Exercise 9.1

Write the first five terms of each of the sequences in Exercises 1 to 6 whose n^{th} terms are:

1. $a_n = n(n+2)$

Ans. Given: $a_n = n(n+2)$

Putting $n = 1, 2, 3, 4$ and 5, we get,

$a_1 = 1(1+2) = 1\times 3 = 3$

$a_2 = 2(2+2) = 2\times 4 = 8$

$a_3 = 3(3+2) = 3\times 5 = 15$

$a_4 = 4(4+2) = 4\times 6 = 24$

$a_5 = 5(5+2) = 5\times 7 = 35$

Therefore, the first five terms are 3, 8, 15, 24 and 35.

---

2. $a_n = \dfrac{n}{n+1}$

Ans. Given: $a_n = \dfrac{n}{n+1}$

Putting $n = 1, 2, 3, 4$ and 5, we get,

$a_1 = \dfrac{1}{1+1} = \dfrac{1}{2}$

$$a_2 = \frac{2}{2+1} = \frac{2}{3}$$

$$a_3 = \frac{3}{3+1} = \frac{3}{4}$$

$$a_4 = \frac{4}{4+1} = \frac{4}{5}$$

$$a_5 = \frac{5}{5+1} = \frac{5}{6}$$

Therefore, the first five terms are and $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ $\frac{5}{6}$.

---

3. $a_n = 2^n$

Ans. Given: $a_n = 2^n$

Putting $n = 1, 2, 3, 4$ and 5, we get,

$$a_1 = 2^1 = 2$$

$$a_2 = 2^2 = 4$$

$$a_3 = 2^3 = 8$$

$$a_4 = 2^4 = 16$$

$$a_5 = 2^5 = 32$$

Therefore, the first five terms are 2, 4, 8, 16 and 32.

---

4. $a_n = \frac{2n-3}{6}$

Ans. Given: $a_n = \frac{2n-3}{6}$

Putting $n = 1, 2, 3, 4$ and 5, we get,

$$a_1 = \frac{2\times1-3}{6} = \frac{2-3}{6} = \frac{-1}{6}$$

$$a_2 = \frac{2\times2-3}{6} = \frac{4-3}{6} = \frac{1}{6}$$

$$a_3 = \frac{2\times3-3}{6} = \frac{6-3}{6} = \frac{3}{6} = \frac{1}{2}$$

$$a_4 = \frac{2\times4-3}{6} = \frac{8-3}{6} = \frac{5}{6}$$

$$a_5 = \frac{2\times5-3}{6} = \frac{10-3}{6} = \frac{7}{6}$$

Therefore, the first five terms are $\frac{-1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}$ and $\frac{7}{6}$.

---

5. $a_n = (-1)^{n-1}.5^{n+1}$

Ans. Given: $a_n = (-1)^{n-1}.5^{n+1}$

Putting $n = 1, 2, 3, 4$ and 5, we get, $a_4 = (-1)^{4-1}.5^{4+1} = (-1)^3.5^5 = -1\times3125 = -3125$

$$a_5 = (-1)^{5-1}.5^{5+1} = (-1)^4.5^6 = 1\times15625 = 15625$$

Therefore, the first five terms are $25, -125, 625, -3125$ and 15625.

---

6. $a_n = n.\frac{n^2+5}{4}$

Ans. Given: $a_n = n.\frac{n^2+5}{4}$

Putting $n = 1, 2, 3, 4$ and 5, we get,

$$a_1 = 1.\frac{1^2+5}{4} = 1.\frac{1+5}{4} = \frac{6}{4} = \frac{3}{2}$$

$$a_2 = 2.\frac{2^2+5}{4} = 2.\frac{4+5}{4} = \frac{18}{4} = \frac{9}{2}$$

$$a_3 = 3.\frac{3^2+5}{4} = 3.\frac{9+5}{4} = 3\times\frac{14}{4} = \frac{42}{4} = \frac{21}{2}$$

$$a_4 = 4.\frac{4^2+5}{4} = 4.\frac{16+5}{4} = \frac{84}{4} = 21$$

$$a_5 = 5.\frac{5^2+5}{4} = 5.\frac{25+5}{4} = 5\times\frac{30}{4} = \frac{150}{4} = \frac{75}{2}$$

Therefore, the first five terms are $\frac{3}{2}, \frac{9}{2}, \frac{21}{2}, 21$ and $\frac{75}{2}$.

Find the indicated terms in each of the sequences in Exercises 7 to 10 where n^{th} terms are:

7. $a_n = 4n - 3;\quad a_{17}, a_{24}$

Ans. Given: $a_n = 4n - 3$

$$\therefore a_{17} = 4\times 17 - 3 = 68 - 3 = 65$$

$$a_{24} = 4\times 24 - 3 = 96 - 3 = 93$$

Therefore, 17^{th} and 24^{th} terms are 65 and 93 respectively.

