PUNA INTERNATIONAL SCHOOL

- **CLASS – 9**
- **SUBJECT - MATHS**
- **CHAPTER - 6**

SAMPLE NOTE-BOOK

SUBJECT: MATHS **STANDARD – 9TH**

CHAPTER – 06

1. In Fig. 6.13, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70^\circ$ and $\angle BOD = 40^\circ$, find $\angle BOE$ and reflex $\angle COE$.

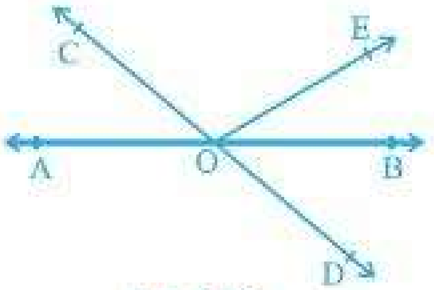


Fig. 6.13

Ans. We are given that $\angle AOC + \angle BOE = 70^\circ$ and $\angle BOD = 40^\circ$.

We need to find $\angle BOE$ and reflex $\angle COE$.

From the given figure, we can conclude that $\angle AOC, \angle COE$ and $\angle BOE$ form a linear pair.

We know that sum of the angles of a linear pair is 180°.

$\angle AOC + \angle COE + \angle BOE = 180^\circ$

$\therefore \angle AOC + \angle BOE + \angle COE = 180^\circ$

$\Rightarrow 70^\circ + \angle COE = 180^\circ$

$\Rightarrow \angle COE = 180^\circ - 70^\circ$

$= 110^\circ$.

Reflex $\angle COE = 360^\circ - \angle COE$

$= 360^\circ - 110^\circ$

$= 250^\circ$.

$\angle AOC = \angle BOD$ (Vertically opposite angles), or

$\angle BOD + \angle BOE = 70^\circ$.

But, we are given that $\angle BOD = 40^\circ$.

$40^\circ + \angle BOE = 70^\circ$

$\angle BOE = 70^\circ - 40^\circ$

$= 30^\circ$.

Therefore, we can conclude that $\text{Reflex } \angle COE = 250^\circ$ and $\angle BOE = 30^\circ$.

2. In Fig. 6.14, lines XY and MN intersect at O. If $\angle POY = 90^\circ$ and *a*:*b* = 2 : 3, find *c*.

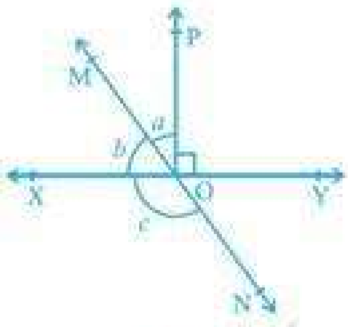


Fig. 6.14

Ans. We are given that $\angle POY = 90^\circ$ and $a : b = 2 : 3$.

We need find the value of *c* in the given figure.

Let *a* be equal to 2*x* and *b* be equal to *3x*.

$\because\ a + b = 90^\circ \Rightarrow 2x + 3x = 90^\circ \Rightarrow 5x = 90^\circ$

$\Rightarrow x = 18^\circ$

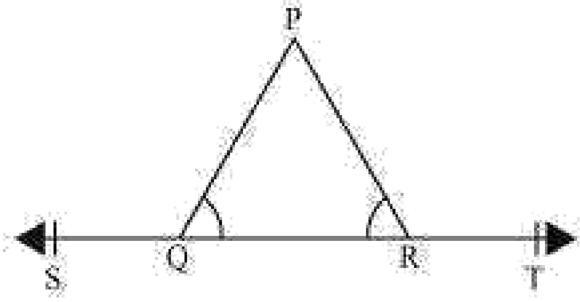
Therefore $b = 3 \times 18^\circ = 54^\circ$

Now $b + c = 180^\circ$ [Linear pair]

$\Rightarrow 54^\circ + c = 180^\circ$

$\Rightarrow c = 180^\circ - 54^\circ = 126^\circ$

3. In the given figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.



Ans. We need to prove that $\angle PQS = \angle PRT$.

We are given that $\angle PQR = \angle PRQ$.

From the given figure, we can conclude that $\angle PQS$ and $\angle PQR$, and $\angle PRS$ and $\angle PRT$ form a linear pair.

We know that sum of the angles of a linear pair is 180°.

$\therefore \angle PQS + \angle PQR = 180^\circ$, and (i)

$\angle PRQ + \angle PRT = 180^\circ$. (ii)

From equations (i) and (ii), we can conclude that

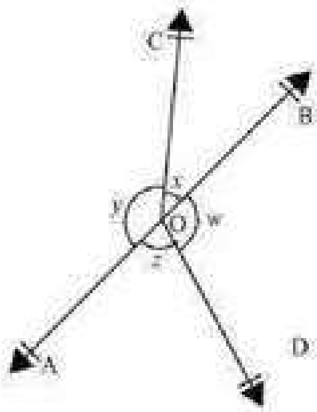
$\angle PQS + \angle PQR = \angle PRQ + \angle PRT$.

But, $\angle PQR = \angle PRQ$.

$\therefore \angle PQS = \angle PRT$.

Therefore, the desired result is proved.

4. In Fig. 6.16, if *x* + *y* = *w* + *z*, then prove that AOB is a line.



Ans. We need to prove that *AOB* is a line.

We are given that $x + y = w + z$.

We know that the sum of all the angles around a fixed point is 360°.

Thus, we can conclude that $\angle AOC + \angle BOC + \angle AOD + \angle BOD = 360^\circ$, or

$y + x + z + w = 360^\circ$.

But, $x + y = w + z$ (Given).

$2(y + x) = 360^\circ$.

$y + x = 180^\circ$.

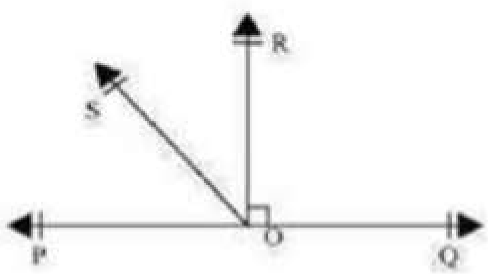
From the given figure, we can conclude that *y* and *x* form a linear pair.

We know that if a ray stands on a straight line, then the sum of the angles of linear pair formed by the ray with respect to the line is 180°.

.

Therefore, we can conclude that *AOB* is a line.

(i) In the given figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.



Ans. We need to prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.

We are given that *OR* is perpendicular to *PQ*, or

$\angle QOR = 90^\circ$.

From the given figure, we can conclude that $\angle POR$ and $\angle QOR$ form a linear pair.

We know that sum of the angles of a linear pair is 180°.

, or

$\angle POR = 90^\circ$.

From the figure, we can conclude that $\angle POR = \angle POS + \angle ROS$.

$\Rightarrow \angle POS + \angle ROS = 90^{\circ}$, or

$\angle ROS = 90^{\circ} - \angle POS$.(*i*)

From the given figure, we can conclude that $\angle QOS$ and $\angle POS$ form a linear pair.

We know that sum of the angles of a linear pair is 180°.

$\angle QOS + \angle POS = 180^{\circ}$, or

$\frac{1}{2}(\angle QOS + \angle POS) = 90^{\circ}$.(*ii*)

Substitute (*ii*) in (*i*), to get

$\angle ROS = \frac{1}{2}(\angle QOS + \angle POS) - \angle POS$

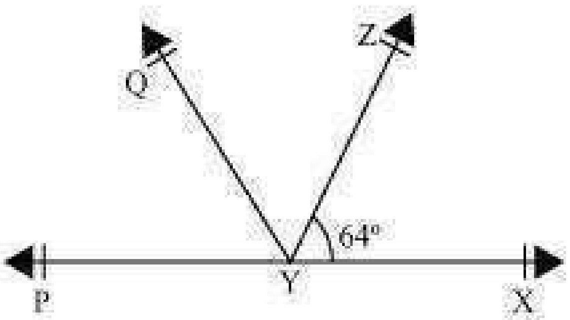
$= \frac{1}{2}(\angle QOS - \angle POS)$.

Therefore, the desired result is proved.

(i) It is given that $\angle XYZ = 64^{\circ}$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$

Ans. We are given that $\angle XYZ = 64^{\circ}$, *XY* is produced to *P* and *YQ* bisects $\angle ZYP$.

We can conclude the given below figure for the given situation:



We need to find $\angle XYQ$ and reflex $\angle QYP$.

From the given figure, we can conclude that $\angle XYZ$ and $\angle ZYP$ form a linear pair.

We know that sum of the angles of a linear pair is 180°.

.

But .

$\Rightarrow 64^{\circ} + \angle ZYP = 180^{\circ}$

$\Rightarrow \angle ZYP = 116^{\circ}.$

Ray *YQ* bisects $\angle ZYP$, or

$\angle QYZ = \angle QYP = \frac{116^{\circ}}{2} = 58^{\circ}.$

$\angle XYQ = \angle QYZ + \angle XYZ$

$= 58^{\circ} + 64^{\circ} = 122^{\circ}.$

Reflex $\angle QYP = 360^{\circ} - \angle QYP$

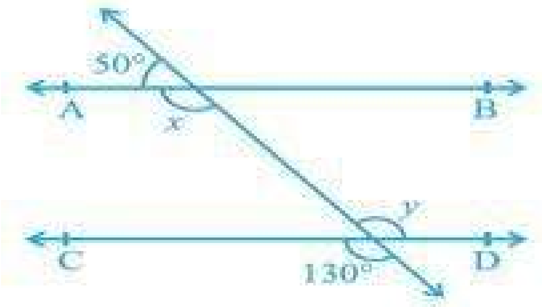
$= 360^{\circ} - 58^{\circ}$

$= 302^{\circ}.$

Therefore, we can conclude that $\angle XYQ = 122^{\circ}$ and Reflex $\angle QYP = 302^{\circ}$

<u>Chapter - 6</u>
<u>Lines and Angles (Ex. 6.2)</u>

1. In the given figure, find the values of x and y and then show that AB || CD.



Ans. We need to find the value of *x* and *y* in the figure given below and then prove that $AB \parallel CD$.

From the figure, we can conclude that

$y = 130^\circ$ (Vertically opposite angles), and

x and 50° form a pair of linear pair.

We know that the sum of linear pair of angles is 180°.

$x + 50^\circ = 180^\circ$

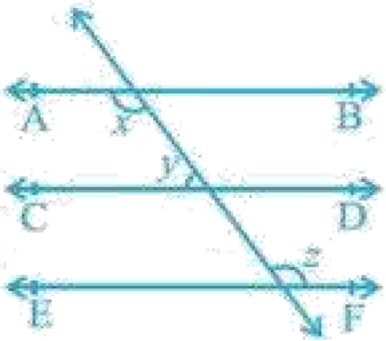
$x = 130^\circ$.

$x = y = 130^\circ$.

From the figure, we can conclude that x and y form a pair of alternate interior angles corresponding to the lines *AB* and *CD*.

