Std: 11 **Chapter – 6** **Sub: Mathematics**

Linear Inequalities

Exercise 6.1

1. Solve 24x < 100 when:

(i) x is a natural number

(ii) **x is an integer Ans.**

Given: 24x < 100

Divide both sides by 24,

$$=> \frac{24x}{24} < \frac{100}{24}$$

$$=> x < \frac{25}{6}$$

(i) When x is a natural number then values of x that make statement true are 1, 2, 3, 4. The solution set of inequality is {1, 2, 3, 4}.

(ii) When **x** is an integer, then values of **x** that make statement true for all (-ve number and 0, 1, 2, 3, 4). So, The solution set of inequality is { -4,-3 - 2, - 1,0,1,2,3,4}.

2. Solve -12x > 30 when:

(i) x is a natural number

(ii) **x is an integer Ans.**

Given: -12x >30

Divide both sides by -12 , then we get,

$$=> \frac{-12x}{-12} < \frac{30}{-12}$$

$$=> x < -\frac{5}{2}$$

(i) There is no natural number less than $-\frac{5}{2}$, so when x is natural number , there is no solution for the given inequality.

(ii) When x is an integer then values of x that make statement true for {..................... ,-5, -4, -3}. The

Solution set of inequality is { ....,-5, -4, -3}

2. **Solve** $-12x > 30$ **when:**

(i) x **is a natural number**

(ii) x **is an integer**

Ans. Given: $-12x > 30$

Dividing both sides by -12 $x < \frac{30}{-12}$

$\Rightarrow x < \frac{-5}{2}$

(i) When x is a natural number then values of x that make statement true are none.

(ii) When x is an integer then values x of that make statement true are ......, $-5, -4, -3$. The solution n set of inequality is $\{........-5, -4, -3\}$.

3. **Solve** $5x - 3 < 7$ **when**

(i) x **is an integer**

(ii) x **is a real number**

Ans. Given: $5x-3<7$

$\Rightarrow 5x < 7+3$

$\Rightarrow 5x < 10$

$\Rightarrow x < 2$

(i) When x is a......, $-3, -2, -1, 0, 1$. n integer then values of x that make statement true are The solution set of inequality is $\{......, -3, -2, -1, 0, 1\}$.

(ii) When x is a real number then solution set of inequality is $x \in (-\infty, 2)$.

4. **Solve** $3x+8>2$ **when:**

(i) x **is an integer**

(ii) x **is a real number**

Ans. Given: $3x+8>2$

$\Rightarrow 3x>2-8$

$\Rightarrow 3x<-6$

$\Rightarrow x>-2$

(i) When x is an integer then values of x that make statement true are $-1, 0, 1, 2, 3, ...$ The solution set of inequality is $\{-1, 0, 1, 2, 3, ...\}$.

(ii) When x is a real number then solution set of inequality is $x \in (-2, \infty)$.

Solve the inequalities in Exercises 5 to 16 for real x:

5. $4x+3<5x+7$

Ans. Here $4x+3<5x+7$

$\Rightarrow 4x-5x<7-3$

$\Rightarrow -x<4$

$\Rightarrow x>-4$

Therefore, the solution set is $(-4, \infty)$.

6. $3x \quad 7 > 5x \quad 1$

Ans. Here $3x-7>5x-1$

$\Rightarrow 3x-5x>-1+7$

$\Rightarrow -2x>6$

$\Rightarrow x<-3$

Therefore, the solution set is $(-\infty,-3)$.

7. $3(x-1)\leq 2(x-3)$

Ans. Here $3(x-1)\leq 2(x-3)$

$\Rightarrow 3x-3\leq 2x-6$

$\Rightarrow 3x-2x\leq -6+3$

$\Rightarrow x\leq -3$

Therefore, the solution set is $(-\infty,-3)$.

