



PUNA
INTERNATIONAL
SCHOOL

- **CLASS – 10**
- **SUBJECT - MATHS**
- **CHAPTER - 1**

SAMPLE
NOTE-BOOK



Chapter Number - 1.

Real Numbers Class 10

Exercise 1.1

Q1 :

Use Euclid's division algorithm to find the HCF of:

- (i) 135 and 225 (ii) 196 and 38220 (iii) 867 and 255

Answer :

(i) 135 and 225

Since $225 > 135$, we apply the division lemma to 225 and

135 to obtain $225 = 135 \times 1 + 90$

Since remainder $90 \neq 0$, we apply the division lemma to 135

and 90 to obtain $135 = 90 \times 1 + 45$

We consider the new divisor 90 and new remainder 45, and apply the division

lemma to obtain $90 = 2 \times 45 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 45,

Therefore, the HCF of 135 and 225 is 45.

(ii) 196 and 38220

Since $38220 > 196$, we apply the division lemma to 38220 and

196 to obtain $38220 = 196 \times 195 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

(iii) 867 and 255

Since $867 > 255$, we apply the division lemma to 867 and

255 to obtain $867 = 255 \times 3 + 102$

Since remainder $102 \neq 0$, we apply the division lemma to 255

and 102 to obtain $255 = 102 \times 2 + 51$

We consider the new divisor 102 and new remainder 51, and apply the

division lemma to obtain $102 = 51 \times 2 + 0$

Since the remainder is zero, the process stops.



Since the divisor at this stage is 51,
Therefore, HCF of 867 and 255 is 51.

Q2 :

Show that any positive odd integer is of the form $6q+1$, or $6q+3$, or $6q+5$, where q is some integer.

Answer :

Let a be any positive integer and $b = 6$. Then, by Euclid's algorithm,
 $a = 6q + r$ for some integer $q \geq 0$, and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, where k_1 is a positive integer

$6q + 3 = (6q + 2) + 1 = 2(3q + 1) + 1 = 2k_2 + 1$, where k_2 is

an integer $6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k_3 + 1$,

where k_3 is an integer Clearly, $6q + 1, 6q + 3, 6q + 5$ are of
the form $2k + 1$, where k is an integer.

Therefore, $6q + 1, 6q + 3, 6q + 5$ are not exactly divisible by 2. Hence, these expressions
of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q$
 $+ 1$, or $6q + 3$, or $6q + 5$

Q3 :

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer :

HCF (616, 32) will give the maximum number of columns in which they can march.

We can use Euclid's algorithm to find the HCF.

$$616 = 32 \times 19 + 8$$

$$32 = 8 \times 4 + 0$$

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

Q4 :



Use Euclid's division lemma to show that the square of any positive integer is either of form $3m$ or $3m + 1$ for some integer m .

[Hint: Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]

Answer :

Let a be any positive integer and $b = 3$.

Then $a = 3q + r$ for some integer $q \geq 0$

And $r = 0, 1, 2$ because $0 \leq r < 3$

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$

Or,

$$\begin{aligned} a^2 &= (3q)^2 \text{ or } (3q+1)^2 \text{ or } (3q+2)^2 \\ a^2 &= (9q^2) \text{ or } 9q^2 + 6q + 1 \text{ or } 9q^2 + 12q + 4 \\ &= 3 \times (3q^2) \text{ or } 3(3q^2 + 2q) + 1 \text{ or } 3(3q^2 + 4q + 1) + 1 \\ &= 3k_1 \text{ or } 3k_2 + 1 \text{ or } 3k_3 + 1 \end{aligned}$$

Where k_1, k_2 , and k_3 are some positive integers

Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m + 1$.

Q5 :

Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

Answer :

Let a be any positive integer and $b = 3$

$a = 3q + r$, where $q \geq 0$ and $0 \leq r < 3$

$$\therefore a = 3q \text{ or } 3q + 1 \text{ or } 3q + 2$$

Therefore, every number can be represented as these three forms. There are three cases.

Case 1: When $a = 3q$,

$$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m,$$

$$3q^3$$

Where m is an integer such

that $m =$ **Case 2:** When $a = 3q$

$$+ 1, a^3 = (3q + 1)^3$$

$$a^3 = 27q^3 + 27q^2 + 9q + 1$$

$$a^3 = 9(3q^3 + 3q^2 + q) + 1$$



$$a^3 = 9m + 1$$

Where m is an integer such that $m = (3q^3 + 3q^2 + q)$

Case 3: When $a = 3q + 2$,

$$a^3 = (3q + 2)^3$$

$$a^3 = 27q^3 + 54q^2 + 36q + 8$$

$$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$$

$$a^3 = 9m + 8$$

Where m is an integer such that $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.