Therefore, we can conclude that $x = 130^\circ, y = 130^\circ$ and $AB \parallel CD$.

2. In the given figure, if AB || CD, CD || EF and y : z = 3: 7, find x.



Ans. We are given that $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$.

We need to find the value of x in the figure given below.

We know that lines parallel to the same line are also parallel to each other.

We can conclude that $AB \parallel CD \parallel EF$.

Let Angles be $y = 3a$ and $z = 7a$.

We know that angles on same side of a transversal are supplementary.

.

$x = z$ (Alternate interior angles)

$z + y = 180^\circ$, or

$7a + 3a = 180^\circ$

$\Rightarrow 10a = 180^\circ$

$a = 18^\circ$.

$z = 7a = 126^\circ$

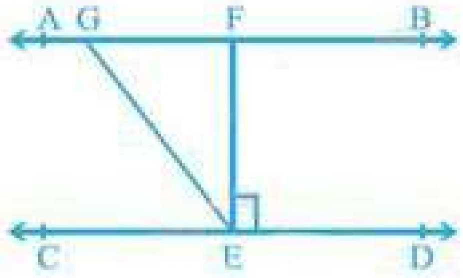
$y = 3a = 54^\circ$.

Now $x + 54^\circ = 180^\circ$

$x = 126^\circ$.

Therefore, we can conclude that $x = 126^\circ$.

3. **In the given figure, If AB || CD, $EF \perp CD$ and $\angle GED = 126^\circ$, find** $\angle AGE, \angle GEF$ and $\angle FGE$.



Ans. We are given that $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126^\circ$.

We need to find the value of $\angle AGE, \angle GEF$ and $\angle FGE$ in the figure given below.

$\angle GED = 126^\circ$

$\angle GED = \angle FED + \angle GEF$.

But, $\angle FED = 90^\circ$.

$126^\circ = 90^\circ + \angle GEF$

$\Rightarrow \angle GEF = 36^\circ$.

$\because \angle AGE = \angle GED$ (Alternate angles)

$\therefore \angle AGE = 126^\circ$.

From the given figure, we can conclude that $\angle FED$ and $\angle FEC$ form a linear pair.

We know that sum of the angles of a linear pair is 180°.

$\angle FED + \angle FEC = 180^\circ$

$\Rightarrow 90^\circ + \angle FEC = 180^\circ$

$\Rightarrow \angle FEC = 90^\circ$

$\angle FEC = \angle GEF + \angle GEC$

$\therefore 90^\circ = 36^\circ + \angle GEC$

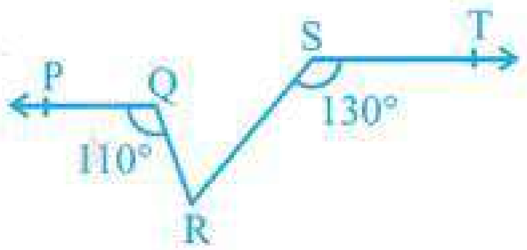
$\Rightarrow \angle GEC = 54^\circ$.

$\angle GEC = \angle FGE = 54^\circ$ (Alternate interior angles)

Therefore, we can conclude that $\angle AGE = 126^\circ, \angle GEF = 36^\circ$ and $\angle FGE = 54^\circ$.

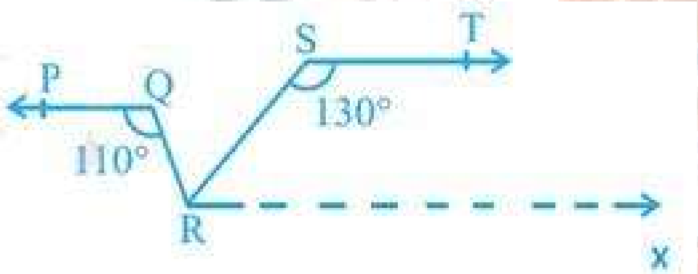
4. **In the given figure, if PQ || ST, $\angle PQR = 110^\circ$ and $\angle RST = 130^\circ$, find $\angle QRS$.**

   [Hint: Draw a line parallel to ST through point R.]



Ans. We are given that $PQ \parallel ST$, $\angle PQR = 110^\circ$ and $\angle RST = 130^\circ$.

We need to find the value of $\angle QRS$ in the figure.



We need to draw a line *RX* that is parallel to the line *ST*, to get

Thus, we have $ST \parallel RX$.

We know that lines parallel to the same line are also parallel to each other.

We can conclude that $PQ \parallel ST \parallel RX$.

(Alternate interior angles)

S0 $\angle QRX = 110^\circ$.

We know that angles on same side of a transversal are supplementary.

$\angle RST + \angle SRX = 180^\circ \Rightarrow 130^\circ + \angle SRX = 180^\circ$

$\Rightarrow \angle SRX = 180^\circ - 130^\circ = 50^\circ.$

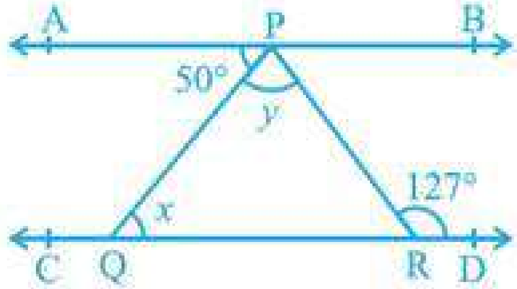
From the figure, we can conclude that

$\angle QRX = \angle SRX + \angle QRS \Rightarrow 110^\circ = 50^\circ + \angle QRS$

$\Rightarrow \angle QRS = 60^\circ.$

Therefore, we can conclude that $\angle QRS = 60^\circ$.

5. In the given figure, if AB || CD, $\angle APQ = 50^\circ$ and $\angle PRD = 127^\circ$, find x and y.



Ans. We are given that $AB \parallel CD$, $\angle APQ = 50^\circ$ and $\angle PRD = 127^\circ$.

We need to find the value of *x* and *y* in the figure.

$\angle APQ = x = 50^\circ$. (Alternate interior angles)

(Alternate interior angles)

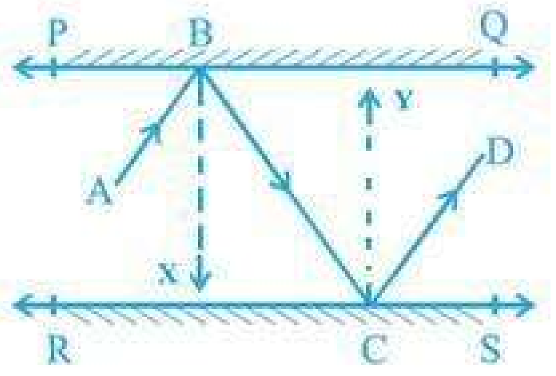
$\angle APR = \angle QPR + \angle APQ.$

$127^\circ = y + 50^\circ \quad \Rightarrow y = 77^\circ.$

Therefore, we can conclude that $x = 50^\circ$ and $y = 77^\circ$.

(i) In the given figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B, the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD. Prove that AB || CD.

Ans. We are given that *PQ* and *RS* are two mirrors that are parallel to each other.



We need to prove that $AB \parallel CD$ in the figure.

Let us draw lines *BX* and *CY* that are parallel to each other, to get $AB \parallel CD$ We know

that according to the laws of reflection

$\angle ABX = \angle CBX$ and $\angle BCY = \angle DCY$.

$\angle BCY = \angle CBX$ (Alternate interior angles)

We can conclude that $\angle ABX = \angle CBX = \angle BCY = \angle DCY$.

From the figure, we can conclude that

and

Therefore, we can conclude that $\angle ABC = \angle DCB$.

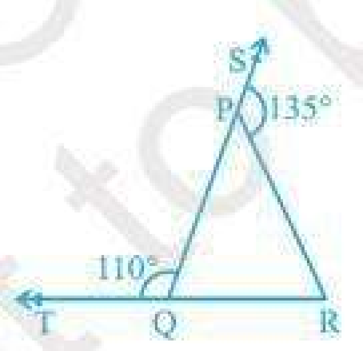
From the figure, we can conclude that $\angle ABC$ and $\angle DCB$ form a pair of alternate interior angles corresponding to the lines *AB* and *CD*, and transversal *BC*.

Therefore, we can conclude that $AB \parallel CD$.

CHAPTER 6

Lines and Angles
(Ex. 6.3)

1. **In the given figure, sides QP and RQ of ΔPQR are produced to points S and T respectively. If ∠ SPR = 135º and ∠ PQT = 110º, find ∠ PRQ.**



Ans. We are given that $\angle SPR = 135^\circ$ and $\angle PQT = 110^\circ$.

We know that the sum of angles of a linear pair is 180°.

$\angle SPR + \angle RPQ = 180^\circ$. (Linear Pair axiom)

and $\angle PQT + \angle PQR = 180^\circ$. (Linear Pair axiom)

$135^\circ + \angle RPQ = 180^\circ$, and $110^\circ + \angle PQR = 180^\circ$.

Or, $\angle RPQ = 45^\circ$, and $\angle PQR = 70^\circ$.

From the figure, we can conclude that

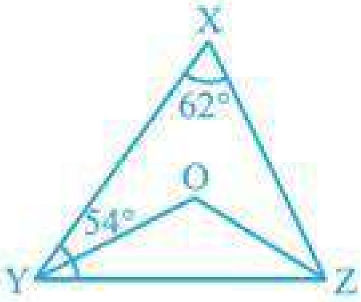
$\angle PQR + \angle RPQ + \angle PRQ = 180^\circ$. (Angle sum property)

$\Rightarrow 70^\circ + 45^\circ + \angle PRQ = 180^\circ \Rightarrow 115^\circ + \angle PRQ = 180^\circ$

$\Rightarrow \angle PRQ = 65^\circ$.

Therefore, we can conclude that $\angle PRQ = 65^\circ$.

2. **In the given figure, $\angle$ X = 62º, $\angle$ XYZ = 54º. If YO and ZO are the bisectors of $\angle$ XYZ and $\angle$ XZY respectively of ΔXYZ, find $\angle$ OZY and $\angle$ YOZ.**



Ans. We are given that $\angle X = 62^\circ, \angle XYZ = 54^\circ$ and *YO* and *ZO* are bisectors of $\angle XYZ$ and $\angle XZY$, respectively.

We need to find $\angle OZY$ and $\angle YOZ$ in the figure.

From the figure, we can conclude that

in ΔXYZ $\angle X + \angle XYZ + \angle XZY = 180^\circ$. (Angle sum property)

$\Rightarrow 62^\circ + 54^\circ + \angle XZY = 180^\circ \Rightarrow 116^\circ + \angle XZY = 180^\circ$

$\Rightarrow \angle XZY = 64^\circ$.

We are given that *OY* and *OZ* are the bisectors of $\angle XYZ$ and $\angle XZY$, respectively.

$\angle XYO = \angle ZYO = \frac{54}{2} = 27^\circ$ and $\angle OZY = \angle XZO = \frac{64}{2} = 32^\circ$.

