial in one variable.

(ii) $y^2 + \sqrt{2}$

We can observe that in the polynomial $y^2 + \sqrt{2}$, we have *y* as the only variable and the powers of *y* in each term are a whole number.

Therefore, we conclude that $y^2 + \sqrt{2}$ is a polynomial in one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$

We can observe that in the polynomial $3\sqrt{t} + t\sqrt{2}$, we have *t* as the only variable and the powers of *t* in each term are not a whole number.

Therefore, we conclude that $3\sqrt{t} + t\sqrt{2}$ is not a polynomial in one variable.

(iv) $y+\frac{2}{y}$

We can observe that in the polynomial $y+\frac{2}{y}$, we have *y* as the only variable and the powers of *y* in each term are not a whole number.

Therefore, we conclude that $y+\frac{2}{y}$ is not a polynomial in one variable.

(v) $x^{10}+y^3+t^{50}$

We can observe that in the polynomial $x^{10}+y^3+t^{50}$, we have *x*, *y* and *t* as the variables and the powers of *x*, *y* and *t* in each term is a whole number.

Therefore, we conclude that $x^{10}+y^3+t^{50}$ is a polynomial but not a polynomial in one variable.

2. Write the coefficients of x^2 in each of the following:

(i) $2+x^2+x$

(ii) $2-x^2+x^3$

(iii) $\frac{\pi}{2}x^2+x$

(iv) $\sqrt{2}x-1$

Ans. (i) $2+x^2+x$

The coefficient of x^2 in the polynomial $2+x^2+x$ is 1.

(ii) $2-x^2+x^3$

The coefficient of x^2 in the polynomial $2-x^2+x^3$ is -1.

(iii) $\frac{\pi}{2}x^2+x$

The coefficient of x^2 in the polynomial $\frac{\pi}{2}x^2+x$ is $\frac{\pi}{2}$.

(iv) $\sqrt{2}x-1$

The coefficient of x^2 in the polynomial $\sqrt{2}x-1$ is 0.

(i) Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Ans. The binomial of degree 35 can be $x^{35}+9$.

The binomial of degree 100 can be t^{100}.

(ii) Write the degree of each of the following polynomials:

(i) $p(x)=5x^3+4x^2+7x$

(ii) $p(y)=4-y^2$

(iii) $f(t)=5t-\sqrt{7}$

(iv) **3**

Ans. (i) $5x^3+4x^2+7x$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We can observe that in the polynomial $5x^3+4x^2+7x$, the highest power of the variable *x* is 3.

Therefore, we conclude that the degree of the polynomial $5x^3+4x^2+7x$ is 3.

(ii) $4-y^2$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We can observe that in the polynomial $4-y^2$, the highest power of the variable *y* is 2.

Therefore, we conclude that the degree of the polynomial $4-y^2$ is 2.

(iii) $5t-\sqrt{7}$

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We observe that in the polynomial $5t-\sqrt{7}$, the highest power of the variable *t* is 1.

Therefore, we conclude that the degree of the polynomial $5t-\sqrt{7}$ is 1.

(iv)3

We know that the degree of a polynomial is the highest power of the variable in the polynomial.

We can observe that in the polynomial 3, the highest power of the assumed variable *x* is 0.

Therefore, we conclude that the degree of the polynomial 3 is 0.

5. Classify the following as linear, quadratic and cubic polynomials:

(i) x^2+x
(ii) $x-x^3$
(iii) $y+y^2+4$
(iv) $1+x$
(i) 3*t*
(ii) r^2
(iii) $7x^3$

Ans. (i) x^2+x

We can observe that the degree of the polynomial x^2+x is 2.

Therefore, we can conclude that the polynomial x^2+x is a quadratic polynomial.

(ii) $x-x^3$

We can observe that the degree of the polynomial $x-x^3$ is 3.

Therefore, we can conclude that the polynomial $x-x^3$ is a cubic polynomial.

(iii) $y+y^2+4$

We can observe that the degree of the polynomial $y+y^2+4$ is 2.

Therefore, the polynomial $y+y^2+4$ is a quadratic polynomial.

(iv) $1+x$

We can observe that the degree of the polynomial $(1+x)$ is 1.

Therefore, we can conclude that the polynomial $1+x$ is a linear polynomial.

(v) $3t$

We can observe that the degree of the polynomial $(3t)$ is 1.

Therefore, we can conclude that the polynomial $3t$ is a linear polynomial.

(vi) r^2

We can observe that the degree of the polynomial r^2 is 2.

Therefore, we can conclude that the polynomial r^2 is a quadratic polynomial.

(vii) $7x^3$

We can observe that the degree of the polynomial $7x^3$ is 3.

Therefore, we can conclude that the polynomial $7x^3$ is a cubic polynomial.

CHAPTER 2

Polynomials (Ex. 2.2)

1. **Find the value of the polynomial** $5x-4x^2+3$ **at**

(i) $x=0$ **(ii)**

$x=-1$ **(iii)**

$x=2$

Ans. (i) Let $f(x)=5x-4x^2+3$.

We need to substitute 0 in the polynomial $f(x)=5x-4x^2+3$ to get $f(0)=5(0)-4(0)^2+3$

0-0+3
=3

Therefore, we conclude that at $x=0$, the value of the polynomial $5x-4x^2+3$ is 3.

(ii) Let $f(x)=5x-4x^2+3$.

We need to substitute -1 in the polynomial $f(x)=5x-4x^2+3$ to get. $f(-1)=5(-1)-4(-1)^2+3$

=-5-4+3

=-6

Therefore, we conclude that at $x=-1$, the value of the polynomial $5x-4x^2+3$ is -6

(iii) Let $f(x)=5x-4x^2+3$.

We need to substitute 2 in the polynomial $f(x)=5x-4x^2+3$ to get $f(2)=5(2)-4(2)^2+3$

=10-16+3

=-3

Therefore, we conclude that at $x=2$, the value of the polynomial $5x-4x^2+3$ is -3.

(iv) Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials:

(i) $p(y)=y^2-y+1$

(ii) $p(t)=2+t+2t^2-t^3$

(iii) $p(x)=x^3$

(iv) $p(x)=(x-1)(x+1)$

Ans. (i) $p(y)=y^2-y+1$

$p(0)=(0)^2-0+1=1$

$p(1)=(1)^2-1+1=1-0=1$

$p(2)=(2)^2-2+1=4-1=3$

(ii) $p(t)=2+t+2t^2-t^3$

$p(0)=2+(0)+2(0)^2-(0)^3$ =2

$p(1)=2+(1)+2(1)^2-(1)^3=2+1+2-1=4$

$p(2)=2+(2)+2(2)^2-(2)^3=4+8-8=4$

(iii) $p(x) = (x)^3$

$p(0) = (0)^3 = 0$

$p(1) = (1)^3 = 1$

$p(2) = (2)^3 = 8$

(vi) $p(x) = (x-1)(x+1)$

$p(0) = (0-1)(0+1) = (-1)(1) = -1$

$p(1) = (1-1)(2+1) = (0)(3) = 0$

$p(2) = (2-1)(2+1) = (1)(3) = 3$

(i) Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x+1, \ x = -\frac{1}{3}$

(ii) $p(x) = 5x-\pi, x = \frac{4}{5}$

(iii) $p(x) = x^2 - 1, \ x = -1, 1$

(iv) $p(x) = (x+1)(x-2), \ x = -1, 2$

(v) $p(x) = x^2, \ x = 0$

(vi) $p(x) = lx + m, \ x = -\frac{m}{l}$

(vii) $p(x) = 3x^2 - 1, \ x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

, $p(x) = 2x+1 x = -\frac{1}{2}$

Ans. (i) $p(x) = 3x+1,\ x = -\frac{1}{3}$

We need to check whether $p(x) = 3x+1$ at $x = -\frac{1}{3}$ is equal to zero or not.

$$p\left(-\frac{1}{3}\right) = 3x+1 = 3\left(-\frac{1}{3}\right)+1 = -1+1 = 0$$

Therefore, we can conclude that $x = -\frac{1}{3}$ is a zero of the polynomial $p(x) = 3x+1$.

(ii) $p(x) = 5x - \pi,\ x = \frac{4}{5}$

We need to check whether $p(x) = 5x - \pi$ at $x = \frac{4}{5}$ is equal to zero or not.

$$p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi = 4 - \pi\, p(x) = 2x+1, x = \frac{1}{2}$$

Therefore, $x = \frac{4}{5}$ is not a zero of the polynomial $p(x) = 5x - \pi$.

(iii) $p(x) = x^2 - 1,\ x = -1, 1$

We need to check whether At $p(x) = x^2 - 1$ at $x = -1, 1$ is equal to zero or not.

$x = -1$

$$p(-1) = (-1)^2 - 1 = 1 - 1 = 0$$

At $x = 1$

$$p(1) = (1)^2 - 1 = 1 - 1 = 0$$

Therefore, $x=-1,1$ are the zeros of the polynomial $p(x)=x^2-1$.

(iv) $p(x)=(x+1)(x-2),\ x=-1,2$

We need to check whether $p(x)=(x+1)(x-2)$ at $x=-1,2$ is equal to zero or not.

At $x=-1$

$$p(-1)=(-1+1)(-1-2)=(0)(-3)=0$$

At $x=2$

$$p(2)=(2+1)(2-2)=(3)(0)=0$$

Therefore, $x=-1,2$ are the zeros of the polynomial $p(x)=(x+1)(x-2)$.

(v) $p(x)=x^2,\ \ x=0$

We need to check whether $p(x)=x^2$ at $x=0$ is equal to zero or not.

$$p(0)=(0)^2=0$$

Therefore, we can conclude that $x=0$ is a zero of the polynomial $p(x)=x^2$.

(vi) $p(x)=lx+m,\ x=-\frac{m}{l}$

We need to check whether $p(x)=lx+m$ at $x=-\frac{m}{l}$ is equal to zero or not.

$$p\left(-\frac{m}{l}\right)=l\left(-\frac{m}{l}\right)+m=m+m=0$$

Therefore, $x=-\frac{m}{l}$ is a zero of the polynomial $p(x)=lx+m$.

(vii) $p(x) = 3x^2 - 1,\quad x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

We need to check whether $p(x) = 3x^2 - 1$ at $x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$ is equal to zero or not.

At $x = \frac{-1}{\sqrt{3}}$

$$p\left(-\frac{1}{\sqrt{3}}\right) = 3\left(-\frac{1}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{1}{3}\right) - 1 = 1 - 1 = 0$$

At $x = \frac{2}{\sqrt{3}}$

$$p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1 = 3\left(\frac{4}{3}\right) - 1 = 4 - 1 = 3$$

Therefore, we can conclude that $x = \frac{-1}{\sqrt{3}}$ is a zero of the polynomial $p(x) = 3x^2 - 1$ but $x = \frac{-1}{\sqrt{3}}$ is not a zero of the polynomial $p(x) = 3x^2 - 1$.

(viii) $p(x) = 2x + 1,\ x = \frac{1}{2}$

We need to check $p(x) = 2x + 1,\ x = \frac{1}{2}$, whether is equal to zero or not.

Therefore $p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 2$, As $p\left(\frac{1}{2}\right) \neq 0$ is not the , $\frac{1}{2}$

zero of P(x)

(iv) Find the zero of the polynomial in each of the following cases:

(i) $p(x) = x + 5$

(ii) $p(x) = x - 5$

(iii) $p(x) = 2x + 5$

(iv) $p(x)=3x-2$

(v) $p(x)=3x$

(vi) $p(x)=ax,\ a\neq 0$

(vii) $p(x)=cx+d, c\neq 0,\ c,d$ **are real numbers.**

Ans. (i) $p(x)=x+5$

$ax+b$, where $a\neq 0$ and $b\neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=x+5$ equal to 0, we get

$x+5=0 \quad \Rightarrow x=-5$

Therefore, we conclude that the zero of the polynomial $p(x)=x+5$ is -5.

(ii) $p(x)=x-5$

$ax+b$, where $a\neq 0$ and $b\neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=x-5$ equal to 0, we get

$x-5=0 \quad \Rightarrow x=5$

Therefore, we conclude that the zero of the polynomial $p(x)=x-5$ is 5.

(iii) $p(x)=2x+5$

$ax+b$, where $a\neq 0$ and $b\neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=2x+5$ equal to 0, we get

$$2x+5=0 \qquad \Rightarrow x=\frac{-5}{2}$$

Therefore, we conclude that the zero of the polynomial $p(x)=2x+5$ is $\frac{-5}{2}$.

(iv) $p(x)=3x-2$

$ax+b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=3x-2$ equal to 0, we get

$$3x-2=0 \quad \Rightarrow x=\frac{2}{3}$$

Therefore, we conclude that the zero of the polynomial $p(x)=3x-2$ is $\frac{2}{3}$.

(v) $p(x)=3x$

$ax+b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=3x$ equal to 0, we get

$$3x=0 \qquad \Rightarrow x=0$$

Therefore, we conclude that the zero of the polynomial $p(x)=3x$ is 0.

(vi) $p(x)=ax, a \neq 0$

$ax+b$, where $a \neq 0$ and $b \neq 0$, and a and b are real numbers, we need to find $p(x)=0$.