Exercise 1.2

Q1 :

Express each number as product of its prime factors:

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Answer :

$$(i) \quad 140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$$

$$(ii) \quad 156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$$

$$(iii) \quad 3825 = 3 \times 3 \times 5 \times 5 \times 17 = 3^2 \times 5^2 \times 17$$

$$(iv) \quad 5005 = 5 \times 7 \times 11 \times 13$$

$$(v) \quad 7429 = 17 \times 19 \times 23$$

Q2 :

Find the LCM and HCF of the following pairs of integers and verify that $\text{LCM} \times \text{HCF} = \text{product of the two numbers}$.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Answer:

- (i) 26 and 91
 $26 = 2 \times 13$
 $91 = 7 \times 13$
HCF = 13
 $\text{LCM} = 2 \times 7 \times 13 = 182$
Product of the two numbers = $26 \times 91 = 2366$
 $\text{HCF} \times \text{LCM} = 13 \times 182 = 2366$

Hence, product of two numbers = HCF \times LCM

- (ii) 510 and 92
 $510 = 2 \times 3 \times 5 \times 17$
 $92 = 2 \times 2 \times 23$
HCF = 2
 $\text{LCM} = 2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$
Product of the two numbers = $510 \times 92 = 46920$
 $\text{HCF} \times \text{LCM} = 2 \times 23460$
 $= 46920$

Hence, product of two numbers = HCF \times LCM

- (iii) 336 and 54
 $336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$
 $336 = 2^4 \times 3 \times 7$
 $54 = 2 \times 3 \times 3 \times 3$
 $54 = 2 \times 3^3$
HCF = $2 \times 3 = 6$
 $\text{LCM} = 2^4 \times 3^3 \times 7 = 3024$
Product of the two numbers = $336 \times 54 = 18144$
 $\text{HCF} \times \text{LCM} = 6 \times 3024 = 18144$

Hence, product of two numbers = HCF \times LCM

Q3 :

Find the LCM and HCF of the following integers by applying the prime factorisation method.

- (i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

Answer :

(i) 12, 15 and 21
 $12 = 2^2 \times 3$
 $15 = 3 \times 5$
 $21 = 3 \times 7$
HCF = 3
 $LCM = 2^2 \times 3 \times 5 \times 7 = 420$

(ii) 17, 23 and 29
 $17 = 1 \times 17$
 $23 = 1 \times 23$
 $29 = 1 \times 29$
HCF = 1
 $LCM = 17 \times 23 \times 29 = 11339$

(iii) 8, 9 and 25
 $8 = 2 \times 2 \times 2$
 $9 = 3 \times 3$
 $25 = 5 \times 5$
HCF = 1
 $LCM = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

Q4 :

Given that $\text{HCF}(306, 657) = 9$, find $\text{LCM}(306, 657)$.

Answer :

$$\text{HCF}(306, 657) = 9$$

We know that, $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$

$$\therefore \text{LCM} \times \text{HCF} = 306 \times 657$$

$$\text{LCM} = \frac{306 \times 657}{\text{HCF}} = \frac{306 \times 657}{9}$$

$$\text{LCM} = 22338$$

Q5 :

Check whether $6n$ can end with the digit 0 for any natural number n . Answer :

If any number ends with the digit 0, it should be divisible by 10 or in other words, it will also be divisible by 2 and 5 as $10 = 2 \times 5$

Prime factorisation of $6n = (2 \times 3)n$

It can be observed that 5 is not in the prime factorisation of $6n$.

Hence, for any value of n , $6n$ will not be divisible by 5.

Therefore, $6n$ cannot end with the digit 0 for any natural number n .

Q6 :

Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Answer :

Numbers are of two types - prime and composite. Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1) = 13 \times (77 + 1)$$

$$= 13 \times 78$$

$$= 13 \times 13 \times 6$$

The given expression has 6 and 13 as its factors. Therefore, it is a composite number.

$$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$$

$$= 5 \times (1008 + 1)$$

$$= 5 \times 1009$$

1009 cannot be factorised further. Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

Q7 :

There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Answer :

It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.

$$18 = 2 \times 3 \times 3$$

$$\text{And, } 12 = 2 \times 2$$

$$\times 3$$

$$\text{LCM of 12 and 18} = 2 \times 2 \times 3 \times 3 = 36$$

Therefore, Ravi and Sonia will meet together at the starting point after 36 minutes.

Exercise 1.3

Q1 :

Prove that $\sqrt{5}$ is irrational.

Answer :

Let $\sqrt{5}$ is a rational number.

$$\sqrt{5} = \frac{a}{b}$$

Therefore, we can find two integers a, b ($b \neq 0$) such that

Let a and b have a common factor other than 1. Then we can divide them by the common factor, and assume that a and b are co-prime.

$$a = \sqrt{5}b$$

$$a^2 = 5b^2$$

Therefore, a^2 is divisible by 5 and it can be said that a is divisible by 5.

Let $a = 5k$, where k is an integer

$(5k)^2 = 5b^2$ This means that b^2 is divisible by 5 and hence, b is divisible by 5.

$b^2 = 5k^2$ This implies that a and b have 5 as a common factor.

And this is a contradiction to the fact that a and b are co-prime.

Hence, $\sqrt{5}$ cannot be expressed as $\frac{p}{q}$ or it can be said that $\sqrt{5}$ is irrational.

Q2 :

Prove that $3+2\sqrt{5}$ is irrational.

Answer :

Let $3+2\sqrt{5}$ is rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$3+2\sqrt{5} = \frac{a}{b}$$

$$2\sqrt{5} = \frac{a}{b} - 3$$

$$\sqrt{5} = \frac{1}{2} \left(\frac{a}{b} - 3 \right)$$

Since a and b are integers, $\frac{1}{2} \left(\frac{a}{b} - 3 \right)$ will also be rational and therefore, $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3+2\sqrt{5}$ is rational is false.