From the figure, we can conclude that in ΔOYZ

$\angle OYZ + \angle OZY + \angle YOZ = 180^\circ$ (Angle sum property)

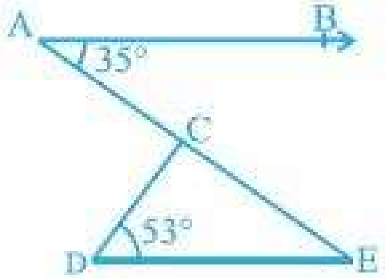
$27^\circ + 32^\circ + \angle YOZ = 180^\circ$

$\Rightarrow 59^\circ + \angle YOZ = 180^\circ$

$\Rightarrow \angle YOZ = 121^\circ$.

Therefore, we can conclude that $\angle YOZ = 121^\circ$ and $\angle OZY = 32^\circ$.

3. In the given figure, if AB || DE, ∠ BAC = 35º and ∠ CDE = 53º, find ∠ DCE.



Ans. We are given that $AB \parallel DE$, $\angle BAC = 35^\circ$ and $\angle CDE = 53^\circ$.

We need to find the value of $\angle DCE$ in the figure given below.

From the figure, we can conclude that

$\angle BAC = \angle CED = 35^\circ$ (Alternate interior)

From the figure, we can conclude that in ΔDCE

$\angle DCE + \angle CED + \angle CDE = 180^\circ$ (Angle sum property)

$\angle DCE + 35^\circ + 53^\circ = 180$

$\Rightarrow \angle DCE + 88^\circ = 180^\circ$

$\Rightarrow \angle DCE = 92^\circ$.

Therefore, we can conclude that $\angle DCE = 92^\circ$.

4. **In the given figure, if lines PQ and RS intersect at point T, such that ∠ PRT = 40º, ∠ RPT = 95º and ∠ TSQ = 75º, find ∠ SQT.**

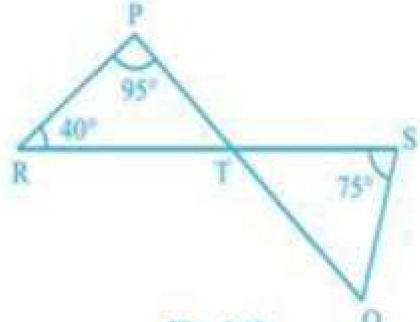


Fig. 6.42

Ans. We are given that $\angle PRT = 40^\circ$, $\angle RPT = 95^\circ$ and $\angle TSQ = 75^\circ$.

We need to find the value of $\angle SQT$ in the figure.

From the figure, we can conclude that in

ΔRTP $\angle PRT + \angle RTP + \angle RPT = 180^\circ$ (Angle sum property)

$40^\circ + \angle RTP + 95^\circ = 180^\circ$

$\Rightarrow \angle RTP + 135^\circ = 180^\circ$

$\Rightarrow \angle RTP = 45^\circ$.

From the figure, we can conclude that

$\angle RTP = \angle STQ = 45^\circ$ (Vertically opposite angles)

From the figure, we can conclude that in ΔSTQ

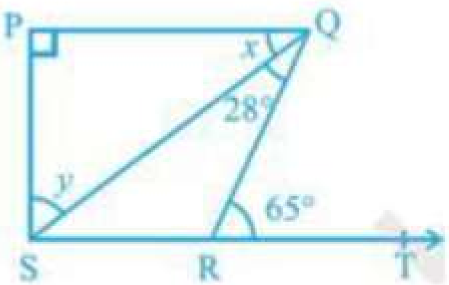
$\angle SQT + \angle STQ + \angle TSQ = 180^\circ$ (Angle sum property)

$\angle SQT + 45^\circ + 75^\circ = 180^\circ \Rightarrow \angle SQT + 120^\circ = 180^\circ$

$\Rightarrow \angle SQT = 60^\circ$.

Therefore, we can conclude that $\angle SQT = 60^\circ$.

(i) In the given figure, if $PQ \perp PS$, PQ || SR, $\angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$, then find the values of x and y.



Ans. We are given that $PQ \perp PS, PQ \parallel SR, \angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$.

We need to find the values of *x* and *y* in the figure.

We know that "If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles."

From the figure, we can conclude that

$\angle SQR + \angle QSR = \angle QRT$, or

$28^\circ + \angle QSR = 65^\circ$

$\Rightarrow \angle QSR = 37^\circ$

From the figure, we can conclude that

$x = \angle QSR = 37^\circ$ (Alternate interior angles) From

the figure, we can conclude that ΔPQS

$\angle PQS + \angle QSP + \angle QPS = 180^\circ$ (Angle sum property)

$\angle QPS = 90^\circ \quad (PQ \perp PS)$

$x + y + 90^\circ = 180^\circ$

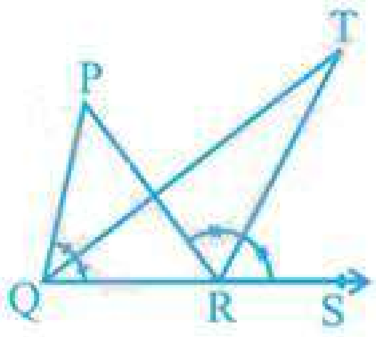
$\Rightarrow y + 37^\circ + 90^\circ = 180^\circ$

$\Rightarrow y + 127^\circ = 180^\circ \Rightarrow y = 53^\circ$

Therefore, we can conclude that $x = 37^\circ y = 53^\circ$,

6. In the given figure, the side QR of ΔPQR is produced to a point S. If the bisectors of

$\angle PQR$ and $\angle PRS$ **meet at point T, then prove that** $\angle QTR = \frac{1}{2} \angle QPR$.



Ans. We need to prove that $\angle QTR = \frac{1}{2}\angle QPR$ in the figure given below.

We know that "If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles."

From the figure, we can conclude that in ΔQTR, $\angle TRS$ is an exterior angle

$\angle QTR + \angle TQR = \angle TRS$, or

$\angle QTR = \angle TRS - \angle TQR$ …(i)

From the figure, we can conclude that in ΔPQR, $\angle PRS$ is an exterior angle

$\angle QPR + \angle PQR = \angle PRS$.

We are given that QT and RT are angle bisectors of $\angle PQR$ and $\angle PRS$.

$\angle QPR + 2\angle TQR = 2\angle TRS$

$\angle QPR = 2(\angle TRS - \angle TQR)$.

We need to substitute equation (*i*) in the above equation, to get

$\angle QPR = 2\angle QTR$, or

$\angle QTR = \frac{1}{2}\angle QPR$.

Therefore, we can conclude that the desired result is prove

PUNA INTERNATIONAL SCHOOL

- **CLASS – 9**
- **SUBJECT - MATHS**
- **CHAPTER - 7**

SAMPLE NOTE-BOOK

Shree Swaminarayan Gurukul, Zundal

पुनना International School

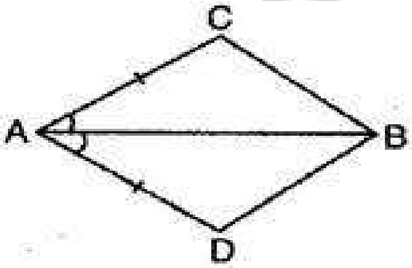
Class 9

CHAPTER 7

Triangles

(Ex. 7.1)

(i) In quadrilateral ABCD (See figure). AC = AD and AB bisects ∠A. Show that ΔABC ≅ ΔABD. What can you say about BC and BD?



Ans. Given: In quadrilateral ABCD, AC = AD and AB bisects ∠A.

To prove: ΔABC ≅ ΔABD

Proof: In ΔABC and ΔABD,

AC = AD [Given]

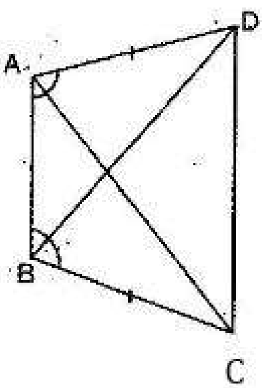
∠BAC = ∠BAD [∵ AB bisects ∠A]

AB = AB [Common]

∴ ΔABC ≅ ΔABD [By SAS congruency]

Thus **BC = BD** [By C.P.C.T.]

(i) ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA. (See figure). Prove that:



(i) Δ**ABD** $\cong$ Δ **BAC**
(i) BD=AC

(ii) $\angle$ **ABD** = $\angle$ **BAC**

Ans. (i) In ΔABC and ΔBAD,

BC = AD [Given]

$\angle$ DAB = $\angle$ CBA [Given]

AB = AB [Common]

$\therefore$ ΔABC $\cong$ Δ ABD [By SAS congruency]

Thus AC = BD [By C.P.C.T.]

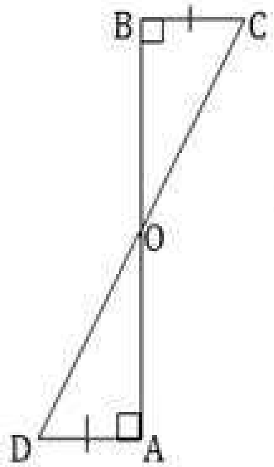
(i) Since ΔABC $\cong$ Δ ABD

$\therefore$ **AC = BD** [By C.P.C.T.]

, Since ΔABC $\cong$ Δ ABD

$\therefore$ $\angle$ **ABD** = $\angle$ **BAC** [By C.P.C.T.]

(iv) AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB (See figure)



Ans. In ΔBOC and ΔAOD,

$\angle$ OBC = $\angle$ OAD = 90° [Given]

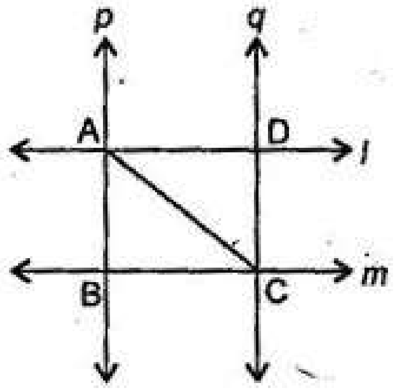
$\angle$ BOC = $\angle$ AOD [Vertically Opposite angles]

BC = AD [Given]

$\therefore$ ΔBOC $\cong$ Δ AOD [By AAS congruency]

$\Rightarrow$ **OB = OA** [By C.P.C.T., Also, OC = OD again by C.P.C.T.]

4. l and m are two parallel lines intersected by another pair of parallel lines p and q (See figure). Show that ΔABC $\cong$ Δ CDA.



Ans. AC being a transversal. [Given]

Therefore ∠DAC = ∠ACB [Alternate angles]

Now p ∥ q [Given]

And AC being a transversal. [Given]

Therefore $\angle$ BAC = $\angle$ ACD [Alternate angles]

Now In ΔABC and ΔADC,

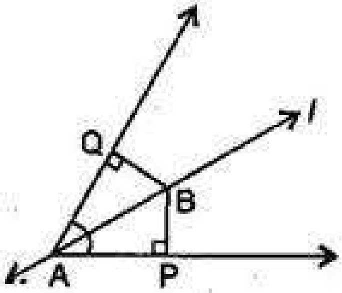
$\angle$ ACB = $\angle$ DAC [Proved above]

$\angle$ BAC = $\angle$ ACD [Proved above]

AC = AC [Common]

$\therefore$ Δ**ABC** $\cong$ Δ **CDA** [By ASA congruency]

5. Line l is the bisector of the angle A and B is any point on BP and BQ are perpendiculars from B to the arms of ∠A. Show that:



(v) △APB ≅ △AQB

(vi) BP = BQ or B is equidistant from the arms of ∠A (See figure). Ans. Given:

Line l bisects ∠A.