8. $3(2-x)\geq 2(1-x)$

Ans. Here $3(2-x)\geq 2(1-x)$

$\Rightarrow 6-3x\geq 2-2x$

$\Rightarrow -3x+2x\geq 2-6$

$\Rightarrow -x\leq -4$

$\Rightarrow x\leq 4$

Therefore, the solution set is $(-\infty,4)$.

9. $x+\frac{x}{2}+\frac{x}{3}<11$

Ans. Here $x+\frac{x}{2}+\frac{x}{3}<11$

$\Rightarrow \frac{6x+3x+2x}{6}<11$

$\Rightarrow \frac{11x}{6}<11$

$\Rightarrow 11x<66$

$\Rightarrow x<6$

Therefore, the solution set is $(-\infty, 6)$.

10. $\frac{x}{3}>\frac{x}{2}+1$

Ans. Here $\frac{x}{3}>\frac{x}{2}+1$

$\Rightarrow \frac{x}{3}-\frac{x}{2}>1$

$\Rightarrow \frac{2x-3x}{6}>1$

$\Rightarrow \frac{-x}{6}>1$

$\Rightarrow x<-6$

Therefore, the solution set is $(-\infty, -6)$.

11. $\frac{3(x-2)}{5}\leq\frac{5(2-x)}{3}$

Ans. Here $\frac{3(x-2)}{5}\leq\frac{5(2-x)}{3}$

$$\Rightarrow \frac{3x-6}{5} \le \frac{10-5x}{3}$$

$$\Rightarrow 9x-18 \le 50-25x$$

$$\Rightarrow 9x+25x \le 50+18$$

$$\Rightarrow 34x \le 68$$

$$\Rightarrow x \le 2$$

Therefore, the solution set is $(-\infty, 2)$.

12. $\frac{1}{2}\left(\frac{3x}{5}+4\right) \ge \frac{1}{3}(x-6)$

Ans. Here $\frac{1}{2}\left(\frac{3x}{5}+4\right) \ge \frac{1}{3}(x-6)$

$$\Rightarrow \frac{3x}{10}+2 \ge \frac{x}{3}-2$$

$$\Rightarrow \frac{3x}{10}-\frac{x}{3} \ge -2-2$$

$$\Rightarrow \frac{9x-10x}{30} \ge -4$$

$$\Rightarrow \frac{-x}{30} \ge -4$$

$$\Rightarrow -x \ge -120$$

$$\Rightarrow x \le 120$$

Therefore, the solution set is $(-\infty, 120)$.

13. $2(2x+3)-10 < 6(x-2)$

Ans. Here $2(2x+3)-10<6(x-2)$

$\Rightarrow 4x+6-10<6x-12$

$\Rightarrow 4x-4<6x-12$

$\Rightarrow 4x-6x<-12+4$

$\Rightarrow -2x<-8$

$\Rightarrow x>4$

Therefore, the solution set is $(4,\infty)$.

14. $37-(3x+5)\geq 9x-8(x-3)$

Ans. Here $37-(3x+5)\geq 9x-8(x-3)$

$\Rightarrow 37-3x-5\geq 9x-8x+24$

$\Rightarrow 32-3x\geq x+24$

$\Rightarrow -3x-x\geq 24-32$

$\Rightarrow -4x\geq -8$

$\Rightarrow x\leq 2$

Therefore, the solution set is $(-\infty,2)$.

15. $\frac{x}{4}<\frac{(5x-2)}{3}-\frac{(7x-3)}{5}$

Ans. Here $\frac{x}{4}<\frac{(5x-2)}{3}-\frac{(7x-3)}{5}$

$\Rightarrow \frac{x}{4}<\frac{5x}{3}-\frac{2}{3}-\frac{7x}{5}+\frac{3}{5}$

$\Rightarrow \frac{x}{4}-\frac{5x}{3}+\frac{7x}{5}<-\frac{2}{3}+\frac{3}{5}$

$$\Rightarrow \frac{15x-100x+84x}{60} < \frac{-10+9}{15}$$

$$\Rightarrow \frac{-x}{60} < \frac{-1}{15} \Rightarrow x > 4$$

Therefore, the solution set is $(4, \infty)$.