Therefore, $3+2\sqrt{5}$ is irrational.

Q3 :

Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6+\sqrt{2}$

Answer :

(i) $\frac{1}{\sqrt{2}}$

Let $\frac{1}{\sqrt{2}}$ is rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$

$$\sqrt{2} = \frac{b}{a}$$

$\frac{b}{a}$ is rational as a and b are integers.

Therefore, $\sqrt{2}$ is rational which contradicts to the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is false and $\frac{1}{\sqrt{2}}$ is irrational.

(ii) $7\sqrt{5}$

Let $7\sqrt{5}$ is rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$7\sqrt{5} = \frac{a}{b} \text{ for some integers } a \text{ and } b$$

$$\therefore \sqrt{5} = \frac{a}{7b}$$

$\frac{a}{7b}$ is rational as a and b are integers.

Therefore, $\sqrt{5}$ should be rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Therefore, our assumption that $7\sqrt{5}$ is rational is false. Hence, $7\sqrt{5}$ is irrational.

(iii) $6 + \sqrt{2}$

Let $6 + \sqrt{2}$ be rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$6 + \sqrt{2} = \frac{a}{b}$$

$$\sqrt{2} = \frac{a}{b} - 6$$

Since a and b are integers, $\frac{a}{b} - 6$ is also rational and hence, $\sqrt{2}$ should be rational. This contradicts the fact that $\sqrt{2}$ is irrational. Therefore, our assumption is false

and hence, $6 + \sqrt{2}$ is irrational.

Exercise 1.4

Q1 :

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$

(ii) $\frac{17}{8}$

(iii) $\frac{64}{455}$

(iv) $\frac{15}{1600}$

(v) $\frac{29}{343}$

(vi) $\frac{23}{2^3 5^2}$

(vii) $\frac{129}{2^2 5^7 7^5}$

(viii) $\frac{6}{15}$

(ix) $\frac{35}{50}$

(x) $\frac{77}{210}$



Answer :

$$(i) \frac{13}{3125}$$

$$3125 = 5^5$$

The denominator is of the form $5m$.

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

$$(ii) \frac{17}{8}$$

$$8 = 2^3$$

The denominator is of the form $2m$.

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

$$(iii) \frac{64}{455}$$

$$455 = 5 \times 7 \times 13$$

Since the denominator is not in the form $2m \times 5n$, and it also contains 7 and 13 as its factors, its decimal expansion will be non-terminating repeating.

$$(iv) \frac{15}{1600}$$

$$1600 = 2^6 \times 5^2$$

The denominator is of the form $2m \times 5n$.

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

$$(v) \frac{29}{343}$$

$$343 = 7^3$$

Since the denominator is not in the form $2m \times 5n$, and it has 7 as its factor, the decimal

expansion of $\frac{29}{343}$ is non-terminating repeating.

$$(vi) \frac{23}{2^3 \times 5^2}$$

The denominator is of the form $2m \times 5n$.

Hence, the decimal expansion of $\frac{23}{2^3 \times 5^2}$ is terminating.

$$(vii) \frac{129}{2^2 \times 5^7 \times 7^5}$$

Since the denominator is not of the form $2m \times 5n$, and it also has 7 as its factor, the decimal expansion

of $\frac{129}{2^2 \times 5^7 \times 7^5}$ is non-terminating repeating.

$$(viii) \frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5}$$

The denominator is of the form $5n$.

Hence, the decimal expansion of $\frac{6}{15}$ is terminating.

$$(ix) \frac{35}{50} = \frac{7 \times 5}{10 \times 5} = \frac{7}{10}$$

$$10 = 2 \times 5$$

The denominator is of the form $2m \times 5n$.

Q2 :

Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Answer :

(i) $\frac{13}{3125} = 0.00416$

$$\begin{array}{r} 0.00416 \\ 3125 \overline{) 13.00000} \\ \underline{0} \\ 130 \\ \underline{0} \\ 1300 \\ \underline{0} \\ 13000 \\ \underline{12500} \\ 5000 \\ 3125 \\ \underline{18750} \\ 18750 \\ \underline{\quad} \\ \times \end{array}$$

(ii) $\frac{17}{8} = 2.125$

$$\begin{array}{r} 2.125 \\ 8 \overline{) 17} \\ \underline{16} \\ 10 \\ 8 \\ \underline{20} \\ 16 \\ \underline{40} \\ 40 \\ \underline{\quad} \\ \times \end{array}$$

$$(iv) \quad \frac{15}{1600} = 0.009375$$

$$\begin{array}{r} 0.009375 \\ 1600 \overline{) 15.000000} \\ \underline{0} \\ 150 \\ \underline{0} \\ 1500 \\ \underline{0} \\ 15000 \\ 14400 \\ \hline 6000 \\ 4800 \\ \hline 12000 \\ 11200 \\ \hline 8000 \\ 8000 \\ \hline \times \end{array}$$

$$(vi) \quad \frac{23}{2^3 \times 5^2} = \frac{23}{200} = 0.115$$

$$\begin{array}{r} 0.115 \\ 200 \overline{) 23.000} \\ \underline{0} \\ 230 \\ 200 \\ \hline 300 \\ 200 \\ \hline 1000 \\ 1000 \\ \hline \times \end{array}$$

$$(viii) \quad \frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5} = 0.4$$

$$\begin{array}{r} 0.4 \\ 5 \overline{) 2.0} \\ \underline{0} \\ 20 \\ \underline{20} \\ \times \end{array}$$