On putting $p(x)=ax$ equal to 0, we get

$ax = 0 \quad \Rightarrow x = 0$

Therefore, we conclude that the zero of the polynomial $p(x) = ax, a \neq 0$ is0.

(vii) $p(x) = cx + d, c \neq 0, c, d$ are real numbers.

$ax + b,$ where $a \neq 0$ and $b \neq 0,$ and a and b are real numbers, we need to find $p(x) = 0$.

On putting $p(x) = cx + d$ equal to 0, we get

$cx + d = 0$

$\Rightarrow \quad x = -\frac{d}{c}.$

Therefore, we conclude that the zero of the polynomial $p(x) = cx + d, c \neq 0, c, d$ are real numbers. is $-\frac{d}{c}$.

CHAPTER 2

Polynomials (Ex. 2.3)

1. **Find the remainder when x^3+3x^2+3x+1 is divided by**

(i) $x+1$

(ii) $x-\frac{1}{2}$

(iii) x

(iv) $x+\pi$

(v) $5+2x$

Ans. (i) $x+1$

We need to find the zero of the polynomial $x+1$.

$x+1=0 \qquad \Rightarrow x=-1$

While applying the remainder theorem, we need to put the zero of the polynomial $x+1$ in the polynomial x^3+3x^2+3x+1, to get

$p(x)=x^3+3x^2+3x+1$

$p(-1)=(-1)^3+3(-1)^2+3(-1)+1$

=-1+3-3+1

=0

Therefore, we conclude that on dividing the polynomial x^3+3x^2+3x+1 by $x+1$, we will get the remainder as 0.

(ii) $x-\frac{1}{2}$

We need to find the zero of the polynomial $x-\frac{1}{2}$.

$x-\frac{1}{2}=0 \qquad \Rightarrow x=\frac{1}{2}$

While applying the remainder theorem, we need to put the zero of the polynomial $x-\frac{1}{2}$ in the polynomial x^3+3x^2+3x+1, to get

$p(x)=x^3+3x^2+3x+1$

$p\left(\frac{1}{2}\right)=\left(\frac{1}{2}\right)^3+3\left(\frac{1}{2}\right)^2+3\left(\frac{1}{2}\right)+1$

$=\frac{1}{8}+3\left(\frac{1}{4}\right)+\frac{3}{2}+1$

$=\frac{1+6+12+8}{8}$

$=\frac{27}{8}$

Therefore, we conclude that on dividing the polynomial x^3+3x^2+3x+1 by $x-\frac{1}{2}$, we will get the remainder as $\frac{27}{8}$.

(iii) x

We need to find the zero of the polynomial x.

$x=0$

While applying the remainder theorem, we need to put the zero of the polynomial x in the polynomial x^3+3x^2+3x+1, to get

$$p(x)=x^3+3x^2+3x+1$$

$$p(0)=(0)^3+3(0)^2+3(0)+1$$

=0+0+0+1

=1

Therefore, we conclude that on dividing the polynomial x^3+3x^2+3x+1 by x, we will get the remainder as 1.

(iv) $x+\pi$

We need to find the zero of the polynomial $x+\pi$.

$$x+\pi=0 \qquad \Rightarrow x=-\pi$$

While applying the remainder theorem, we need to put the zero of the polynomial $x+\pi$ in the polynomial x^3+3x^2+3x+1, to get

$$p(x)=x^3+3x^2+3x+1$$

$$p(-\pi)=(-\pi)^3+3(-\pi)^2+3(-\pi)+1$$

$$=-\pi^3+3\pi^2-3\pi+1.$$

Therefore, we conclude that on dividing the polynomial x^3+3x^2+3x+1 by $x+\pi$, we will get the remainder as $-\pi^3+3\pi^2-3\pi+1$.

(v) $5+2x$

We need to find the zero of the polynomial $5+2x$.

$$5+2x=0 \qquad \Rightarrow x=-\frac{5}{2}$$

While applying the remainder theorem, we need to put the zero of the polynomial $5+2x$ in the polynomial x^3+3x^2+3x+1, to get

$$p(x)=x^3+3x^2+3x+1$$

$$p\left(-\frac{5}{2}\right)=\left(-\frac{5}{2}\right)^3+3\left(-\frac{5}{2}\right)^2+3\left(-\frac{5}{2}\right)+1$$

$$=-\frac{125}{8}+3\left(\frac{25}{4}\right)-\frac{15}{2}+1$$

$$=-\frac{125}{8}+\frac{75}{4}-\frac{15}{2}+1$$

$$=\frac{-125+150-60+8}{8}$$

$$=-\frac{27}{4}.$$

Therefore, we conclude that on dividing the polynomial x^3+3x^2+3x+1 by $5+2x$, we will get the remainder as $-\frac{27}{4}$.

(v) Find the remainder when x^3-ax^2+6x-a is divided by $x-a$.

Ans. We need to find the zero of the polynomial $x-a$.

$$x-a=0 \qquad \Rightarrow x=a$$

While applying the remainder theorem, we need to put the zero of the polynomial $x-a$ in the polynomial x^3-ax^2+6x-a, to get

$$p(x)=x^3-ax^2+6x-a$$

$$p(a)=(a)^3-a(a)^2+6(a)-a$$

$$= a^3 - a^3 + 6a - a$$

= 5a

Therefore, we conclude that on dividing the polynomial $x^3 - ax^2 + 6x - a$ by $x - a$, we will get the remainder as $5a$.

3. Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.

Ans. We know that if the polynomial $7 + 3x$ is a factor of $3x^3 + 7x$, then on dividing the polynomial $3x^3 + 7x$ by $7 + 3x$, we must get the remainder as 0.

We need to find the zero of the polynomial $7 + 3x$.

$$7 + 3x = 0 \qquad \Rightarrow x = -\frac{7}{3}$$

While applying the remainder theorem, we need to put the zero of the polynomial $7 + 3x$ inthe polynomial $3x^3 + 7x$, to get

$$p(x) = 3x^3 + 7x$$

$$= 3\left(-\frac{7}{3}\right)^3 + 7\left(-\frac{7}{3}\right) = 3\left(-\frac{343}{27}\right) - \frac{49}{3}$$

$$= -\frac{343}{9} - \frac{49}{3} = \frac{-343 - 147}{9}$$

$$= \frac{-490}{9}.$$

We conclude that on dividing the polynomial $3x^3 + 7x$ by $7 + 3x$, we will get the remainder as $\frac{-490}{9}$, which is not 0.

Therefore, we conclude that $7 + 3x$ is not a factor of $3x^3 + 7x$.

CHAPTER 2

Polynomials (Ex. 2.4)

1. Determine which of the following polynomials has $(x+1)$ a factor:

(i) x^3+x^2+x+1
(ii)

(iii) $x^4+3x^3+3x^2+x+1$
(iv) $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$

Ans. (i) x^3+x^2+x+1

While applying the factor theorem, we get

$p(x)=x^3+x^2+x+1$

$p(-1)=(-1)^3+(-1)^2+(-1)+1$

=-1+1-1+1 =0

We conclude that on dividing the polynomial x^3+x^2+x+1 by $(x+1)$, we get the remainder as 0.

Therefore, we conclude that $(x+1)$ is a factor of x^3+x^2+x+1.

(ii) $x^4+x^3+x^2+x+1$

While applying the factor theorem, we get

$p(x)=x^4+x^3+x^2+x+1$

$p(-1)=(-1)^4+(-1)^3+(-1)^2+(-1)+1$

=1-1+1-1+1 =1

We conclude that on dividing the polynomial $x^4+x^3+x^2+x+1$ by $(x+1)$, we will get the remainder as1, which is not 0.

Therefore, we conclude that $(x+1)$ is not a factor of $x^4+x^3+x^2+x+1$.

(iii) $x^4+3x^3+3x^2+x+1$

While applying the factor theorem, we get

$$p(x)=x^4+3x^3+3x^2+x+1$$
$$p(-1)=(-1)^4+3(-1)^3+3(-1)^2+(-1)+1$$
=1-3+3-1+1 =1

We conclude that on dividing the polynomial $x^4+3x^3+3x^2+x+1$ by $(x+1)$, we will get the remainder as 1, which is not 0.

Therefore, we conclude that $(x+1)$ is not a factor of $x^4+3x^3+3x^2+x+1$.

(iv) $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$

While applying the factor theorem, we get

$$p(x)=x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$$
$$p(-1)=(-1)^3-(-1)^2-\left(2+\sqrt{2}\right)(-1)+\sqrt{2}$$
$$=-1-1+2+\sqrt{2}+\sqrt{2}$$
$$=2\sqrt{2}.$$

We conclude that on dividing the polynomial $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$ by $(x+1)$, we will get the remainder as $2\sqrt{2}$, which is not 0.

Therefore, we conclude that $(x+1)$ is not a factor of $x^3-x^2-\left(2+\sqrt{2}\right)x+\sqrt{2}$.

(ii) Use the Factor Theorem to determine whether *g*(*x*) is a factor of *p*(*x*) in each of the following cases:

(i) $p(x)=2x^3+x^2-2x-1,\ g(x)=x+1$

(ii) $p(x)=x^3+3x^2+3x+1,\ g(x)=x+2$

(iii) $p(x)=x^3-4x^2+x+6,\ g(x)=x-3$

Ans. (i) $p(x)=2x^3+x^2-2x-1,\ g(x)=x+1$

We know that according to the factor theorem, $(x-a)$ is a factor of $p(x)$, if $p(a)=0$. We can conclude that $g(x)$ is a factor of $p(x)$, if $p(-1)=0$.

$$p(-1)=2(-1)^3+(-1)^2-2(-1)-1$$

= 2+1-1-2 = 0

Therefore, we conclude that the $g(x)$ is a factor of $p(x)$.

(ii) $p(x)=x^3+3x^2+3x+1, g(x)=x+2$

We know that according to the factor theorem, $(x-a)$ is a factor of $p(x)$, if $p(a)=0$. We can conclude that $g(x)$ is a factor of $p(x)$, if $p(-2)=0$.

$$p(-2)=(-2)^3+3(-2)^2+3(-2)+1$$

= -8+12-6+1 = -1

Therefore, we conclude that the $g(x)$ is not a factor of $p(x)$.

(iii) $p(x)=x^3-4x^2+x+6, g(x)=x-3$

We know that according to the factor theorem, $(x-a)$ is a factor of $p(x)$, if $p(a)=0$. We can conclude that $g(x)$ is a factor of $p(x)$, if $p(3)=0$.

$$p(3)=(3)^3-4(3)^2+(3)+6$$

= 27-36+3+6 = 0

Therefore, we conclude that the $g(x)$ is a factor of $p(x)$.

(vi) Find the value of k, if $x-1$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x)=x^2+x+k$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

(iii) $p(x)=kx^2-\sqrt{2}x+1$

(iv) $p(x)=kx^2-3x+k$

Ans. (i) $p(x)=x^2+x+k$

We know that according to the factor theorem

$p(a)=0$, if $x-a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of $p(x)=x^2+x+k$, then $p(1)=0$.

$p(1)=(1)^2+(1)+k=0,$

or

$k+2=0$

$k=-2$

Therefore, we can conclude that the value of *k* is -2.

(ii) $p(x)=2x^2+kx+\sqrt{2}$

We know that according to the factor theorem

$p(a)=0$, if $x-a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of $p(x)=2x^2+kx+\sqrt{2}$, then $p(1)=0$.

$p(1)=2(1)^2+k(1)+\sqrt{2}=0,$

or

$2+k+\sqrt{2}=0$

$k=-\left(2+\sqrt{2}\right).$

Therefore, we can conclude that the value of *k* is $-\left(2+\sqrt{2}\right)$.

(iii) $p(x)=kx^2-\sqrt{2}x+1$

We know that according to the factor theorem

$p(a)=0$, if $x-a$ is a factor of $p(x)$.

We conclude that if $(x-1)$ is a factor of $p(x)=kx^2-\sqrt{2}x+1$, then $p(1)=0$.

$p(1) = k(1)^2 - \sqrt{2}(1) + 1 = 0,$

or

$k - \sqrt{2} + 1 = 0$

$k = \sqrt{2} - 1.$

Therefore, we can conclude that the value of *k* is $\sqrt{2} - 1$.

(iv) $p(x) = kx^2 - 3x + k$

We know that according to the factor theorem is a

$p(a) = 0,$ if $x - a$ factor of p(x)

We conclude that if $(x-1)$ is a factor of $p(x) = kx^2 - 3x + k$, then $p(1) = 0$.

$p(1) = k(1)^2 - 3(1) + k$

or $2k - 3 = 0 \quad \Rightarrow k = \frac{3}{2}$

Therefore, we can conclude that the value of *k* is $\frac{3}{2}$.

4. Factorize:

(i) $12x^2 - 7x + 1$

(ii)

(iii)

(iv)

Ans. (i) $12x^2 - 7x + 1$

$12x^2 - 7x + 1 = 12x^2 - 3x - 4x + 1$

$= 3x(4x - 1) - 1(4x - 1)$

$= (3x - 1)(4x - 1).$

Therefore, we conclude that on factorizing the polynomial $12x^2 - 7x + 1$, we get $(3x - 1)(4x - 1)$.

(ii) $2x^2 + 7x + 3$

$2x^2 + 7x + 3 = 2x^2 + 6x + x + 3$

$= 2x(x+3)+1(x+3)$
$= (2x+1)(x+3)$.

