PUNA INTERNATIONAL SCHOOL

- **CLASS – 10**
- **SUBJECT - MATHS**
- **CHAPTER - 13**

SAMPLE NOTE-BOOK

Chapter - 13

Surface Areas and Volumes

Exercise 13.1

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. **cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.**

Ans. Volume of cube = $(Side)^3$

According to question, $(Side)^3 = 64$

$\Rightarrow (Side)^3 = 4^3$

$\Rightarrow$ Side = 4 cm

For the resulting cuboid, length (l) = 4 + 4 = 8 cm, breadth (b) = 4 cm and height (h) = 4 cm

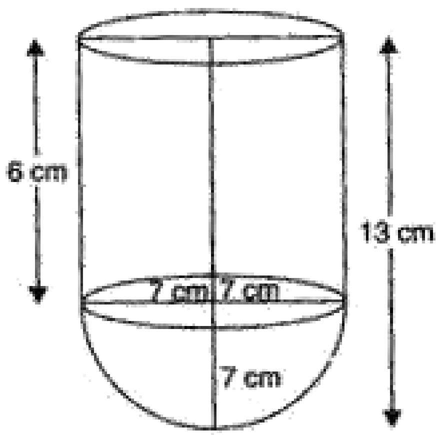
Surface area of resulting cuboid = $2(lb+bh+hl)$ = $2(8\times4+4\times4+4\times8)$ =2(32+16+32)

= $2\times80 = 160\ cm^2$

2. **A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.**

Ans. $\because$ Diameter of the hollow hemisphere = 14 cm

$\therefore$ Radius of the hollow hemisphere = $\frac{14}{2}$ = 7 cm



Total height of the vessel = 13 cm

∴ Height of the hollow cylinder = 13 – 7 = 6 cm ∴

Inner surface area of the vessel

Inner surface area of the hollow hemisphere + Inner surface area of the hollow cylinder

$$2\pi(7)^2 + 2\pi(7)(6)$$

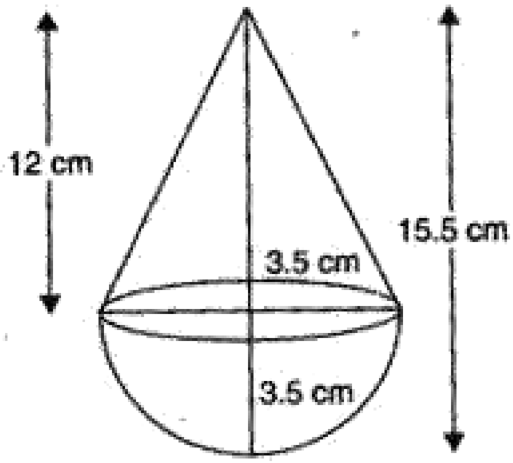
$$98\pi + 84\pi = 182\pi$$

$$182 \times \frac{22}{7} = 26 \times 22 = 572\ cm^2$$

3. **A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.**

Ans. Radius of the cone = 3.5 cm

∴ Radius of the hemisphere = 3.5 cm



Total height of the toy = 15.5 cm

∴ Height of the cone = 15.5 – 3.5 = 12 cm

Slant height of the cone = $\sqrt{(3.5)^2 + (12)^2}$

= $\sqrt{12.25 + 144}$

= $\sqrt{156.25}$ = 12.5 cm

∴ TSA of the toy = CSA of hemisphere + CSA of cone =

$2\pi r^2 + \pi r l$

= $2\pi(3.5)^2 + \pi(3.5)(12.5)$

= $24.5\pi + 43.75\pi$ = 68.25π

= $68.25 \times \frac{22}{7}$ = $214.5\ cm^2$

4. **A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.**

Ans. Greatest diameter of the hemisphere = Side of the cubical block = 7 cm

∴ TSA of the solid = External surface area of the cubical block + CSA of hemisphere

$$= \left\{6(7)^2 - \pi\left(\frac{7}{2}\right)^2\right\} + 2\pi\left(\frac{7}{2}\right)^2$$

$$> \left(294 - \frac{49}{4}\pi\right) - \frac{49}{2}\pi$$

$$= 294 + \frac{49}{4}\pi$$

$$= 294 - \frac{49}{4} \times \frac{22}{7}$$

$$= 294 + \frac{77}{2}$$

= 294 + 38.5 = $332.5\ cm^2$

5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.

Ans. $\because$ Diameter of the hemisphere = l, therefore radius of the hemisphere = Also, length $\frac{l}{2}$

of the edge of the cube = l

$\therefore$ Surface area of the remaining solid = total surface area of cubical block + curved surface area of hemispherical - area of circular base

$$= 2\pi\left(\frac{l}{2}\right)^2 + 6l^2 - \pi\left(\frac{l}{2}\right)^2$$

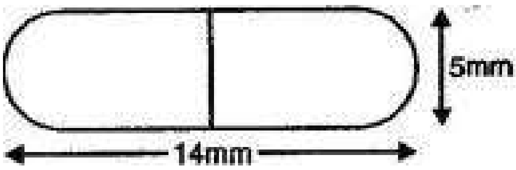
$$= \pi\left(\frac{l}{2}\right)^2 + 6l^2$$

$$= \frac{\pi l^2}{4} + 6l^2$$

$$= \frac{1}{4}l^2(\pi + 24)$$

6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each

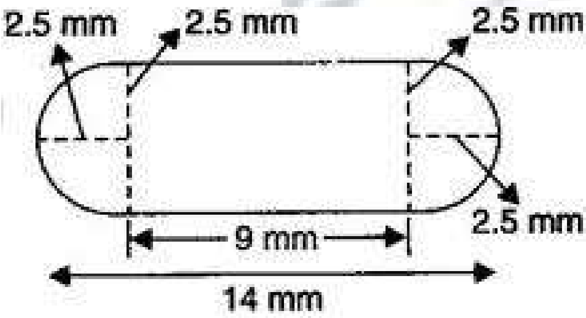
of its ends (see figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.



Ans. Radius of the hemisphere = $\frac{5}{2}$ mm

Let radius = r = 2.5 mm

Cylindrical height = Total height – Diameter of sphere = h = 14 – (2.5 + 2.5) = 9 mm Surface area of the capsule = CSA of cylinder + curved Surface area of 2 hemispheres



= $2\pi rh + 2\left(2\pi r^2\right)$

= $2\pi\left(\frac{5}{2}\right)(9) + 2\left\{2\pi\left(\frac{5}{2}\right)^2\right\}$

= $45\pi + 25\pi$

= $70\pi = 70 \times \frac{22}{7}$ = $220\ mm^2$

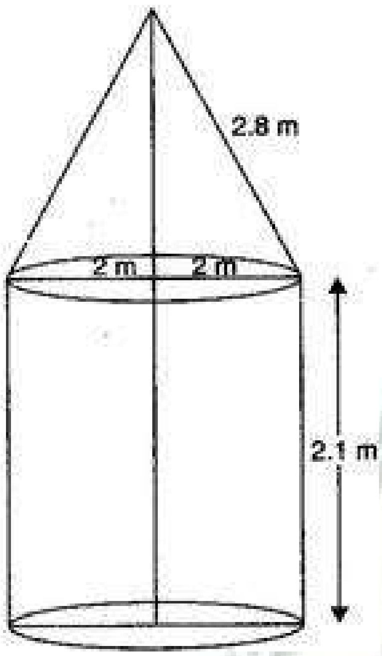
7. A tent is in the shape of a cylinder surmounted buy a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost

of the canvas of the tent at the rate of Rs. 500 per m^2. (Note that the base of the tent will not be covered with canvas.)