∴ ∠BAP = ∠BAQ

(i) In △ABP and △ABQ,

∠BAP = ∠BAQ [Given]

∠BPA = ∠BQA = 90° [Given]

AB = AB [Common]

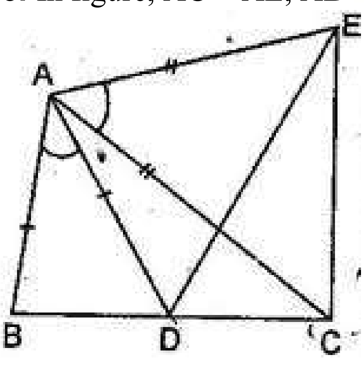
∴ △APB ≅ △AQB [By AAS congruency]

(ii) Since △APB ≅ △AQB

∴ BP = BQ [By C.P.C.T.]

⇒ B is equidistant from the arms of ∠A.

6. In figure, AC = AE, AB = AD and ∠BAD = ∠EAC. Show that BC = DE.



Ans. Given that $\angle BAD = \angle EAC$

Adding $\angle DAC$ on both sides, we get

$\angle BAD + \angle DAC = \angle EAC + \angle DAC$

$\Rightarrow \angle BAC = \angle EAD$ ……….(i)

Now in ΔABC and ΔADE,

AB = AD [Given]

AC = AE [Given]

$\angle BAC = \angle DAE$ [From eq. (i)]

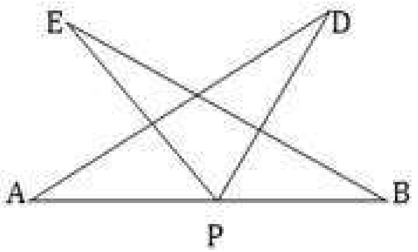
$\therefore \Delta ABC \cong \Delta ADE$ [By SAS congruency]

$\Rightarrow$ BC = DE [By C.P.C.T.]

(ii) AB is a line segment and P is the mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$. Show that:

(iii) $\Delta DAP \cong \Delta EBP$

(iv) AD = BE (See figure)



Ans. Given that $\angle EPA = \angle DPB$

Adding $\angle EPD$ on both sides, we get

$\angle EPA + \angle EPD = \angle DPB + \angle EPD$

$\Rightarrow \angle APD = \angle BPE$ ……….(i)

Now in ΔAPD and ΔBPE,

$\angle$ PAD = $\angle$ PBE [$\because$ $\angle$ BAD = $\angle$ ABE (given),

$\therefore$ $\angle$ PAD = $\angle$ PBE]

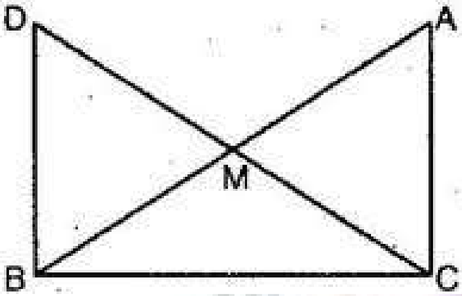
AP = PB [P is the mid-point of AB]

$\angle$ APD = $\angle$ BPE [From eq. (i)]

$\therefore$ ΔDAP $\cong$ Δ EBP [By ASA congruency]

$\Rightarrow$ AD = BE [By C.P.C.T.]

(iv) In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B. (See figure)



Show that:

(v) ΔAMC $\cong$ Δ BMD

(vi) $\angle$ DBC is a right angle.

(vii) ΔDBC $\cong$ Δ ACB

(iv) CM = $\frac{1}{2}$ AB

Ans. (i) In ΔAMC and ΔBMD,

AM = BM [M is the mid-point of AB]

$\angle$ AMC = $\angle$ BMD [Vertically opposite angles]

CM = DM [Given]

$\therefore$ ΔAMC $\cong$ Δ BMD [By SAS congruency]

$\therefore$ $\angle$ ACM = $\angle$ BDM ……….(i)

∠ CAM = ∠ DBM and AC = BD [By C.P.C.T.]

5. For two lines AC and DB and transversal DC, we have,

∠ ACD = ∠ BDC [Alternate angles]

∴ AC || DB

Now for parallel lines AC and DB and for transversal BC.

$\angle DBC + \angle ACB = 180^0$[cointerior angles].....(ii)

But Δ ABC is a right angled triangle, right angled at C.

∴ ∠ ACB = 90^0 ……….(iii)

Therefore ∠ DBC = 90^0 [Using eq. (ii) and (iii)]

⇒ ∠ DBC is a right angle.

6. Now in Δ DBC and Δ ABC,

DB = AC [Proved in part (i)]

∠ DBC = ∠ ACB = 90^0 [Proved in part (ii)]

BC = BC [Common]

∴ Δ DBC ≅ Δ ACB [By SAS congruency]

7. Since Δ DBC ≅ Δ ACB [Proved above]

∴ DC=AB

⇒ DM+CM=AB

⇒ CM+CM=AB[∵ DM=CM]

⇒ 2CM = AB

⇒ CM= $\frac{1}{2}$AB

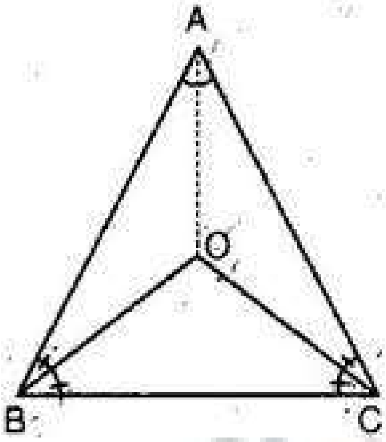
Ex. 7.2

(ii) In an isosceles triangle ABC, with AB = AC, the bisectors of ∠B and ∠C intersect each other at O. Join A to O. Show that:

(ii) OB=OC

(iii) AO bisects ∠A.

Ans. (i) ABC is an isosceles triangle in which AB = AC.



∴ ∠C = ∠B [Angles opposite to equal sides]

⇒ ∠OCA + ∠OCB = ∠OBA + ∠OBC

∵ OB bisects ∠B and OC bisects ∠C

∴ ∠OBA = ∠OBC and ∠OCA = ∠OCB

⇒ ∠OCB + ∠OCB = ∠OBC + ∠OBC

⇒ 2∠OCB=2∠OBC

⇒ ∠OCB = ∠OBC

Now in △OBC,

$\angle$OCB = $\angle$OBC [Proved above]

$\therefore$ OB = OC [Sides opposite to equal angles]

(iii) In ΔAOB and ΔAOC,

AB = AC [Given]

OA = OA [Common]

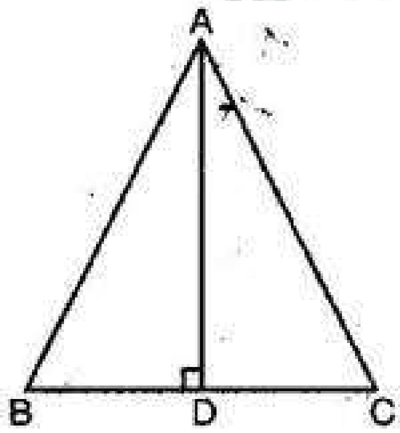
OB = OC [Prove above]

$\therefore$ ΔAOB $\cong$ ΔAOC [By SSS congruency]

$\Rightarrow$ $\angle$OAB = $\angle$OAC [By C.P.C.T.]

Hence AO bisects $\angle$A.

(ii) In ΔABC, AD is the perpendicular bisector of BC (See figure). Show that ΔABC is an isosceles triangle in which AB = AC.



Ans. In ΔADB and ΔADC,

BD = CD [AD bisects BC]

$\angle$ADB = $\angle$ADC = 90° [AD $\perp$ BC]

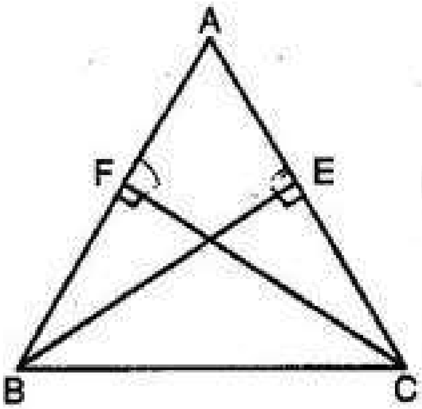
AD = AD [Common]

$\therefore$ ΔABD $\cong$ ΔACD [By SAS congruency]

$\Rightarrow$ AB = AC [By C.P.C.T.]

Therefore, ABC is an isosceles triangle with AB = AC. Hence, proved.

, ABC is an isosceles triangle in which altitudes BE and CF are drawn to equal sides AC and AB respectively (See the given figure). Show that these altitudes are equal.



Ans. In Δ ABE and Δ ACF,

$\angle$A= $\angle$A [Common]

$\angle$AEB = $\angle$AFC = 90° [Given]

AB = AC [Given]

$\therefore$ Δ ABE $\cong$ Δ ACF [By AAS congruency]

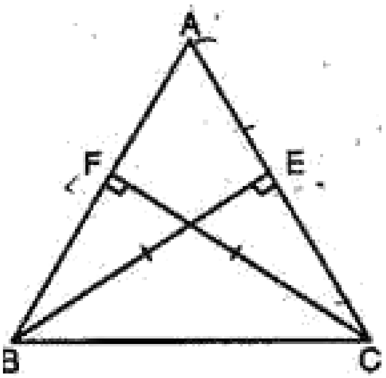
$\Rightarrow$ BE = CF [By C.P.C.T.]

$\Rightarrow$ **Altitudes are equal.**

(v) ABC is a triangle in which altitudes BE and CF to sides AC and AB are equal (See figure). Show that:

(vii) Δ ABE $\cong$ Δ ACF

(viii) AB = AC or Δ ABC is an isosceles triangle.



Ans. (i) In Δ ABE and Δ ACF,

$\angle$A= $\angle$A [Common]

$\angle$AEB = $\angle$AFC = 90° [Given]

BE = CF [Given]

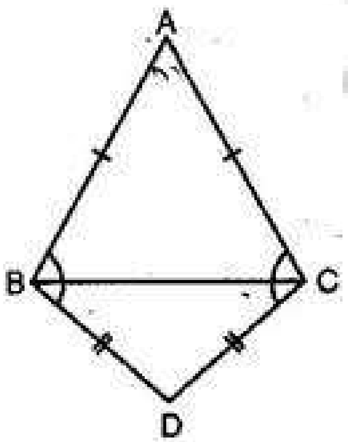
$\therefore$ Δ ABE $\cong$ Δ ACF [By AAS congruency]

(iii) Since Δ ABE $\cong$ Δ ACF

$\Rightarrow$ BE = CF [By C.P.C.T.]