16. $\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

Ans. Here $\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

$$\Rightarrow \frac{2x}{3} - \frac{1}{3} \geq \frac{3x}{4} - \frac{2}{4} - \frac{2}{5} + \frac{x}{5}$$

$$\Rightarrow \frac{2x}{3} - \frac{3x}{4} - \frac{x}{5} \geq -\frac{2}{4} - \frac{2}{5} + \frac{1}{3}$$

$$\Rightarrow \frac{40x-45x-12x}{60} \geq \frac{-30-24+20}{60}$$

$$\Rightarrow \frac{-17x}{60} \geq \frac{-34}{60}$$

$$\Rightarrow x \leq 2$$

Therefore, the solution set is $(-\infty, 2)$.

Solve the inequalities in Exercises 17 to 20 and show the graph of the solution in each case on numberline:

17. $3x-2 < 2x+1$

Ans. Here $3x-2 < 2x+1$

$$\Rightarrow 3x-2x < 1+2$$

$$\Rightarrow x < 3$$

$\therefore$

The solution set is $(-\infty, 3)$.

18. $5x-3 \geq 3x-5$

Ans. Here $5x-3 \geq 3x-5$

$\Rightarrow 5x-3x \geq -5+3$

$\Rightarrow 2x \geq -2$

$\Rightarrow x \geq -1$

The solution set is $(-1, \infty)$.

19. $3(1-x) < 2(x+4)$

Ans. Here $3(1-x) < 2(x+4)$

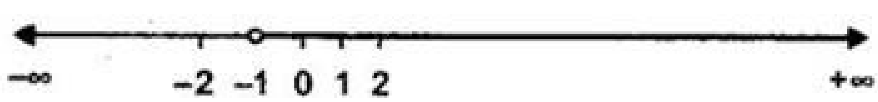
$\Rightarrow 3-3x < 2x+8$

$\Rightarrow -3x-2x < 8-3$

$\Rightarrow -5x < 5$

$\Rightarrow x > -1$

The solution set is $(-1, \infty)$.



20. $\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$

Ans. Here $\frac{x}{2} \geq \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$

$$\Rightarrow \frac{x}{2} \geq \frac{5x}{3} - \frac{2}{3} - \frac{7x}{5} + \frac{3}{5}$$

$$\Rightarrow \frac{x}{2} - \frac{5x}{3} + \frac{7x}{5} \geq -\frac{2}{3} + \frac{3}{5}$$

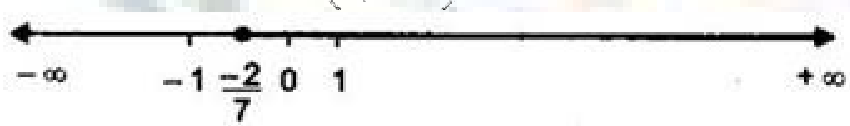
$$\Rightarrow \frac{15x - 50x + 42x}{30} \geq \frac{-10+9}{15}$$

$$\Rightarrow \frac{7x}{30} \geq \frac{-1}{15}$$

$$\Rightarrow 7x \geq -2$$

$$\Rightarrow x \geq \frac{-2}{7}$$

$\therefore$ The solution set is $\left(\frac{-2}{7}, \infty\right)$.



21. Ravi obtained 70 and 75 marks in first two unit tests. Find the minimum marks he should get in the third test to have an average of at least 60 marks.

Ans. Let the marks obtained by Ravi in third test be x. According to question,

$$\frac{70+75+x}{3} \geq 60$$

$$\Rightarrow \frac{145+x}{3} \geq 60$$

$$\Rightarrow 145 + x \geq 180$$

Therefore, minimum marks needed to be obtained by Ravi are 35.

22. To receive Grade 'A' in a course, one must obtain an average 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Sunita must obtain in fifth examination to get Grade 'A' in the course.