Ans. Diameter of the cylindrical part = 4 cm

∴ Radius of the cylindrical part = 2 cm

TSA of the tent = CSA of the cylindrical part + CSA of conical top



= $2\pi(2)(2.1)+\pi(2)(2.8)$

=

= 14π

= $14\times\frac{22}{7}$

= $44\ m^2$

∴ Cost of the canvas of the tent of 1 m^2 = Rs. 500

cost of canvas of the tent of 44 m^2 =

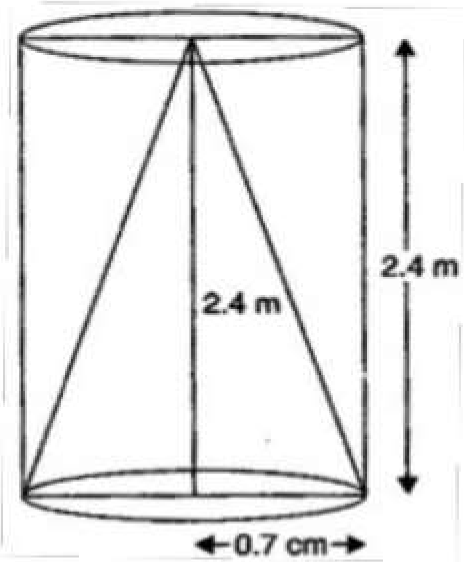
= 44×500 = Rs. 22000

8.From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

Ans. Diameter of the solid cylinder = 1.4 cm

∴ Radius of the solid cylinder = 0.7 cm

∴ Radius of the base of the conical cavity = 0.7 cm



Height of the solid cylinder = 2.4 cm

$\therefore$ Height of the conical cavity = 2.4 cm

$\therefore$ Slant height of the conical cavity = $\sqrt{(0.7)^2+(2.4)^2}$

= $\sqrt{0.49+5.76}$

= $\sqrt{6.25}$ = 2.5 cm

$\therefore$ TSA of remaining solid = curved surface area of cylinder + area of upper circular part + curved surface area of conical part

= $2\pi(0.7)(2.4)+\pi(0.7)^2+\pi(0.7)(2.5)$

= $3.36\pi+0.49\pi+1.75\pi$

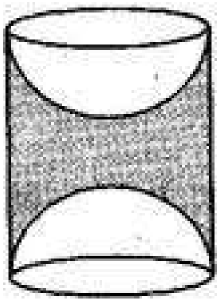
= 5.6π

$5.6\times\frac{22}{7}$ = 17.6 cm^2

= $18\ cm^2$ (to the nearest cm^2)

9. A wooden article was made by scooping out a hemisphere from each end of a solid

cylinder as shown in figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the article.



Ans. TSA of the article = $2\pi rH + 2\left(2\pi r^2\right)$ = curved surface area of cylinder + curved surface area of 2 hemispheres

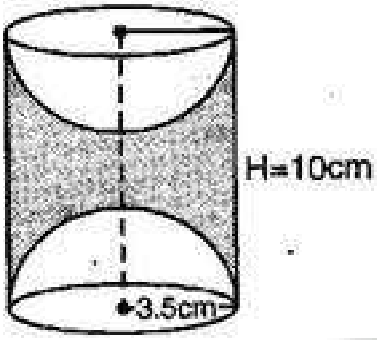
= $2\pi(3.5)(10)+2\left[2\pi(3.5)^2\right]$

= $70\pi + 49\pi$

= 119π

= $119\times\frac{22}{7}$

= $374\ cm^2$



Exercise 13.2

Unless stated otherwise, take $\pi = \frac{22}{7}$.

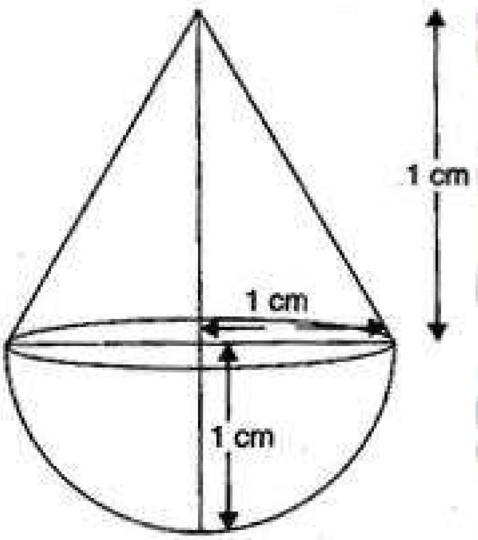
1 **A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of** π.

Ans. For hemisphere, Radius (r) = 1 cm

Volume = $\frac{2}{3}\pi r^3$

= $\frac{2}{3}\pi(1)^3$

= $\frac{2}{3}\pi$ cm^3



For cone, Radius of the base (r) = 1 cm

Height (h) = 1 cm

Volume = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3}\pi(1)^2 \times 1$

= $\frac{1}{3}\pi$ cm^3

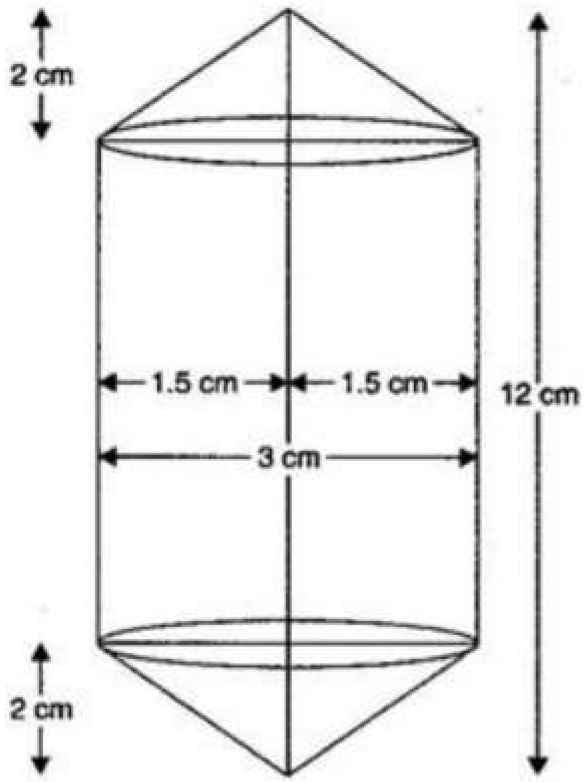
$\therefore$ Volume of the solid = Volume of hemisphere + Volume of cone

= $\frac{2}{3}\pi$ + $\frac{1}{3}\pi$ = $\pi\ cm^3$

2 Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)

Ans. For upper conical portion, Radius of the base (r) = 1.5 cm

Height (h_1) = 2 cm



Volume = $\frac{1}{3}\pi r^2 h_1$

$= \frac{1}{3}\pi(1.5)^2 \times 2$

$= 1.5\pi cm^3$

For lower conical portion, Volume = $1.5\pi cm^3$

For central cylindrical portion:

Radius of the base (r) = 1.5 cm

Height (h_2) = 12 – (2 + 2) = 8 cm

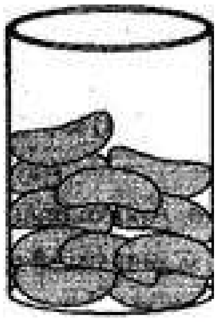
Volume = $\pi r^2 h_2$ = $\pi(1.5)^2 \times 8$ = $18\pi cm^3$

$\therefore$ Volume of the model = 1.5π + 1.5π + 18π = volume of top cone + volume of bottom cone + volume of cylindrical part

$= 21\pi$

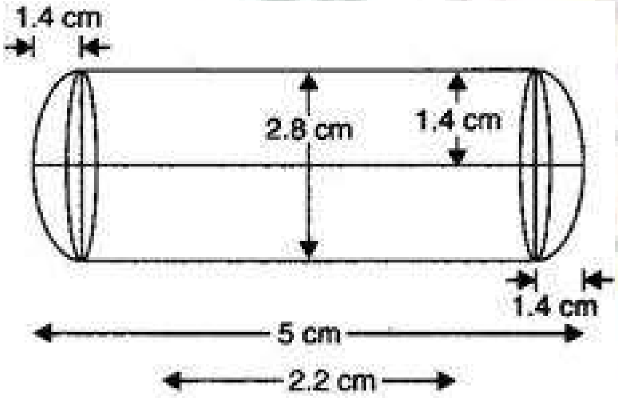
$_{12}21\times\frac{22}{7} = 66\ cm^3$

3.A *gulab jamun*, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 *gulab jamuns*, each shaped like a cylinder with two hemispherical ends, with length 5 cm and diameter 2.8 cm (see figure).



