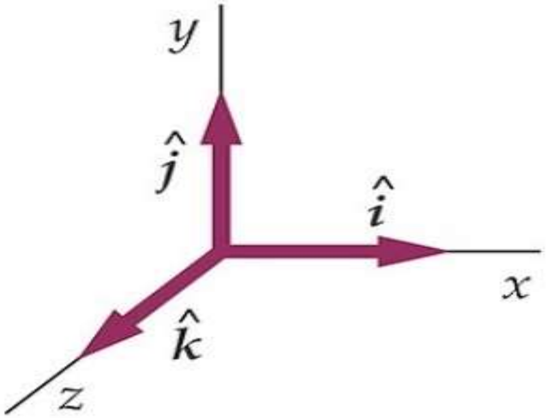
Unit Vectors in Rectangular Coordinates



(a)

Vectors with Rectangular Unit Vectors

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

Dot Product - Scalar

In 2 dimensions

$$\vec{A} \bullet \vec{B} = A\,B\cos(\Theta)$$

In any number of dimensions

$$\vec{A} \bullet \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

The dot product multiplies the portion of A that is *parallel* to B with B

Dot Product - Scalar

The dot product multiplies the portion of A that is *parallel* to B with B

$$\hat{i} \bullet \hat{i} = 1$$

$$\hat{i} \bullet \hat{j} = 0$$

$$\hat{i} \bullet \hat{k} = 0$$

	$\hat{i}$	$\hat{j}$	$\hat{k}$
$\hat{i}$	1	0	0
$\hat{j}$	0	1	0
$\hat{k}$	0	0	1

Cross Product - Vector

The cross product multpilies the portion of A that is *perpendicular* to B with B

$$\hat{i} \times \hat{i} = 0$$

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$

	$\hat{i}$	$\hat{j}$	$\hat{k}$
$\hat{i}$	0	$\hat{k}$	$-\hat{j}$
$\hat{j}$	$-\hat{k}$	0	$\hat{i}$
$\hat{k}$	$\hat{j}$	$-\hat{i}$	0

Cross Product - Vector

In 2 dimensions

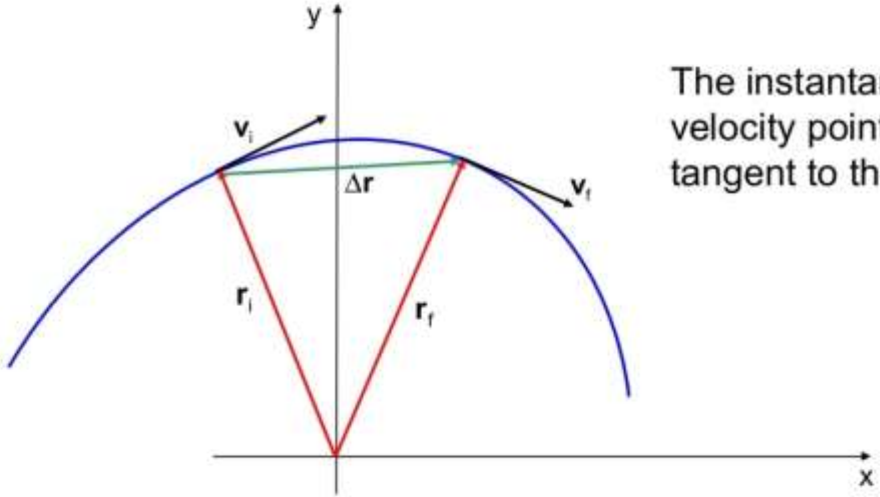
$$\left|\vec{A} \times \vec{B}\right| = A\,B\sin(\Theta)$$

In any number of dimensions

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{bmatrix} \begin{aligned} &= (A_y B_z - A_z B_y)\,\hat{i} \\ &+ (A_x B_z - A_z B_x)\,\hat{j} \\ &+ (A_x B_y - A_x B_y)\,\hat{k} \end{aligned}$$

Velocity

A particle moves along the curved path as shown. At time t_1 its position is $\mathbf{r}_i$ and at time t_2 its position is $\mathbf{r}_f$.