---

8. $a_n = \frac{n^2}{2^n};\quad a_7$

Ans. Given: $a_n = \frac{n^2}{2^n}$

$\therefore a_7 = \frac{7^2}{2^7} = \frac{49}{128}$

Therefore, 7^{th} term is $\frac{49}{128}$.

---

9. $a_n = (-1)^{n-1} n^3; \quad a_9$

Ans. Given: $a_n = (-1)^{n-1} n^3$

$\therefore a_9 = (-1)^{9-1} \times (9)^3 = (-1)^8 \times 729 = 729$

Therefore, 9^{th} term is 729.

---

10. $a_n = \frac{n(n-2)}{n+3}; \quad a_{20}$

Ans. Given: $a_n = \frac{n(n-2)}{n+3}$

$a_1 = (-1)^{1-1} .5^{1+1} = (-1)^0 .5^2 = 1 \times 25 = 25$

$a_2 = (-1)^{2-1} 5^{2+1} = (-1)^1 .5^3 = -1 \times 125 = -125$

$a_3 = (-1)^{3-1} .5^{3+1} = (-1)^2 .5^4 = 1 \times 625 = 625$

$\therefore a_{20} = \frac{20(20-2)}{20+3} = \frac{20 \times 18}{23} = \frac{360}{23}$

Therefore, 20^{th} term is $\frac{360}{23}$.

Write the first five terms of each of the sequences in Exercises 11 to 13 and obtain the

corresponding series:

11. $a_1 = 3, \ a_n = 3a_{n-1} + 2$ **for all** $n > 1$

Ans. Given: $a_1 = 3, \ a_n = 3a_{n-1} + 2$ for all $n > 1$

Putting $n = 2, 3, 4$ and 5, we get

$a_2 = 3a_{2-1} + 2 = 3a_1 + 2 = 3 \times 3 + 2 = 9 + 2 = 11$

$a_3 = 3a_{3-1} + 2 = 3a_2 + 2 = 3 \times 11 + 2 = 33 + 2 = 35$

$a_4 = 3a_{4-1} + 2 = 3a_3 + 2 = 3 \times 35 + 2 = 105 + 2 = 107$

$a_5 = 3a_{5-1} + 2 = 3a_4 + 2 = 3 \times 107 + 2 = 321 + 2 = 323$

Hence the first five terms are 3,11,35,107,323.

Therefore, corresponding series is 3 + 11 + 35 + 107 + 323 + ……….

---

12. $a_1 = -1, \ a_n = \frac{a_{n-1}}{n}, n \geq 2$

Ans. Given: $a_1 = -1, \ a_n = \frac{a_{n-1}}{n}, n \geq 2$

Putting $n = 2, 3, 4$ and 5, we get

$$a_2 = \frac{a_{2-1}}{2} = \frac{a_1}{2} = \frac{-1}{2}$$

$$a_3 = \frac{a_{3-1}}{3} = \frac{a_2}{3} = \frac{-1/2}{3} = \frac{-1}{6}$$

$$a_4 = \frac{a_{4-1}}{4} = \frac{a_3}{4} = \frac{-1/6}{4} = \frac{-1}{24}$$

$$a_5 = \frac{a_{5-1}}{5} = \frac{a_4}{5} = \frac{-1/24}{5} = \frac{-1}{120}$$

Hence the first five terms are $-1, \frac{-1}{2}, \frac{-1}{6}, \frac{-1}{24}, \frac{-1}{120}$

$\therefore$ Correspondingseriesis $-1 + \left(\frac{-1}{2}\right) + \left(\frac{-1}{6}\right) + \left(\frac{-1}{24}\right) + \left(\frac{-1}{120}\right)$.............

13. $a_1 = a_2 = 2,\ a_n = a_{n-1} - 1, n > 2$

Ans. Given: $a_1 = a_2 = 2,\ a_n = a_{n-1} - 1, n > 2$

Putting $n = 3, 4$ and 5, weget

$$a_3 = a_{3-1} - 1 = a_2 - 1 = 2 - 1 = 1$$

$$a_4 = a_{4-1} - 1 = a_3 - 1 = 1 - 1 = 0$$

$$a_5 = a_{5-1} - 1 = a_4 - 1 = 0 - 1 = -1$$

Hence the first five terms are 2,2,1,0,-1.

Therefore, correspondingseries is $2 + 2 + 1 + 0 + (-1) + \ldots\ldots$

14. The Fibonacci sequence is defined by $1 = a_1 = a_2$ and $a_{n-1} + a_{n-2}, n > 2$. Find for $\frac{a_{n+1}}{a_n}$,

$n = 1, 2, 3, 4, 5$.

Ans. Given: $a_1 = a_2 = 1$ and $a_{n-1} + a_{n-2}, n > 2$

Exercise 9.2

1. Find the sum of odd integers from 1 to 2001.

Ans. Odd integers from 1 to 2001 are 1, 3, 5, 7,................., 2001.