Ans. Volume of a gulab jamun = $\frac{2}{3}\pi r^3 + \pi r^2 h + \frac{2}{3}\pi r^3$ = volume of 2hemisphere + volume of cylinder

$= \frac{2}{3}\pi(1.4)^3 + \pi(1.4)^2\times 2.2 + \frac{2}{3}\pi(1.4)^3$



$= \frac{4}{3}\pi(1.4)^3 + \pi(1.4)^2\times 2.2$

$= \pi(1.4)^2\left[\frac{4\times 1.4}{3} + 2.2\right]$

$= \pi \times 1.96\left[\frac{5.6+6.6}{3}\right] = \frac{1.96\times 12.2}{3}\pi cm^3$

$\therefore$ Volume of 45 gulab jamuns

$= 45\times\frac{1.96\times 12.2}{3}\pi$

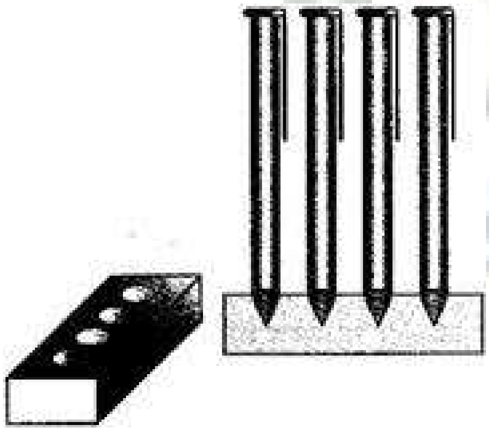
$= 15\times 1.96\times 12.2\times\frac{22}{7}$

$= 1127.28\ cm^3$

$\therefore$ Volume of syrup = $1127.28\times\frac{30}{100}$ = 30% of volume of 45 gulab jamun

$= 338.184\ cm^3 = 338\ cm^3$ (approx.)

6. **A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth id 1.4 cm. Find the volume of wood in the entire stand (see figure).**



Ans: For Cuboid:

l=15 cm

b=10 cm

h=3.5 cm

Volume of the cuboid = $l \times b \times h$

= $15 \times 10 \times 3.5$

= $525\ cm^3$

For Cone: r = 0.5 cm

h = 1.4 cm

Volume of conical depression = $\frac{1}{3}\pi r^2 h$

= $\frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$

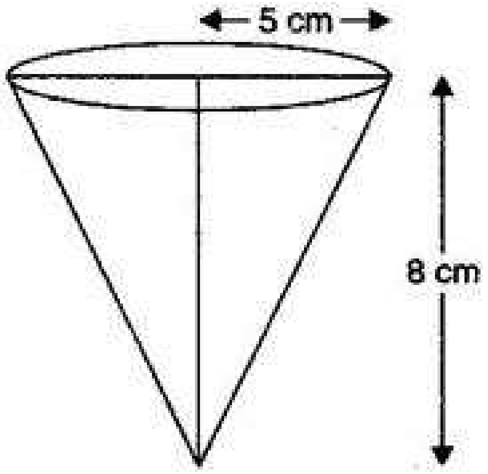
= $\frac{11}{30} cm^3$

$\therefore$ Volume of four conical depressions = $4 \times \frac{11}{30}$ = $1.47\ cm^3$

$\therefore$ Volume of the wood in the entire stand = volume of cuboid - volume of 4 conical depression =525 – 1.47 = $523.53\ cm^3$

7. **A vessel is in the form of inverted cone. Its height is 8 cm and the radius of the top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.**

Ans. For cone, Radius of the top (r) = 5 cm and height (h) = 8 cm



Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(5)^2 \times 8$

$= \frac{200}{3}\pi$ cm^3

For spherical lead shot, Radius (R) = 0.5 cm

Volume of spherical lead shot = $\frac{4}{3}\pi R^3$

$= \frac{4}{3}\pi(0.5)^3$

$= \frac{\pi}{6} cm^3$

Volume of water that flows out = $\frac{1}{4}$ Volume of the cone

$= \frac{1}{4} \times \frac{200\pi}{3} = \frac{50\pi}{3} cm^3$

Let the number of lead shots dropped in the vessel be n. n *

volume of spherical shot = volume of water flows out

$\therefore \; n \times \frac{\pi}{6} = \frac{50\pi}{3}$

$\Rightarrow \; n = \frac{50\pi}{3} \times \frac{6}{\pi}$

$\Rightarrow n$ = 100

8. **A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the**

mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass. $(\text{Use } \pi = 3.14)$

Ans. For lower cylinder, Base radius (r) = $\frac{24}{2}$ = 12 cm

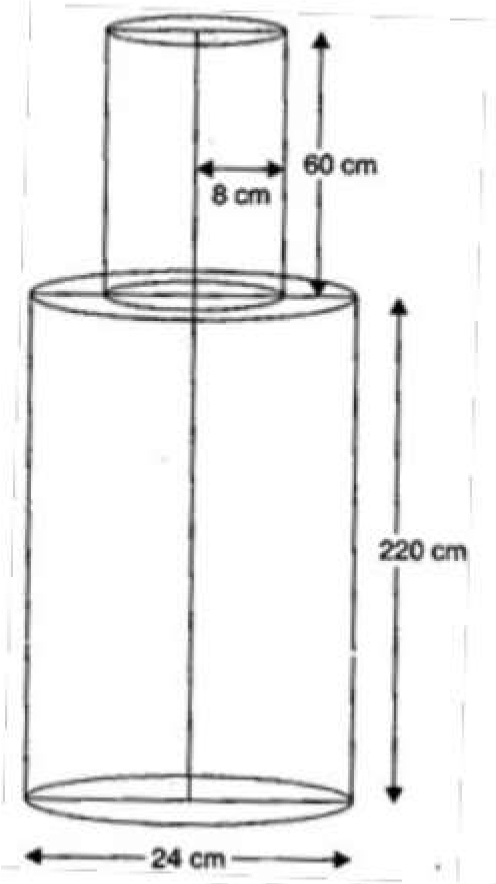
And Height (h) = 220 cm

Volume = $\pi r^2 h$

= $\pi(12)^2 \times 220$

= $31680\pi \; cm^3$

For upper cylinder, Base Radius (R) = 8 cm



And Height (H) = 60 cm

Volume = $\pi R^2 H$

= $\pi(8)^2 \times 60$

=

$\therefore$ Volume of the solid Iron pole

V of lower cylinder + V of upper cylinder

$31680\pi + 3840\pi = 35520\pi$

$35520 \times 3.14 = 111532.8\ cm^3$

mass of 1 cm^3 iron = 8 gm

mass of 111532.8 cm^3 iron = 8* 111532.8 = 892262.4 gm = 892.2624 kg

9. **A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.**

Ans. For right circular cone, Radius of the base (r) = 60 cm

And Height (h_1) = 120 cm

Volume = $\frac{1}{3}\pi r^2 h_1$

= $\frac{1}{3}\pi(60)^2 \times 120$

= $144000\pi cm^3$

For Hemisphere, Radius of the base (r) = 60 cm

Volume = $\frac{2}{3}\pi r^3$

= $\frac{2}{3}\pi(60)^3$

= $144000\pi cm^3$

For right circular cylinder, Radius of the base (r) = 60 cm

And Height (h_2) = 180 cm

And Height (H) = 60 cm

Volume = $\pi R^2 H$

= $\pi(8)^2 \times 60$

=

$\therefore$ Volume of the solid Iron pole

V of lower cylinder + V of upper cylinder

31680π + 3840π = 35520π

35520×3.14 = 111532.8 cm^3

mass of 1 cm^3 iron = 8 gm

mass of 111532.8 cm^3 iron = 8* 111532.8 = 892262.4 gm = 892.2624 kg

10. **solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.**

Ans. For right circular cone, Radius of the base (r) = 60 cm

And Height (h_1) = 120 cm

Volume = $\frac{1}{3}\pi r^2 h_1$

= $\frac{1}{3}\pi(60)^2\times 120$

= $144000\pi\, cm^3$

For Hemisphere, Radius of the base (r) = 60 cm

Volume = $\frac{2}{3}\pi r^3$

= $\frac{2}{3}\pi(60)^3$

= $144000\pi\, cm^3$

Exercise 13.3

Unless stated otherwise, take $\pi = \frac{22}{7}$.

1. **A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.**

Ans. For sphere, Radius (r) = 4.2 cm

Volume = $\frac{4}{3}\pi r^3$ = $\frac{4}{3}\pi(4.2)^3\ cm^3$

For cylinder, Radius (R) = 6 cm

Let the height of the cylinder be H cm.