$\Rightarrow$ ABC is an isosceles triangle.

(iii) ABC and DBC are two isosceles triangles on the same base BC (See figure). Show that $\angle$ABD = $\angle$ACD.



Ans. In isosceles triangle ABC,

AB = AC [Given]

$\angle$ACB = $\angle$ABC …….(i) [Angles opposite to equal sides]

Also in Isosceles triangle BCD.

BD=DC

$\therefore$ $\angle$BCD = $\angle$CBD ……….(ii) [Angles opposite to equal sides]

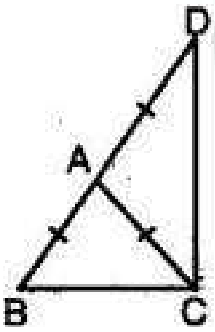
Adding eq. (i) and (ii),

$\angle$ACB + $\angle$BCD = $\angle$ABC + $\angle$CBD

$\Rightarrow$ $\angle$ACD = $\angle$ABD

Or $\angle$ABD = $\angle$ACD

(v) ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB. Show that $\angle$BCD is a right angle (See figure).



Ans. In isosceles triangle ABC,

AB = AC [Given]

$\angle$ACB = $\angle$ABC …….(i) [Angles opposite to equal sides]

Now AD = AB [By construction]

But AB = AC [Given]

$\therefore$ AD=AB=AC

$\Rightarrow$ AD=AC

Now in triangle ADC,

AD=AC

$\Rightarrow \angle$ ADC = $\angle$ ACD ………(ii) [Angles opposite to equal sides]

In triangle BCD,

$\Rightarrow \angle ABC + \angle BCD + \angle CDA = 180^0$ [Angle sum property]

$\Rightarrow \angle ACB + \angle BCD + \angle CDA = 180^0$ [Because $\angle$ ACB = $\angle$ ABC, see (i)]

$\Rightarrow \angle ACB + \angle ACB + \angle ACD + \angle CDA = 180^0$ [Because $\angle BCD = \angle ACB + \angle ACD$]

$\Rightarrow 2\angle ACB + \angle ACD + \angle CDA = 180^0$

$\Rightarrow 2\angle ACB + \angle ACD + \angle ACD = 180^0$ [Because $\angle$ ADC = $\angle$ ACD, see (ii)]

$\Rightarrow 2\angle ACB + 2\angle ACD = 180^0$

$\Rightarrow 2(\angle ACB + \angle ACD) = 180^0$ [Taking out 2 common]

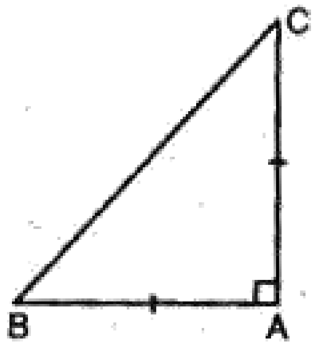
$\Rightarrow 2\angle BCD = 180^0$ [Because, $\angle ACD + \angle ACB = \angle BCD$]

$\Rightarrow \angle$ BCD = 90°

Hence $\angle$ BCD is a right angle.

(v) ABC is a right angled triangle in which $\angle$ A = 90° and AB = AC. Find $\angle$ B and $\angle$ C. Ans.

ABC is a right triangle in which,



$\angle$A = 90° And AB = AC

In ΔABC,

AB=AC

$\Rightarrow \angle$C = $\angle$B ……….(i)

We know that, in ΔABC,

$\angle$A+ $\angle$B+ $\angle$C= [Angle sum property]

$180^\circ \Rightarrow 90^\circ + \angle$B+

$\angle$B=

[$\angle$A = 90° (given) and $\angle$B = $\angle$C (from eq. (i)]

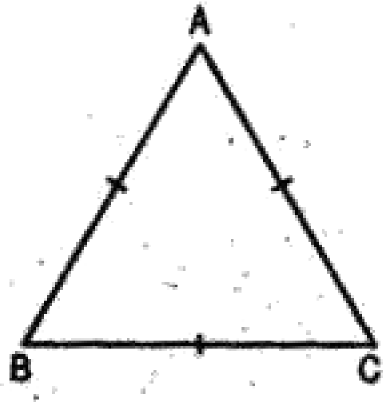
$\Rightarrow 2\angle B = 90^\circ$

$\Rightarrow \angle B = 45^\circ$

Also $\angle$C = 45° [$\angle$B = $\angle$C]

(viii) Show that the angles of an equilateral triangle are 60° each.

Ans. Let ABC be an equilateral triangle.



$\therefore$ AB=BC=AC

$\Rightarrow$ AB=BC

$\Rightarrow \angle C = \angle A$ ……….(i)

Similarly, AB = AC

$\Rightarrow \angle C = \angle B$ ……….(ii)

From eq. (i) and (ii),

$\angle A = \angle B = \angle C$ ……….(iii)

Now in Δ ABC

$\angle A + \angle B + \angle C = 180^\circ$ ……….(iv)

$\Rightarrow \angle A + \angle A + \angle A = 180^\circ$

$\Rightarrow 3\angle A = 180^\circ$

$\Rightarrow \angle A = 60^\circ$

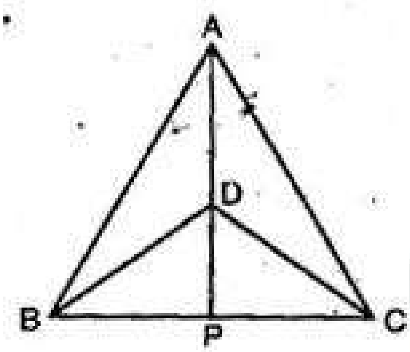
Since $\angle A = \angle B = \angle C$ [From eq. (iii)]

$\therefore \angle A = \angle B = \angle C =$

Hence each angle of equilateral triangle is 60°.

Ex. 7.3

(iii) ΔABC and ΔDBC are two isosceles triangles on the same base BC and vertices A and D are on the same side of BC (See figure). If AD is extended to intersect BC at P, show that:



(iv) $\Delta ABD \cong \Delta ACD$

(v) $\Delta ABP \cong \Delta ACP$

(vi) AP bisects $\angle$A as well as $\angle$D.

(vii) AP is the perpendicular bisector of BC. Ans. (i)

ΔABC is an isosceles triangle.

$\therefore$ AB=AC

ΔDBC is an isosceles triangle.

$\therefore$ BD=CD

Now in ΔABD and ΔACD,

AB = AC [Given]

BD = CD [Given] AD

= AD [Common]

$\therefore \Delta$ ABD $\cong \Delta$ ACD [By SSS congruency]

$\Rightarrow \angle$ BAD = $\angle$ CAD [By C.P.C.T.] ……….(i)

(iv) Now in ΔABP and ΔACP,

AB = AC [Given]

$\angle$ BAD = $\angle$ CAD [From eq. (i)]

AP = AP [Common]

$\therefore \Delta$ ABP $\cong \Delta$ ACP [By SAS congruency]

Also, BP = CP [By C.P.C.T.].............(ii)

(iii) Since Δ ABP $\cong \Delta$ ACP [From part (ii)]

$\Rightarrow \angle$ BAP = $\angle$ CAP [By C.P.C.T.]

$\Rightarrow$ AP bisects $\angle$ A.

In ΔBDP and ΔCDP,

BD = CD [Given]

DP = DP [Commmon] BP

= CP [From eqn (ii)]

Therefore, $\Delta BDP \cong \Delta CDP$ [By SSS Conruency]

=> $\angle BDP = \angle CDP$ [By C.P.C.T.].............(iii)

and $\angle BPD = \angle CPD$ [By C.P.C.T.] ....(iv)

Hence, AP bisects $\angle D$ from (iii)

, Since=> $\angle BPD = \angle CPD$[By eqn (iv)]

Now $\angle$ BPD + $\angle$ CPD = 180° [Linear pair]

$\Rightarrow \angle BPD + \angle BPD = 180^\circ$ [Using eq. (iii)]

$\Rightarrow 2\angle BPD = 180^\circ$

$\Rightarrow \angle BPD = 90^\circ$

$\Rightarrow AP \perp BC$ ……….(v)

From eq. (iv) and (v), we have AP = BP and AP $\perp$ BC. So, collectively AP is perpendicular bisector of BC.

(vi) AD is an altitude of an isosceles triangle ABC in which AB = AC. Show that:

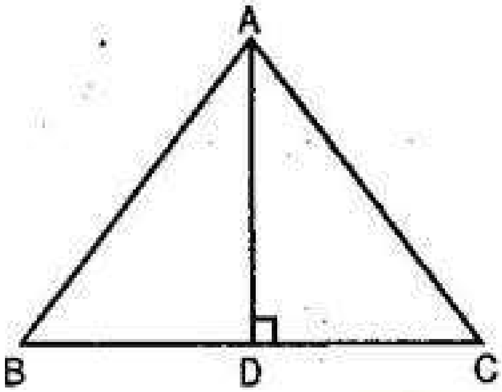
(i) AD bisects BC. (ii)

AD bisects $\angle A$.

Ans. In ΔABD and ΔACD,

AB = AC [Given]

$\angle ADB = \angle ADC = 90^\circ$ [AD $\perp$ BC]



AD = AD [Common]

$\therefore \Delta ABD \cong \Delta ACD$ [RHS rule of congruency]

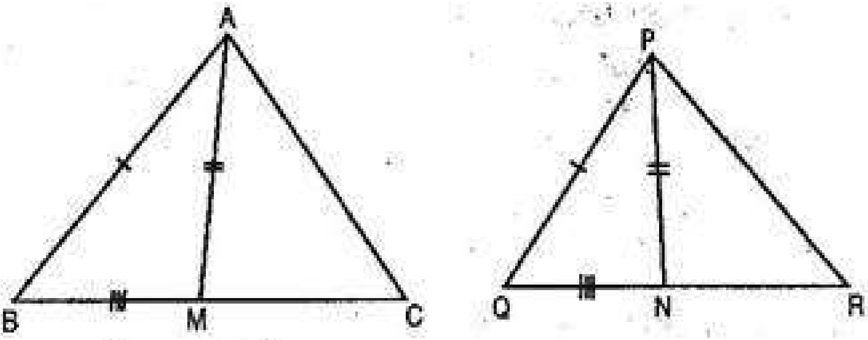
$\Rightarrow$ BD = DC [By C.P.C.T.]

$\Rightarrow$ AD bisects BC

Also $\angle BAD = \angle CAD$ [By C.P.C.T.]

$\Rightarrow$ AD bisects $\angle A$. Hence proved.

(ix) Two sides AB and BC and median AM of the triangle ABC are respectively equal to side PQ and QR and median PN of ΔPQR (See figure). Show that:



(iv) $\Delta ABM \cong \Delta PQN$

(v) $\Delta ABC \cong \Delta PQR$

Ans. AM is the median of ΔABC.