Therefore, we conclude that on factorizing the polynomial $2x^2+7x+3$, we get $(2x+1)(x+3)$.

(iii) $6x^2+5x-6$

$6x^2+5x-6 = 6x^2+9x-4x-6$
$= 3x(2x+3)-2(2x+3)$
$= (3x-2)(2x+3)$.

Therefore, we conclude that on factorizing the polynomial $6x^2+5x-6$, we get $(3x-2)(2x+3)$.

(iv) $3x^2-x-4$

$3x^2-x-4 = 3x^2+3x-4x-4$
$= 3x(x+1)-4(x+1)$
$= (3x-4)(x+1)$.

Therefore, we conclude that on factorizing the polynomial $3x^2-x-4$, we get $(3x-4)(x+1)$.

5. Factorize:

(i) x^3-2x^2-x+2
(ii) $x^3 - 3x^2 - 9x - 5$
(iii) $x^3+13x^2+32x+20$
(iv) $2y^3+y^2-2y-1$

Ans. (i) x^3-2x^2-x+2

We need to consider the factors of 2, which are $\pm 1, \pm 2$.

Let us substitute x=1 in the polynomial x^3-2x^2-x+2, to get

$(1)^3-2(1)^2-(1)+2$ =1-1-2+2=0,so we can say that P(1)=0

Thus, according to factor theorem, we can conclude that $(x-1)$ is a factor of the polynomial

x^3-2x^2-x+2.

Let us divide the polynomial x^3-2x^2-x+2 by $(x-1)$, to get

$$\begin{array}{r} x^2-x-2 \\ x-1\overline{)x^3-2x^2-x+2} \\ x^3-x^2 \\ \underline{-\quad+} \\ -x^2-x \\ -x^2+x \\ \underline{+\quad-} \\ -2x+2 \\ -2x+2 \\ \underline{+\quad-} \\ 0 \end{array}$$

$x^3-2x^2-x+2=(x-1)(x^2-x-2)$.

$x^3-2x^2-x+2=(x-1)(x^2-x-2)$.

$=(x-1)(x^2+x-2x-2)$

$=(x-1)[x(x+1)-2(x+1)]$

$=(x-1)(x-2)(x+1)$.

Therefore, we can conclude that on factorizing the polynomial x^3-2x^2-x+2, we get $(x-1)(x-2)(x+1)$.

(ii) x^3-3x^2-9x-5

We need to consider the factors of -5, which are $\pm1, \pm5$.

Let us substitute x= -1 in the polynomial x^3-3x^2-9x-5 to get ,

$(-1)^3-3(-1)^2-9(-1)-5=-1-3+9-5=0$, so p(-1)=0

Thus, according to factor theorem, we can conclude that $(x+1)$ is a factor of the polynomial x^3-3x^2-9x-5.

Let us divide the polynomial x^3-3x^2-9x-5 by $(x+1)$, to get

$$x^2-4x-5$$

$$x+1\overline{\big)x^3-3x^2-9x-5}$$

$$\underline{\underset{-}{x^3}+\underset{-}{x^2}}$$

$$-4x^2-9x$$

$$\underline{\underset{+}{-4x^2}\ \underset{+}{-4x}}$$

$$-5x-5$$

$$\underline{\underset{+}{-5x}\underset{+}{-5}}$$

$$\underline{\ \ 0\ \ }$$

$$x^3-3x^2-9x-5=(x+1)\left(x^2-4x-5\right)$$

$$=(x+1)\left(x^2+x-5x-5\right)$$

$$=(x+1)\left[x(x+1)-5(x+1)\right]$$

$$=(x+1)(x-5)(x+1).$$

Therefore, we can conclude that on factorizing the polynomial x^3-3x^2-9x-5, we get $(x+1)(x-5)(x+1)$.

(iii) $x^3+13x^2+32x+20$

We need to consider the factors of 20, which are $\pm5, \pm4, \pm2, \pm1$.

Let us substitute x= -1 in the polynomial $x^3+13x^2+32x+20$, to get

$(-1)^3+13(-1)^2+32(-1)+20$ =-1+13-32+20=-20+20=0

Thus, according to factor theorem, we can conclude that $(x+1)$ is a factor of the polynomial $x^3+13x^2+32x+20$.

Let us divide the polynomial $x^3+13x^2+32x+20$ by $(x+1)$, to get

$$
\begin{array}{r|l}
 & x^2+12x+20 \\
x+1 & x^3+13x^2+32x+20 \\
 & \underline{x^3+x^2} \\
 & 12x^2+32x \\
 & \underline{12x^2+12x} \\
 & 20x+20 \\
 & \underline{20x+20} \\
 & \underline{\quad 0 \quad}
\end{array}
$$

$$x^3+13x^2+32x+20=(x+1)(x^2+12x+20)$$
$$=(x+1)(x^2+2x+10x+20)$$
$$=(x+1)[x(x+2)+10(x+2)]$$
$$=(x+1)(x+10)(x+2).$$

Therefore, we can conclude that on factorizing the polynomial $x^3+13x^2+32x+20$, we get $(x+1)(x-10)(x+2)$.

(iv) $2y^3+y^2-2y-1$

We need to consider the factors of -1, which are ±1.

Let us substitute y=1 in the polynomial $2y^3+y^2-2y-1$, to get

$2(1)^3+(1)^2-2(1)-1$ =2+1-2-1=3-3=0,so y(1)=0

Thus, according to factor theorem, we can conclude that $(y-1)$ is a factor of the polynomial $2y^3+y^2-2y-1$.

Let us divide the polynomial $2y^3+y^2-2y-1$ by $(y-1)$, to get

$$
\begin{array}{r|l}
 & 2y^2 + 3y + 1 \\
\hline
y-1 & 2y^3 + y^2 - 2y - 1 \\
 & 2y^3 - 2y^2 \\
 & \underline{-\quad +} \\
 & 3y^2 - 2y \\
 & 3y^2 - 3y \\
 & \underline{-\quad +} \\
 & y - 1 \\
 & y - 1 \\
 & \underline{-\quad +} \\
 & 0
\end{array}
$$

$$2y^3 + y^2 - 2y - 1 = (y-1)(2y^2 + 3y + 1)$$
$$= (y-1)(2y^2 + 2y + y + 1)$$
$$= (y-1)[2y(y+1) + 1(y+1)]$$
$$= (y-1)(2y+1)(y+1).$$

Therefore, we can conclude that on factorizing the polynomial $2y^3 + y^2 - 2y - 1$, we get $(y-1)(2y+1)(y+1)$.

CHAPTER 2

Polynomials (Ex. 2.5)

1. Use suitable identities to find the following products:

(i) $(x+4)(x+10)$

(ii) $(x+8)(x-10)$

(iii) $(3x+4)(3x-5)$

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

(v) $(3-2x)(3+2x)$

Ans. (i) $(x+4)(x+10)$

We know that $(x+a)(x+b)=x^2+(a+b)x+ab$.

We need to apply the above identity to find the product

$(x+4)(x+10)=x^2+(4+10)x+(4\times10)$

$=x^2+14x+40$

Therefore, we conclude that the product $(x+4)(x+10)$ is $x^2+14x+40$.

(ii) $(x+8)(x-10)$

We know that $(x+a)(x+b)=x^2+(a+b)x+ab$.

We need to apply the above identity to find the product

$(x+8)(x-10)=x^2+\left[8+(-10)\right]x+\left[8\times(-10)\right]$

$=x^2-2x-80.$

Therefore, we conclude that the product $(x+8)(x-10)$ is $x^2-2x-80$.

(iii) $(3x+4)(3x-5)$

We know that $(x+a)(x+b)=x^2+(a+b)x+ab$.

We need to apply the above identity to find the product $(3x+4)(3x-5)$

$(3x+4)(3x-5)=(3x)^2+[4+(-5)]3x+[4\times(-5)]$

$=9x^2-3x-20.$

Therefore, we conclude that the product $(3x+4)(3x-5)$ is $9x^2-3x-20$.

(iv) $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

We know that $(x+y)(x-y)=x^2-y^2$.

We need to apply the above identity to find the product $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

$\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$

$=(y^2)^2-\left(\frac{3}{2}\right)^2=y^4-\frac{9}{4}.$

Therefore, we conclude that the product $\left(y^2+\frac{3}{2}\right)\left(y^2-\frac{3}{2}\right)$ is $\left(y^4-\frac{9}{4}\right)$.

(v) $(3+2x)(3-2x)$

We know that $(x+y)(x-y)=x^2-y^2$.

We need to apply the above identity to find the product $(3+2x)(3-2x)$

$(3+2x)(3-2x)=(3)^2-(2x)^2$

$=9-4x^2.$

Therefore, we conclude that the product $(3+2x)(3-2x)$ is $(9-4x^2)$.

2. Evaluate the following products without multiplying directly:

(i) 103×107

(ii) 98 x 96

(iii) 104 x 96

Ans. (i) 103×107

103×107 can also be written as $(100+3)(100+7)$.

We can observe that, we can apply the identity $(x+a)(x+b)=x^2+(a+b)x+ab$

$(100+3)(100+7)=(100)^2+(3+7)(100)+3\times7$

=10000+1000+21

=11021

Therefore, we conclude that the value of the product 103×107 is 11021.

(ii) 95×96

95×96 can also be written as $(100-5)(100-4)$

We can observe that, we can apply the identity $(x+a)(x+b)=x^2+(a+b)x+ab$

$(100-5)(100-4)=(100)^2+[(-5)+(-4)](100)+(-5)\times(-4)$

=10000-900+20 =9120

Therefore, we conclude that the value of the product 95×96 is 9120.

(iii) 104×96

104×96 can also be written as $(100+4)(100-4)$.

We can observe that, we can apply the identity $(x+y)(x-y)=x^2-y^2$ with respect to the expression $(100+4)(100-4)$, to get

$(100+4)(100-4)=(100)^2-(4)^2$

=10000-16

=9984

Therefore, we conclude that the value of the product 104×96 is 9984.

---

3. Factorize the following using appropriate identities:

(i) $9x^2+6xy+y^2$

(ii) $4y^2-4y+1$

(iii) $x^2-\frac{y^2}{100}$

Ans. (i)

$$9x^2+6xy+y^2=(3x)^2+2\times3x\times y+(y)^2$$

We can observe that, we can apply the identity $(x+y)^2=x^2+2xy+y^2$

$\Rightarrow (3x)^2+2\times3x\times y+(y)^2=(3x+y)^2.$

$\Rightarrow (3x+y)(3x+y)$

(ii) $4y^2-4y+1$

$$4y^2-4y+1=(2y)^2-2\times2y\times1+(1)^2$$

We can observe that, we can apply the identity $(x-y)^2=x^2-2xy+y^2$

$\Rightarrow (2y)^2-2\times2y\times1+(1)^2=(2y-1)^2.$

$\Rightarrow (2y-1)(2y-1)$

(iii) $x^2-\frac{y^2}{100}$

We can observe that, we can apply the identity $(x)^2-(y)^2=(x+y)(x-y)$

$\Rightarrow (x)^2-\left(\frac{y}{10}\right)^2=\left(x+\frac{y}{10}\right)\left(x-\frac{y}{10}\right).$

4. Expand each of the following, using suitable identities:

(i) $(x+2y+4z)^2$

(ii) $(2x-y+z)^2$

(iii) $(-2x+3y+2z)^2$

(iv) $(3a-7b-c)^2$

(v) $(-2x+5y-3z)^2$

(vi) $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2$

Ans. (i) $(x+2y+4z)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(x+2y+4z)^2$.

$$(x+2y+4z)^2 = (x)^2+(2y)^2+(4z)^2+2\times x\times 2y+2\times 2y\times 4z+2\times 4z\times x$$
$$= x^2+4y^2+16z^2+4xy+16yz+8zx.$$

(ii) $(2x-y+z)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(2x-y+z)^2$.

$$(2x-y+z)^2 = \left[2x+(-y)+z\right]^2$$
$$= (2x)^2+(-y)^2+(z)^2+2\times 2x\times(-y)+2\times(-y)\times z+2\times z\times 2x$$
$$= 4x^2+y^2+z^2-4xy-2yz+4zx.$$

(iii) $(-2x+3y+2z)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(-2x+3y+2z)^2$.

$$(-2x+3y+2z)^2 = \left[(-2x)+3y+2z\right]^2$$
$$= (-2x)^2+(3y)^2+(2z)^2+2\times(-2x)\times 3y+2\times 3y\times 2z+2\times 2z\times(-2x)$$
$$= 4x^2+9y^2+4z^2-12xy+12yz-8zx$$

(iv) $(3a-7b-c)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(3a-7b-c)^2$.

$$(3a-7b-c)^2 = \left[3a+(-7b)+(-c)\right]^2$$
$$= (3a)^2+(-7b)^2+(-c)^2+2\times 3a\times(-7b)+2\times(-7b)\times(-c)+2\times(-c)\times 3a$$
$$= 9a^2+49b^2+c^2-42ab+14bc-6ac.$$

(v) $(-2x+5y-3z)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

We need to apply the above identity to expand the expression $(-2x+5y-3z)^2$.