Ans. Let the marks obtained by Sunita in fifth examination be x. According to question,

$$\frac{87+92+94+95+x}{5} \geq 90$$

$$\Rightarrow \frac{368+x}{5} \geq 90$$

$$\Rightarrow 368+x \geq 90$$

$$\Rightarrow x \geq 82$$

Therefore, minimum marks needed to be obtained by Sunita is 82.

23. Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

Ans. Let x and $x+2$ be two consecutive odd positive integers.

$\Rightarrow x+2<10$ and $x+x+2>11$

$\Rightarrow x<8$ and $2x>9$

$\Rightarrow x<8$ and $x>\frac{9}{2}$

$$\Rightarrow \frac{9}{2}<x<8$$

$\Rightarrow x=5$ and 7

Therefore, the required pairs of odd positive integers are (5, 7) and (7, 9).

24. Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.

Ans. Let x and $x+2$ be two consecutive even positive integers.

$\Rightarrow x > 5$ and $x + x + 2 < 23$

$\Rightarrow x > 5$ and $2x < 21$

$\Rightarrow x > 5$ and $x < \frac{21}{2}$

$\Rightarrow 5 < x < \frac{21}{2}$

$\Rightarrow x = 6, 8$ and 10

Therefore, the required pairs of even positive integers are (6,8), (8 10) and (10, 12).

25. The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm. Find the minimum length of the shortest side

Ans. Let the length of the shortest side be x cm.

$\therefore$ Length of longest side = $3x$ cm and length of third side = $(3x-2)$ cm

$\Rightarrow$ Perimeter of triangle = $x + 3x + 3x - 2$ = $(7x-2)$ cm

Now, $7x - 2 \geq 61$

$\Rightarrow 7x \geq 63$

$\Rightarrow x \geq 9$

Therefore, the minimum length of shortest side is 9 cm.

26.A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5 cm longer than the second?

Ans. Let the length of the shortest side be x cm.

$\therefore$ Length of the second board = $(x+3)$ cm and length of third side = $2x$ cm

Now, $x+x+3+2x\leq 91$ and $2x\geq x+3+5$

$\Rightarrow 4x+3\leq 91$ and $2x-x\geq 5+3$

$\Rightarrow 4x\leq 88$ and $x\geq 8$

$\Rightarrow x\leq 22$ and $x\geq 8$

Therefore, minimum length of shortest board is 8 cm and maximum length is 22 cm.

Exercise 6.2

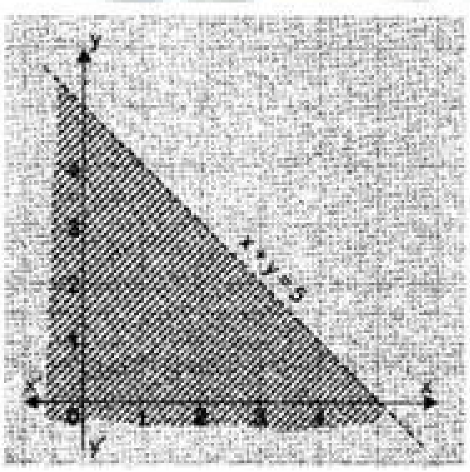
Solve the following inequalities graphically in two dimensional planes:

1. X + Y 5

Ans. Given: $x+y<5$

Draw a dashed boundary line satisfying the equation $x+y=5$

x	1	2
y	4	3



Putting (0, 0) in the given inequality $x+y<5$ we get,

$0+0<5$

Which is true

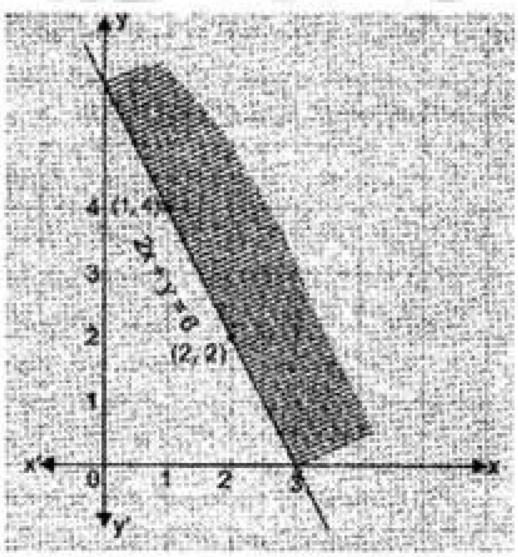
Therefore, Half plane of $x + y < 5$ towards origin.