$\therefore$ BM = MC = $\frac{1}{2}$ BC ……….(i)

PN is the median of ΔPQR.

$\therefore$ QN = NR = $\frac{1}{2}$ QR ……….(ii)

Now BC = QR [Given] $\Rightarrow \frac{1}{2}$ BC = $\frac{1}{2}$ QR

$\therefore$ BM = QN ……….(iii)

(iv) Now in ΔABM and

ΔPQN, AB = PQ [Given]

AM = PN [Given]

BM = QN [From eq. (iii)]

$\therefore \Delta ABM \cong \Delta PQN$ [By SSS congruency]

$\Rightarrow \angle B = \angle Q$ [By C.P.C.T.] ……….(iv)

(v) In ΔABC and ΔPQR,

AB = PQ [Given]

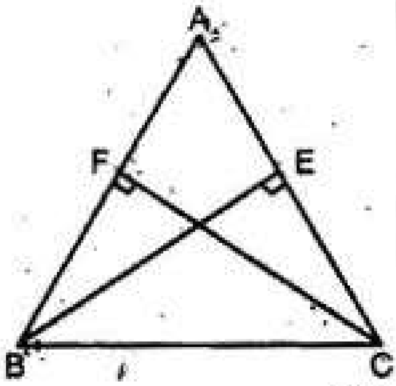
$\angle B = \angle Q$ [Prove above]

BC = QR [Given]

$\therefore \Delta ABC \cong \Delta PQR$ [By SAS congruency]

(vi) BE and CF are two equal altitudes of a triangle ABC. Using RHS congruence rule, prove that the triangle ABC is isosceles.

Ans. In ΔBEC and ΔCFB,



$\angle BEC = \angle CFB$ [Each 90°]

BC = BC [Common]

BE = CF [Given]

$\therefore \Delta BEC \cong \Delta CFB$ [RHS congruency]

$\Rightarrow$ EC = FB [By C.P.C.T.] …..(i)

Now In ΔAEB and ΔAFC

$\angle AEB = \angle AFC$ [Each 90°]

$\angle A = \angle A$ [Common]

BE = CF [Given]

$\therefore \Delta AEB \cong \Delta AFC$ [AAS congruency]

$\Rightarrow$ AE = AF [By C.P.C.T.] …………(ii)

Adding eq. (i) and (ii), we get,

EC+AE=FB+AF

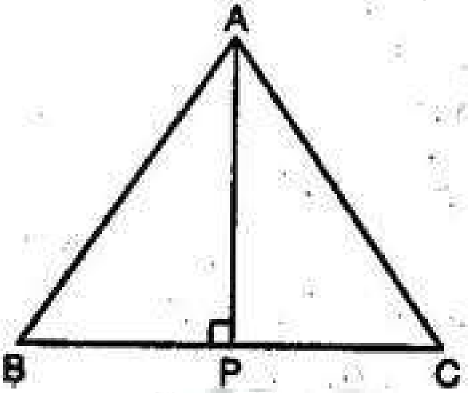
$\Rightarrow$ AB=AC

$\Rightarrow$ ABC is an isosceles triangle.

Hence proved.

(vi) ABC is an isosceles triangles with AB = AC. Draw AP $\perp$ BC and show that $\angle$B = $\angle$C. Ans.

Given: ABC is an isosceles triangle in which AB = AC



To prove: $\angle$B = $\angle$C

Construction: Draw AP $\perp$ BC

Proof: In ΔABP and ΔACP

$\angle$APB = $\angle$APC = 90° [By construction]

AB = AC [Given]

AP = AP [Common]

$\therefore$ ΔABP $\cong$ ΔACP [RHS congruency]

$\Rightarrow$ $\angle$B = $\angle$C [By C.P.C.T.]

Hence proved.

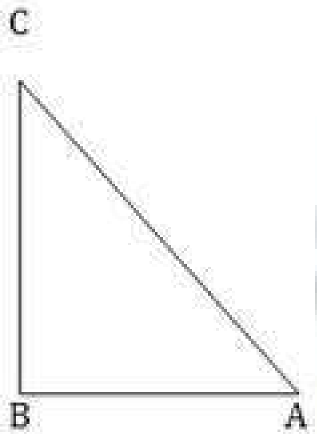
Ex. 7.4

1. Show that in a right angles triangle, the hypotenuse is the longest side.

Ans. Given: Let ABC be a right angled triangle, right angled at B.

To prove: Hypotenuse AC is the longest side.

Proof: In right angled triangle ABC,



$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle A + 90^\circ + \angle C = 180^\circ$ [$\because \angle B = 90^\circ$]

$\Rightarrow \angle A + \angle C = 180^\circ - 90^\circ$

=> $+ \quad C \quad 90$

And $\angle B = 90^\circ$

$\Rightarrow \angle B > \angle C$ and $\angle B > \angle A$

Since the greater angle has a longer side opposite to it.

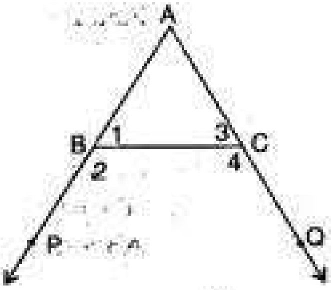
$\Rightarrow$ AC > AB and AC > BC

Therefore $\angle B$ being the greatest angle has the longest opposite side AC, i.e. hypotenuse.

Hence, proved.

2. In figure, sides AB and AC of ΔABC are extended to points P and Q respectively. Also

∠ PBC < ∠ QCB. Show that AC > AB.



Ans. Given: In ΔABC, ∠ PBC < ∠ QCB

To prove: AC > AB

Proof: In the given figure,

∠ 4 > ∠ 2 [Given]

Now ∠ 1 + ∠ 2 = 180° [Linear pair]

=> $\angle 1 = 180^0 - \angle 2$

And, ∠ 3 + ∠ 4 = 180°

=> $\angle 3 = 180^0 - \angle 4$

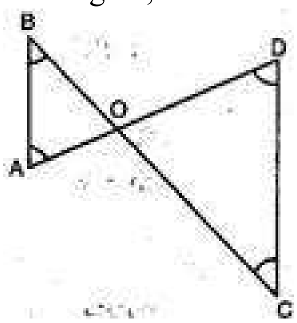
Because, ∠ 4 is greater than ∠ 2, therefore when we will subtract it from 180° we will get a value which would be lesser than the quantity obtained on deducting ∠ 2 from 180°.

∴ ∠ 1 > ∠ 3

$\Rightarrow$ AC > AB [Side opposite to greater angle is longer]

Hence, proved.

3. In figure, ∠ B < ∠ A and ∠ C < ∠ D. Show that AD < BC.



Ans. In ΔAOB,

$\angle A > \angle B$ [Given]

$\Rightarrow$ OB > OA ……….(i) [Side opposite to greater angle is longer]

Similarly, In ΔCOD,

$\angle D > \angle C$ [Given]

$\Rightarrow$ OC > OD ……….(ii) [Side opposite to greater angle is longer]

Adding eq. (i) and (ii),

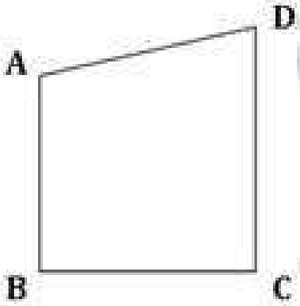
$OB+OC>OA+OD$

$\Rightarrow BC>AD$

$\Rightarrow AD<BC$

Hence, proved.

(iv) AB and CD are respectively the smallest and longest sides of a quadrilateral ABCD (See figure). Show that $\angle A > \angle C$ and $\angle B > \angle D$.

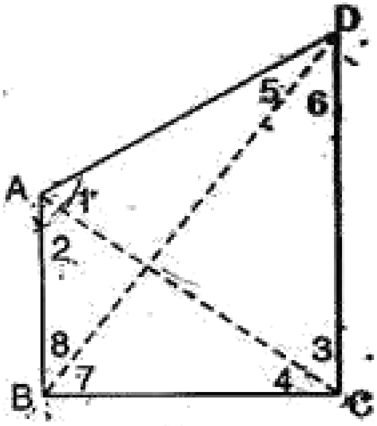


Ans. Given: ABCD is a quadrilateral with AB as smallest and CD as longest side.

To prove: (i) $\angle A > \angle C$ (ii) $\angle B > \angle D$

Construction: Join AC and BD.

Proof: (i) In ΔABC, AB is the smallest side.



$\therefore \angle 4 < \angle 2$ ……….(i)

[Angle opposite to smaller side is smaller]

In ADC, DC is the longest side.

$\therefore \angle 3 < \angle 1$ ……….(ii)

[Angle opposite to smaller side is smaller]

Adding eq. (i) and (ii),

$\angle 4 + \angle 3 < \angle 1 + \angle 2$

$\Rightarrow \angle C < \angle A$

=> $\angle A > \angle C$

(viii) In ΔABD, AB is the smallest side.

$\therefore \angle 5 < \angle 8$ ……….(iii)

[Angle opposite to smaller side is smaller] In

ΔBDC, DC is the longest side.

$\therefore \angle 6 < \angle 7$ ……….(iv)

[Angle opposite to smaller side is smaller]

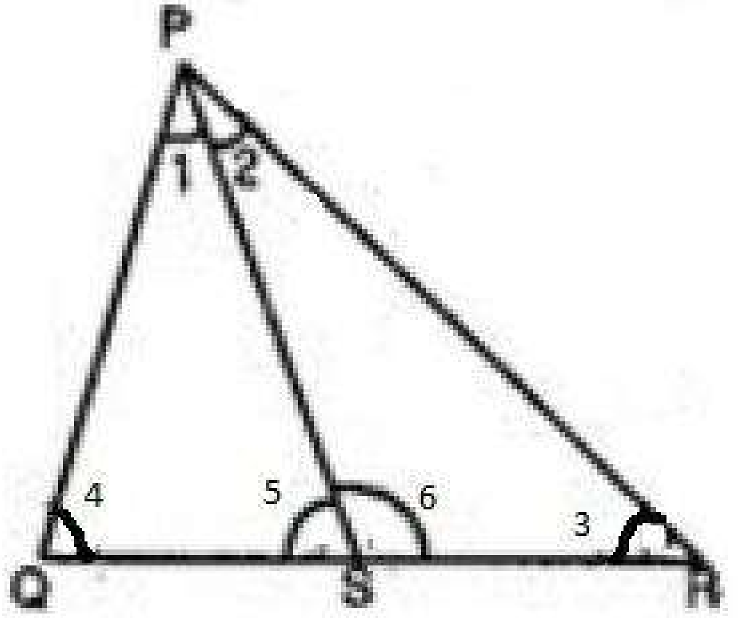
Adding eq. (iii) and (iv),

$\angle 5 + \angle 6 < \angle 7 + \angle 8$

$\Rightarrow \angle D < \angle B$

=> $\angle B > \angle D$

5. In figure, PR > PQ and PS bisects $\angle$ QPR. Prove that $\angle$ PSR > $\angle$ PSQ.



Ans. In ΔPQR, PR > PQ [Given]

$\therefore \angle 4 > \angle 3$ …..(i) [Angle opposite to longer side is greater]

Again $\angle 1 = \angle 2$…..(ii) [$\because$ PS is the bisector of $\angle$ P]

Now, $\angle 6$ is exterior angle of ΔPQS,

=> $\angle 6 = \angle 4 + \angle 1$ ……………………..(iii)

Again, $\angle 5$ is exterior angle of ΔPSR

=> $\angle 5 = \angle 2 + \angle 3$ …………… ………… (iv)

Adding (i) and (ii), we get :-

=> $\angle 4 + \angle 1 > \angle 2 + \angle 3$

=> $\angle 6 > \angle 5$ [From, (iii) and (iv)]

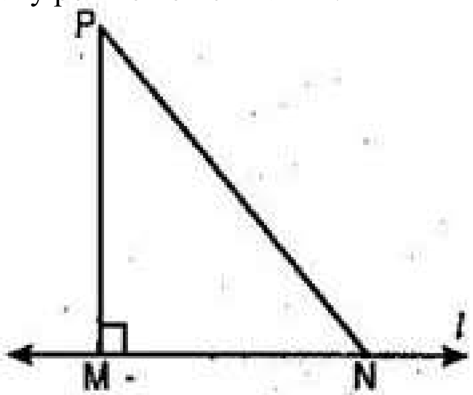
i.e. $\angle$ PSR > $\angle$ PSQ

Hence, Proved.