$(-2x+5y-3z)^2 = [(-2x)+5y+(-3z)]^2$

$= (-2x)^2+(5y)^2+(-3z)^2+2\times(-2x)\times 5y+2\times 5y\times(-3z)+2\times(-3z)\times(-2x)$

$= 4x^2+25y^2+9z^2-20xy-30yz+12zx.$

(vi) $\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2$

We know that $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$.

$\left(\frac{1}{4}a-\frac{1}{2}b+1\right)^2 = \left[\frac{a}{4}+\left(-\frac{b}{2}\right)+1\right]^2$

$= \left(\frac{a}{4}\right)^2+\left(-\frac{b}{2}\right)^2+(1)^2+2\times\frac{a}{4}\times\left(-\frac{b}{2}\right)+2\times\left(-\frac{b}{2}\right)\times 1+2\times 1\times\frac{a}{4}$

$= \frac{a^2}{16}+\frac{b^2}{4}+1-\frac{ab}{4}-b+\frac{a}{2}.$

5. Factorize:

(i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

(ii) $2x^2+y^2+8z^2-2\sqrt{2}xy+4\sqrt{2}yz-8xz$

Ans. (i) $4x^2+9y^2+16z^2+12xy-24yz-16xz$

The expression $4x^2+9y^2+16z^2+12xy-24yz-16xz$ can also be written as

$(2x)^2+(3y)^2+(-4z)^2+2\times 2x\times 3y+2\times 3y\times(-4z)+2\times(-4z)\times 2x.$

We can observe that, we can apply the identity $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$ with respect to the expression

$(2x)^2+(3y)^2+(-4z)^2+2\times 2x\times 3y+2\times 3y\times(-4z)+2\times(-4z)\times 2x,$

We get $(2x+3y-4z)^2 = (2x+3y-4z)(2x+3y-4z)$

Therefore, we conclude that after factorizing the expression

$4x^2+9y^2+16z^2+12xy-24yz-16xz$, we get.

$(2x + 3y - 4z)(2x + 3y - 4z)$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

We need to factorize the expression $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$.

The expression $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$ can also be written as

$\left(-\sqrt{2}x\right)^2 + (y)^2 + \left(2\sqrt{2}z\right)^2 + 2\times\left(-\sqrt{2}x\right)\times y + 2\times y\times\left(2\sqrt{2}z\right) + 2\times\left(2\sqrt{2}z\right)\times\left(-\sqrt{2}x\right)$.

We can observe that, we can apply the identity $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ with respect to the expression

$\left(-\sqrt{2}x\right)^2 + (y)^2 + \left(2\sqrt{2}z\right)^2 + 2\times\left(-\sqrt{2}x\right)\times y + 2\times y\times\left(2\sqrt{2}z\right) + 2\times\left(2\sqrt{2}z\right)\times\left(-\sqrt{2}x\right)$, to get

$\left(-\sqrt{2}x + y + 2\sqrt{2}z\right)^2$

Therefore, we conclude that after factorizing the expression

$2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$,

we get $(-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$

6. Write the following cubes in expanded form:

(i) $(2x+1)^3$

(ii) $(2a-3b)^3$

(iii) $\left(\frac{3}{2}x+1\right)^3$

(iv) $\left(x-\frac{2}{3}y\right)^3$

Ans. (i) $(2x+1)^3$

We know that $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$.

$\therefore (2x+1)^3 = (2x)^3 + (1)^3 + 3\times 2x\times 1(2x+1)$

$= 8x^3 + 1 + 6x(2x+1)$

$= 8x^3 + 12x^2 + 6x + 1.$

Therefore, the expansion of the expression $(2x+1)^3$ is $8x^3+12x^2+6x+1$.

(ii) $(2a-3b)^3$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$$\therefore (2a-3b)^3 = (2a)^3 - (3b)^3 - 3\times 2a\times 3b(2a-3b)$$
$$= 8a^3 - 27b^3 - 18ab(2a-3b)$$
$$= 8a^3 - 36a^2b + 54ab^2 - 27b^3.$$

Therefore, the expansion of the expression $(2a-3b)^3$ is $8a^3-36a^2b+54ab^2-27b^3$.

(iii) $\left(\frac{3}{2}x+1\right)^3$

We know that $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$.

$$\left(\frac{3}{2}x+1\right)^3 = \left(\frac{3}{2}x\right)^3 + (1)^3 + 3\times\frac{3}{2}x\times 1\left(\frac{3}{2}x+1\right)\therefore$$
$$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x\left(\frac{3}{2}x+1\right)$$
$$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1.$$

Therefore, the expansion of the expression $\left(\frac{3}{2}x+1\right)^3$ is $\frac{27}{8}x^3+\frac{27}{4}x^2+\frac{9}{2}x+1$.

(iv) $\left(x-\frac{2}{3}y\right)^3$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$\therefore\left(x-\frac{2}{3}y\right)^3=(x)^3-\left(\frac{2}{3}y\right)^3-3\times x\times\frac{2}{3}y\left(x-\frac{2}{3}y\right)$

$=x^3-\frac{8}{27}y^3-2xy\left(x-\frac{2}{3}y\right)$

$=x^3-2x^2y+\frac{4}{3}xy^2-\frac{8}{27}y^3.$

Therefore, the expansion of the expression $\left(x-\frac{2}{3}y\right)^3$ is $x^3-2x^2y+\frac{4}{3}xy^2-\frac{8}{27}y^3.$

7. Evaluate the following using suitable identities:

(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Ans. (i) $(99)^3$

$(99)^3$ can also be written as $(100-1)^3$.

Using identity, $(x-y)^3=x^3-y^3-3xy(x-y)$

$(100-1)^3=(100)^3-(1)^3-3\times100\times1(100-1)$

=1000000-1-300(99)

=999999-29700

=970299

(ii) $(102)^3$

$(102)^3$ can also be written as $(100+2)^3$.

Using identity $(x+y)^3=x^3+y^3+3xy(x+y)$

$(100+2)^3=(100)^3+(2)^3+3\times100\times2(100+2)$

=1000000+8+600(102)

=1000008+61200

=1061208

(iii) $(998)^3$

$(998)^3$ can also be written as $(1000-2)^3$.

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

$(1000-2)^3 = (1000)^3 - (2)^3 - 3\times1000\times2(1000-2)$

=1000000000-8-6000(998)

=999999992-5988000

=994011992

8. Factorize each of the following:

(i) $8a^3+b^3+12a^2b+6ab^2$

(ii)

(iii)

(iv) $64a^3-27b^3-144a^2b+108ab^2$

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

Ans. (i) $8a^3+b^3+12a^2b+6ab^2$

The expression $8a^3+b^3+12a^2b+6ab^2$ can also be written as

$=(2a)^3+(b)^3+3\times2a\times2a\times b+3\times2a\times b\times b$

$=(2a)^3+(b)^3+3\times2a\times b(2a+b)$.

Using identity

$(x+y)^3 = x^3+y^3+3xy(x+y)$ with respect to the expression $(2a)^3+(b)^3+3\times2a\times b(2a+b)$, we get $(2a+b)_3=(2a+b)(2a+b)(2a+b)$

Therefore, after factorizing the expression

$8a^3+b^3+12a^2b+6ab^2$, we get $(2a+b)(2a+b)(2a+b)$.

(ii) $8a^3-b^3-12a^2b+6ab^2$

The expression $8a^3 - b^3 - 12a^2b + 6ab^2$ can also be written as

$= (2a)^3 - (b)^3 - 3\times 2a\times 2a\times b + 3\times 2a\times b\times b$

$= (2a)^3 - (b)^3 - 3\times 2a\times b(2a-b).$

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$ with respect to the expression

$(2a)^3 - (b)^3 - 3\times 2a\times b(2a-b)$, we get $(2a - b)^3 = (2a - b)(2a - b)(2a - b)$

Therefore, after factorizing the expression

,

we get $(2a - b)(2a - b)(2a - b)$

(iii) $27 - 125a^3 - 135a + 225a^2$

The expression $27 - 125a^3 - 135a + 225a^2$ can also be written as

$= (3)^3 - (5a)^3 - 3\times 3\times 3\times 5a + 3\times 3\times 5a\times 5a$

$= (3)^3 - (5a)^3 + 3\times 3\times 5a(3-5a).$

Using identity

$(x-y)^3 = x^3 - y^3 - 3xy(x-y)$ with respect to the expression

$(3)^3 - (5a)^3 + 3\times 3\times 5a(3-5a)$, we get $(3-5a)^3$.

Therefore, after factorizing the expression

,we get $5a)(3-5a)$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

The expression $64a^3 - 27b^3 - 144a^2b + 108ab^2$ can also be written as

$= (4a)^3 - (3b)^3 - 3\times 4a\times 4a\times 3b + 3\times 4a\times 3b\times 3b$

$= (4a)^3 - (3b)^3 - 3\times 4a\times 3b(4a-3b).$

Using identity $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

with respect to the expression

$(4a)^3 - (3b)^3 - 3\times 4a\times 3b(4a-3b)$,

we get $(4a - 3b)^3 = (4a - 3b)(4a - 3b)(4a - 3b)$

Therefore, after factorizing the expression

$64a^3 - 27b^3 - 144a^2b + 108ab^2$

we get $(4a-3b)(4a-3b)(4a-3b)$.

(v) $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$

The expression $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$ can also be written as

$$=(3p)^3-\left(\frac{1}{6}\right)^3-3\times3p\times3p\times\frac{1}{6}+3\times3p\times\frac{1}{6}\times\frac{1}{6}$$

$$=(3p)^3-\left(\frac{1}{6}\right)^3-3\times3p\times\frac{1}{6}\left(3p-\frac{1}{6}\right).$$

Using identity $(x-y)^3=x^3-y^3-3xy(x-y)$

with respect to the expression $(3p)^3-\left(\frac{1}{6}\right)^3-3\times3p\times\frac{1}{6}\left(3p-\frac{1}{6}\right)$,

to get $\left(3p-\frac{1}{6}\right)^3=\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)$

Therefore, after factorizing the expression $27p^3-\frac{1}{216}-\frac{9}{2}p^2+\frac{1}{4}p$,

we get $\left(3p-\frac{1}{6}\right)^3=\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)\left(3p-\frac{1}{6}\right)$

(iii) Verify:

(i) $x^3+y^3=(x+y)(x^2-xy+y^2)$

(ii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

Ans.

(i) $x^3+y^3=(x+y)(x^2-xy+y^2)$

We know that $(x+y)^3=x^3+y^3+3xy(x+y)$.

$$\Rightarrow x^3+y^3=(x+y)^3-3xy(x+y)$$

$$=(x+y)\left[(x+y)^2-3xy\right]$$

$\because$ We know that $(x+y)^2=x^2+2xy+y^2$

$$\therefore x^3+y^3=(x+y)(x^2+2xy+y^2-3xy)$$

$= (x+y)(x^2 - xy + y^2)$

Therefore, the desired result has been verified.

(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

We know that $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$.

$\Rightarrow x^3 - y^3 = (x-y)^3 + 3xy(x-y)$

$= (x-y)\left[(x-y)^2 + 3xy\right]$

$\because$ We know that $(x-y)^2 = x^2 - 2xy + y^2$

$\therefore x^3 - y^3 = (x-y)(x^2 - 2xy + y^2 + 3xy)$

$= (x-y)(x^2 + xy + y^2)$

Therefore, the desired result has been verified.

10. Factorize:

(i) $27y^3 + 125z^3$

(ii)

Ans. (i) $27y^3 + 125z^3$

The expression $27y^3 + 125z^3$ can also be written as $(3y)^3 + (5z)^3$.

We know that $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$.

$(3y)^3 + (5z)^3 = (3y + 5z)\left[(3y)^2 - 3y \times 5z + (5z)^2\right]$

$= (3y + 5z)(9y^2 - 15yz + 25z^2)$.

(ii) $64m^3 - 343n^3$

The expression $64m^3 - 343n^3$ can also be written as $(4m)^3 - (7n)^3$.

We know that $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$.

$(4m)^3 - (7n)^3 = (4m - 7n)\left[(4m)^2 + 4m \times 7n + (7n)^2\right]$

.

$= (4m - 7n)(16m^2 + 28mn + 49n^2)$

Therefore, we conclude that after factorizing the expression $64m^3 - 343n^3$, we get $(4m-7n)(16m^2+28mn+49n^2)$.

(vii) Factorize:

$27x^3+y^3+z^3-9xyz$ **Ans.** $27x^3+y^3+z^3-9xyz$ can also be written as

$(3x)^3+(y)^3+(z)^3-3\times 3x\times y\times z.$

We know that $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx).$

$$\therefore (3x)^3+(y)^3+(z)^3-3\times 3x\times y\times z=(3x+y+z)\left[(3x)^2+(y)^2+(z)^2-3x\times y-y\times z-z\times 3x\right]$$

$$=(3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz).$$

Therefore, we conclude that after factorizing the expression $27x^3+y^3+z^3-9xyz$, we get $(3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz)$.

(vi) Verify that $x^3+y^3+z^3-3xyz=\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$

Ans.

LHS is $x^3+y^3+z^3-3xyz$ and RHS is $\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right].$

We know that $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx).$

And also, we know that $(x-y)^2=x^2-2xy+y^2.$

$$\frac{1}{2}(x+y+z)\left[(x-y)^2+(y-z)^2+(z-x)^2\right]$$

$$\frac{1}{2}(x+y+z)\left[(x^2-2xy+y^2)+(y^2-2yz+z^2)+(z^2-2xz+x^2)\right]$$

$$\frac{1}{2}(x+y+z)(2x^2+2y^2+2z^2-2xy-2yz-2zx)$$

$$(x+y+z)(x^2+y^2+z^2-xy-yz-zx).$$

Therefore, we can conclude that the desired result is verified.