2. $2x + y \geq 6$

Ans. Given: $2x + y \geq 6$

Draw a solid boundary line satisfying the equation $2x + y = 6$

x	1	2
y	4	2



Putting (0, 0) in the given inequality $2x + y \geq 6$ we get

$2 \times 0 + 0 \geq 6$

$\Rightarrow 0 \geq 6$

which is false.

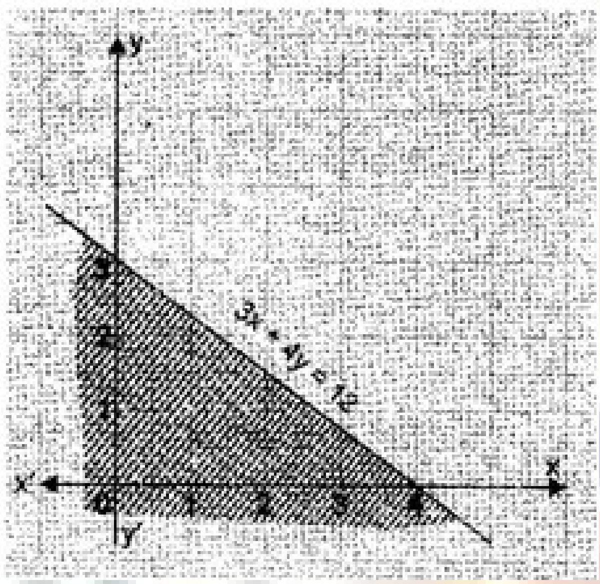
Therefore, Half plane of $2x + y \geq 6$ away from the origin.

3. $3x + 4y \leq 12$

Ans. Given: $3x + 4y \leq 12$

Draw a solid boundary line satisfying the equation $3x + 4y = 12$

x	0	4
y	3	0



Putting (0, 0) in the given inequality $3x + 4y \leq 12$ we get,

$3 \times 0 + 4 \times 0 \leq 12$

$\Rightarrow 0 \leq 12$

which is true.

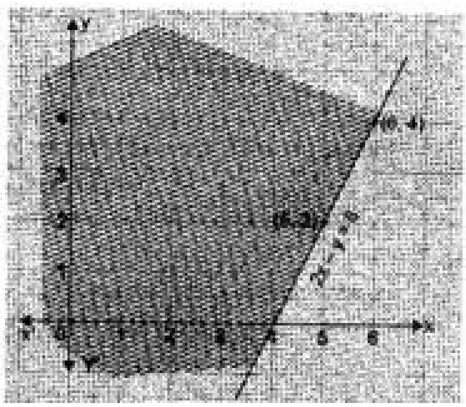
Therefore, Half plane of $3x + 4y \leq 12$ towards the origin.

4. $y + 8 \geq 2x$

Ans. Given: $y + 8 \geq 2x$

Draw a solid boundary line satisfying the equation $y + 8 = 2x$

x	5	6
y	2	4



Putting (0, 0) in the given inequality $y + 8 \geq 2x$ we get,

$$0 + 8 \geq 0$$

$$8 \geq 0 \Rightarrow 0 \leq 8$$

which is true.