(v) Show that all the line segments drawn from a given point not on it, the perpendicular line segment is the shortest.

Ans. Given: l is a line and P is point not lying on l. PM $\perp l$ N is

any point on other than l M.



To prove: PM < PN

Proof: In ΔPMN $\angle$M is the right angle.

$\therefore$ N is an acute angle. (Angle sum property of Δ)

$\therefore \angle M > \angle N$

$\therefore$ PN > PM [Side opposite greater angle]

$\Rightarrow$ PM<PN

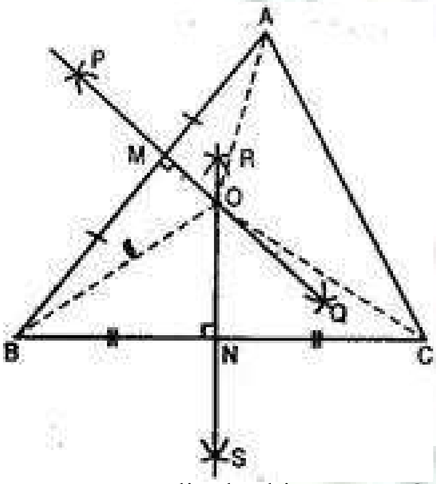
Hence of all line segments drawn from a given point not on it, the perpendicular is the shortest.

Ex. 7.5

(v) ABC is a triangle. Locate a point in the interior of ΔABC which is equidistant from all the vertices of ΔABC.

Ans. The point which is equidistant from all the vertices of a triangle is known as the circum-centre of the triangle. This point acts as the centre of a circle which can be drawn by passing through the vertices of the given triangle. And to find out the circum-centre we usually, draw the perpendicular bisectors of any two sides, their point of intersection is the required point which is equidistant from the vertices (being the radius). So we will proceed with drawing a circum-centre.

Let ABC be a triangle.



Draw perpendicular bisectors PQ and RS of sides AB and BC respectively of triangle ABC. Let PQ bisects AB at M and RS bisects BC at point N.

Let PQ and RS intersect at point O.

Join OA, OB and OC.

Now in ΔAOM and Δ BOM,

AM = MB [By construction]

$\angle$ AMO = $\angle$ BMO = 90° [By construction]

OM = OM [Common]

$\therefore$ ΔAOM $\cong$ Δ BOM [By SAS congruency]

⇒ OA = OB [By C.P.C.T.] …..(i)

Similarly, ΔBON $\cong$ ΔCON

⇒ OB = OC [By C.P.C.T.] …..(ii)

From eq. (i) and (ii),

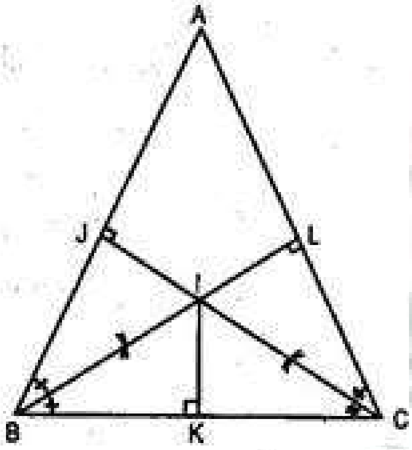
OA=OB=OC

Hence O, the point of intersection of perpendicular bisectors of any two sides of ΔABC equidistant from its vertices.

(ix) In a triangle locate a point in its interior which is equidistant from all the sides of the triangle.

Ans. The point which is equidistant from all the sides of a triangle is known as its in-centre and is the point of intersection of the angle bisectors. Hence we will proceed with finding the in-centre of the given triangle.

Let ABC be a triangle.



Draw bisectors of $\angle$ B and $\angle$ C.

Let these angle bisectors intersect each other at point I.
Draw IK $\perp$ BC

Also draw IJ $\perp$ AB and IL $\perp$ AC.

Join AI.

In ΔBIK and ΔBIJ,

$\angle$ IKB = $\angle$ IJB = 90° [By construction]

$\angle$ IBK = $\angle$ IBJ

[$\because$ BI is the bisector of $\angle$B (By construction)] BI =

BI [Common]

$\therefore$ ΔBIK $\cong$ ΔBIJ [ASA criteria of congruency]

$\therefore$ IK = IJ [By C.P.C.T.] ……….(i)

Similarly, ΔCIK $\cong$ ΔCIL

$\therefore$ IK = IL [By C.P.C.T.] ……….(ii)

From eq (i) and (ii),

IK=IJ=IL

Hence, I is the point of intersection of angle bisectors of any two angles of ΔABC equidistant from its sides.

(vi) In a huge park, people are concentrated at three points (See figure).

A: where there are different slides and swings for children.

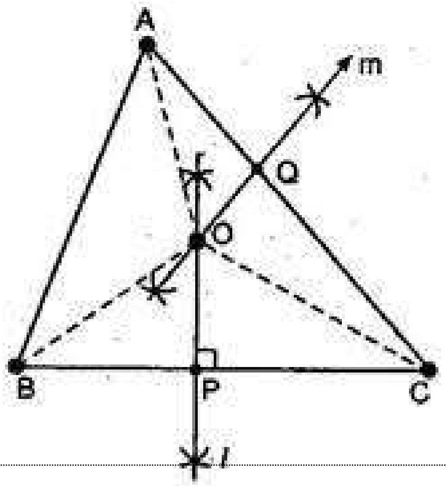
B: near which a man-made lake is situated.

C: which is near to a large parking and exit.

Where should an ice cream parlour be set up so that maximum number of persons can approach it?

Ans. The parlour should be equidistant from A, B and C. So we should find out the circum-centre of the triangle obtained by joining A, B and C respectively.

For this let us draw perpendicular bisector say l of line joining points B and C also draw perpendicular bisector say m of line joining points A and C.



Let l and m intersect each other at point O. O is the required point.

Proof that O is the required point:

Join OA, OB and OC.

Proof: In ΔBOP and ΔCOP,

OP = OP [Common]

$\angle$OPB = $\angle$OPC = 90°

BP = PC [P is the mid-point of BC]

$\therefore$ ΔBOP $\cong$ ΔCOP [By SAS congruency]

$\Rightarrow$ OB = OC [By C.P.C.T.] …..(i)

Similarly, ΔAOQ $\cong$ ΔCOQ
$\Rightarrow$ OA = OC [By C.P.C.T.] …..(ii)

From eq. (i) and (ii),

OA=OB=OC

Therefore, O is the required point as it is equidistant from the given points. Thus, ice cream parlour should be set up at point O, the point of intersection of perpendicular bisectors of any two sides out of three formed by joining these points.

(iv) Complete the hexagonal rangoli and the star rangolies (See figure) but filling them with as many equilateral triangles of side 1 cm as you can. Count the number of triangles in each case. Which has more triangles?

Ans. In hexagonal rangoli, Number of equilateral triangles each of side 5 cm are 6.

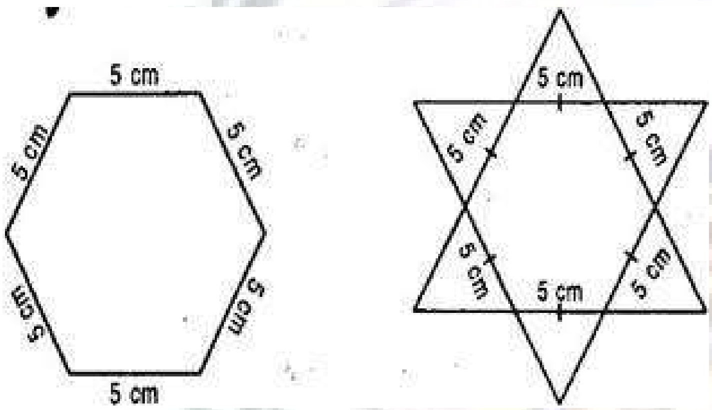
Area of equilateral triangle = $\frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(5)^2 = \frac{\sqrt{3}}{4} \times 25$ sq. cm

Area of hexagonal rangoli = 6 x Area of an equilateral triangle

= $6 \times \frac{\sqrt{3}}{4} \times 25 = 150 \times \frac{\sqrt{3}}{4}$ sq. cm ……….(i)

Now area of equilateral triangle of side 1 cm = = $\frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}(1)^2 = \frac{\sqrt{3}}{4}$ sq. cm …..(ii)

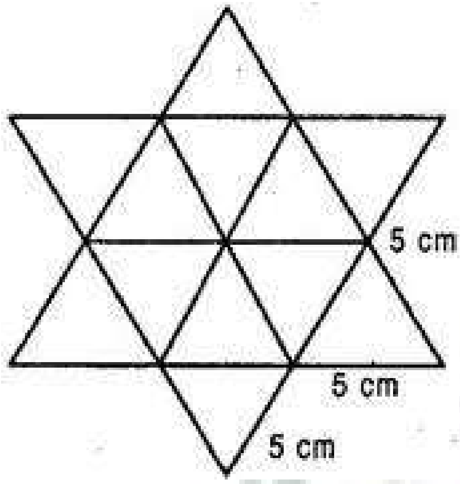
Number of equilateral triangles each of side 1 cm in hexagonal rangoli



$= 150\times\frac{\sqrt{3}}{4} \div \frac{\sqrt{3}}{4} \quad 150\times\frac{\sqrt{3}}{4} \times\frac{4}{\sqrt{3}}$ = 150 …..(iii)

Now in Star rangoli,

Number of equilateral triangles each of side 5 cm = 12



Therefore, total area of star rangoli = 12 $\times$ Area of an equilateral triangle of side 5 cm

$= 12\times\left(\frac{\sqrt{3}}{4}(5)^2\right)$

$= 12\times\frac{\sqrt{3}}{4}\times 25$

, $300\frac{\sqrt{3}}{4}$ sq. cm ……….(iv)

Number of equilateral triangles each of side 1 cm in star rangoli

$= 300\frac{\sqrt{3}}{4} \div\frac{\sqrt{3}}{4}$

$= 300\frac{\sqrt{3}}{4} \times\frac{4}{\sqrt{3}}$

= 300 ……….(v)

From eq. (iii) and (v), we observe that star rangoli has more equilateral triangles each of side 1 cm

PUNA INTERNATIONAL SCHOOL

- **CLASS – 9**
- **SUBJECT - MATHS**
- **CHAPTER - 15**

SAMPLE NOTE-BOOK

CHAPTER 15

Probability
(Ex. 15.1)

(i) In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.