13. If $x+y+z=0$, show that $x^3+y^3+z^3=3xyz$

Ans. We know that $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$.

We need to substitute x + y + z = 0

$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$, to get

$x^3+y^3+z^3-3xy=(0)\times(x_2+y^2+z^2\_xy\_yz\_zx)$

$x^3+y^3+z^3-3xyz=0$

$\Rightarrow x^3+y^3+z^3=3xyz.$

Therefore, the desired result is verified.

14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$

(ii) $(28)^3+(-15)^3+(-13)^3$

Ans. (i) $(-12)^3+(7)^3+(5)^3$

Let $a=-12, b=7$ and $c=5$

We know that, if $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Here, $a+b+c=-12+7+5=0$

$\therefore (-12)^3+(7)^3+(5)^3=3(-12)(7)(5)$

$=-1260$

(ii) $(28)^3+(-15)^3+(-13)^3$

Let $a=28, b=-15$ and $c=-13$

We know that, if $a+b+c=0$, then $a^3+b^3+c^3=3abc$

Here, $a+b+c=28-15-13=0$

$\therefore (28)^3+(-15)^3+(-13)^3=3(28)(-15)(-13)$

$= 16380$

15.Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i) Area : $25a^2-35a+12$

(ii) Area : $35y^2+13y-12$

Ans. (i) Area : $25a^2-35a+12$

The expression $25a^2-35a+12$ can also be written as $25a^2-15a-20a+12$.

$25a^2-15a-20a+12=5a(5a-3)-4(5a-3)$

$=(5a-4)(5a-3)$.

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $25a^2-35a+12$ is $\text{Length}=(5a-4)$ and $\text{Breadth}=(5a-3)$.

(ii) Area : $35y^2+13y-12$

The expression $35y^2+13y-12$ can also be written as $35y^2+28y-15y-12$.

$35y^2+28y-15y-12=7y(5y+4)-3(5y+4)$

$=(7y-3)(5y+4)$.

Therefore, we can conclude that a possible expression for the length and breadth of a rectangle of area $35y^2+13y-12$ is $\text{Length}=(7y-3)$ and $\text{Breadth}=(5y+4)$.

16.What are the possible expressions for the dimensions of the cuboids whose volumes are given below ?

(i) $Volume: 3x^2 - 12x$

(ii) $Volume: 12ky^2 + 8ky - 20k$

Ans. (i) $\text{Volume}: 3x^2 - 12x$

The expression $3x^2 - 12x$ can also be written as $3 \times x \times (x-4)$.

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $3x^2 - 12x$ is $3, x \text{ and } (x-4)$ .so we get length=3,breadth=x, height=(x-4)

(ii) $\text{Volume}: 12ky^2 + 8ky - 20k$

The expression $12ky^2 + 8ky - 20k$ can also be written as $k(12y^2 + 8y - 20)$.

$$k(12y^2 + 8y - 20) = k(12y^2 - 12y + 20y - 20)$$

$$= k[12y(y-1) + 20(y-1)]$$

$$= k(12y + 20)(y-1)$$

$$= 4k \times (3y+5) \times (y-1).$$

Therefore, we can conclude that a possible expression for the dimension of a cuboid of volume $12ky^2 + 8ky - 20k$ is $4k, (3y+5) \text{ and } (y-1)$.

so we ger length=4k, breadth= 3y+5, height= y-1

PUNA INTERNATIONAL SCHOOL

- **CLASS – 9**
- **SUBJECT - MATHS**
- **CHAPTER - 4**

SAMPLE NOTE-BOOK

CHAPTER - 4

Linear Equations in Two Variables

(Ex. 4.1)

(i) The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this statement.

(Take the cost of a notebook to be Rs *x* and that of a pen to be Rs *y*).

Ans. Let the cost of a notebook be $Rs.\ x$.

Let the cost of a pen be $Rs.\ y$.

We need to write a linear equation in two variables to represent the statement, "Cost of a notebook is twice the cost of a pen".

Therefore, we can conclude that the required statement will be $x = 2y$ = x -2y = 0

(i) Express the following linear equations in the form *ax* + *by* + *c* = 0 and indicate the values of *a*, *b* and *c* in each case:

(i) $2x+3y=9.3\overline{5}$

(ii) $x-\frac{y}{5}-10=0$

(iii) $-2x+3y=6$

(iv) $x=3y$

(v) $2x=-5y$

(vi) $3x+2=0$

(vii) $y-2=0$

(viii) $5 = 2x$

Ans. (i) $2x + 3y = 9.3\overline{5}$

We need to express the linear equation $2x + 3y = 9.3\overline{5}$ in the form *ax* + *by* + *c* = 0 and indicate the values of *a*, *b* and *c*.

$2x + 3y = 9.3\overline{5}$ can also be written as $2x + 3y - 9.3\overline{5} = 0$.

We need to compare the equation $2x + 3y - 9.3\overline{5} = 0$ with the general equation *ax* + *by* + *c* = 0, to get the values of *a*, *b* and *c*.

Therefore, we can conclude that $a = 2, b = 3 \text{ and } c = -9.3\overline{5}$.

(ii) $x - \frac{y}{5} - 10 = 0$

We need to express the linear equation $x - \frac{y}{5} - 10 = 0$ in the form *ax* + *by* + *c* = 0 and indicate the values of *a*, *b* and *c*.

$x - \frac{y}{5} - 10 = 0$ can also be written as $1 \cdot x - \frac{y}{5} - 10 = 0$.

We need to compare the equation $1 \cdot x - \frac{y}{5} - 10 = 0$ with the general equation *ax* + *by* + *c* = 0, to get the values of *a*, *b* and *c*.

Therefore, we can conclude that $a = 1, b = -\frac{1}{5} \text{ and } c = -10$.

(iii) $-2x + 3y = 6$

We need to express the linear equation $-2x + 3y = 6$ in the form *ax* + *by* + *c* = 0 and indicate the values of *a*, *b* and *c*.

$-2x + 3y = 6$ can also be written as $-2x + 3y - 6 = 0$.

We need to compare the equation -2x + 3y – 6 = 0 with the general equation *ax* + *by* + *c* = 0, to get the values of *a*, *b* and *c*.

Therefore, we can conclude that $a = -2, b = 3 \text{ and } c = -6$.

$$-2x + 3y - 6 = 0$$

(iv) $x = 3y$

We need to express the linear equation $x = 3y$ in the form *ax* + *by* + *c* = 0 and indicate the values of *a*, *b* and *c*.

$x = 3y$ can also be written as $x - 3y + 0 = 0$.

We need to compare the equation $x - 3y + 0 = 0$ with the general equation *ax* + *by* + *c* = 0, to get the values of *a*, *b* and *c*.

Therefore, we can conclude that $a = 1, b = -3 \text{ and } c = 0$.

(v) $2x = -5y$

We need to express the linear equation $2x = -5y$ in the form *ax* + *by* + *c* = 0 and indicate the values of *a*, *b* and *c*.

$2x = -5y$ can also be written as $2x + 5y + 0 = 0$.

We need to compare the equation $2x + 5y + 0 = 0$ with the general equation *ax* + *by* + *c* = 0, to get the values of *a*, *b* and *c*.

Therefore, we can conclude that $a = 2, b = 5 \text{ and } c = 0$.

(vi) $3x + 2 = 0$

We need to express the linear equation $3x + 2 = 0$ in the form *ax* + *by* + *c* = 0 and indicate the values of *a*, *b* and *c*.

$3x + 2 = 0$ can also be written as $3x + 0 \cdot y + 2 = 0$.

We need to compare the equation $3x + 0 \cdot y + 2 = 0$ with the general equation *ax* + *by* + *c* = 0, to get the values of *a*, *b* and *c*.

Therefore, we can conclude that $a = 3, b = 0$ and $c = 2$.

(vii) $y - 2 = 0$

We need to express the linear equation $y - 2 = 0$ in the form $ax + by + c = 0$ and indicate the values of a, b and c.

$y - 2 = 0$ can also be written as $0 \cdot x + 1 \cdot y - 2 = 0$.

We need to compare the equation $0 \cdot x + 1 \cdot y - 2 = 0$ with the general equation $ax + by + c = 0$, to get the values of a, b and c.

Therefore, we can conclude that $a = 0, b = 1$ and $c = -2$.

(viii) $5 = 2x$

We need to express the linear equation $5 = 2x$ in the form $ax + by + c = 0$ and indicate the values of a, b and c.

$5 = 2x$ can also be written as $-2x + 0 \cdot y + 5 = 0$.

We need to compare the equation $-2x + 0 \cdot y + 5 = 0$ with the general equation $ax + by + c = 0$, to get the values of a, b and c.

Therefore, we can conclude that $a = -2, b = 0$ and $c = 5$.

CHAPTER 4

Linear Equations in Two Variables

(Ex. 4.2)

1. **Which one of the following options is true, and why?**

   $y = 3x + 5$ **has**

(i) a unique solution, (ii)

only two solutions,

(iii) Infinitely many solutions

Ans. We need to the number of solutions of the linear equation $y = 3x + 5$. We know

that any linear equation has infinitely many solutions. Justification:

If $x = 0$ then $y = 3\times 0 + 5 = 5$

If $x = 1$ then $y = 3\times 1 + 5 = 8$

If $x = -2$ then $y = 3\times(-2) + 5$ $= -1$

Similarly, we can find infinite many solutions by putting the values of x so correct answer is (iii)

2. **Write four solutions for each of the following equations:**

(i) $2x + y = 7$

(ii) $\pi x + y = 9$

(iii) $x = 4y$

Ans. $2x+y=7$

We know that any linear equation has infinitely many solutions.

Let us put $x=0$ in the linear equation $2x+y=7$, to get

$2(0)+y=7 \qquad \Rightarrow y=7.$

Thus, we get first pair of solution as $(0,7)$.

Let us put $x=2$ in the linear equation $2x+y=7$, to get

$2(2)+y=7 \Rightarrow y+4=7 \Rightarrow y=3.$

Thus, we get second pair of solution as $(2,3)$.

Let us put $x=4$ in the linear equation $2x+y=7$, to get

$2(4)+y=7 \quad \Rightarrow y+8=7 \Rightarrow y=-1.$

Thus, we get third pair of solution as $(4,-1)$.

Let us put $x=6$ in the linear equation $2x+y=7$, to get

$2(6)+y=7 \Rightarrow y+12=7 \Rightarrow y=-5.$

Thus, we get fourth pair of solution as $(6,-5)$.

Therefore, we can conclude that four solutions for the linear equation $2x+y=7$ are $(0,7),(2,3),(4,-1) \text{ and } (6,-5)$.

(ii) $\pi x+y=9$

We know that any linear equation has infinitely many solutions.

Let us put $x=0$ in the linear equation $\pi x+y=9$, to get

$\pi(0)+y=9 \qquad \Rightarrow y=9$

Thus, we get first pair of solution as $(0,9)$.

Let us put $y=0$ in the linear equation $\pi x+y=9$, to get

$\pi x+(0)=9 \qquad \Rightarrow x=\frac{9}{\pi}$.

Thus, we get second pair of solution as $\left(\frac{9}{\pi},0\right)$.

Let us put $x=1$ in the linear equation $\pi x+y=9$, to get

$\pi(1)+y=9 \Rightarrow y=9-\pi$

Thus, we get third pair of solution as.$(1$ Let us

put $y=2$ in the linear equation $\pi x+y=9$, to get

$\pi x+2=9 \qquad \Rightarrow \pi x=7 \Rightarrow x=\frac{7}{\pi}$

Thus, we get fourth pair of solution as $\left(\frac{7}{\pi},2\right)$.

Therefore, we can conclude that four solutions for the linear equation $\pi x+y=9$ are

$(0,9),\left(\frac{9}{\pi},0\right),(1,9-\pi),\left(\frac{7}{\pi},2\right)$.

(iii) $x=4y$

We know that any linear equation has infinitely many solutions.

Let us put $y=0$ in the linear equation $x=4y$, to get

$x=4(0) \qquad \Rightarrow x=0$

Thus, we get first pair of solution as $(0,0)$.

Let us put $y=2$ in the linear equation $x=4y$, to get

$x = 4(2) \qquad \Rightarrow \ x = 8$

Thus, we get second pair of solution as $(8, 2)$.

Let us put $y = 4$ in the linear equation $x = 4y$, to get

$x = 4(4) \qquad \Rightarrow \ x = 16$

Thus, we get third pair of solution as $(16, 4)$.

Let us put $y = 6$ in the linear equation $x = 4y$, to get

$x = 4(6) \qquad \Rightarrow \ x = 24$

Thus, we get fourth pair of solution as $(24, 6)$.

Therefore, we can conclude that four solutions for the linear equation $x = 4y$ are $(0, 0), (8, 2), (16, 4)$ and $(24, 6)$.