Then, Volume = $\pi R^2 H$ = $\pi(6)^2 H\ cm^3$

According to question, Volume of sphere = Volume of cylinder

$\Rightarrow \frac{4}{3}\pi(4.2)^3 = \pi(6)^2 H$

$\Rightarrow$ H = $\frac{4(4.2)^3}{3(6)^2}$

$\Rightarrow$ H = 2.744 cm

2. **Metallic spheres of radii 6 cm, 8 cm and 10 cm respectively are melted to form a single solid sphere. Find the radius of the resulting sphere.**

Ans. Let the volume of resulting sphere be r cm.

According to question,

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 + \frac{4}{3}\pi(8)^3 + \frac{4}{3}\pi(10)^3$$

$$\Rightarrow r^3 = (6)^3 + (8)^3 + (10)^3$$

$\Rightarrow r^3$ = 216 + 512 + 1000

$\Rightarrow r^3$ = 1728

$\Rightarrow r$ = 12 cm

3. **A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.**

Ans. Diameter of well = 7 m

∴ Radius of well $(r) = \frac{7}{2}$ M

And Depth of earth dug (h) = 20 m

Length of platform (l) = 22 m, Breadth of platform (b) = 14 m

Let height of the platform be h' M

According to question,

Volume of earth dug = Volume of platform

$$\Rightarrow \pi r^2 h = l \times b \times h'$$

$$\Rightarrow \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 = 22 \times 14 \times h'$$

$$\Rightarrow h' = \frac{22 \times 7 \times 7 \times 20}{28 \times 22 \times 14}$$

$\Rightarrow h'$ = 2.5 m

4. **A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.**

Ans. Diameter of well = 3 m

$\therefore$ Radius of well (r) = $\frac{3}{2}$ m and Depth of earth dug (h) = 14 m

Width of the embankment = 4 m

$\therefore$ Radius of the well with embankment r' = $\frac{3}{2}+4=\frac{11}{2}$ m

Let the height of the embankment be h' m

According to the question,

Volume of embankment = Volume of the earth dug

$$\Rightarrow \pi\left[(r')^2-r^2\right]h' = \pi r^2 h$$

$$\Rightarrow \left[\left(\frac{11}{2}\right)^2-\left(\frac{3}{2}\right)^2\right]h' = \left(\frac{3}{2}\right)^2\times 14$$

$$\Rightarrow \left[\frac{121}{4}-\frac{9}{4}\right]h'=\frac{9}{4}\times 14$$

$$\Rightarrow \frac{112}{4}\times h'=\frac{9}{4}\times 14$$

$$\Rightarrow h'=\frac{9\times 14\times 4}{4\times 112}$$

$\Rightarrow h'$ = 1.125 m

5. A container shaped like a right circular cylinder having diameter 12 cm and height 15

cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

Ans. For right circular cylinder, Diameter = 12 cm

$\therefore$ Radius (r) = $\frac{12}{2}$ = 6 cm and height (h) = 15 cm

For cone & Hemisphere , Diameter = 6 cm

$\therefore$ Radius (r_1) = $\frac{6}{2}$ = 3 cm and height (h_1) = 12 cm

Let n cones be filled with ice cream.

Then, According to question,

Volume of n (cones +Hemisphere) = Volume of right circular cylinder

$\Rightarrow n\times\left(\frac{1}{3}\pi(r_1)^2(h)+\frac{2}{3}\pi(r_1)^3\right)=\pi r^2 h$

$\Rightarrow n\left(\frac{1}{3}\pi(3)^2(12)+\frac{2}{3}\pi(3)^3\right)=\frac{22}{7}\times(6)^2\times 15$

$\Rightarrow n=\frac{22\times36\times15\times3\times7}{(7\times22\times9\times12+7\times44\times27)}=\frac{249480}{24948}$

$\Rightarrow n$ = 10

11. How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions $5.5\text{ cm}\times10\text{ cm}\times3.5\text{ cm}$?

Ans. For silver coin, Diameter = 1.75 cm

$\therefore$ Radius $(r)=\frac{1.75}{2}$ = $\frac{7}{8}$ cm and Thickness (h) = 2 mm = $\frac{1}{5}$ cm

For cuboid, Length (l) = 5.5 cm, Breadth (b) = 10 cm and Height (h') = 3.5 cm

Let n coins be melted.

Then, According to question,

Volume of n coins = Volume of cuboid

$\Rightarrow n \times \pi r^2 h = l \times b \times h'$

$\Rightarrow n \times \pi \left(\frac{7}{8}\right)^2 \times \left(\frac{1}{5}\right) = 5.5 \times 10 \times 3.5$

$\Rightarrow n \times \frac{22}{7} \times \frac{49}{64} \times \frac{1}{5} = 5.5 \times 10 \times 3.5$

$\Rightarrow n = \frac{5.5 \times 10 \times 3.5 \times 7 \times 64 \times 5}{22 \times 49}$

$\Rightarrow n = 400$

12. **A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.**

Ans. For cylindrical bucket, Radius of the base (r) = 18 cm and height (h) = 32 cm

$\therefore$ Volume = $\pi r^2 h = \pi (18)^2 \times 32$

$= 10368\pi \ cm^3$

For conical heap, Height (h') = 24 cm

Let the radius be r_1 cm.

Then, Volume = $\frac{1}{3} \pi r_1^2 h'$

$= \frac{1}{3} \times \pi \times r_1^2 \times 24 = 8\pi r_1^2 \ cm^3$

According to question, Volume of bucket = Volume of conical heap

$\Rightarrow 10368\pi = 8\pi r_1^2$

$\Rightarrow r_1^2 = \frac{10368\pi}{8\pi}$ = 1296

$\Rightarrow r_1$ = 36 cm

Now, Slant height $(l) = \sqrt{(r_1)^2 + (h')^2}$

$= \sqrt{(36)^2 + (24)^2} = \sqrt{1296 + 576}$

$= \sqrt{1872} = 12\sqrt{13}$ cm

(a) Water in a canal 6 m wide and 1.5 m deep is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Ans. For canal, Width = 6 m and Depth = 1.5 m = $\frac{3}{2}$ m

Speed of flow of water = 10 km/h

$= 10 \times 1000\ m/h$ = 10000 m/h

$= \frac{10000}{60}$ m/min = $\frac{500}{3}$ m/min

$\therefore$ Speed of flow of water in 30 minutes

5. $\frac{500 \times 30}{3}$ m/min = 5000 m/min

$\therefore$ Volume of water that flows in 30 minutes

$= 6 \times \frac{3}{2} \times 5000 = 45000\ m^3$

$\therefore$ The area it will irrigate = $\dfrac{45000}{\left(\dfrac{8}{100}\right)} = \dfrac{4500000}{8}$

= $562500\, m^2$

7. $\dfrac{562500}{10000}$ hectares = 56.25 hectares

(b) A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

Ans. For cylindrical tank, Diameter = 10 m

$\therefore$ Radius $(r) = \dfrac{10}{2}$ = 5 m and Depth (h) = 2 m

$\therefore$ Volume = $\pi r^2 h$ = $\pi(5)^2 \times 2$ = $50\pi m^3$

Rate of flow of water (h') = 3 km/h = 3000 m/h = $\dfrac{3000}{60}$ m/min = 50 m/min

For pipe, Internal diameter = 20 cm, therefore radius (r_1) = 10 cm = 0.1 m

$\therefore$ Volume of water that flows per minute = $\pi(r_1)^2 h'$

= $\pi(0.1)^2 \times 50$ = $\dfrac{\pi}{2} m^3$

$\therefore$ Required time = $\dfrac{50\pi}{\pi/2}$ = 100 minutes

Exercise 13.5

1. **A copper wire, 3 mm in diameter is wound about a cylinder whose length is 12 cm and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and**

mass of the wire, assuming the density of copper to be 8.88 g per cm^3 .