The instantaneous velocity points tangent to the path.

$$\mathbf{v}_{av} = \frac{\Delta \mathbf{r}}{\Delta t}$$ Points in the direction of $\Delta\mathbf{r}$

A displacement over an interval of time is a velocity

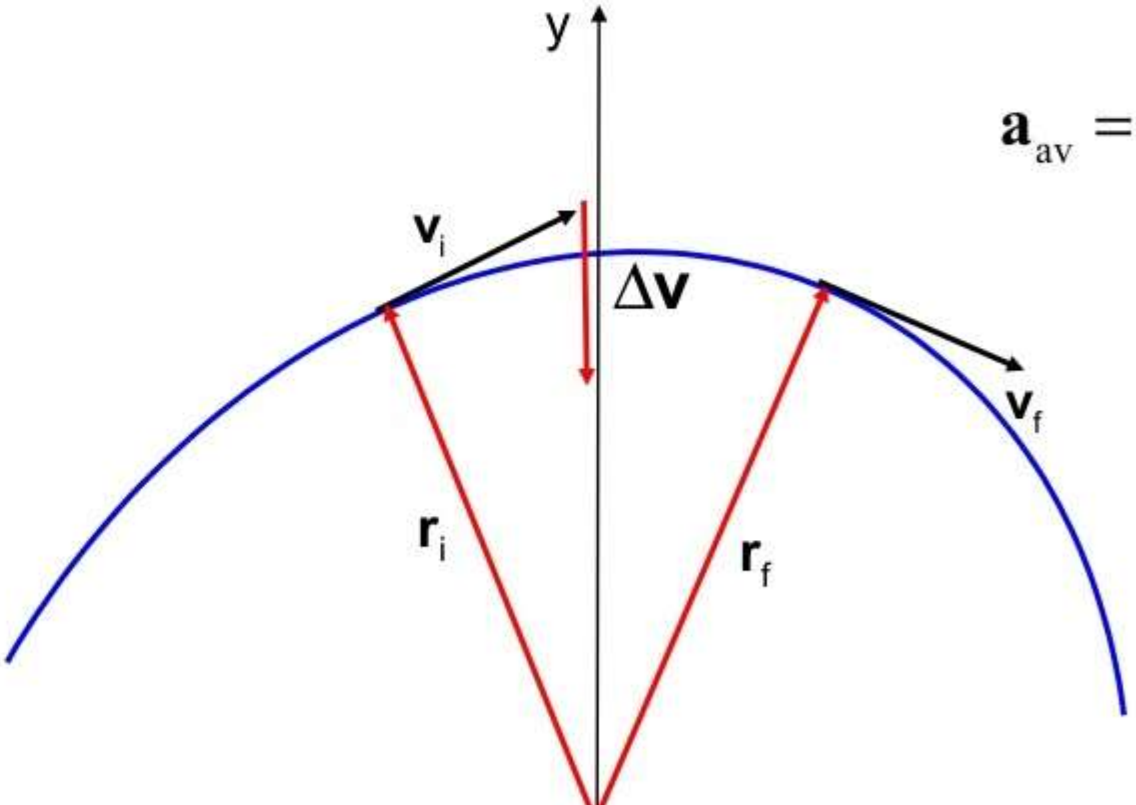
$$\text{Average velocity} = \mathbf{v}_{\text{av}} = \frac{\Delta \mathbf{r}}{\Delta t} \quad \left(\text{The x - component would be}: v_{\text{av},x} = \frac{\Delta x}{\Delta t} \right)$$

$$\text{Instantaneous velocity} = \mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t}$$

The instantaneous velocity is represented by the slope of a line tangent to the curve on the graph of an

Acceleration

A particle moves along the curved path as shown. At time t_1 its position is $\mathbf{r}_0$ and at time t_2 its position is $\mathbf{r}_f$.



$$\mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{\Delta t}$$

Points in the direction of $\Delta\mathbf{v}$.

A nonzero acceleration changes an object's state of motion

$$\text{Average acceleration} = \mathbf{a}_{\text{av}} = \frac{\Delta \mathbf{v}}{\Delta t}$$

$$\text{Instantaneous acceleration} = \text{a} = \lim_{\Delta t \to 0} \frac{\Delta \text{v}}{\Delta \text{t}}$$