(i) Check which of the following are solutions of the equation $x - 2y = 4$ and which are not:

(i) $(0, 2)$

(ii) $(2, 0)$

(iii) $(4, 0)$

(iv) $(\sqrt{2}, 4\sqrt{2})$

(v) $(1, 1)$

Ans. (i) $(0, 2)$

We need to put $x = 0$ and $y = 2$ in the L.H.S. of linear equation $x - 2y = 4$, to get

$(0)-2(2)=-4$

$\therefore$ L.H.S. $\neq$ R.H.S.

Therefore, we can conclude that $(0,2)$ is not a solution of the linear equation $x-2y=4$.

(ii) $(2,0)$

We need to put $x=2$ and $y=0$ in the L.H.S. of linear equation $x-2y=4$, to get

$(2)-2(0)=2$

$\therefore$ L.H.S. $\neq$ R.H.S.

Therefore, we can conclude that $(2,0)$ is not a solution of the linear equation $x-2y=4$.

(iii) $(4,0)$

We need to put $x=4$ and $y=0$ in the linear equation $x-2y=4$, to get

$(4)-2(0)=4$

$\therefore$ L.H.S. $=$ R.H.S.

Therefore, we can conclude that $(4,0)$ is a solution of the linear equation $x-2y=4$.

(iv) $\left(\sqrt{2},4\sqrt{2}\right)$

We need to put $x=\sqrt{2}$ and $y=4\sqrt{2}$ in the linear equation $x-2y=4$, to get

$\left(\sqrt{2}\right)-2\left(4\sqrt{2}\right)=-7\sqrt{2}, L.H.S = -7\sqrt{2}, R.H.S = 4$

$\therefore$ L.H.S. $\neq$ R.H.S.

Therefore, we can conclude that $\left(\sqrt{2},4\sqrt{2}\right)$ is not a solution of the linear equation $x-2y=4$.

(v) $(1,1)$

We need to put $x = 1 \text{ and } y = 1$ in the linear equation $x - 2y = 4$, to get

$(1) - 2(1) = -1$

$\therefore$ L.H.S. $\neq$ R.H.S.

Therefore, we can conclude that $(1,1)$ is not a solution of the linear equation $x - 2y = 4$.

4. Find the value of *k*, if *x* = 2, *y* = 1 is a solution of the equation $2x + 3y = k$.

Ans. We know that, if $x = 2 \text{ and } y = 1$ is a solution of the linear equation $2x + 3y = k$, then on substituting the respective values of *x* and *y* in the linear equation $2x + 3y = k$, the LHS and RHS of the given linear equation will not be effected.

$\therefore\ 2(2) + 3(1) = k \ \Rightarrow k = 4 + 3 \Rightarrow k = 7$

Therefore, we can conclude that the value of *k*, for which the linear equation $2x + 3y = k$ has $x = 2 \text{ and } y = 1$ as one of its solutions is 7.

CHAPTER 4

Linear Equations in Two Variables

(Ex. 4.3)

1. Draw the graph of each of the following linear equations in two variables:

(i) $x + y = 4$

(ii) x – y = 2

(iii) y = 3x

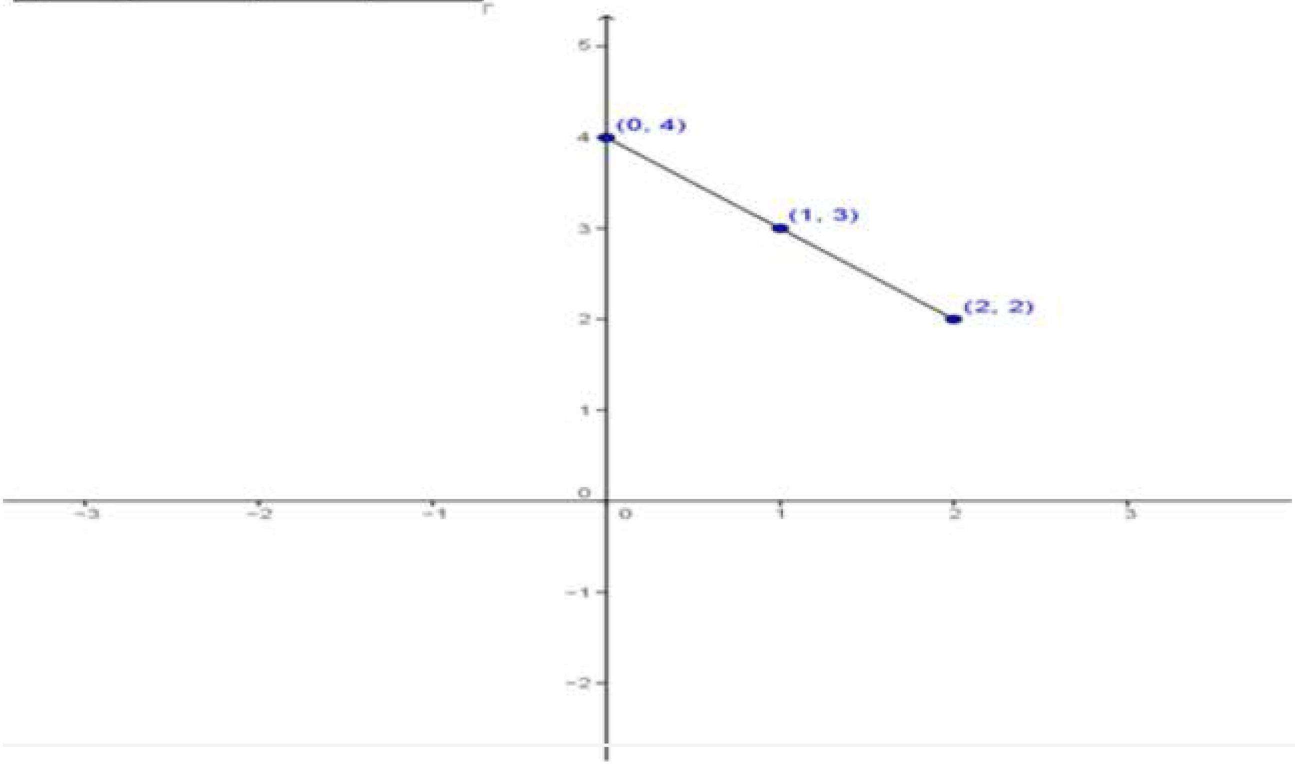
(iv) 3 = 2x + y

(i) $x + y = 4$

Ans. We can conclude that $x = 0, y = 4; x = 1, y = 3$ and $x = 2, y = 2$ are the solutions of the linear equation $x + y = 4$.

We can optionally consider the given below table for plotting the linear equation $x + y = 4$ on the graph.

X	0	1	2
y	4	3	2

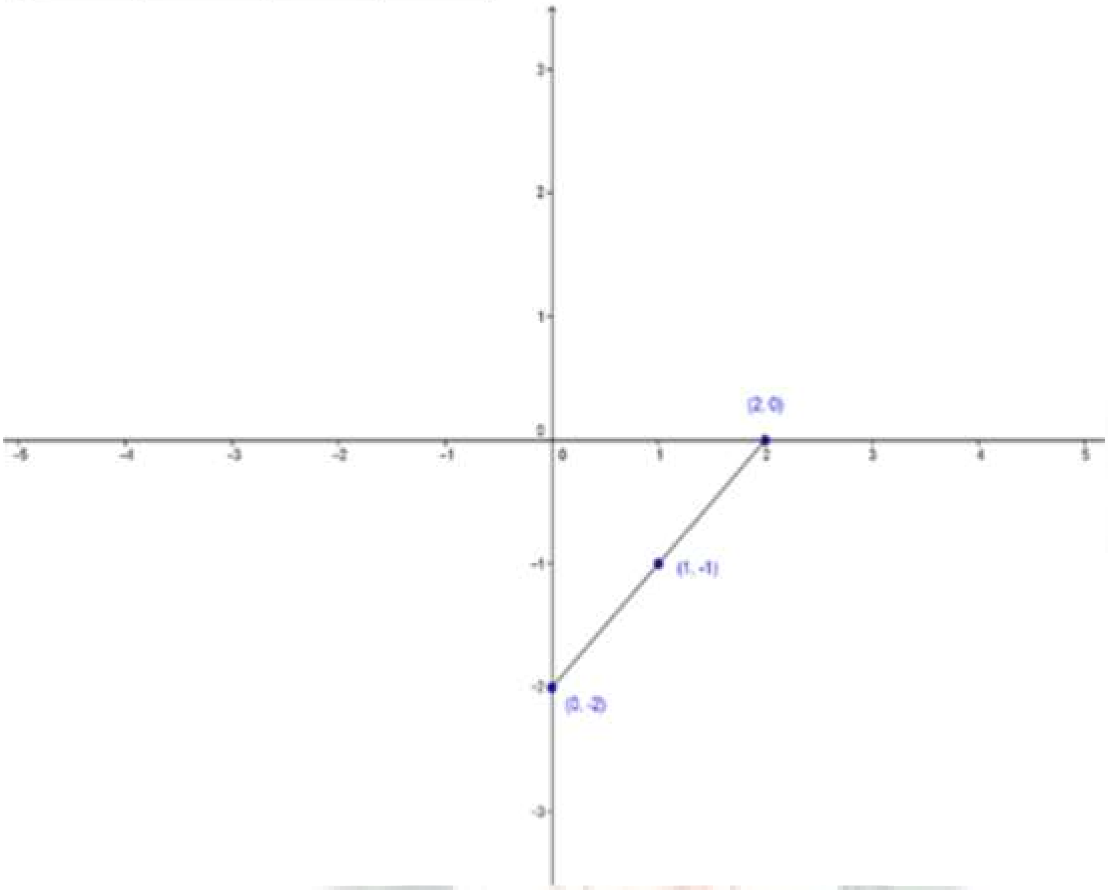


(ii) $x-y=2$

We can conclude that $x=0, y=-2; x=1, y=-1$ and $x=2, y=0$ are the solutions of the linear equation $x-y=2$.

We can optionally consider the given below table for plotting the linear equation $x-y=2$ on the graph.

X	0	1	2
y	-2	-1	0



(iii) $y=3x$

We can conclude that $x=0, y=0; x=1, y=3$ and $x=2, y=6$ are the solutions of the linear equation $y=3x$.

We can optionally consider the given below table for plotting the linear equation $y=3x$ on the graph.

X	0	1	2
y	0	3	6

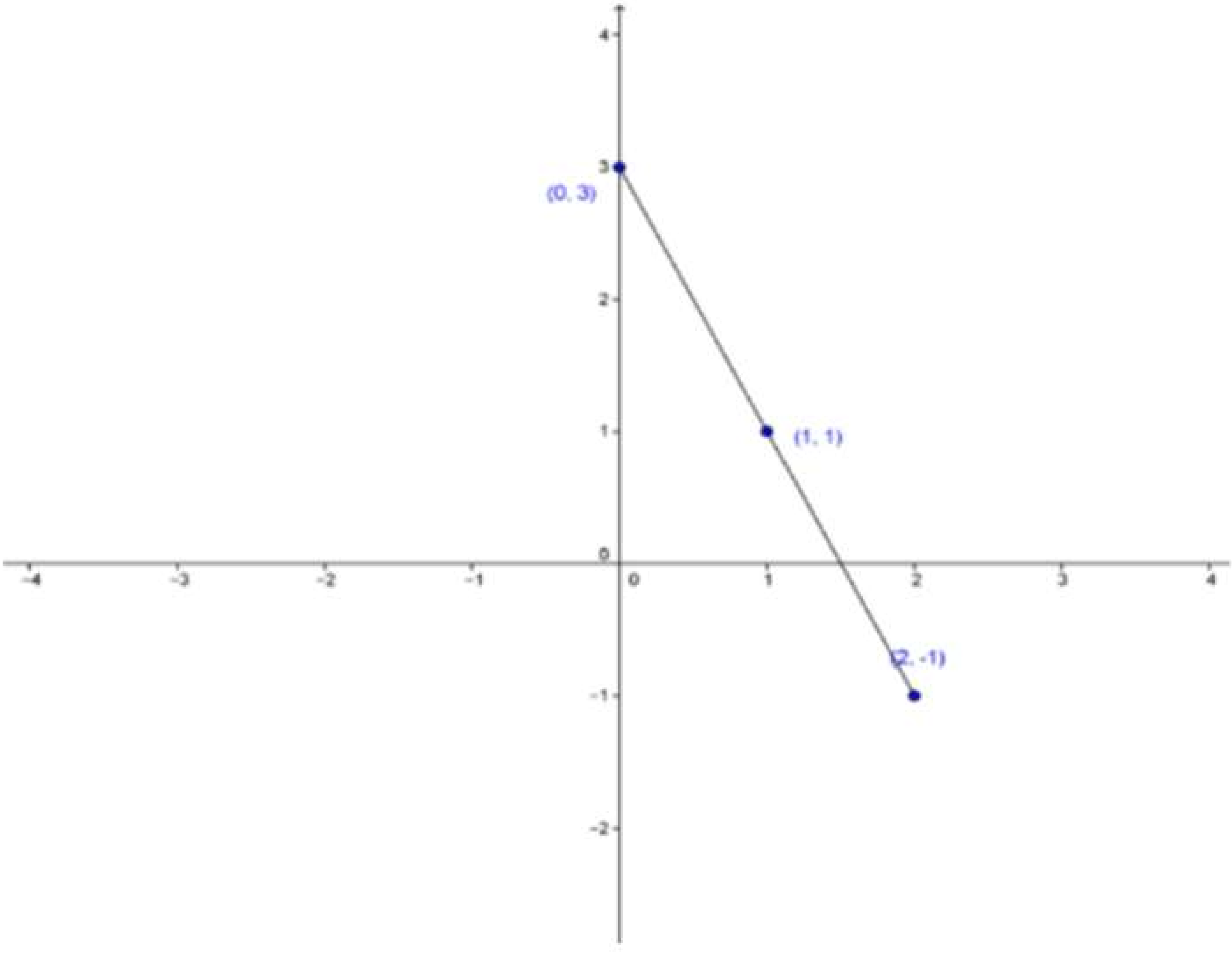
so

(iv) $3 = 2x + y$

We can conclude that $x = 0, y = 3; x = 1, y = 1$ and $x = 2, y = -1$ are the solutions of the linear equation $3 = 2x + y$.