Here, $a = 1, d = 3 - 1 = 2$ and $a_n = 2001$

$\therefore\ a_n = a + (n-1)d$

$\Rightarrow\ 2001 = 1 + (n-1) \times 2$

$\Rightarrow\ 2001 - 1 = (n-1) \times 2$

$\Rightarrow\ \frac{2000}{2} = (n-1)$

$\Rightarrow n = 1000 + 1 = 1001$

Now, $S_n = \frac{n}{2}(a + l)$

$\Rightarrow S_{1001} = \frac{1001}{2}(1 + 2001)$

$\Rightarrow S_{1001} = \frac{1001}{2} \times 2002 = 1002001$

2. Find the sum of all natural numbers lying between 100 and 1000 which are multiples of 5.

Ans. According to question, series is 105, 110, 115, 120, ..., 995

Here $a = 105, d = 110 - 105 = 5$ and $a_n = 995$

$\therefore\ a_n = a + (n-1)d$

$\Rightarrow 995 = 105 + (n-1) \times 5$

$\Rightarrow 995 - 105 = (n-1) \times 5$

$\Rightarrow \frac{890}{5} = (n-1)$

$\Rightarrow n = 178 + 1 = 179$

Now, $S_n = \frac{n}{2}(a + l)$

$\Rightarrow S_{179} = \frac{179}{2}(105 + 995)$

$\Rightarrow S_{179} = \frac{179}{2} \times 1100 = 98450$

---

3. In an A.P. the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20$^{\text{th}}$ term is -112.

Ans. According to question, a=2 and $S_5 = \frac{1}{4}[S_{10} - S_5]$

$\Rightarrow 4S_5 = S_{10} - S_5$

$\Rightarrow 5S_5 = S_{10}$

$\Rightarrow 5\left[\frac{5}{2}\{2 \times 2 + (5-1)d\}\right] = \frac{10}{2}[2 \times 2 + (10-1)d]$ since $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\Rightarrow \frac{25}{2}[4+4d]=5[4+9d]$$

$$\Rightarrow 25[4+4d]=10[4+9d]$$

$$\Rightarrow 100+100d=40+90d$$

$$\Rightarrow 10d=-60$$

$$\Rightarrow d=-6$$

Now, $a_n = a+(n-1)d$

$$\Rightarrow a_{20}=2+(20-1)\times(-6)$$

$$\Rightarrow a_{20}=2-114=-112 \qquad -6, \frac{-11}{2}, -5, \ldots\ldots\ldots$$

---

4. How many terms of the A.P., are needed to give the sum -25 ?

Ans. Here, $a=-6, d=\frac{-11}{2}-(-6)=\frac{-11}{2}+6=\frac{1}{2}$

$$\therefore\ S_n=\frac{n}{2}[2a+(n-1)d]$$

$$\Rightarrow -25=\frac{n}{2}\left[2\times(-6)+(n-1)\times\frac{1}{2}\right]$$

$$\Rightarrow -50=n\left[-12+\frac{n-1}{2}\right]$$

$$\Rightarrow -50=n\left[\frac{-24+n-1}{2}\right]$$

$$\Rightarrow -100=n^2-25n$$

$$\Rightarrow n^2-25n+100=0$$

$$\Rightarrow (n-20)(n-5)=0$$

Exercise 9.3

---

1. Find the 20th and terms of the G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots\ldots$

Ans. Here and

$$\therefore\ a_n = ar^{n-1}$$

$$\Rightarrow a_{20} = \frac{5}{2}\times\left(\frac{1}{2}\right)^{20-1}$$

$$\Rightarrow a_{20} = \frac{5}{2}\times\left(\frac{1}{2}\right)^{19} = \frac{5}{2^{20}}$$

And $a_n = \frac{5}{2}\times\left(\frac{1}{2}\right)^{n-1} = \frac{5}{2\times 2^{n-1}} = \frac{5}{2^n}$

---

2. Find the 12th term of a G.P. whose 8th term is 192 and the common ratio is 2.

Ans. Let a be the first term of given G.P. Here $r = 2$ and $a_8 = 192$

$$\therefore\ a_n = ar^{n-1}$$

$$\Rightarrow a_8 = a\times(2)^{8-1} = 192$$

$$\Rightarrow a\times(2)^7 = 192$$

$$\Rightarrow a\times 128 = 192$$

$$\Rightarrow a = \frac{192}{128} = \frac{3}{2}$$

$\therefore a_{12} = ar^{12-1}$

$\Rightarrow a_{12} = \frac{3}{2} \times 2^{11} = 3 \times 2^{10}$

$= 3 \times 1024 = 3072$

---

3. The 5th, 8th and 11th terms of a G.P. are p, q and s respectively. Show that $q^2 = ps$.

Ans. Let a be the first term and r be the common ratio of given G.P.