(ix) $\frac{35}{50} = 0.7$

$$\begin{array}{r} 0.7 \\ 50 \overline{)35.0} \\ \underline{0} \\ 350 \\ \underline{350} \\ \times \end{array}$$

Q3 :

The following real numbers have decimal expansions as given below. In each case, decide whether they are

rational or not. If they are rational, and $\frac{p}{q}$, what can you say about the prime factor of q ?

of the form (i) 43.123456789 (ii)

0.120120012000120000... (iii)

Answer :

(i) 43.123456789

Since this number has a terminating decimal expansion, it is a rational number of the form $2^m \times 5^n$

$\frac{p}{q}$ and q is of the

i.e., the prime factors of q will be either 2 or 5 or both.

(ii) 0.120120012000120000 ...

The decimal expansion is neither terminating nor recurring. Therefore, the given number is an irrational number.

(iii) $\overline{43.123456789}$

Since the decimal expansion is non-terminating recurring, the given number is a rational number of the form $\frac{p}{q}$ and q is not of the form $2^m \times 5^n$ i.e., the prime factors of q will also have a factor other than 2 or 5.







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- **SUBJECT - MATHS**
- **CHAPTER - 2**

SAMPLE
NOTE-BOOK



CHAPTER - 2

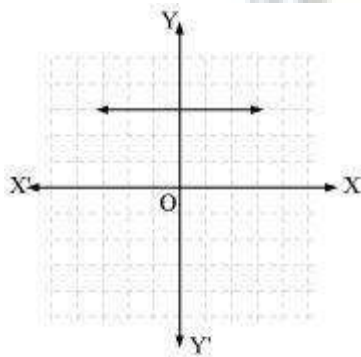
Polynomials

Exercise 2.1

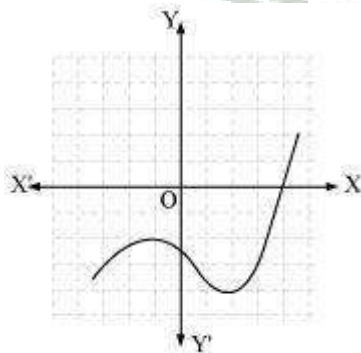
Question 1:

The graphs of $y = p(x)$ are given in following figure, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

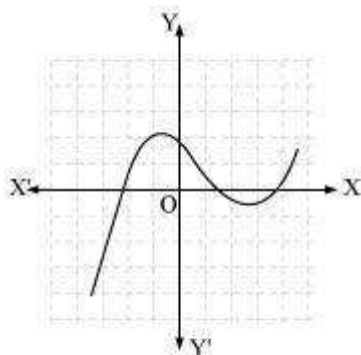
(i)

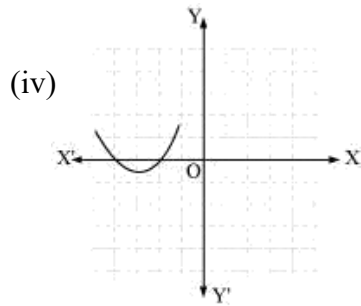


(ii)

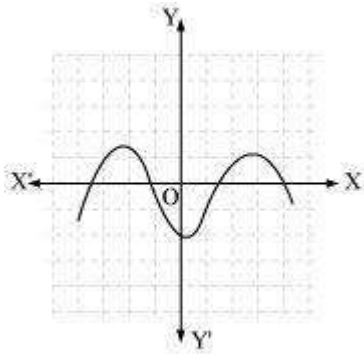


(iii)

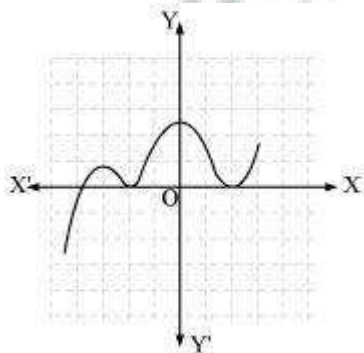




(v)



(v)



Answer:

= The number of zeroes is 0 as the graph does not cut the x-axis at any point.

= The number of zeroes is 1 as the graph intersects the x-axis at only 1 point.

= The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

= The number of zeroes is 2 as the graph intersects the x-axis at 2 points.

= The number of zeroes is 4 as the graph intersects the x-axis at 4 points.

= The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

Exercise 2.2

Question 1:

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$ (ii) $4s^2 - 4s + 1$ (iii) $6x^2 - 3 - 7x$

(iv) $4u^2 + 8u$ (v) $t^2 - 15$ (vi) $3x^2 - x - 4$

Answer:

(i) $x^2 - 2x - 8 = (x - 4)(x + 2)$

The value of $x^2 - 2x - 8$ is zero when $x - 4 = 0$ or $x + 2 = 0$, i.e., when $x = 4$ or $x = -2$

Therefore, the zeroes of $x^2 - 2x - 8$ are 4 and -2.