We can optionally consider the given below table for plotting the linear equation $3 = 2x + y$ on the graph.

X	0	1	2
y	3	1	-1



(ii) Give the equations of two lines passing through (2, 14). How many more such lines are there, and why?

Ans. We need to give the two equations of the line that passes through the point $(2,14)$.

We know that infinite number of lines can pass through any given point.

We can consider the linear equations $7x-y=0$ and $2x+y=18$.

We can conclude that on putting the values $x=2$ and $y=14$ in the above mentioned linear equations, we get LHS=RHS.

Therefore, we can conclude that the line of the linear equations $7x-y=0$ and $28x-4y=0$ will pass through the point $(2,14)$. so infinitely many lines can be drawn through (2,14)

3. If the point (3, 4) lies on the graph of the equation $3y=ax+7$, find the value of *a*.

Ans. We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

We can conclude that $(3,4)$ is a solution of the linear equation $3y=ax+7$.
We need to substitute $x=3$ and $y=4$ in the linear equation $3y=ax+7$, to get

$$3(4)=a(3)+7 \Rightarrow 12=3a+7$$

$$\Rightarrow 3a=12-7 \quad \Rightarrow 3a=5 \Rightarrow a=\frac{5}{3}$$

Therefore, we can conclude that the value of *a* will be $\frac{5}{3}$.

(ii) The taxi fare in a city is as follows: For the first kilometre, the fare is Rs 8 and for the subsequent distance it is Rs 5 per km. Taking the distance covered as *x* km and total fare as Rs *y*, write a linear equation for this information, and draw its graph.

Ans. From the given situation, we can conclude that the distance covered at the rate Rs 5 per km will be $(x-1)$, as first kilometer is charged at Rs 8 per km.
We can conclude that the linear equation for the given situation will be:

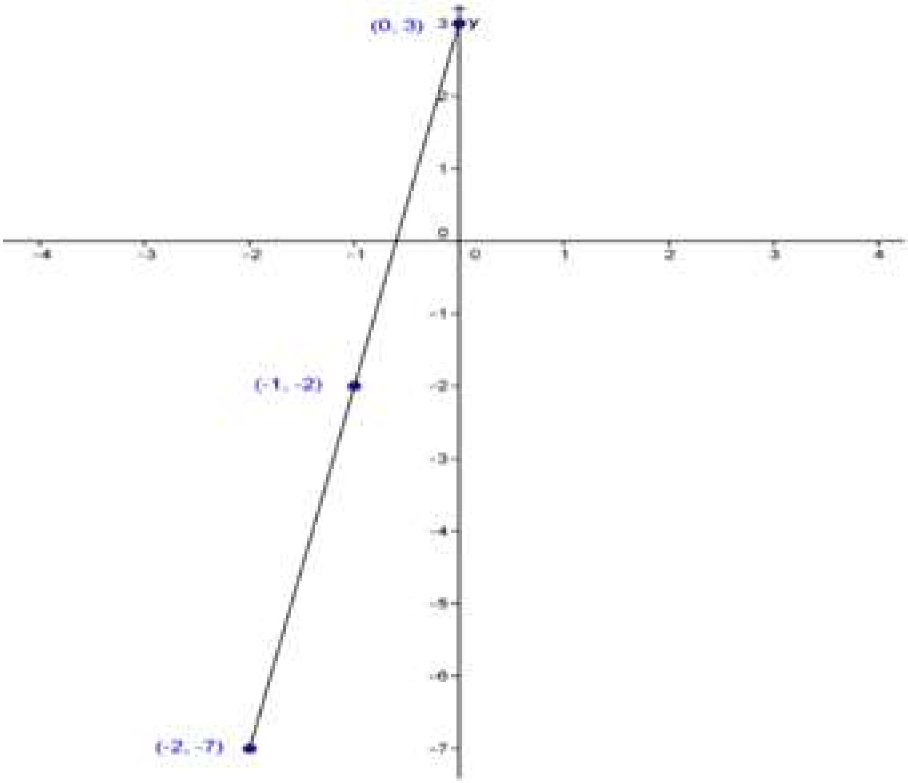
$$8+5(x-1)=y \Rightarrow 8+5x-5=y \Rightarrow 3+5x=y.$$

We need to draw the graph of the linear equation $3+5x=y$.

We can conclude that $x = 0, y = 3; x = 1, y = 1$ and $x = 2, y = -1$ are the solutions of the linear equation $3 + 5x = y$.

We can optionally consider the given below table for plotting the linear equation $3 + 5x = y$ on the graph.

X	0	-1	-2
y	3	-2	-7



(ii) From the choices given below, choose the equation whose graphs are given in the given figures.

For the first figure

(i) y = x

(ii) x + y = 0

(iii) y = 2x

(iv) 2 + 3y = 7x

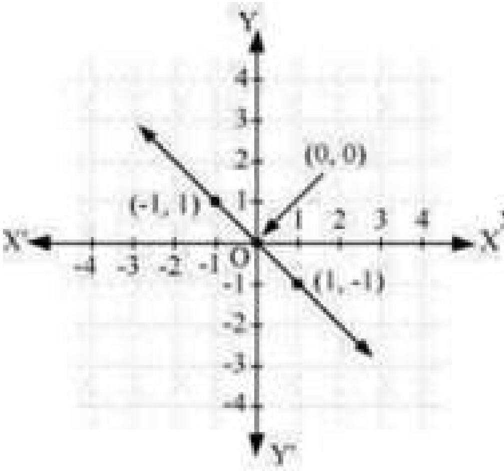
For the second figure

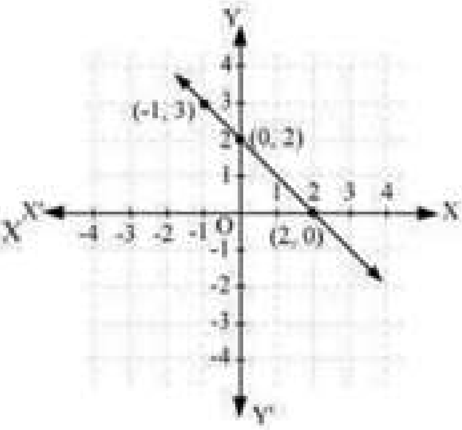
(i) y = x +2

(ii) y = x – 2

(iii) y = – x + 2

(iv) x + 2y = 6





Ans. For First figure

(i) $y = x$

We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

Let us check whether $x = -1, y = 1; x = 0, y = 0$ and $x = 1, y = -1$ are the solutions of the linear equation $y = x$.

For $x = -1, y = 1$, we get

$$y = x \qquad \Rightarrow \quad -1 \neq 1$$

Therefore, the given graph does not belong to the linear equation $y = x$.

(ii) $x + y = 0$

We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

For $x = -1, y = 1$, we get

$$-1 + 1 = 0 \qquad \Rightarrow \quad 0 = 0.$$

For $x = 0, y = 0$, we get

$$0 + 0 = 0 \qquad \Rightarrow \quad 0 = 0.$$

For $x = 1, y = -1$, we get

$$1 + (-1) = 0 \quad \Rightarrow 1 - 1 = 0 \Rightarrow 0 = 0.$$

Therefore, the given graph belongs to the linear equation $x+y=0$.

(iii) $y=2x$

We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

For $x=-1, y=1$, we get

$y=2x \qquad \Rightarrow -1=2(1) \Rightarrow -1 \neq 2.$

Therefore, the given graph does not belong to the linear equation $y=2x$.

(iv) $2+3y=7x$

We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

For $x=-1, y=1$, we get

$2+3(1)=7(-1) \Rightarrow 2+3=-7 \quad \Rightarrow 5 \neq -7.$

Therefore, the given graph does not belong to the linear equation $2+3y=7x$.

For Second figure

(i) $y=x+2$

We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

For $x=-1, y=3$, we get

$3=-1+2 \qquad \Rightarrow 3 \neq 1.$

Therefore, the given graph does not belong to the linear equation $y=x+2$.

(ii) $y=x-2$

We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

For $x=-1, y=3$, we get

$3=-1-2 \qquad \Rightarrow 3 \neq -3.$

Therefore, the given graph does not belong to the linear equation $y=x-2$.

(iii) $y=-x+2$

We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

For $x=-1, y=3$, we get

$3=-(-1)+2 \quad \Rightarrow 3=1+2 \Rightarrow 3=3.$

For $x=0, y=2$, we get

$2=-(0)+2 \quad \Rightarrow 2=2.$

For $x=2, y=0$, we get

$0=-(2)+2 \quad \Rightarrow 0=0.$

Therefore, hat the given graph belongs to the linear equation $y=-x+2$.

(iv) $x+2y=6$

We know that if any point lie on the graph of any linear equation, then that point is the solution of that linear equation.

For $x=-1, y=3$, we get

$(-1)+2(3)=6 \quad \Rightarrow -1+6=6 \Rightarrow 5 \neq 6.$

Therefore, the given graph does not belong to the linear equation $x+2y=6$.

(i) If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units. Also read from the graph the work done when the distance travelled by the body is:

, **2 units**

, **0 units**

Ans. We are given that the work done by a body on application of a constant force is directly proportional to the distance travelled by the body.

Let the work done be *W* and let constant force be *F*.

Let distance travelled by the body be *D*.

According to the question,

$W \propto D \qquad \Rightarrow W = F \cdot D.$

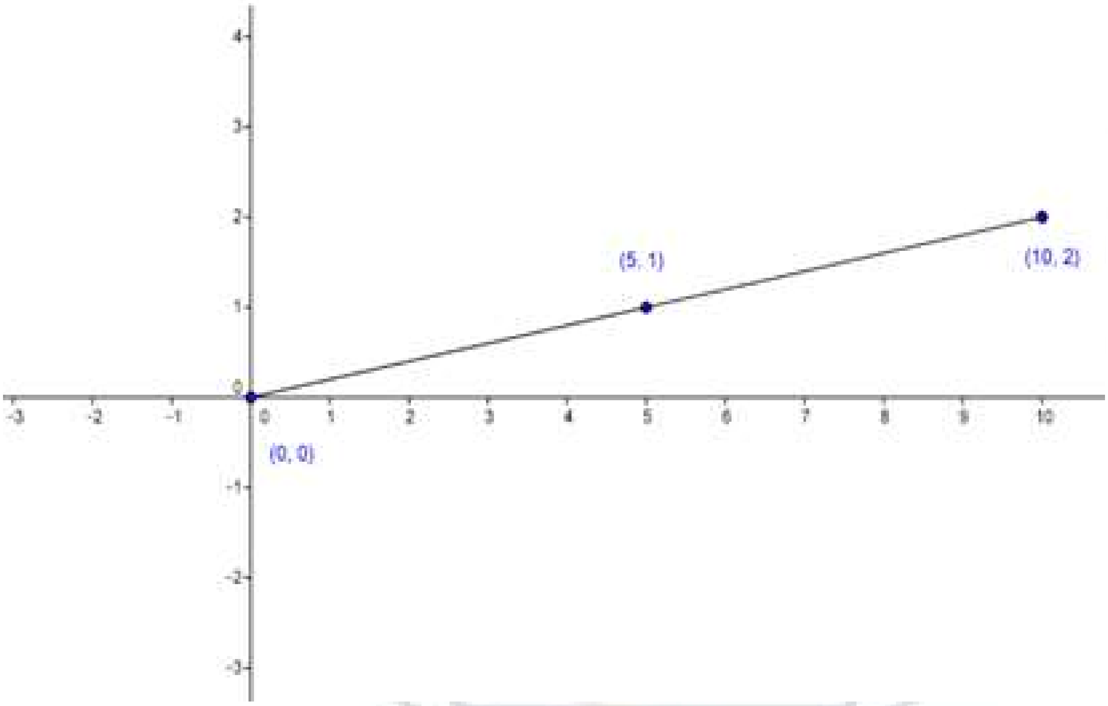
We need to draw the graph of the linear equation $W = F \cdot D$, when the force is constant as 5 units, i.e., $W = 5D$.

Work done W is along x-axis and distance D is along y-axis.

We can conclude that W=0,D=0

W=5,D=1 and W=10,D=2 are the solutions of the linear equation $W = 5D$.

W	0	5	10
D	0	1	2



Therefore, we can conclude from the above mentioned graph, the work done by the body, when the distance is 2 units will be 10 units and when the distance is 0 units, the work done will be 0 unit.