$\therefore a_5 = p \Rightarrow ar^4 = p$ ....(i)

$a_8 = q \Rightarrow ar^7 = q$ ....(ii)

$a_{11} = s \Rightarrow ar^{10} = s$ ....(iii)

Squaring both sides of eq. (ii), we get $q^2 = (ar^7)^2$

$\Rightarrow q^2 = a^2 r^{14}$

$\Rightarrow q^2 = (ar^4)(ar^{10})$

$\Rightarrow q^2 = ps$ [From eq. (i) and (iii)]

---

4. The 4th term of a G.P. is square of its second term and the first term is -3. Determine its 7th term.

Ans. Let a be the first term and r be the common ratio of given G.P.

Here $a = -3$ and $a_4 = (a_2)^2$

Now, $a_4 = (a_2)^2$

$\Rightarrow ar^3 = (ar)^2$

= $-3 \times 729 = -2187$

---

1. Which term of the following sequences:

(a) $2, 2\sqrt{2}, 4, \ldots\ldots$ **is 128?**

(b) $\sqrt{3}, 3, 3\sqrt{3}, \ldots\ldots$ **is 729?**

(c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots\ldots$ **is** $\frac{1}{19683}$?

Ans. (a) Here $a = 2$, $r = \frac{2\sqrt{2}}{2} = \sqrt{2}$ and $a_n = 128$

$\therefore\ a_n = ar^{n-1}$

$\Rightarrow 128 = 2 \times \left(\sqrt{2}\right)^{n-1}$

$\Rightarrow 64 = \left(\sqrt{2}\right)^{n-1}$

$\Rightarrow \left(\sqrt{2}\right)^{12} = \left(\sqrt{2}\right)^{n-1}$

$\Rightarrow n - 1 = 12$

$\Rightarrow n = 13$

Therefore, 13^{th} term of the given G.P. is 128.

(b) Here $a = \sqrt{3}$, $r = \frac{3}{\sqrt{3}} = \sqrt{3}$ and $a_n = 729$

$\therefore a_n = ar^{n-1}$

$\Rightarrow 729 = \sqrt{3} \times \left(\sqrt{3}\right)^{n-1}$

$\Rightarrow \left(\sqrt{3}\right)^{12} = \left(\sqrt{3}\right)^{n}$

$\Rightarrow n = 12$

Therefore, 12th term of the given G.P. is 729.

(c) Here $a = \frac{1}{3}, r = \frac{1}{9} \div \frac{1}{3} = \frac{1}{3}$ and $a_n = \frac{1}{19683}$

$\therefore a_n = ar^{n-1}$

$\Rightarrow \frac{1}{19683} = \frac{1}{3} \times \left(\frac{1}{3}\right)^{n-1}$

$\Rightarrow \left(\frac{1}{3}\right)^{9} = \left(\frac{1}{3}\right)^{n}$

$\Rightarrow n = 9$

Therefore, 9th term of the given G.P. is $\frac{1}{19683}$.

---

6. For what values of x, the numbers $\frac{-2}{7}, x, \frac{-7}{2}$ are in G.P.?

Ans. Given: $\frac{-2}{7}, x, \frac{-7}{2}$ are in G.P.

$\therefore \frac{x}{\frac{-2}{7}} = \frac{\frac{-7}{2}}{x}$

$\Rightarrow x^2 = \frac{-2}{7} \times \frac{-7}{2}$

$\Rightarrow x^2 = 1$

$\Rightarrow x = \pm 1$

Therefore for $x = \pm 1$ the given numbers are in G.P

Find the sum to indicated number of terms in each of the geometric progression in Exercises 7 to 10:

7. 0.15,0.015, 0.0015, ... 20 terms

Ans. Here, $a = 0.15$ and $r = \frac{0.015}{0.15} = \frac{1}{10}$

$\therefore S_n = \frac{a(1-r^n)}{1-r}$ when $r < 1$

$\Rightarrow S_{20} = \frac{0.15\left[1-\left(\frac{1}{10}\right)^{20}\right]}{1-\frac{1}{10}}$

$\Rightarrow S_{20} = \frac{15}{100} \times \frac{10}{9}\left[1-(0.1)^{20}\right]$

$\Rightarrow S_{20} = \frac{1}{6}\left[1-(0.1)^{20}\right]$

$\sqrt{7}, \sqrt{21}, 3\sqrt{7}, \ldots\ldots\ldots n$

8. **terms**

Ans. Here, $a=\sqrt{7}$ and $r=\frac{\sqrt{21}}{\sqrt{7}}=\sqrt{3}$

$$\therefore \ S_n=\frac{a\left(r^n-1\right)}{r-1} \text{ when } r>1$$

$$\Rightarrow S_n=\frac{\sqrt{7}\left[\left(\sqrt{3}\right)^n-1\right]}{\sqrt{3}-1}$$

$$\Rightarrow S_n=\frac{\sqrt{7}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}\left[(3)^{\frac{n}{2}}-1\right]$$

$$\Rightarrow S_n=\frac{\sqrt{7}\left(\sqrt{3}+1\right)}{2}\left[(3)^{\frac{n}{2}}-1\right]$$