Ans. Number of rounds to cover 12 cm, i.e. 120 mm = $\frac{120}{3}$ = 40

Here, Diameter = 10 cm, Radius $(r) = \frac{10}{2}$ cm

Length of the wire used in taking one round

= $2\pi r = 2\pi \times 5 = 10\pi$ cm

Length of the wire used in taking 40 rounds

= $10\pi \times 40 = 400\pi$ cm

Radius of the copper wire = $\frac{3}{2}$ mm

$\frac{\frac{3}{2}}{20}$ cm

$\therefore$ Volume of wire = $\pi\left(\frac{3}{20}\right)^2(400\pi)$

3. $9\pi^2 cm^3$--------------***

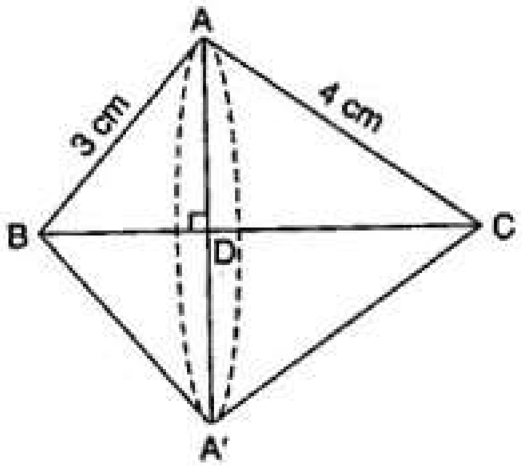
$\therefore$ Mass of the wire = $9\times(3.14)^2\times 8.88$

= 787.98 gm

= **A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. (Choose value of π as found appropriate)**

Ans. Hypotenuse = $\sqrt{3^2+4^2}$ = 5 cm

In figure, Δ ADB $\sim \Delta$ CAB [AA similarity]



$\therefore \frac{AD}{CA} = \frac{AB}{CB}$

$\Rightarrow \frac{AD}{4} = \frac{3}{5}$

$\Rightarrow$ AD = $\frac{12}{5}$ cm

Also, $\frac{DB}{AB} = \frac{AB}{CB}$

$\Rightarrow \frac{DB}{3} = \frac{3}{5}$

$\Rightarrow$ DB = $\frac{9}{5}$ cm

$$5-\frac{9}{5}=\frac{16}{5}$$

$\therefore$ CD = BC – DB = cm

Volume of the double cone

$= \frac{1}{3}\pi\left(\frac{12}{5}\right)^2\left(\frac{9}{5}\right)+\frac{1}{3}\pi\left(\frac{12}{5}\right)^2\left(\frac{16}{5}\right)$

$= \frac{1}{3}\times 3.14\times\frac{12}{5}\times\frac{12}{5}\times 5 = 30.14\ cm^3$

Surface area of the double cone

$= \pi\times\frac{12}{5}\times 3+\pi\times\frac{12}{5}\times 4$

$= \pi\times\frac{12}{5}(3+4) = 3.14\times\frac{12}{5}\times 7$

$= 52.75\ cm^2$

3. A cistern, internally measuring $150\text{ cm}\times 120\text{ cm}\times 110\text{ cm}$ has 129600 cm^3 of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each brick being $22.5\text{ cm}\times 7.5\text{ cm}\times 6.5\text{ cm}$?

Ans. Volume of cistern = $150\times 120\times 110 = 1980000\ cm^3$

Volume of water = $129600\ cm^3$

$\therefore$ Volume of cistern to be filled

= 1980000 – 129600 = $1850400\ cm^3$

Volume of a brick =

=

Let n bricks be needed.

Then, water absorbed by n bricks = $n \times \frac{1096.875}{17} cm^3$

$\therefore n = \frac{1850400 \times 17}{16 \times 1096.875}$ = 1792 (approx.)

---

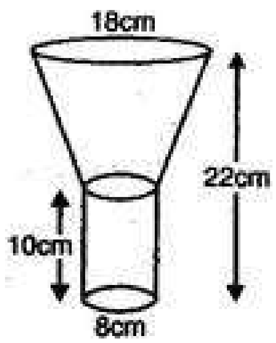
4. In one fortnight of a given month, there was a rainfall of 10 cm in a river valley. If the area of the valley is 7280 km^2, show that the total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.

Ans. Volume of rainfall = $7280 \times \frac{10}{100 \times 1000}$ = 0.728 km^2

Volume of three rivers = $3 \times 1072 \times \frac{75}{1000} \times \frac{3}{1000}$ = 0.7236 km^2

Hence, the amount of rainfall is approximately equal to the amount of water in three rivers.

5.An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see figure).



Ans. Slant height of the frustum of the cone

$(l) = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{(22-10)^2 + \left(\frac{18}{2} - \frac{8}{2}\right)^2}$ = 13 cm

Area of the tin sheet required

= CSA of cylinder + CSA of the frustum

$= 2\pi(4)(10) + \pi(4+9)13$

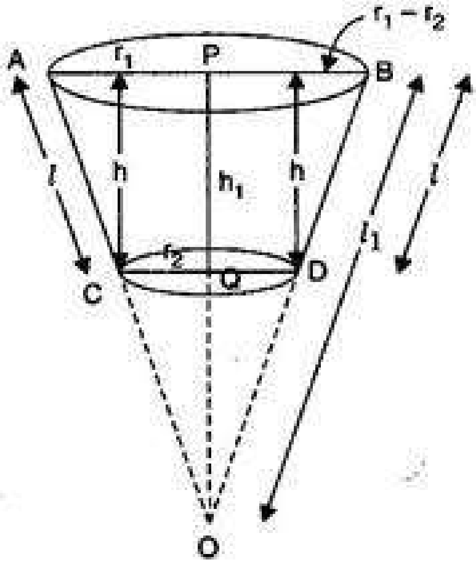
$= 80\pi + 169\pi$

$= 249\pi = 249 \times \frac{22}{7} = 782\frac{4}{7} cm^2$

= **Derive the formula for the volume of the frustum of a cone, given to you in Section**

6. 5, using the symbols as explained.

Ans. According to the question, the frustum is the difference of the two cones OAB and OCD (in figure).



For frustum, height = h, slant height = l and radii of the bases = r_1 and r_2 $(r_1 > r_2)$

OP= h_1, OA=OB= l

$\therefore$ Height of the cone OCD = $h_1 - h$

$\because \Delta_{OQD} \sim \Delta_{OPB}$ [By, AA similarity]

$\therefore \dfrac{h_1 - h}{h_1} = \dfrac{r_2}{r_1}$

$\Rightarrow 1 - \dfrac{h}{h_1} = \dfrac{r_2}{r_1}$

$\Rightarrow 1 - \dfrac{r_2}{r_1} = \dfrac{h}{h_1}$

$\Rightarrow h_1 = \dfrac{hr_1}{r_1 - r_2}$ ……….(i)

$\therefore$ height of the cone OCD = $h_1 - h$

$= \dfrac{hr_1}{r_1 - r_2} - h = \dfrac{hr_2}{r_1 - r_2}$ ……….(ii)

$\therefore$ V of the frustum

8. V of cone OAB – V of cone OCD

9. $\dfrac{1}{3}\pi r_1^2 h_1 - \dfrac{1}{3}\pi r_2^2 (h_1 - h)$

$= \dfrac{\pi}{3}\left[r_1^2 \cdot \dfrac{hr_1}{r_1 - r_2} - r_2^2 \cdot \dfrac{hr_2}{r_1 - r_2} \right]$

[From eq. (i) & (ii)]

$= \dfrac{\pi h}{3}\left(\dfrac{r_1^3 - r_2^3}{r_1 - r_2} \right)$

$= \dfrac{1}{3}\pi h \left(r_1^2 + r_2^2 + r_1 r_2 \right)$

If $A_1 and A_2$ are the surface areas of two circular bases, then

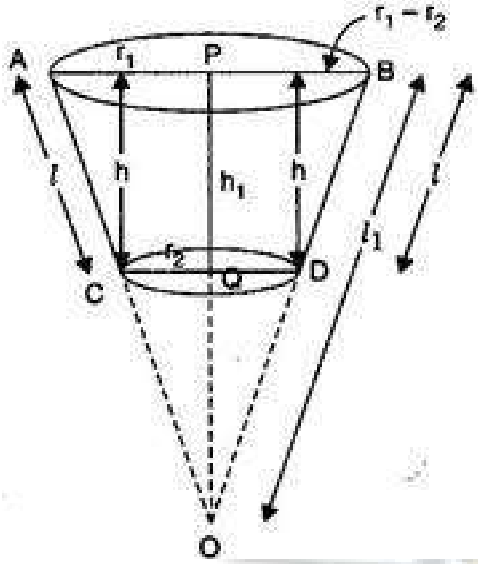
$A_1 = \pi r_1^2$ and $A_2 = \pi r_2^2$

$\therefore$ V of the frustum

$$= \frac{h}{3}\left(\pi r_1^2 + \pi r_2^2 + \sqrt{\pi r_1^2}.\sqrt{\pi r_2^2}\right)$$

$$= \frac{h}{3}\left(A_1 + A_2 + \sqrt{A_1 A_2}\right)$$

7..Derive the formula for the curved surface area and total surface area of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.