Ans. Probability = $\dfrac{\text{Favourable outcomes}}{\text{Total outcomes}}$

Number of times on boundary is not hit = 30 – 6 = 24

$\therefore$ P (did not hit a boundary) = $\dfrac{24}{30} = \dfrac{4}{5}$

(i) 1500 families with 2 children were selected randomly and the following data were recorded:

No. of girls in a family	No. of families
2	475
1	814
0	211

Compute the probability of a family, chosen at random, having:

(i) 2 girls (ii) 1 girl (iii) No girl

Also check whether the sum of these probabilities is 1.

Ans. (i) Total number of families = 1500

No. of families having 2 girls = 475

$\therefore$ P (Family having 2 girls) = $\dfrac{475}{1500} = \dfrac{19}{60}$

(i) No of families having 1 girl = 814 $\therefore$

P (Family having 1 girl) = $\dfrac{814}{1500} = \dfrac{407}{750}$

(i) No. of families having no girl = 211 ∴

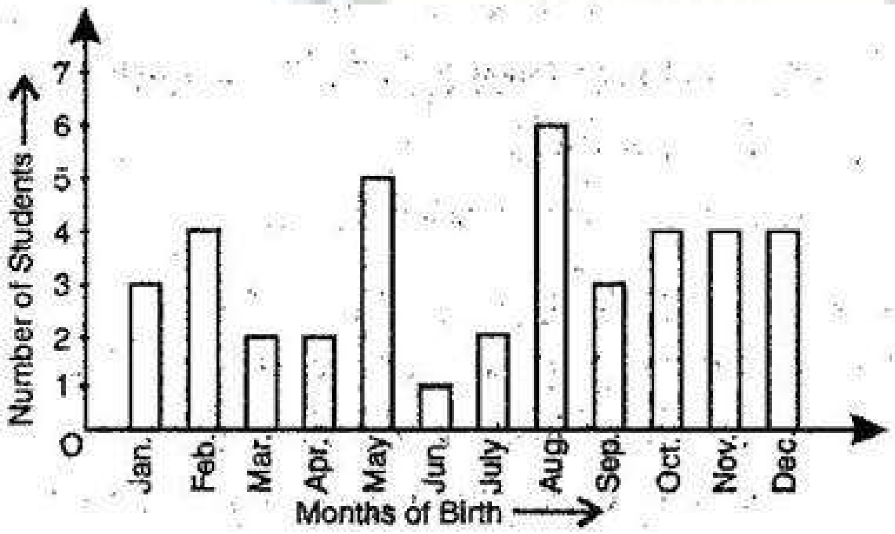
P (Family having no girl) = $\frac{211}{1500}$

Checking: Sum of all probabilities = $\frac{19}{60}+\frac{407}{750}+\frac{211}{1500}$

$=\frac{475+814+211}{1500}=\frac{1500}{1500}$ = 1

Yes, the sum of probabilities is 1.

, In a particular section of Class IX, 40 students were asked about the months of their birth and the following graph was prepared for the data so obtained:



Find the probability that a student of the class was born in August.

Ans. From the bar graph, we observe,

Total no. of students of Class IX = 40

No. of students of Class IX born in August = 6

∴ P (A student born in August) = $\frac{6}{40}=\frac{3}{20}=0.15$

(iv) Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes:

Outcomes	Frequency
3 heads	23
2 heads	72
1 head	77
No head	28

If the three coins are simultaneously tossed again, compute the probability of 2 heads coming up.

Ans. No. of 2 heads = 72

Total number of outcomes = 23 + 72 + 77 + 28 = 200

$\therefore$ P (2 heads) = $\frac{72}{200} = \frac{9}{25}$

(v) An organization selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a family. The information gathered is listed in the table below:

Monthly income (in Rs.)	Vehicles per family			
	0	1	2	Above 2
Less than 7000	10	160	25	0
7000 – 10000	0	305	27	2
10000 – 13000	1	535	29	1
13000 – 16000	2	469	59	25
16000 or more	1	579	82	88

Suppose a family is chosen. Find the probability that the family chosen is:

(i) earning Rs. 10000 – 13000 per month and owning exactly 2 vehicles.

(ii) earning Rs. 16000 or more per month and owning exactly 1 vehicle.

(iii) earning less than Rs. 7000 per month and does not own any vehicle.

(iv) earning Rs. 13000 – 16000 per month and owning more than 2 vehicles.

(v) not more than 1 vehicle.

Ans. (i) P (earning Rs. 10000 – 13000 per month and owning exactly 2 vehicles) = $\frac{29}{2400}$

(ii) P (earning Rs. 16000 or more per month and owning exactly 1 vehicles) = $\frac{579}{2400}$

(iii) P (earning Rs. 7000 per month and does not own any vehicles) = $\frac{10}{2400} = \frac{1}{240}$

(iv) P (earning Rs. 13000 – 16000 per month and owning more than 2 vehicles) = $\frac{25}{2400} = \frac{1}{96}$

(v) Number of families owning not more than 1 vehicle = 10 + 160 + 0 + 305 + 1 + 532 + 2 + 469 + 1 579 = 2062

Therefore, P (owning not more than 1 vehicle) = $\frac{2062}{2400} = \frac{1031}{1200}$

(vi) A teacher analyses the performance of two sections of students in a mathematics test of 100 marks given in the following table:

Marks	No. of students
0 – 20	7
20 – 30	10
30 – 40	10
40 – 50	20
50 – 60	20
60 – 70	15
70 and above	8
Total	90

(iv) Find the probability that a student obtained less than 20% in the mathematics test.

(v) Find the probability that a student obtained 60 or above.

Ans. (i) No. of students obtaining marks less than 20 out of 100, i.e. 20% = 7 Total

students in the class = 90

∴ P (A student obtained less than 20%) = $\frac{7}{90}$

(v) No. of students obtaining marks 60 or above = 15 + 8 = 23 ∴ P

(A student obtained marks 60 or above) = $\frac{23}{90}$

5. To know the opinion of the students about the subject statistics, a survey of 200 students was conducted. The data is recorded in the following table:

Opinion	No. of students
likes	135
dislikes	65

Find the probability that a student chosen at random:

(i) likes statistics (ii) dislikes it.

Ans. Total no. of students on which the survey about the subject of statistics was conducted = 200

7. No. of students who like statistics = 135 ∴ P

(a student likes statistics) = $\frac{135}{200} = \frac{27}{40}$

8. No. of students who do not like statistics = 65 ∴ P

(a student does not like statistics) = $\frac{65}{200} = \frac{13}{40}$

8. Refer Q.2, Exercise 14.2. What is the empirical probability than an engineer lives:

(i) less than 7 km from her place of work?

(ii) more than or equal to 7 km from her place of work?

(iii) within $\frac{1}{2}$ km from her place of work?

Ans. Total number of engineers = 40

(i) No. of engineers living less than 7 km from her place of work = 9 $\therefore$ P

(Engineer living less than 7 km from her place of work) = $\frac{9}{40}$

(ii) No. of engineers living more than or equal to 7 km from her place of work = 40 - 9 = 31 $\therefore$ P

(Engineer living more than or equal to 7 km from her place of work) = $\frac{31}{40}$

(iii) No. of engineers living within $\frac{1}{2}$ km from her place of work = 0

$\therefore$ P (Engineer living within $\frac{1}{2}$ km from her place of work) = $\frac{0}{40}$ = 0

9. Activity: Note the frequency of two wheelers, three wheelers and four wheelers going past during a time interval, in front of your school gate. Find the probability that any one vehicle out of the total vehicles you have observed is a two wheeler.

Ans. Let you noted the frequency of types of wheelers after school time (i.e. 3 pm to 3.30 pm) for half an hour.

Let the following table shows the frequency of wheelers.

Type of wheelers	Frequency of wheelers
Two wheelers	125
Three wheelers	45
Four wheelers	30

Probability that a two wheelers passes after this interval = $\frac{125}{200}=\frac{5}{8}$

10. Activity: Ask all the students in your class room to write a 3-digit number. Choose any student from the room at random. What is the probability that the number written by him is divisible by 3, if the sum of its digits is divisible by 3.

Ans. Let number of students in your class is 24.

Let 3-digit number written by each of them is as follows:

837, 172, 643, 371, 124, 512, 432, 948, 311, 252, 999, 557, 784, 928, 867, 798, 665, 245, 107, 463, 267, 523, 944, 314

Numbers divisible by 3 are = 837, 432, 948, 252, 999, 867, 798 and 267 Number

of 3-digit numbers divisible by 3 = 8

$\therefore$ P (3-digit numbers divisible by 3) = $\frac{8}{24}=\frac{1}{3}$

11. Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of four (in kg): 4.97, 5.05, 5.08, 5.03, 5.00, 5.06, 5.08, 4.98, 5.04, 5.07, 5.00

Find the probability that any of these bags chosen at random contains more than 5 kg of flour.

Ans. Number of bags containing more than 5 kg of wheat flour = 7

Total number of wheat flour bags = 11

$\therefore$ P (a bag containing more than 5 kg of wheat flour) = $\frac{7}{11}$

12. In Q.5, Exercise 14.2, you were asked to prepare a frequency distribution table, regarding the concentration of Sulphur dioxide in the air in parts per million of a certain city for 30 days. Using this table, find the probability of the concentration of

Sulphur dioxide in the interval 0.12 – 0.16 on any of these days.

Ans. From the frequency distribution table we observe that:

No. of days during which the concentration of Sulphur dioxide lies in interval 0.12 – 0.16 = 2 Total no. of days during which concentration of Sulphur dioxide recorded = 30

$\therefore$ P (day when concentration of Sulphur dioxide (in ppm) lies in 0.12 – 0.16) = $\frac{2}{30} = \frac{1}{15}$

13. In Q.1, Exercise 14.1 you were asked to prepare a frequency distribution table regarding the blood groups of 30 students of a class. Use this table to determine the probability that a student of this class selected at random has blood group AB.

Ans. From the frequency distribution table we observe that:

Number of students having blood group AB = 3

Total number of students whose blood group were recorded = 30

$\therefore$ P (a student having blood group AB) = $\frac{3}{30} = \frac{1}{10}$