---

9. $1,-a,a^2,-a^3,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots n$**terms(if** $a\neq-1$**)**

Ans. Here, $a=1$ and $r=\frac{-\alpha}{1}=-\alpha$

$$\therefore \ S_n=\frac{a\left(1-r^n\right)}{1-r} \text{ when } r<1$$

$$\Rightarrow S_n=\frac{1\left[1-(-\alpha)^n\right]}{1-(-\alpha)}$$

$$\Rightarrow S_n=\frac{1}{1+\alpha}\left[1-(-\alpha)^n\right]$$

---

10. $x^3, x^5, x^7, \ldots\ldots n$ **terms (if $x \neq \pm 1$)**

Ans. Here, $a = x^3$ and $r = \frac{x^5}{x^3} = x^2$

$\therefore S_n = \frac{a(1-r^n)}{1-r}$ when $r < 1$

$\Rightarrow S_n = \frac{x^3\left[1-(x^2)^n\right]}{1-x^2}$

$\Rightarrow S_n = \frac{x^3}{1-x^2}\left[1-x^{2n}\right]$

---

11. Evaluate: $\sum_{k=1}^{11}\left(2+3^k\right)$

Ans. Given: $\sum_{k=1}^{11}\left(2+3^k\right)$

$= \left(2+3^1\right)+\left(2+3^2\right)+\left(2+3^3\right)+\ldots\ldots+\left(2+3^{11}\right)$

$= \left(2+2+2+\ldots\ldots 11 \text{ times}\right)+\left(3+3^2+3^3+\ldots\ldots+3^{11}\right)$

$= 22+\left(3+3^2+3^3+\ldots\ldots+3^{11}\right)$ ……………(i)

Here $3, 3^2, 3^3, \ldots\ldots, 3^{11}$ is in G.P.

$\therefore a = 3$ and $r = \frac{3^2}{3} = 3$

$\therefore S_n = \frac{3\left(3^{11}-1\right)}{3-1} = \frac{3}{2}\left(3^{11}-1\right)$

Putting the value of S_n in eq. (i), we get $\sum_{k=1}^{11}\left(2+3^k\right)=22+\frac{3}{2}\left(3^{11}-1\right)$

---

12. The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

Ans. Let $\frac{a}{r}, a, r$ be first three terms of the given G.P.

According to question, $\frac{a}{r}+a+ar=\frac{39}{10}$............(i)

And $\frac{a}{r}\times a\times r=1$

$\Rightarrow a^3=1$

$\Rightarrow a=1$

Putting value of a in eq. (i), $\frac{1}{r}+1+r=\frac{39}{10}$

$\Rightarrow 10+10r+10r^2=39r$

$\Rightarrow 10r^2-29r+10=0$

$$\Rightarrow r=\frac{-(-29)\pm\sqrt{(-29)^2-4\times10\times10}}{2\times10}$$

$$\Rightarrow r=\frac{29\pm\sqrt{841-400}}{20}$$

$$\Rightarrow r=\frac{29\pm21}{20}$$

Taking $r = \frac{29+21}{20} = \frac{50}{20} = \frac{5}{2}$ and Taking $r = \frac{29-21}{20} = \frac{8}{20} = \frac{2}{5}$

When $r = \frac{5}{2}$, then first three terms are $\frac{1}{5/2}, 1, 1\times\frac{5}{2}$

$\Rightarrow \frac{2}{5}, 1, \frac{5}{2}$

When $r = \frac{2}{5}$ then first three terms are $\frac{1}{2/5}, 1, 1\times\frac{2}{5}$

$\Rightarrow \frac{5}{2}, 1, \frac{2}{5}$

---

11. How many terms of G.P. $3, 3^2, 3^3, \ldots\ldots\ldots\ldots$ are needed to give the sum 120 ?

Ans. Here, $\therefore a = 3$ and $r = \frac{3^2}{3} = 3$

$\therefore S_n = \frac{a(r^n - 1)}{r-1}$ when $r > 1$

$\Rightarrow 120 = \frac{3(3^n - 1)}{3-1}$

$\Rightarrow 120 = \frac{3}{2}(3^n - 1)$

$\Rightarrow 120 \times \frac{2}{3} = 3^n - 1$

$\Rightarrow 3^n = 81$

$\Rightarrow 3^n = (3)^4$

$\Rightarrow n = 4$

Therefore, the sum of 4 terms of the given G.P. is 120.