Ans. 

For frustum, height = h, slant height = l and radii of the bases = r_1 and $r_2\left(r_1 > r_2\right)$

OP= h_1, OA=OB= l

Again, from ΔDEB, $l = \sqrt{h^2 + \left(r_1 - r_2\right)^2}$

$\because \Delta OQD \sim \Delta OPB$ [AA similarity]

$\therefore \frac{l_1 - l}{l_1} = \frac{r_2}{r_1}$

$\Rightarrow l_1 = \frac{lr_1}{r_1 - r_2}$ ……….(iii)

$\therefore l_1 - l = \frac{lr_1}{r_1 - r_2} - l = \frac{lr_2}{r_1 - r_2}$ ……….(iv)

Hence, CSA of the frustum of the cone = $\pi r_1 l_1 - \pi r_2 (l_1 - l)$

= $\pi r_1 . \frac{lr_1}{r_1 - r_2} - \pi r_2 \frac{lr_2}{r_1 - r_2}$ [From eq. (i) and (ii)]

= $\pi l \left(\frac{r_1^2 - r_2^2}{r_1 - r_2} \right)$ = $\pi l (r_1 + r_2)$,

where $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$\therefore$ TSA of the frustum of the cone

=

PUNA INTERNATIONAL SCHOOL

- **CLASS – 10**
- **SUBJECT - MATHS**
- **CHAPTER - 15**

SAMPLE NOTE-BOOK

Chapter - 15
Probability

Exercise 15.1

1. Complete the statements:

1. **Probability of event E + Probability of event "not E" = ________________**
2. **The probability of an event that cannot happen is ________________. Such an event is called ________________.**
3. **The probability of an event that is certain to happen is ________________. Such an event is called ________________.**
4. **The sum of the probabilities of all the elementary events of an experiment is ________________.**
5. **The probability of an event is greater than or equal to ________________ and less than or equal to ________________.**

Ans. 1 1

2 0, impossible event

3. 1, sure or certain event

4. 1

5. 0, 1

2. **Which of the following experiments have equally likely outcomes? Explain.**

(i) A driver attempts to start a car. The car starts or does not start.

(ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.
(iii) A trial is made to answer a true-false question. The answer is right or wrong .
(iv) A baby is born. It is a boy or a girl.

Ans. (i) In the experiment, "A driver attempts to start a car. The car starts or does not start", we are not justified to assume that each outcome is as likely to occur as the other. Thus, the experiment has no equally likely outcomes.

(II) In the experiment, "A player attempts to shoot a basketball. She/he shoots or misses the shot", the outcome depends upon many factors e.g. quality of player. Thus, the experiment has no equally likely outcomes.

(ii) In the experiment, "A trial is made to answer a true-false question. The answer is right or wrong." We know, in advance, that the result can lead to one of the two possible ways – either right or wrong. We can reasonably assume that each outcome, right or wrong, is likely to occur as the other. Thus, the outcomes right or wrong are equally likely.

(iii) In the experiment, "A baby is born, It is a boy or a girl, we know, in advance that there are only two possible outcomes – either a boy or a girl. We are justified to assume that each
outcome, boy or girl, is likely to occur as the other. Thus, the outcomes boy or girl are equally likely.

3. **Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?**

Ans. The tossing of a coin is considered to be a fair way of deciding which team should get the ball at the beginning of a football game as we know that the tossing of the coin only land in one of two possible ways – either head up or tail up. It can reasonably be assumed that each outcome, head or tail, is as likely to occur as the other, i.e., the outcomes head and tail are equally likely. So the result of the tossing of a coin is completely unpredictable.

4. **Which of the following cannot be the probability of an event:**

(A) $\frac{2}{3}$

(B) -1.5

(C) 15%

(D) 0.7

Ans. (B) Since the probability of an event E is a number P(E) such that

$0 \leq P(E) \leq 1$

$\therefore -1.5$ cannot be the probability of an event.

5. **If P(E) = 0.05, what is the probability of 'not E'?**

   Ans. Since P(E) + P (not E) = 1

$\therefore$ P (not E) = 1 – P(E) = 1 – 0.05 = 0.95

6. **A bag contains lemon flavoured candles only. Malini takes out one candy without looking into the bag. What is the probability that she takes out:**

(i) an orange flavoured candy?

(ii) a lemon flavoured candy?

Ans. (i) Consider the event related to the experiment of taking out of an orange flavoured candy from a bag containing only lemon flavoured candies. Since no outcome gives an orange flavoured candy, therefore, it is an impossible event. So its probability is 0.

(ii) Consider the event of taking a lemon flavoured candy out of a bag containing only lemon flavoured candies. This event is a certain event. So its probability is 1.

7. **It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?**

Ans. Let E be the event of having the same birthday

$\Rightarrow$ P(E) = 0.992

But $P(E) + P(\overline{E}) = 1$

$\therefore P(\overline{E}) = 1 - P(E) = 1 - 0.992 = 0.008$

8. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is:

(i) red?

(ii) not red?

Ans. There are 3 + 5 = 8 balls in a bag. Out of these 8 balls, one can be chosen in 8 ways. $\therefore$ Total number of elementary events = 8

(i)Since the bag contains 3 red balls, therefore, one red ball can be drawn in 3 ways.

$\therefore$ Favourable number of elementary events = 3

Hence P (getting a red ball) = $\frac{3}{8}$

(ii) Since the bag contains 5 black balls along with 3 red balls, therefore one black (not red) ball can be drawn in 5 ways.

$\therefore$ Favourable number of elementary events = 5

Hence P (getting a black ball) = $\frac{5}{8}$

9. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be:

(i) red?

(ii) white?

(iii) not green?

Ans. Total number of marbles in the box = 5 + 8 + 4 = 17

$\therefore$ Total number of elementary events = 17

(i) There are 5 red marbles in the box.

$\therefore$ Favourable number of elementary events = 5

$\therefore$ P (getting a red marble) = $\frac{5}{17}$

(ii) There are 8 white marbles in the box.

$\therefore$ Favourable number of elementary events = 8

$\therefore$ P (getting a white marble) = $\frac{8}{17}$

(iii)There are 5 + 8 = 13 marbles in the box, which are not green.

$\therefore$ Favourable number of elementary events = 13

$\therefore$ P (not getting a green marble) = $\frac{13}{17}$

10. **A piggy bank contains hundred 50 p coins, fifty Re. 1 coins, twenty Rs. 2coins and ten Rs. 5 coins. If it is equally likely that of the coins will fall out when the bank is turned upside down, what is the probability that the coin:**

(i) will be a 50 p coin?

(ii) will not be a Rs.5 coin?

Ans. Total number of coins in a piggy bank = 100 + 50 + 20 + 10 = 180

$\therefore$ Total number of elementary events = 180

There are one hundred 50 coins in the piggy bank.

∴ Favourable number of elementary events = 100

∴ P (falling out of a 50 p coin) = $\frac{100}{180} = \frac{5}{9}$

(ii)There are 100 + 50 + 20 = 170 coins other than Rs. 5 coin.

∴ Favourable number of elementary events = 170

∴ P (falling out of a coin other than Rs. 5 coin) = $\frac{170}{180} = \frac{17}{18}$

11. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fishes and 8 female fishes (see figure). What is the probability that the fish taken out is a male fish?

Ans. Total number of fish in the tank = 5 + 8 = 13

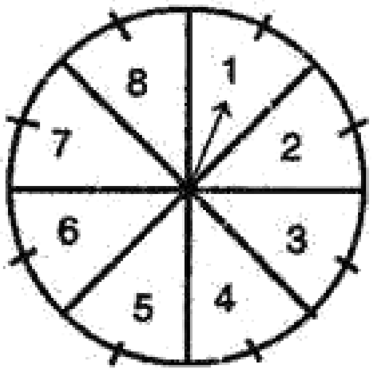
∴ Total number of elementary events = 13

There are 5 male fishes in the tank.