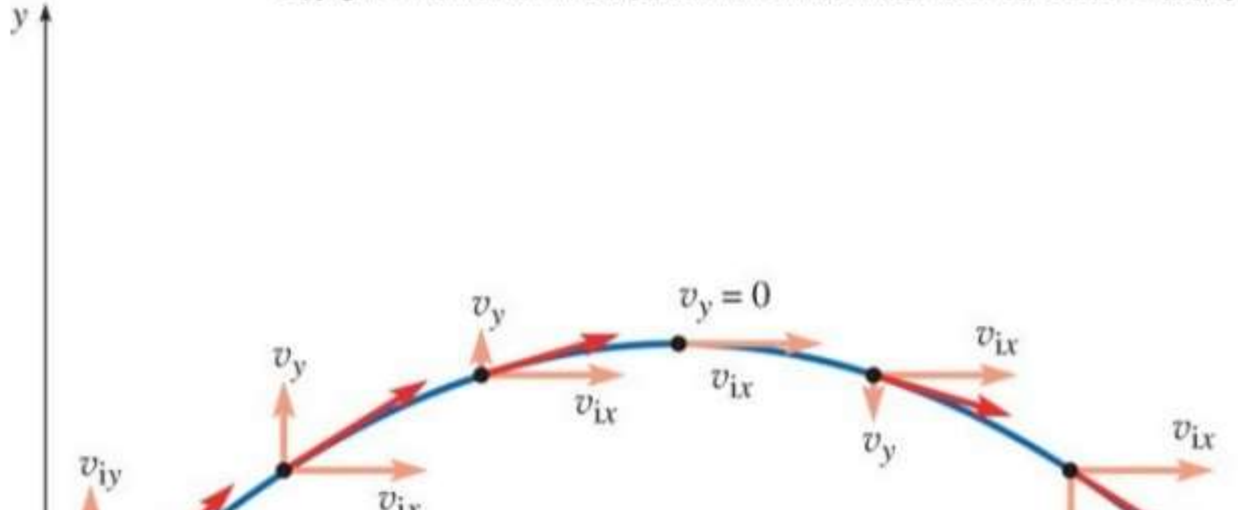
Motion in a Plane with Constant Acceleration - Projectile

What is the motion of a struck baseball? Once it leaves the bat (if air resistance is negligible) only the force of gravity acts on the baseball.

Acceleration due to gravity has a constant value near the surface of the earth. We call it $g = 9.8\ m/s^2$

Projectile Motion

The baseball has $a_x = 0$ and $a_y = -g$, it moves with constant velocity along the x-axis and with a changing velocity along the y-axis.



Example:

An object is projected from the origin. The initial velocity components are v_{ix} = 7.07 m/s, and v_{iy} = 7.07 m/s.

Determine the x and y position of the object at 0.2 second intervals for 1.4 seconds. Also plot the results.

$$\Delta y = y_f - y_i = v_{iy}\Delta t + \frac{1}{2}a_y\Delta t^2$$

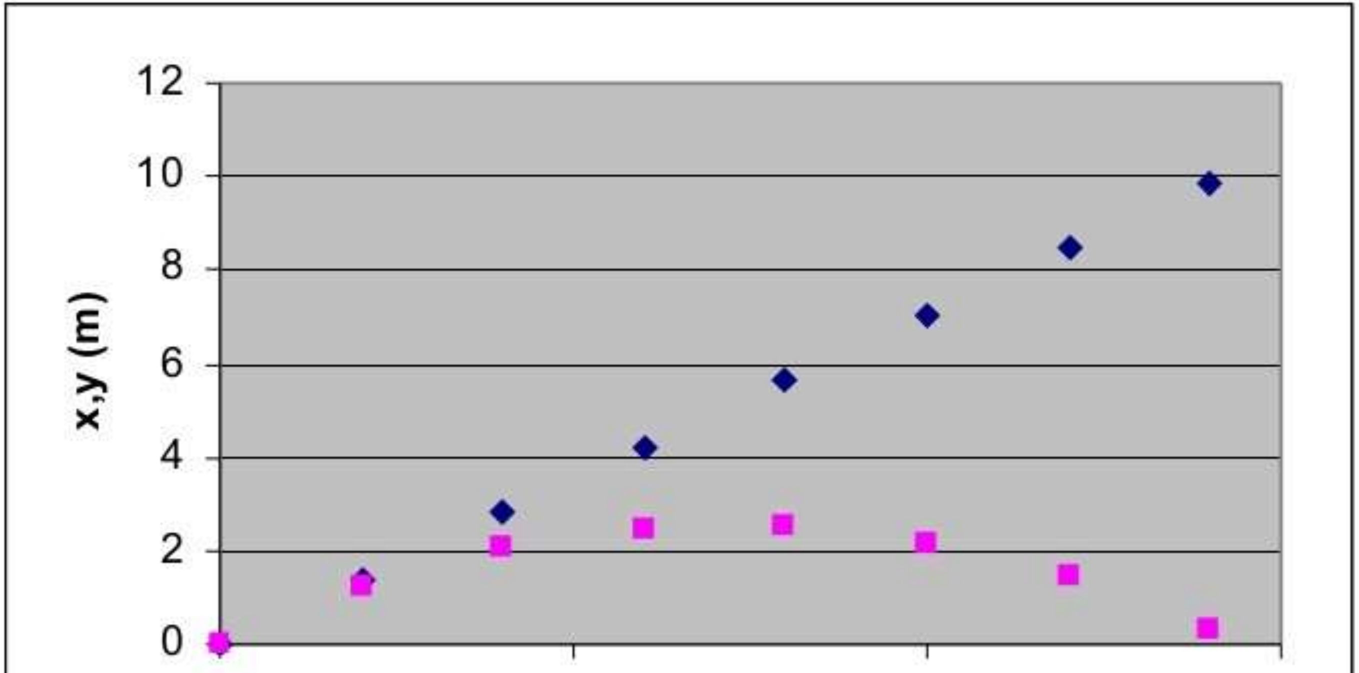
$$\Delta x = x_f - x_i = v_{ix}\Delta t$$