---

11. The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common n

NA. Let a be the first term and r be the common ratio of given G.P.

$$\therefore\ a + ar + ar^2 = 16$$

$$\Rightarrow a\left(1 + r + r^2\right) = 16 \text{ ....(i)}$$

And $ar^3 + ar^4 + ar^5 = 128$

$$\Rightarrow ar^3\left(1 + r + r^2\right) = 128 \text{.........(ii)}$$

Putting the value from eq. (i) into eq. (ii), we get

$$16r^3 = 128$$

$$\Rightarrow r^3 = 8$$

$$\Rightarrow r = 2$$

Putting value of r in eq. (i), we get $a\left(1 + 2 + 2^2\right) = 16$

$$\Rightarrow a = \frac{16}{7}$$

$$\therefore\ S_n = \frac{a\left(r^n - 1\right)}{r - 1} \text{ when } r > 1$$

$$\Rightarrow\ S_n = \frac{\frac{16}{7}\left(2^n - 1\right)}{2 - 1} = \frac{16}{7}\left(2^n - 1\right)$$

---

15. Given a G.P. with a = 729 and 7$^{\text{th}}$ term 64, determine S_7.

a $a_7 = 64$

Ans. Given: = 729 and

$\Rightarrow ar^6 = 64$

$\Rightarrow 729r^6 = 64$

$\Rightarrow r^6 = \frac{64}{729} = \left(\frac{2}{3}\right)^6$

$\Rightarrow r = \frac{2}{3}$

$\therefore S_n = \frac{a(1-r^n)}{1-r}$ when $r < 1$

$\Rightarrow S_7 = \frac{729\left[1-\left(\frac{2}{3}\right)^7\right]}{1-\frac{2}{3}} = \frac{729\left[1-\frac{128}{2187}\right]}{\frac{3-2}{3}}$

$\Rightarrow S_7 = 729 \times 3\left(\frac{2187-128}{2187}\right)$

$\Rightarrow S_7 = \frac{729 \times 3 \times 2059}{2187} = 2059$

16. Find a G.P. for which sum of the first two terms is -4 and the fifth term is 4 times the third term.

Ans. Let a be the first term and r be the common ratio of given G.P. Given:

$a + ar = -4$

$\Rightarrow a(1+r) = -4$ ....(i)

And $a_5 = 4a_3$

$\Rightarrow ar^4 = 4ar^2$

Putting X= 2in eq. (i), we get

$\Rightarrow r^2 = 4$

$a(1+2) = -4$

$\Rightarrow r = \pm 2$

$r = 2$ $a(1+2) = -4$

Therefore, required G.P. is $\frac{-4}{3}, \frac{-8}{3}, \frac{-16}{3}, \ldots\ldots$

Putting $r=-2$ in eq. (i), we get $a(1-2)=-4$

$\Rightarrow a=4$

Therefore, required G.P. is $4, -8, 16, -32, \ldots\ldots\ldots$

---

15. If the 4th, 10th and 16th terms of a G.P. are x, y and z respectively. Prove that x, y, z are in G.P.

Ans. Let a be the first term and r be the common ratio of given G.P.

$\therefore\ a_4 = x$

$\Rightarrow ar^3 = x$ ……….(i)

$a_{10} = y$

$\Rightarrow ar^9 = y$ ……….(ii)

$a_{16} = z$

$\Rightarrow ar^{15} = z$ ……….(iii)

$\Rightarrow (ar^9)^2 = y^2$

$\Rightarrow y^2 = (ar^3)(ar^{15})$

From eq. (ii), $ar^9 = y$

$\Rightarrow y^2 = xz$

[From eq. (i) and (iii)]

$\Rightarrow a = \frac{-4}{3}$

$\therefore\ x, y, z$ are in G.P.

18. Find the sum to n terms of the sequences 8, 88, 888, 8888, …… Ans.

Here $S_n = 8 + 88 + 888 + 8888 + ...$ up to n terms

$$\Rightarrow S_n = 8(1 + 11 + 111 + 1111 + ... \text{ up to } n \text{ terms})$$

$$\Rightarrow S_n = \frac{8}{9}(9 + 99 + 999 + 9999 + ... \text{ up to } n \text{ terms})$$

$$\Rightarrow S_n = \frac{8}{9}\left[(10-1) + (10^2 - 1) + (10^3 - 1) + ... \text{ up to } n \text{ terms}\right]$$

$$\Rightarrow S_n = \frac{8}{9}\left[(10 + 10^2 + 10^3 + ... \text{ up to } n \text{ terms}) - (1 + 1 + 1 + ... \text{ up to } n \text{ terms})\right]$$

$$\Rightarrow S_n = \frac{8}{9}\left[\frac{10 \times (10^n - 1)}{10 - 1} - n\right]$$

$$= \frac{8}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$$

$$= \frac{80}{81}(10^n - 1) - \frac{8}{9}n$$

---

17. Find the sum of the product of the corresponding terms of the sequences 2, 4, 8,16, 32 and 128, 32, 8,2, $\frac{1}{2}$.

Ans. Multiplying the corresponding terms of the given sequences 2, 4, 8,16, 32 and 128, 32, 8, 2, $\frac{1}{2}$.

$$(2 \times 128), (4 \times 32), (8 \times 8), (16 \times 2), \left(32 \times \frac{1}{2}\right)$$

$\Rightarrow 256, 128, 64, 32, 16$ are in G.P.