Sum of zeroes = $4 - 2 = 2 = \frac{-(-2)}{1} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes = $4 \times (-2) = -8 = \frac{(-8)}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(ii) $4s^2 - 4s + 1 = (2s - 1)^2$

The value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, i.e., $s = \frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are and .

Sum of zeroes =

$$\text{Product of zeroes} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$$

$$(iii) \quad 6x^2 - 3 - 7x = 6x^2 - 7x - 3 = (3x+1)(2x-3)$$

The value of $6x^2 - 3 - 7x$ is zero when $3x + 1 = 0$ or $2x - 3 = 0$, i.e., $x = \frac{-1}{3}$ or $x = \frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

$$\text{Sum of zeroes} = \frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$(iv) \quad 4u^2 + 8u = 4u^2 + 8u + 0 \\ = 4u(u+2)$$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$, i.e., $u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and -2.

$$\text{Sum of zeroes} = 0 + (-2) = -2 = \frac{-(8)}{4} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$$

$$\text{Product of zeroes} = 0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$$

$$(v) \quad t^2 - 15 \\ = t^2 - 0t - 15 \\ = (t - \sqrt{15})(t + \sqrt{15})$$

The value of $t^2 - 15$ is zero when $t - \sqrt{15} = 0$ or $t + \sqrt{15} = 0$, i.e., when $t = \sqrt{15}$ or $t = -\sqrt{15}$

Therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

$$\text{Sum of zeroes} = \sqrt{15} + (-\sqrt{15}) = 0 = \frac{-0}{1} = \frac{-(\text{Coefficient of } t)}{(\text{Coefficient of } t^2)}$$

$$\text{Product of zeroes} = (\sqrt{15})(-\sqrt{15}) = -15 = \frac{-15}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\begin{aligned} \text{(vi)} \quad & 3x^2 - x - 4 \\ & = (3x - 4)(x + 1) \end{aligned}$$

The value of $3x^2 - x - 4$ is zero when $3x - 4 = 0$ or $x + 1 = 0$, i.e., when $x = \frac{4}{3}$ or $x = -1$

Therefore, the zeroes of $3x^2 - x - 4$ are $\frac{4}{3}$ and -1 .

$$\text{Sum of zeroes} = \frac{4}{3} + (-1) = \frac{1}{3} = \frac{-(-1)}{3} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \frac{4}{3}(-1) = \frac{-4}{3} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Question 2:

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

$$\text{(i)} \quad \frac{1}{4}, -1 \quad \text{(ii)} \quad \sqrt{2}, \frac{1}{3} \quad \text{(iii)} \quad 0, \sqrt{5}$$

$$\text{(iv)} \quad 1, 1 \quad \text{(v)} \quad -\frac{1}{4}, \frac{1}{4} \quad \text{(vi)} \quad 4, 1$$

Answer:

(i) $\frac{1}{4}, -1$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β .

$$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$$

$$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$$

If $a = 4$, then $b = -1$, $c = -4$

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii) $\sqrt{2}, \frac{1}{3}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β .

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$

$$\alpha\beta = \frac{1}{3} = \frac{c}{a}$$

If $a = 3$, then $b = -3\sqrt{2}$, $c = 1$

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) $0, \sqrt{5}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β .

$$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$$

If $a = 1$, then $b = 0$, $c = \sqrt{5}$

Therefore, the quadratic polynomial is $x^2 + \sqrt{5}$.

(iv) 1, 1

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β .

$$\alpha + \beta = 1 = \frac{1}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If $a = 1$, then $b = -1$, $c = 1$

Therefore, the quadratic polynomial is $x^2 - x + 1$.

(v) $-\frac{1}{4}, \frac{1}{4}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β .

$$\alpha + \beta = -\frac{1}{4} = \frac{-b}{a}$$

$$\alpha \times \beta = \frac{1}{4} = \frac{c}{a}$$

If $a = 4$, then $b = 1$, $c = 1$

Therefore, the quadratic polynomial is $4x^2 + x + 1$.

(vi) 4, 1

Let the polynomial be $ax^2 + bx + c$.

$$\alpha + \beta = 4 = \frac{4}{1} = \frac{-b}{a}$$

$$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$$

If $a = 1$, then $b = -4$, $c = 1$

Therefore, the quadratic polynomial is $x^2 - 4x + 1$.

Exercise 2.3

Question 1:

Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$

Answer:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$
 $q(x) = x^2 - 2$

$$\begin{array}{r} x-3 \\ x^2-2 \overline{) x^3-3x^2+5x-3} \\ \underline{x^3 \quad -2x} \\ -3x^2+7x-3 \\ \underline{-3x^2 \quad +6} \\ + -9 \\ \hline 7x-9 \end{array}$$