(iv) Yamini and Fatima, two students of Class IX of a school, together contributed Rs 100 towards the Prime Minister's Relief Fund to help the earthquake victims. Write a linear equation which satisfies this data. (You may take their contributions as Rs x and Rs y.) Draw the graph of the same.

Ans. The contribution made by Yamini is Rs x and the contribution made by Fatime is Rs y.

We are given that together they both contributed Rs 100.

We get the given below linear equation from the given situation.

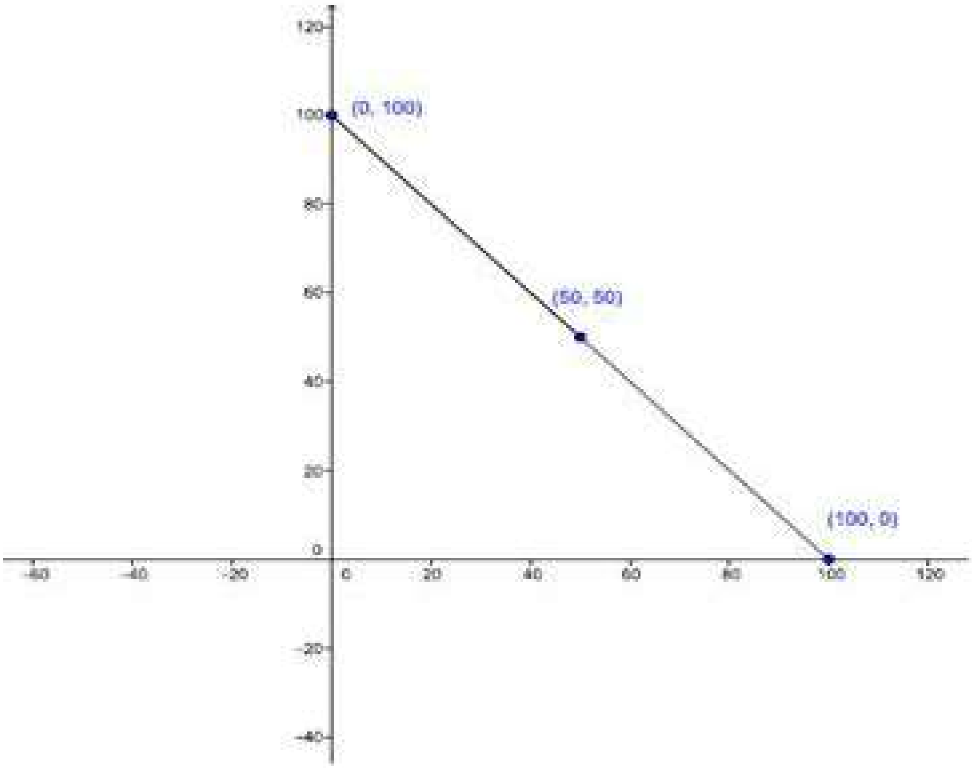
$x + y = 100$.

We need to consider any 3 solutions of the linear equation $x + y = 100$, to plot the graph of the linear equation $x + y = 100$.

We can conclude that x=0,y=100,x=50,y=50 and x=100,y=0 are the solutions of the linear equation $x + y = 100$.

We can optionally consider the given below table for plotting the linear equation $x + y = 100$ on the graph.

X	0	50	100
y	100	50	0



(v) In countries like USA and Canada, temperature is measured in Fahrenheit, whereas in countries like India, it is measured in Celsius. Here is a linear equation that converts Fahrenheit to Celsius:

$$F = \left(\frac{9}{5}\right)C + 32$$

(i) Draw the graph of the linear equation above using Celsius for x-axis and Fahrenheit for y-axis.

(ii) If the temperature is $30°C$, what is the temperature in Fahrenheit ?

(iii) If the temperature is $95°F$, what is the temperature in Celsius ?

(iv) If the temperature is $0°C$, what is the temperature in Fahrenheit and if the temperature is $0°F$, what is the temperature in Celsius ?

(v) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it.

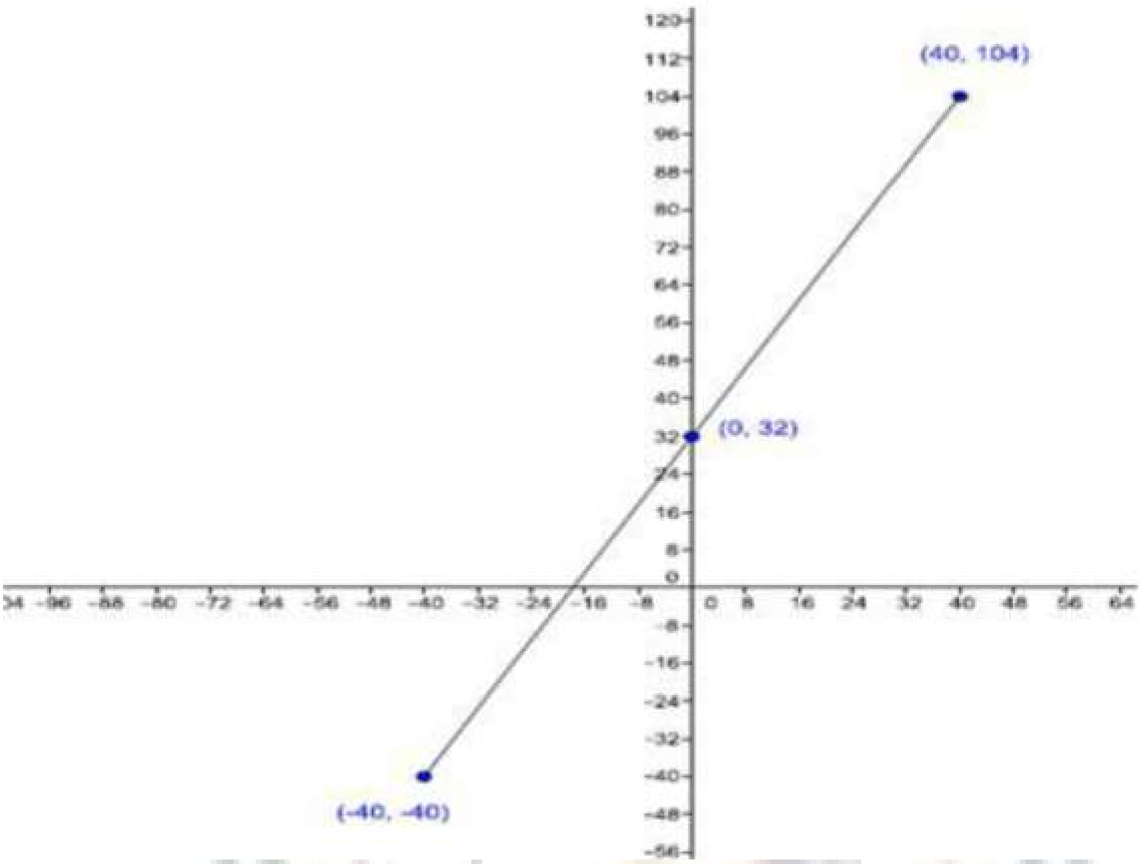
Ans. We are given a linear equation that converts the temperature in Fahrenheit into degree Celsius.

$$F = \left(\frac{9}{5}\right)C + 32$$

(i) We need to consider any 3 solutions of the linear equation $F = \left(\frac{9}{5}\right)C + 32$, to plot the graph of the linear equation $F = \left(\frac{9}{5}\right)C + 32$.

We can conclude that C=-40,F=-40,C=0,F=32 and C=40,F=104 are the solutions of the linear equation $F = \left(\frac{9}{5}\right)C + 32$.

C	-40	0	40
F	-40	32	104



(ii) We need to find the temperature in Fahrenheit, when the temperature in degree Celsius is 30°.when C= 30°

$$F=\left(\frac{9}{5}\right)(30)+32 \; =9\times 6+32=86^{\circ}$$

Therefore, we can conclude that the temperature in Fahrenheit will be $86^{\circ}F$.

(iii) We need to find the temperature in degree Celsius, when the temperature in Fahrenheit is . 95°

$$95=\left(\frac{9}{5}\right)C+32 \Rightarrow \frac{9}{5}C=95-32 \Rightarrow C=63\times\frac{5}{9} \; =35^{\circ}.$$

Therefore, we can conclude that the temperature in degree Celsius will be 35° .

(iv) We need to find the temperature in Fahrenheit, when the temperature in degree Celsius is . 0°

$$F=\left(\frac{9}{5}\right)(0)+32=32^{\circ}$$

Therefore, we can conclude that the temperature in Fahrenheit will be 32° .

We need to find the temperature in degree Celsius, when the temperature in Fahrenheit is 0° .

$$0=\left(\frac{9}{5}\right)C+32 \Rightarrow \frac{9}{5}C=0-32 \Rightarrow C=-32\times\frac{5}{9}=-17.77^\circ.$$

Therefore, we can conclude that the temperature in degree Celsius will be -17.77°

(v) We need to find a temperature that is numerically same in both Fahrenheit and degree Celsius.So

F=C

$$F=\left(\frac{9}{5}\right)F+32 \Rightarrow F-\frac{9F}{5}=32 \Rightarrow -\frac{4F}{5}=32 \Rightarrow F=-40^\circ.$$

Therefore, we can conclude that the temperature that is numerically same in Fahrenheit and Degree Celsius will be -40° .

CHAPTER 4

Linear Equations in Two Variables

(Ex. 4.4)

(iii) Give the geometric representations of y = 3 as an equation

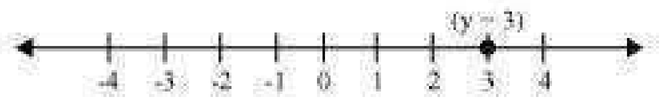
(i) In one variable (ii)

In two variables

Ans. (i) We need to represent the linear equation y=3 geometrically in one variable.

We can conclude that in one variable, the geometric representation of the linear equation y=3 will be same as representing the number 3 on a number line.

Given below is the representation of number 3 on the number line.



(iii) We need to represent the linear equation y=3 geometrically in two variables. We know that

the linear equation y=3 can also be written as 0.x + y = 3

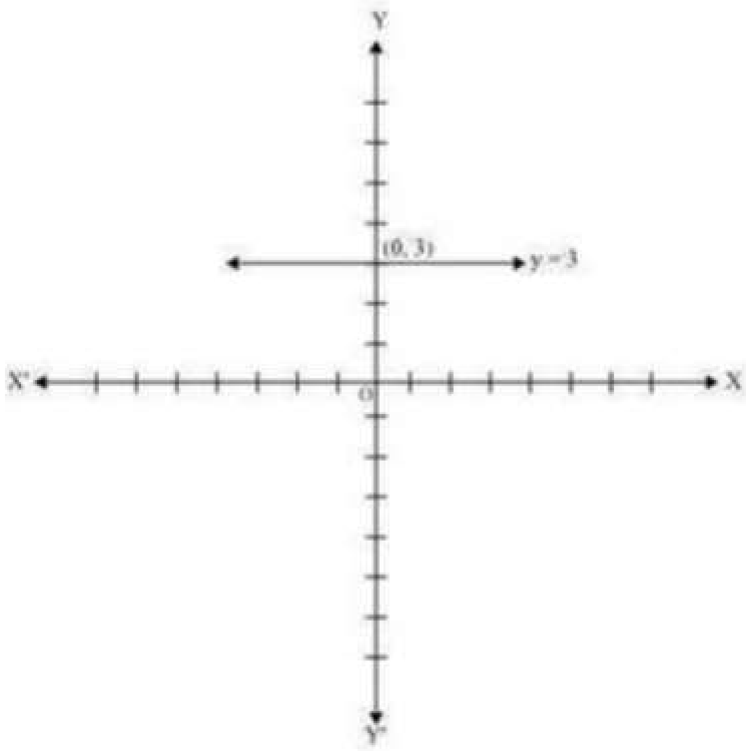
We can conclude that in two variables, the geometric representation of the linear equation y=3 will be same

as representing the graph of linear equation 0.x + y = 3

Given below is the representation of the linear equation 0.x + y = 3 on a graph.

We can optionally consider the given below table for plotting the linear equation 0.x + y = 3 on the graph.

X	1	0
y	3	3



(iii) **Give the geometric representations of 2x + 9 = 0 as an equation**

(i) In one variable (ii)

In two variables

Ans. (i) We need to represent the linear equation 2x + 9 = 0 geometrically in one variable.

We know that the linear equation 2x+9 = 0 can also be written as $x = -\frac{9}{2}$ or x = -4.5. We can conclude that in one variable, the geometric representation of the linear equation 2x + 9 = 0 will be same as representing the number -4.5 on a number line.

Given below is the representation of number -4.5 on the number line.

(ii) We need to represent the linear equation 2x+9 = 0 geometrically in two variables. We know that the linear equation 2x+9 = 0 can also be written as 2x+ 0.y + 9 = 0

We can conclude that in two variables, the geometric representation of the linear equation $2x+9 = 0$ will be same as representing the graph of linear equation $2x+ 0.y + 9 = 0$.

Given below is the representation of the linear equation $2x+ 0.y + 9 = 0$ on a graph.

We can optionally consider the given below table for plotting the linear equation

$2x+ 0.y + 9 = 0$ on the graph.

x	-4.5	-4.5	-4.5	-4.5	-4.5
y	0	1	2	-1	-2