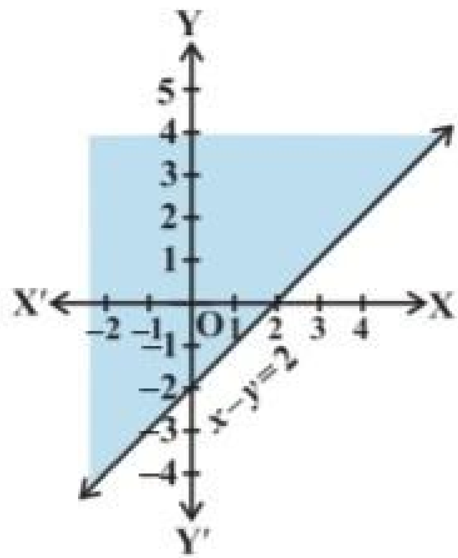
Therefore, Halfplane of $y + 8 \geq 2x$ towards the origin.

5. $x - y \leq 2$

Ans. Given: $x - y \leq 2$

Draw a solid boundary line satisfying the equation $x - y = 2$

x	2	0
y	0	-2



Putting (0, 0) in the given inequality $x - y \leq 2$ we get,

$\Rightarrow 0 - 0 \leq 2$

$\Rightarrow 0 \leq 2$

which is true.

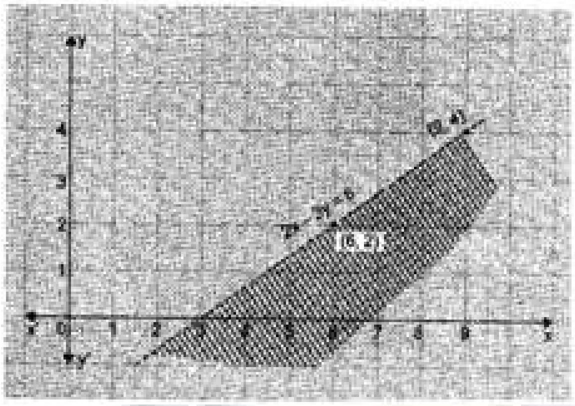
Therefore, Half plane of $x - y \leq 2$ towards the origin.

6 . $2x - 3y > 6$

Ans. Given: $2x - 3y > 6$

Draw a dashed boundary line satisfying the equation $2x - 3y = 6$

x	6	9
y	2	4



Putting (0, 0) in the given inequality $2x - 3y > 6$ we get

$2\times0-3\times0>6$

$\Rightarrow 0>6$

which is false.

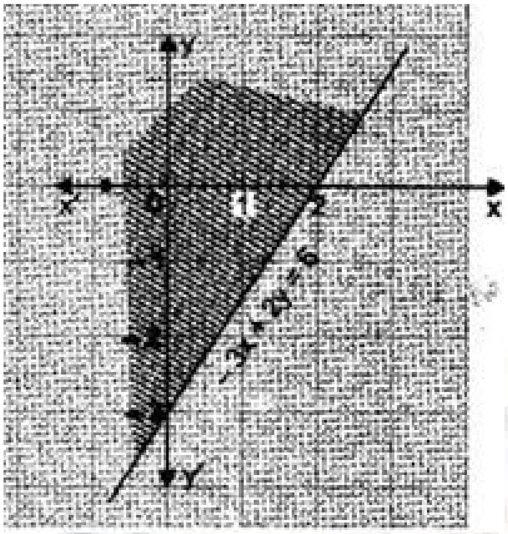
Therefore, Half plane of $2x-3y>6$ away from the origin.

7. $-3x+2y\geq -6$

Ans. Given: $-3x+2y\geq -6$

Draw a solid boundary line which satisfying the equation $-3x+2y=-6$

x	2	0
y	0	-3



Putting (0, 0) in the given inequality $-3x + 2y \geq -6$ we get,

$-3 \times 0 + 2 \times 0 \geq -6$

$\Rightarrow 0 \geq -6$

Which is true.