Quotient = $x - 3$

Remainder = $7x - 9$

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

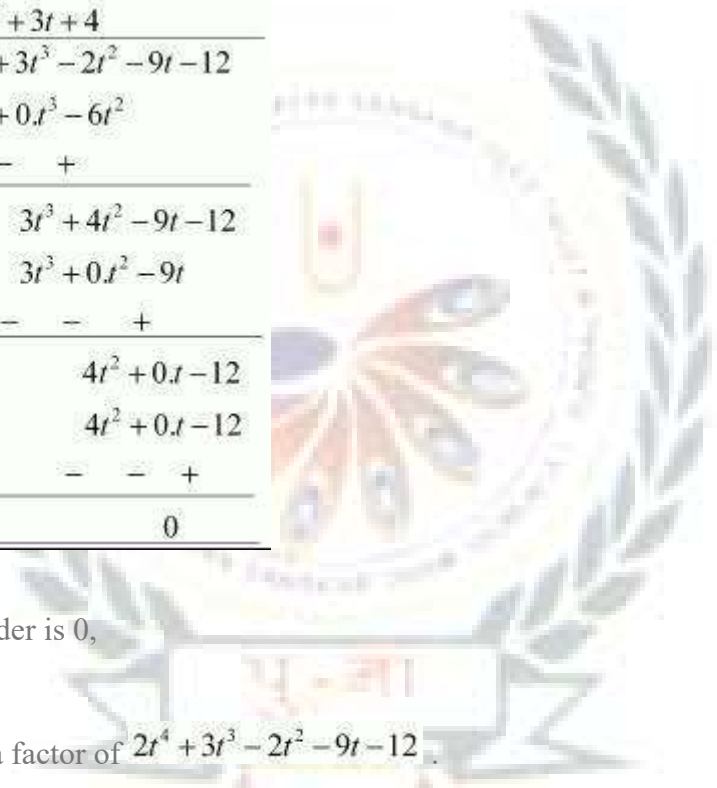
(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Answer:

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

$$t^2 - 3 = t^2 + 0t - 3$$


$$\begin{array}{r} 2t^2 + 3t + 4 \\ t^2 + 0t - 3 \overline{) 2t^4 + 3t^3 - 2t^2 - 9t - 12} \\ \underline{2t^4 + 0t^3 - 6t^2} \\ - - + \\ 3t^3 + 4t^2 - 9t - 12 \\ 3t^3 + 0t^2 - 9t \\ \underline{ - - + } \\ 4t^2 + 0t - 12 \\ 4t^2 + 0t - 12 \\ \underline{ - - + } \\ 0 \end{array}$$

Since the remainder is 0,

Hence, $t^2 - 3$ is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

$$\begin{array}{r}
 3x^2 - 4x + 2 \\
 x^2 + 3x + 1 \overline{) 3x^4 + 5x^3 - 7x^2 + 2x + 2} \\
 \underline{3x^4 + 9x^3 + 3x^2} \\
 -4x^3 - 10x^2 + 2x + 2 \\
 \underline{-4x^3 - 12x^2 - 4x} \\
 + + \\
 \underline{2x^2 + 6x + 2} \\
 \underline{2x^2 + 6x + 2} \\
 0
 \end{array}$$

Since the remainder is 0,

Hence, $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

(iii) $x^3 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

$$\begin{array}{r}
 x^2 - 1 \\
 x^3 - 3x + 1 \overline{) x^5 - 4x^3 + x^2 + 3x + 1} \\
 \underline{x^5 - 3x^3 + x^2} \\
 -x^3 + 1 \\
 \underline{-x^3 - 1} \\
 + + \\
 \underline{2}
 \end{array}$$

Since the remainder $\neq 0$,

Hence, $x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

Question 3:

Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are

$$\sqrt{\frac{5}{3}} \text{ and } -\sqrt{\frac{5}{3}}.$$

Answer:

$$p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$,

$\therefore \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \left(x^2 - \frac{5}{3}\right)$ is a factor of $3x^4 + 6x^3 - 2x^2 - 10x - 5$.

Therefore, we divide the given polynomial by $x^2 - \frac{5}{3}$.

$$\begin{array}{r} x^2 + 0x - \frac{5}{3} \overline{) 3x^4 + 6x^3 - 2x^2 - 10x - 5} \\ \underline{3x^4 + 0x^3 - 5x^2} \\ 6x^3 + 3x^2 - 10x - 5 \\ \underline{6x^3 + 0x^2 - 10x} \\ 3x^2 + 0x - 5 \\ \underline{3x^2 + 0x - 5} \\ 0 \end{array}$$

$$\begin{aligned} 3x^4 + 6x^3 - 2x^2 - 10x - 5 &= \left(x^2 - \frac{5}{3}\right)(3x^2 + 6x + 3) \\ &= 3\left(x^2 - \frac{5}{3}\right)(x^2 + 2x + 1) \end{aligned}$$

We factorize $x^2 + 2x + 1$

$$= (x + 1)^2$$

Therefore, its zero is given by $x + 1 = 0$

$$x = -1$$

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at $x = -1$.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, -1 and -1 .