∴ Favourable number of elementary events = 5

Hence, P (taking out a male fish) = $\frac{5}{13}$

12. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see figure) and these are equally likely outcomes. What is the probability that it will point at:



(i) 8?

(ii) an odd number?

(iii)a number greater than 2?

(IV) a number less than 9?

Ans. Out of 8 numbers, an arrow can point any of the numbers in 8 ways.

∴ Total number of possible outcomes = 8

(i) Favourable number of outcomes = 1

Hence, P (arrow points at 8) = 1/8 $\frac{1}{8}$

(ii) Favourable number of outcomes = 4

Hence, P (arrow points at an odd number) = $\frac{4}{8}=\frac{1}{2}$

(iii)Favourable number of outcomes = 6

Hence, P (arrow points at a number›2) = $\frac{6}{8}=\frac{3}{4}$

(iv)Favourable number of outcomes = 8

Hence, P (arrow points at a number < 9) = $\frac{8}{8}$ =1

13. A dice is thrown once. Find the probability of getting:

(i) a prime number.

(ii) a number lying between 2 and 6. (iii)

an odd number.

Ans. Total number of Possible outcomes of throwing a dice = 6

(i) On a dice, the prime numbers are 2, 3 and 5.

Therefore, favourable outcomes = 3

Hence P (getting a prime number) = $\frac{3}{6} = \frac{1}{2}$

(ii) On a dice, the number lying between 2 and 6 are 3, 4, 5.

Therefore, favourable outcomes = 3

Hence P (getting a number lying between 2 and 6) = $\frac{3}{6} = \frac{1}{2}$

(iii) On a dice, the odd numbers are 1, 3 and 5.

Therefore, favourable outcomes = 3

Hence P (getting an odd number) = $\frac{3}{6} = \frac{1}{2}$

14. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting:

(i) a king of red colour

(ii) a face card

(iii)a red face card

(iv) the jack of hearts

(v) a spade

(vi) the queen of diamonds.

Ans. Total number of possible outcomes = 52

(i) There are two suits of red cards, i.e., diamond and heart. Each suit contains one king. $\therefore$ Favourable outcomes = 1

Hence, P (a king of red colour) = $\frac{2}{52} = \frac{1}{26}$

(ii) There are 12 face cards in a pack.

$\therefore$ Favourable outcomes = 12

Hence, P (a face card) = $\frac{12}{52} = \frac{3}{13}$

(iii) There are two suits of red cards, i.e., diamond and heart. Each suit contains 3 face cards. $\therefore$ Favourable outcomes = 2×3 = 6

Hence, P (a red face card) = $\frac{6}{52} = \frac{3}{26}$

(iv) There are only one jack of heart.

$\therefore$ Favourable outcome = 1

Hence, P (the jack of hearts) = $\frac{1}{52}$

(v) There are 13 cards of spade.

$\therefore$ Favourable outcomes = 13

Hence, P (a spade) = $\frac{13}{52} = \frac{1}{4}$

(vi) There is only one queen of diamonds.

$\therefore$ Favourable outcome = 1

Hence, P (the queen of diamonds) = $\frac{1}{52}$

15. Five cards – then ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.

(i) What is the probability that the card is the queen?

(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

Ans. Total number of possible outcomes = 5

(i) There is only one queen.

$\therefore$ Favourable

outcome = 1

Hence, P (the queen) = $\frac{1}{5}$

(ii) In this situation, total number of favourable outcomes = 4

(a) Favourable outcome = 1

Hence, P (an ace) = $\frac{1}{4}$

(b) There is no card as queen.

$\therefore$ Favourable outcome =0

Hence, P (the queen) = $\frac{0}{4} = 0$

16. **12 defective pens are accidently mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.**

Ans. Total number of possible outcomes = 132 + 12 = 144

Number of favourable outcomes = 132

Hence, P (getting a good pen) = $\frac{132}{144} = \frac{11}{12}$

17. **(i) A lot of 20 bulbs contains 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?**

(ii) **Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?**

Ans. (i) Total number of possible outcomes = 20

Number of favourable outcomes = 4

Hence P (getting a defective bulb) = $\frac{4}{20} = \frac{1}{5}$

(ii) Now total number of possible outcomes = 20 – 1 = 19

Number of favouroable outcomes = 19 – 4 = 15

Hence P (getting a non-defective bulb) = $\frac{15}{19}$

18. **A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.**

Ans. Total number of possible outcomes = 90

(i) Number of two-digit numbers from 1 to 90 are 90 – 9 = 81

∴ Favourable outcomes = 81

Hence, P (getting a disc bearing a two-digit number) = $\frac{81}{90} = \frac{9}{10}$

(ii) From 1 to 90, the perfect squares are 1, 4, 9, 16, 25, 36, 49, 64 and 81.

∴ Favourable outcomes = 9

Hence P (getting a perfect square) = $\frac{9}{90} = \frac{1}{10}$

(iii) The numbers divisible by 5 from 1 to 90 are 18.

∴ Favourable outcomes = 18

Hence P (getting a number divisible by 5) = $\frac{18}{90} = \frac{1}{5}$

19. A child has a die whose six faces show the letters as given below:

ABCDEA

The die is thrown once. What is the probability of getting:

(i) A?

(ii) D?

Ans. Total number of possible outcomes = 6

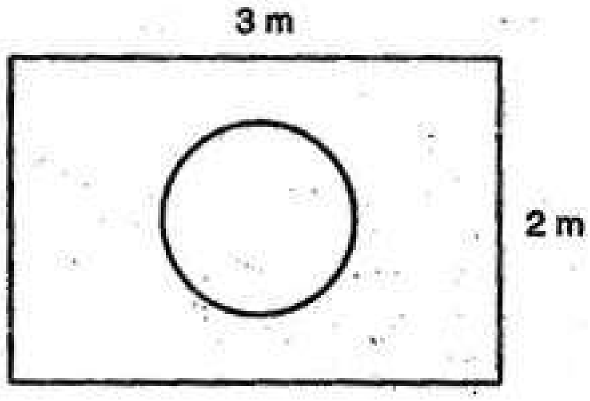
(i) Number of favourable outcomes = 2

Hence P (getting a letter A) = $\frac{2}{6} = \frac{1}{3}$

(ii) Number of favourable outcomes = 1

Hence P (getting a letter D) = $\frac{1}{6}$

20. Suppose you drop a die at random on the rectangular region shown in the figure given on the next page. What is the probability that it will land inside the circle with diameter 1 m?



Ans. Total area of the given figure (rectangle) = $3 \times 2 = 6\ m^2$

And Area of circle = πr^2 = $\pi\left(\frac{1}{2}\right)^2$ = $\frac{\pi}{4}$ m^2

Hence, P (die to land inside the circle) = $\frac{\pi/4}{6} = \frac{\pi}{24}$

21. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that:

(i) she will buy it?

(ii) she will not buy it?

Ans. Total number of possible outcomes = 144

(i) Number of non-defective pens = 144 – 20 = 124

$\therefore$ Number of favourable outcomes = 124

Hence P (she will buy) = P (a non-defective pen) = $\frac{124}{144} = \frac{31}{36}$

(ii) Number of favourable outcomes = 20

Hence P (she will not buy) = P (a defective pen) = $\frac{20}{144} = \frac{5}{36}$

22. Refer to example 13.

(i) Complete the following table:

Event: Sum of 2 dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$						$\frac{5}{36}$				$\frac{1}{36}$

(ii) A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Therefore each of them has a probability $\frac{1}{11}$ Do you agree with this argument? Justify your answer.