Here $a = 256$, $r = \frac{128}{256} = \frac{1}{2}$ and $n = 5$

$\therefore S_n = \frac{a(1-r^n)}{1-r}$ when $r < 1$

$\Rightarrow S_5 = \frac{256\left[1-\left(\frac{1}{2}\right)^5\right]}{1-\frac{1}{2}} = 256 \times 2\left(1-\frac{1}{32}\right)$

$\Rightarrow S_5 = 256 \times 2 \times \frac{31}{32} = 496$

---

15. Show that the products of the corresponding terms of the sequences $a, ar, ar^2, \ldots\ldots ar^{n-1}$ **and** $A, AR, AR^2, \ldots\ldots AR^{n-1}$ **form a G.P. and find the common ratio.**

Ans. Multiplying the corresponding terms of the given sequences, we have

$(a \times A), (ar \times AR), (ar^2 \times AR^2), \ldots\ldots\ldots\ldots, (ar^{n-1} \times AR^{n-1})$

$\Rightarrow (aA), (aArR), (aAr^2R^2), \ldots\ldots\ldots\ldots, (aAr^{n-1}R^{n-1})$ are in G.P.

Now $\frac{a_2}{a_1} = \frac{aArR}{aA} = rR and \frac{a_3}{a_2} = \frac{aAr^2R^2}{aArR} = rR$

Since the ratio of the two succeeding terms are same, the resulting sequence is also in G.P

and common ratio = $\frac{aArR}{aA} = rR$

---

16. Find four numbers forming a geometric progression in which the third term is greater than the first term by 9 and the second term is greater than by 4th by 18.

Ans. Let the four numbers in G.P. be a, ar, ar^2, ar^3

$\therefore ar^2 = a + 9$ and $ar = ar^3 + 18$

Now, $ar^2 - a = 9$

$\Rightarrow a\left(r^2-1\right)=9$ ……….(i)

And $ar - ar^3 = 18$

$\Rightarrow ar\left(1-r^2\right)=18$

$\Rightarrow -ar\left(r^2-1\right)=18$ ……….(ii)

Dividing eq. (ii) by eq. (i), we have

$$\frac{-ar\left(r^2-1\right)}{a\left(r^2-1\right)}=\frac{18}{9}$$

$\Rightarrow r=-2$

Putting value of r in eq. (i), we get

$a\left(4-1\right)=9$

$\Rightarrow a=3$

$\therefore\ ar=3\times(2)=-6$

$ar^2=3\times(-2)^2=12$ $\qquad ar^3=3\times(-2)^3=-24$

Therefore, the required numbers are $3, -6, 12, -24$.

---

15. If the p^{th}, q^{th} and r^{th} terms of a G.P. are a, b and c respectively. Prove that $a^{q-r}b^{r-p}c^{p-q}=1$.

Ans. Let A be the first term and R be the common ratio of given G.P.

$\therefore a_p = a$

$a_q = b$

$a_r = c$

$\Rightarrow AR^{r-1}=c$

$\Rightarrow AR^{p-1} = a$ ……….(i)

Now, L.H.S. =

$\Rightarrow AR^{q-1} = b$ ……….(ii)

$= A^{q-r} R^{(p-1)(q-r)} . A^{r-p} R^{(q-1)(r-p)} . A^{p-q} R^{(r-1)(p-q)}$

……….(iii)

$$a^{q-r} b^{r-p} c^{p-q} = \left(AR^{p-1}\right)^{q-r} . \left(AR^{q-1}\right)^{r-p} . \left(AR^{r-1}\right)^{p-q}$$

$= A^{q-r+r-p+p-q} R^{pq-pr-q+r+qr-pq-r+p+pr-qr-p+q}$

$= A^0 R^0 = 1 \times 1 = 1 =$ R.H.S.

15. If the first and the n^{th} term of a G.P. are a and b respectively and if P is the product of n terms, prove that $P^2 = (ab)^n$.

Ans. Let r be the common ratio of the given G.P

Here, first term of G.P. is a

and $a_n = b$

$\Rightarrow ar^{n-1} = b$ ……….(i)

Given: P =

$\Rightarrow a.ar.ar^2.ar^3 \ldots\ldots ar^{n-1}$ P = $a^n . r^{1+2+3+\ldots+n-1}$

$\Rightarrow p = a^n r^{\frac{n(n-1)}{2}}$

$\Rightarrow p^2 = a^{2n} r^{n(n-1)} = \left[aar^{n-1}\right]$ [Squaring both sides]

$\Rightarrow P^2 = (ab)^n$ [From eq. (i)]

Hence proved

15. Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$.

Ans. Let a be the first term and r be the common ratio of given G.P.