Question 4:

On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Answer:

$$p(x) = x^3 - 3x^2 + x + 2 \quad (\text{Dividend})$$

$$g(x) = ? \quad (\text{Divisor})$$

$$\text{Quotient} = (x - 2)$$

$$\text{Remainder} = (-2x + 4)$$

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$x^3 - 3x^2 + x + 2 = g(x) \times (x - 2) + (-2x + 4)$$

$$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x - 2)$$

$$x^3 - 3x^2 + 3x - 2 = g(x)(x - 2)$$

$g(x)$ is the quotient when we divide $(x^3 - 3x^2 + 3x - 2)$ by $(x - 2)$

$$\begin{array}{r}
 x^2 - x + 1 \\
 x-2 \overline{) x^3 - 3x^2 + 3x - 2} \\
 \underline{x^3 - 2x^2} \\
 -x^2 + 3x - 2 \\
 \underline{-x^2 + 2x} \\
 +x - 2 \\
 \underline{+x - 2} \\
 0
 \end{array}$$

$$\therefore g(x) = (x^2 - x + 1)$$

Question 5:

Give examples of polynomial $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

$$= \deg p(x) = \deg q(x)$$

$$= \deg q(x) = \deg r(x)$$

$$= \deg r(x) = 0$$

Answer:

According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that

$$p(x) = g(x) \times q(x) + r(x),$$

where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

$$(i) \deg p(x) = \deg q(x)$$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2 + 2x + 2$ by 2.

$$\text{Here, } p(x) = 6x^2 + 2x + 2$$

$$g(x) = 2$$

$$q(x) = 3x^2 + x + 1 \text{ and } r(x) = 0$$

Degree of $p(x)$ and $q(x)$ is the same i.e., 2.

Checking for division algorithm,

$$p(x) = g(x) \times q(x) + r(x)$$

$$6x^2 + 2x + 2 = 2(3x^2 + x + 1)$$

$$= 6x^2 + 2x + 2$$

Thus, the division algorithm is satisfied.

$$(ii) \text{ deg } q(x) = \text{deg } r(x)$$

Let us assume the division of $x^3 + x$ by x^2 ,

$$\text{Here, } p(x) = x^3 + x$$

$$g(x) = x^2$$

$$q(x) = x \text{ and } r(x) = x$$

Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1.

Checking for division algorithm,

$$p(x) = g(x) \times q(x) + r(x)$$

$$x^3 + x = (x^2) \times x + x$$

$$x^3 + x = x^3 + x$$

Thus, the division algorithm is satisfied.

$$(iii) \deg r(x) = 0$$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^3 + 1$ by x^2 .

$$\text{Here, } p(x) = x^3 + 1$$

$$g(x) = x^2$$

$$q(x) = x \text{ and } r(x) = 1$$

Clearly, the degree of $r(x)$ is 0.

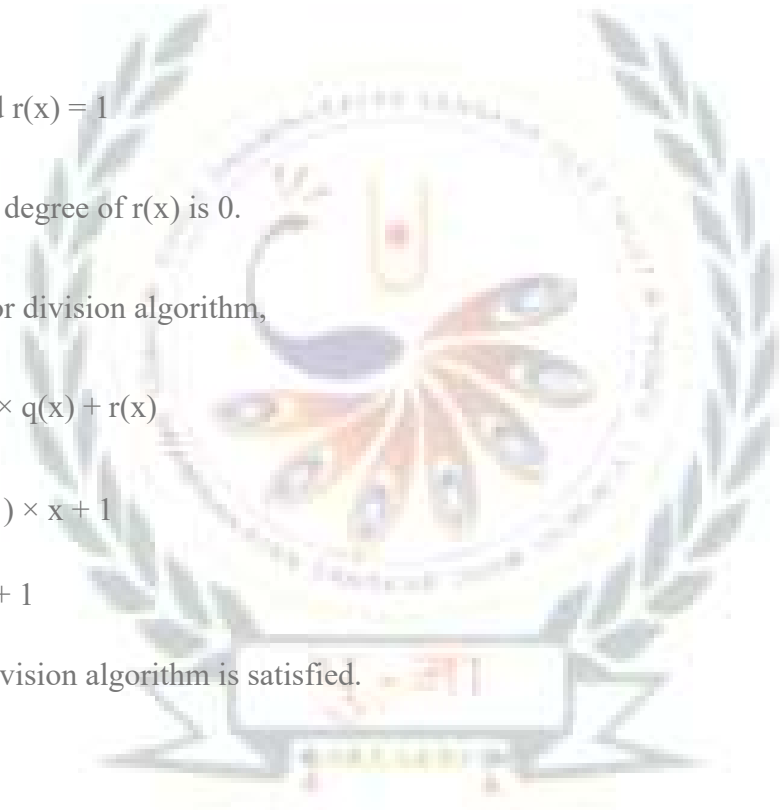
Checking for division algorithm,

$$p(x) = g(x) \times q(x) + r(x)$$

$$x^3 + 1 = (x^2) \times x + 1$$

$$x^3 + 1 = x^3 + 1$$

Thus, the division algorithm is satisfied.



Exercise 2.4

Question 1:

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2$; $\frac{1}{2}, 1, -2$

(ii) $x^3 - 4x^2 + 5x - 2$; $2, 1, 1$



Answer:

(i) $p(x) = 2x^3 + x^2 - 5x + 2.$

Zeroes for this polynomial are $\frac{1}{2}, 1, -2$

$$\begin{aligned} p\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2 \\ &= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} p(1) &= 2 \times 1^3 + 1^2 - 5 \times 1 + 2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} p(-2) &= 2(-2)^3 + (-2)^2 - 5(-2) + 2 \\ &= -16 + 4 + 10 + 2 = 0 \end{aligned}$$

Therefore, $\frac{1}{2}$, 1, and -2 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain $a = 2, b = 1, c = -5, d = 2$

We can take $\alpha = \frac{1}{2}, \beta = 1, \gamma = -2$

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(-2)}{2} = \frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$

Zeroes for this polynomial are 2, 1, 1.

$$p(2) = 2^3 - 4(2^2) + 5(2) - 2$$

$$= 8 - 16 + 10 - 2 = 0$$

$$p(1) = 1^3 - 4(1)^2 + 5(1) - 2$$

$$= 1 - 4 + 5 - 2 = 0$$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain $a = 1$, $b = -4$, $c = 5$, $d = -2$.