Example continued:

t (sec)	x (meters)	y (meters)
0	0	0
0.2	1.41	1.22
0.4	2.83	2.04
0.6	4.24	2.48
0.8	5.66	2.52
1.0	7.07	2.17
1.2	8.48	1.43

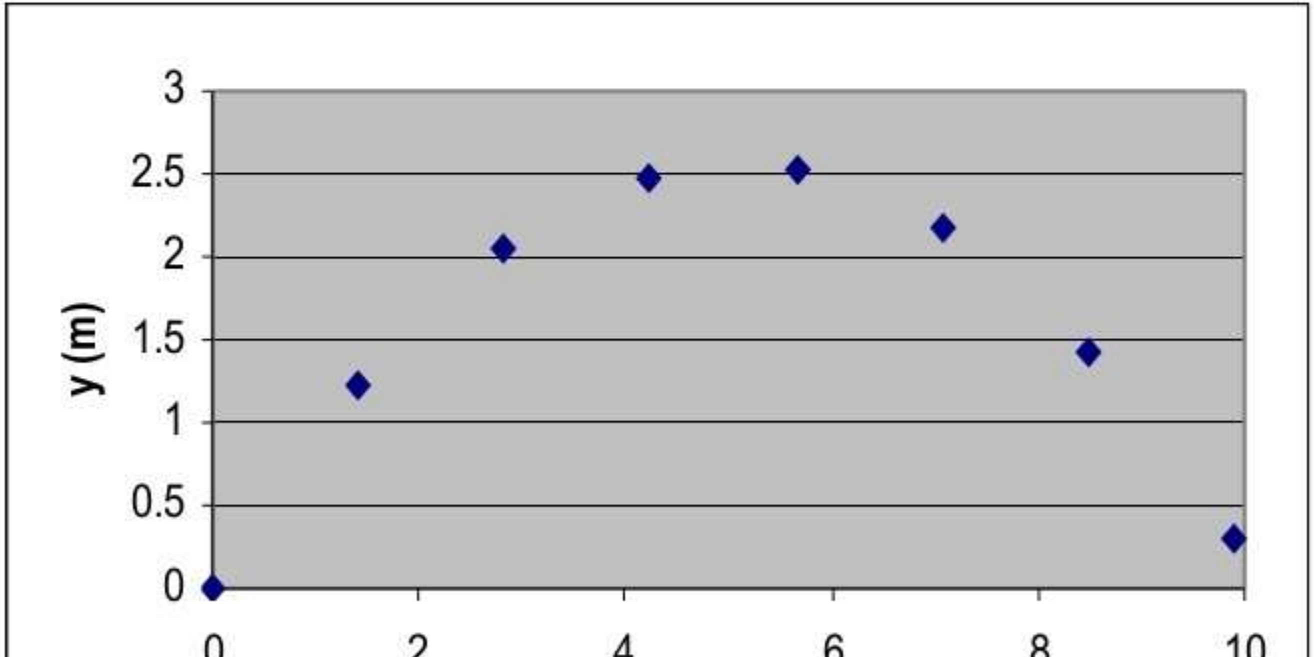
Example continued:

This is a plot of the x position (black points) and y position (red points) of the object as a function of time.