Then $\frac{\text{Sum of first } n \text{ terms}}{\text{Sum of terms from } (n+1)^{th} \text{ to } (2n)^{th}}$

$= \frac{a+ar+ar^2+.................+ar^{n-1}}{ar^n+ar^{n+1}+.................+ar^{2n-1}}$

$= \frac{a+ar+ar^2+..........+ar^{n-1}}{r^n\left[a+ar+ar^2+..........+ar^{n-1}\right]} = \frac{1}{r^n}$

25. If a,b,c and d are in G.P., show that $(a^2+b^2+c^2)(b^2+c^2+d^2)=(ab+bc+cd)^2$.

Ans. Given a,b,c,d are in G.P

Let r be the common ratio of given G.P.

Then $b=ar$, $c=ar^2$ and $d=ar^3$

Now, L.H.S. = $(a^2+b^2+c^2)(b^2+c^2+d^2)$

$= (a^2+a^2r^2+a^2r^4)(a^2r^2+a^2r^4+a^2r^6)$

$= a^2(1+r^2+r^4)a^2r^2(1+r^2+r^4) = a^4r^2(1+r^2+r^4)^2$

R.H.S.= $(ab+bc+cd)^2$

$= (a.ar+ar.ar^2+ar^2.ar^3)^2$

$= (a^2r+a^2r^3+a^2r^5)^2$

$= (a^2r)^2(1+r^2+r^4)^2 = a^4r^2(1+r^2+r^4)^2$

Therefore, L.H.S. = R.H.S.

26. Insert two numbers between 3 and 81 so that the resulting sequence us G.P.

Ans. Let $G_1 and\ G_2$ be two numbers between 3 and 81 such that $3,\ G_1,\ G_2,\ 81$ are in G.P. Let r be the common ratio

Here $a = 3$ and $a_4 = 81$

$\Rightarrow\ ar^3 = 81$

$\Rightarrow 3 \times r^3 = 81$

$\Rightarrow r^3 = 27$

$\Rightarrow r = 3$

$\therefore\ G_1 = ar = 3 \times 3 = 9$

And $G_2 = ar^2 = 3 \times (3)^2 = 27$

Therefore, the required numbers are 9 and 27.

27. Find the value of n so that $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b

Ans. Since, G.M. between two numbers a and b is $\sqrt{ab}$. According to question, $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n} = \sqrt{ab}$

$$\Rightarrow \frac{a^{n+1}+b^{n+1}}{a^n+b^n} = a^{\frac{1}{2}}b^{\frac{1}{2}}$$

$$\Rightarrow a^{n+1} + b^{n+1} = (a^n + b^n)a^{\frac{1}{2}}b^{\frac{1}{2}}$$

$$\Rightarrow a^{n+1} + b^{n+1} = a^{n+\frac{1}{2}}b^{\frac{1}{2}} + a^{\frac{1}{2}}b^{n+\frac{1}{2}}$$

$$\Rightarrow a^{n+1} - a^{n+\frac{1}{2}}b^{\frac{1}{2}} = a^{\frac{1}{2}}b^{n+\frac{1}{2}} - b^{n+1}$$

$$\Rightarrow a^{n+\frac{1}{2}}\left(a^{\frac{1}{2}} - b^{\frac{1}{2}}\right) = b^{n+\frac{1}{2}}\left(a^{\frac{1}{2}} - b^{\frac{1}{2}}\right)$$

Ans. Since, G.M. between two numbers a and b is $\sqrt{ab}$. According to question,

$$\frac{a^{n+1}+b^{n+1}}{a^n+b^n}=\sqrt{ab}$$

$$\Rightarrow \frac{a^{n+1}+b^{n+1}}{a^n+b^n}=a^{\frac{1}{2}}b^{\frac{1}{2}}$$

$$\Rightarrow a^{n+1}+b^{n+1}=(a^n+b^n)a^{\frac{1}{2}}b^{\frac{1}{2}}$$

$$\Rightarrow a^{n+1}+b^{n+1}=a^{n+\frac{1}{2}}b^{\frac{1}{2}}+a^{\frac{1}{2}}b^{n+\frac{1}{2}}$$

$$\Rightarrow a^{n+1}-a^{n+\frac{1}{2}}b^{\frac{1}{2}}=a^{\frac{1}{2}}b^{n+\frac{1}{2}}-b^{n+1}$$

$$\Rightarrow a^{n+\frac{1}{2}}\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)=b^{n+\frac{1}{2}}\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)$$

$$\Rightarrow a^{n+\frac{1}{2}}=b^{n+\frac{1}{2}}$$

$$\Rightarrow \frac{a^{n+\frac{1}{2}}}{b^{n+\frac{1}{2}}}=1$$

$$\Rightarrow \left(\frac{a}{b}\right)^{n+\frac{1}{2}}=\left(\frac{a}{b}\right)^0$$

$$\Rightarrow n+\frac{1}{2}=0$$

$$\Rightarrow n=-\frac{1}{2}$$