Verification of the relationship between zeroes and coefficients is verified. Coefficient of the given polynomial

$$\text{Sum of zeroes} = 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$$

Multiplication of zeroes taking two at a time = $(2)(1) + (1)(1) + (2)(1) = 2 + 1 + 2 = 5$

$$= \frac{(5)}{1} = \frac{c}{a}$$

$$\text{Multiplication of zeroes} = 2 \times 1 \times 1 = 2 = \frac{-(-2)}{1} = \frac{-d}{a}$$

Hence, the relationship between the zeroes and the coefficients is verified. Coefficient is verified.

Question 2:

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, -7, -14 respectively.

Answer:

Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be α, β , and γ .

It is given that

$$\alpha + \beta + \gamma = \frac{2}{1} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-7}{1} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$$

If $a = 1$, then $b = -2$, $c = -7$, $d = 14$

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

Question 3:

If the zeroes of polynomial $x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find a and b .

Answer:

$$p(x) = x^3 - 3x^2 + x + 1$$

Zeroes are $a - b, a, a + b$

Comparing the given polynomial with $px^3 + qx^2 + rx + t$, we obtain

$$p = 1, q = -3, r = 1, t = 1$$

$$\text{Sum of zeroes} = a - b + a + a + b$$

$$\frac{-q}{p} = 3a$$

$$\frac{-(-3)}{1} = 3a$$

$$3 = 3a$$

$$a = 1$$

The zeroes are $1 - b, 1, 1 + b$.

Multiplication of zeroes = $1(1-b)(1+b)$

$$\frac{-t}{p} = 1 - b^2$$

$$\frac{-1}{1} = 1 - b^2$$

$$1 - b^2 = -1$$

$$1 + 1 = b^2$$

$$b = \pm\sqrt{2}$$

Hence, $a = 1$ and $b = \sqrt{2}$ or $-\sqrt{2}$.

Question 4:

It two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Answer:

Given that $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of the given polynomial.

Therefore, $(x - 2 - \sqrt{3})(x - 2 + \sqrt{3}) = x^2 + 4 - 4x - 3$

$= x^2 - 4x + 1$ is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4 - 6x^3 - 26x^2 + 138x - 35$ by $x^2 - 4x + 1$.

$$\begin{array}{r}
 \overline{x^2 - 2x - 35} \\
 x^2 - 4x + 1 \overline{) x^4 - 6x^3 - 26x^2 + 138x - 35} \\
 \underline{x^4 - 4x^3 + } \\
 -2x^3 - 27x^2 + 138x - 35 \\
 \underline{-2x^3 + 8x^2 - 2x } \\
 +35x^2 + 140x - 35 \\
 \underline{+35x^2 + 140x - 35} \\
 0
 \end{array}$$

Clearly, $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

It can be observed that $(x^2 - 2x - 35)$ is also a factor of the given polynomial.

And $(x^2 - 2x - 35) = (x - 7)(x + 5)$

Therefore, the value of the polynomial is also zero when $x - 7 = 0$ or $x + 5 = 0$

Or $x = 7$ or -5

Hence, 7 and -5 are also zeroes of this polynomial.

Question 5:

If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a .

Answer:

By division algorithm,

Dividend = Divisor \times Quotient + Remainder

Dividend – Remainder = Divisor × Quotient

$x^4 - 6x^3 + 16x^2 - 25x + 10 - x - a = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ will be perfectly divisible by $x^2 - 2x + k$.

Let us divide $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ by $x^2 - 2x + k$

$$\begin{array}{r}
 x^2 - 4x + (8 - k) \\
 x^2 - 2x + k \overline{) x^4 - 6x^3 + 16x^2 - 26x + 10 - a} \\
 \underline{x^4 - 2x^3 + kx^2} \\
 - 4x^3 + (16 - k)x^2 - 26x \\
 \underline{- 4x^3 + 8x^2 - 4kx} \\
 (8 - k)x^2 - (26 - 4k)x + 10 - a \\
 \underline{(8 - k)x^2 - (16 - 2k)x + (8k - k^2)} \\
 - 10 + 2k)x + (10 - a - 8k + k^2)
 \end{array}$$

It can be observed that $(-10 + 2k)x + (10 - a - 8k + k^2)$ will be 0.

Therefore, $(-10 + 2k) = 0$ and $(10 - a - 8k + k^2) = 0$

For $(-10 + 2k) = 0$,

$$2k = 10$$

And thus, $k = 5$

For $(10 - a - 8k + k^2) = 0$

$$10 - a - 8 \times 5 + 25 = 0$$

$$10 - a - 40 + 25 = 0$$

$-5 - a = 0$ Therefore, $a = -5$ Hence,

$k = 5$ and $a = -5$