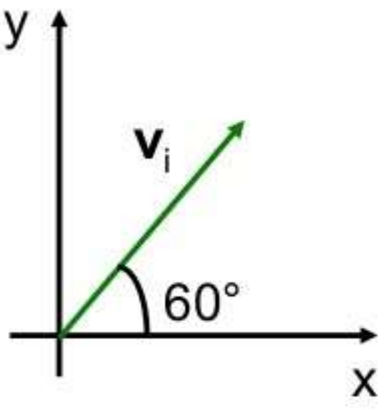
Example continued:

This is a plot of the y position versus x position for the object (its trajectory). The object's path is a parabola.



Example (text problem 3.50): An arrow is shot into the air with $\theta = 60°$ and $v_i = 20.0$ m/s.

(a) What are v_x and v_y of the arrow when t = 3 sec?

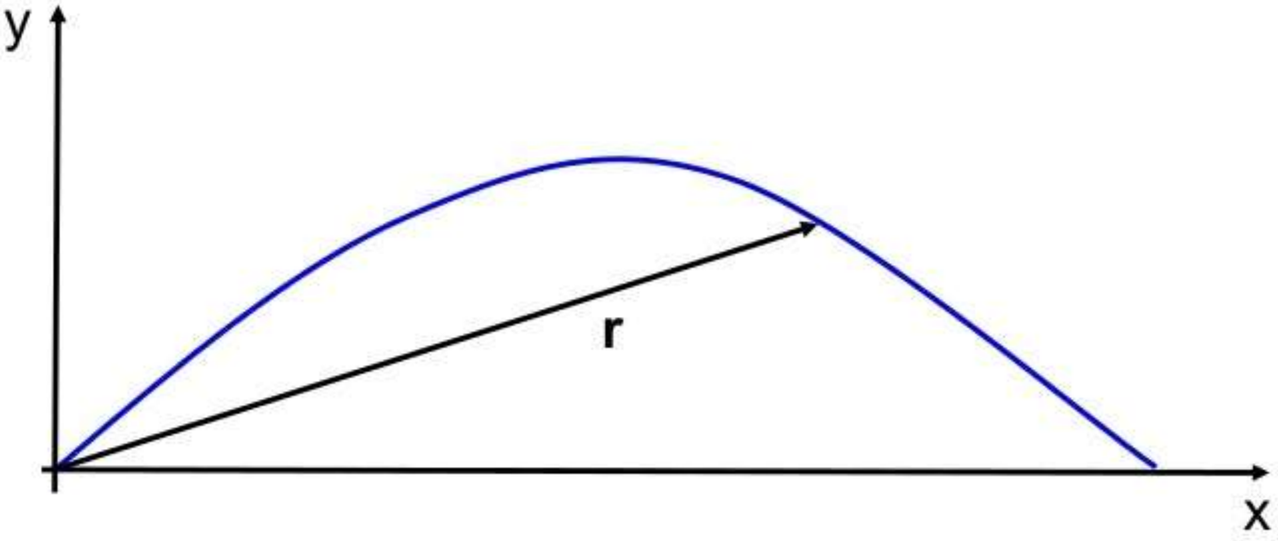


The components of the initial velocity are:

$$v_{ix} = v_i \cos\theta = 10.0 \text{ m/s}$$
$$v_{iy} = v_i \sin\theta = 17.3 \text{ m/s}$$

Example continued:

(b) What are the x and y components of the displacement of the arrow during the 3.0 sec interval?



$$\Delta r_x = \Delta x = x_f - x_i = v_{ix}\Delta t + \frac{1}{2}a_x\Delta t^2 = v_{ix}\Delta t + 0 = 30.0\ m$$

Example: How far does the arrow in the previous example land from where it is released?

The arrow lands when $\Delta y = 0$. $$\Delta y = v_{iy}\Delta t - \frac{1}{2}g\Delta t^2 = 0$$

$$\Delta y = (v_{iy} - \frac{1}{2}g\Delta t)\Delta t = 0$$

Solving for Δt: $$\Delta t = \frac{2v_{iy}}{g} = 3.53\,\text{sec}$$

Summary

- Adding and subtracting vectors (graphical method & component method)
- Velocity
- Acceleration
- Projectile motion (here $a_x = 0$ and $a_y = -g$)

Projectiles Examples

- Problem solving strategy
- Symmetry of the motion
- Dropped from a plane
- The home run