Ans. Total possible outcomes of throwing two dice are:

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

$\therefore$ Total number of favourable outcomes = 36

(i) Favourable outcomes of getting the sum as 3 = 2 Hence

P (getting the sum as 3) = $\frac{2}{36} = \frac{1}{18}$

Favourable outcomes of getting the sum as 4 = 3

Hence P (getting the sum as 4) = $\frac{3}{36} = \frac{1}{12}$

Favourable outcomes of getting the sum as 5 = 4

Hence P (getting the sum as 5) = $\frac{4}{36} = \frac{1}{9}$

Favourable outcomes of getting the sum as 6 = 5

Hence P (getting the sum as 6) = $\frac{5}{36}$

Favourable outcomes of getting the sum as 7 = 6

Hence P (getting the sum as 7) = $\frac{6}{36} = \frac{1}{6}$

Favourable outcomes of getting the sum as 9 = 4

Hence P (getting the sum as 9) = $\frac{4}{36} = \frac{1}{9}$

Favourable outcomes of getting the sum as 10 = 3

Hence P (getting the sum as 10) = $\frac{3}{36} = \frac{1}{12}$

Favourable outcomes of getting the sum as 11 = 2

Hence P (getting the sum as 11) = $\frac{2}{36} = \frac{1}{18}$

Event: Sum of 2 dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(ii) I do not agree with the argument given here. Justification has already been given in part (i).

23. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result, i.e., three heads or three tails and loses otherwise. Calculate the probability that Hanif will lose the game.

Ans. The outcomes associated with the experiment in which a coin is tossed thrice:

HHH, HHT, HTH, THH, TTH, HTT, THT, TTT

Therefore, Total number of possible outcomes = 8

Number of favourable outcomes = 6

Hence required probability = $\frac{6}{8} = \frac{3}{4}$

24. A die is thrown twice. What is the probability that:

(i) 5 will not come up either time? (ii) 5

will come up at least once?

Ans. (i) The outcomes associated with the experiment in which a dice is thrown is twice: (1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

Therefore, Total number of possible outcomes = 36

Now consider the following events:

A = first throe shows 5 and B = second throw shows 5

Therefore, the number of favourable outcomes = 6 in each case.

$\therefore$ P(A) = $\frac{6}{36}$ and P(B) = $\frac{6}{36}$

$\Rightarrow P\left(\overline{A}\right) = 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6}$ and $P\left(\overline{B}\right) = \frac{5}{6}$

$\therefore$ Required probability = $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

(ii) Let S be the sample space associated with the random experiment of throwing a die twice. Then, n(S) = 36

$\therefore$ A $\cap$ B = first and second throw shoe 5, i.e. getting 5 in each throw.

We have, A = (5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

And B = (1, 5) (2, 5) (3, 5) (4, 5) (5, 5) (6, 5)

$\therefore$ P(A) = $\frac{6}{36}$, P(B) = $\frac{6}{36}$ and P(A $\cap$ B) = $\frac{1}{36}$

$\therefore$ Required probability = Probability that at least one of the two throws shows 5

= P (A $\cup$ B) = P(A) + P(B) – P(A $\cap$ B)

$= \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}$

25. Which of the following arguments are correct and which are not correct? Give reasons for your answer:

(i) If two coins are tossed simultaneously there are three possible outcomes – two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is $\frac{1}{3}$.

(ii) If a die is thrown, there are two possible outcomes – an odd number or an even number. Therefore, the probability of getting an odd number is $\frac{1}{2}$.

Ans. (i) Incorrect: We can classify the outcomes like this but they are not then, 'equally likely'. Reason is that 'one of each' can result in two ways – from a head on first coin and tail on the second coin or from a tail on the first coin and head on the second coin. This makes it twice as likely as two heads (or two tails).

(iv) Correct: The two outcomes considered in the question are equally likely.

Exercise 15.2

1. **Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on (i) the same day? (ii) consecutive days? (iii) different days?**

Ans. Total favourable outcomes associated to the random experiment of visiting a particular shop in the same week (Tuesday to Saturday) by two customers Shyam and Exta are:

(T, T) (T, W) (T, TH) (T, F) (T, S)

(W, T) (W, W) (W, TH) (W, F) (W, S)

(TH, T) (TH, W) (TH, TH) (TH, F) (TH, S)

(F, T) (F, W) (F, TH) (F, F) (F, S)

(S, T) (S, W) (S, TH) (S, F) (S< S)

where T = Tuesday, W = Wednesday, Th = Thursday, F = Friday, S = Saturday

$\therefore$ Total number of favourable outcomes = 5 x 5 = 25

= The favourable outcomes of visiting on the same day are (T, T), (W, W), (TH, TH), (F, F) and (S, S).

$\therefore$ Number of favourable outcomes = 5

Hence required probability = $\frac{5}{25} = \frac{1}{5}$

The favourable outcomes of visiting on consecutive days are (T, W), (W, T), (W, TH), (TH, W), (TH, F), (F, TH), (S, F) and (F, S).

$\therefore$ Number of favourable outcomes = 8

Hence required probability = $\frac{8}{25}$

Number of favourable outcomes of visiting on different days are 25 – 5 = 20 $\therefore$ Number of favourable outcomes = 20

Hence required probability = $\frac{20}{25} = \frac{4}{5}$

2. **A die is numbered in such a way that its faces show the numbers 1, 2, 2, 3, 3, 6. It is thrown two times and the total score in two throws is noted. Complete the following table which gives a few values of the total score on the two throws:**

Number in first throw

Number in second throw

+	1	2	2	3	3	6
1	2	3	3	4	4	7
2	3	4	4	5	5	8
2					5	
3						
3			5			9
6	7	8	8	9	9	12

What is the probability that the total score is

What is the probability that the total score is:

(i) even

(ii) 6

(iii) at least 6?

Ans. Complete table is as under:

Number in first throw

Number in second throw

+	1	2	2	3	3	6
1	2	3	3	4	4	7
2	3	4	4	5	5	8
2	3	4	4	5	5	8
3	4	5	5	6	6	9
3	4	5	5	6	6	9
6	7	8	8	9	9	12

It is clear that total number of favourable outcomes = 6×6 = 36

(i) Even scores are: 2, 4, 4, 4, 4, 8, 4, 4, 8, 4, 6, 4, 6, 6, 8, 8, 12

Number of favourable outcomes of getting total score even are 18

Hence P (getting total score even) = $\frac{18}{36} = \frac{1}{2}$

(ii) Number of favourable outcomes of getting total score 6 are 4

Hence P (getting total score 6) = $\frac{4}{36} = \frac{1}{9}$

(iii) Total score at least 6 = 7, 8, 8, 6, 6, 9, 6, 6, 9, 7, 8, 8, 9, 9, 12

Number of favourable outcomes of getting total score at least 6 are 15

Hence P (getting total score at least 6) = $\frac{15}{36} = \frac{5}{12}$

3. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag.

Ans. Let there be x blue balls in the bag.

$\therefore$ Total number of balls in the bag = $5 + x$

Now, P_1 = Probability of drawing a blue ball = $\frac{x}{5+x}$

And P_2 = Probability of drawing a red ball = But

according to question, $P_1 = 2P_2$

$$\Rightarrow \frac{x}{5+x} = 2\times\frac{5}{5+x}$$

$$\Rightarrow \frac{x}{5+x}\times\frac{5+x}{5} = 2$$

$$\Rightarrow x = 10$$

Hence, there are 10 blue balls in the bag.

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4. **A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball?**

If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x.

Ans. There are 12 balls in the box.

Therefore, total number of favourbale outcomes = 12

The number of favourable outcomes (Black balls) = x

Therefore P_1 = P (getting a black ball) = $\frac{x}{12}$

If 6 more balls put in the box, then

Total number of favourable outcomes = 12 + 6 = 18

And Number of favourable outcomes = $x+6$

$\therefore$ P_2 = P (getting a black ball) = $\frac{x+6}{18}$

According to question, $P_2 = 2P_1$

$\Rightarrow \frac{x+6}{18} = 2 \times \frac{x}{12}$

$\Rightarrow \frac{x+6}{18} = \frac{x}{6}$

$\Rightarrow 6x + 36 = 18x$

$\Rightarrow 18x - 6x = 36$

$\Rightarrow 12x = 36$

$\Rightarrow x = 3$

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5. **A jar contains 24 marbles, some are green and others are at blue. If a marble is drawn Find random from the jar, the probability that it is green is balls in $\frac{2}{3}$. the number if blue the jar.**

Ans. Here, Total number of favourable outcomes = 24 Let there be x green marbles.

Therefore, Favourable number of outcomes =

$x \therefore$ P(Green ball) = $\frac{x}{24}$

But P(Green ball) = $\frac{2}{3}$

$\therefore \frac{x}{24} = \frac{2}{3}$

$\Rightarrow x = 16$

Therefore, number of green marbles are 16

And number of blue marbles = 24 – 16 = 8