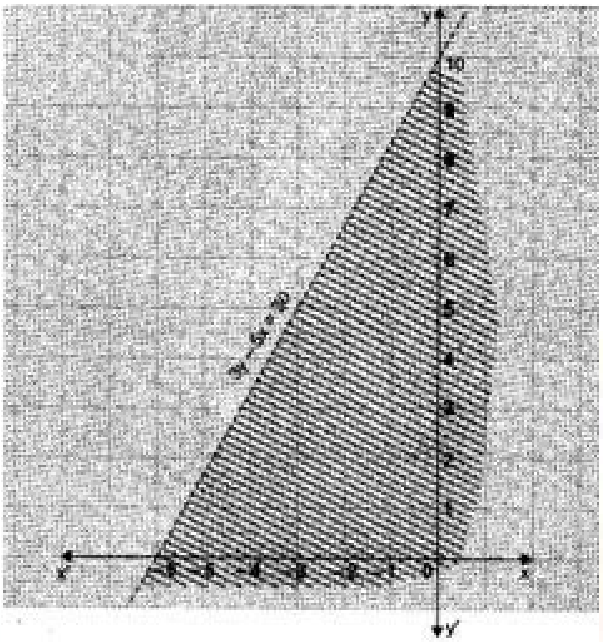
Therefore, Halfplane of $-3x + 2y \geq -6$ towards the origin.

8. $3y - 5x < 30$

Ans. Given: $3y - 5x < 30$

Draw a dashed boundary line satisfying the equation $3y - 5x = 30$

x	–6	0
y	0	10



Putting (0, 0) in the given inequality $3y \quad -5x \quad <30$ we get,

$-3\times 0-5\times 0<30$

$\Rightarrow 0<30$

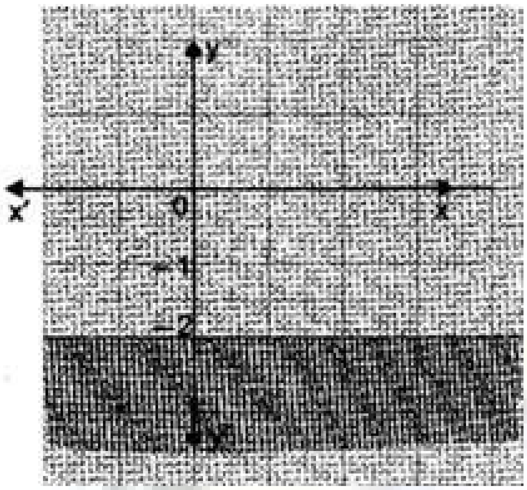
which is true.

Therefore, Half plane of $3y-5x<30$ towards the origin.

9. $y<-2$

Ans. Given: $y<-2$

Draw a dashed boundary line satisfying the equation $y=-2$



Putting (0, 0) in the given inequality $y < -2$ we get,

$0 < -2$

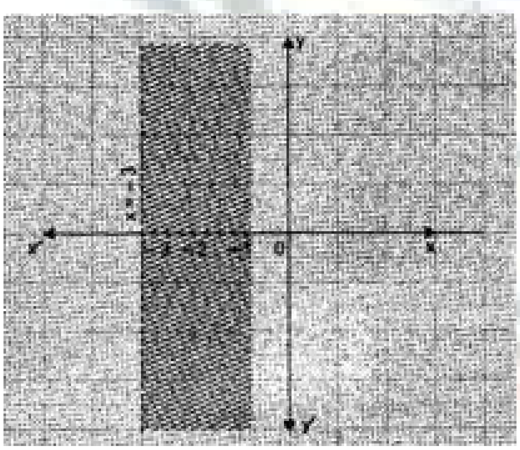
which is false.

Therefore, Half plane of $y < -2$ away from the origin.

10. $x > -3$

Ans. Given: $x > -3$

Draw a dashed boundary line which satisfying the equation $x = -3$



Putting (0, 0) in the given inequality $x > -3$ we get

$0 > -3$

which is true

, therefore, Half plane of x -3 towards the origin.

Exercise 6.3

Solve the following systems of inequalities graphically:

1. $x \geq 3, y \geq 2$

Ans. Given: $x \geq 3, y \geq 2$

Putting (0, 0) in $x \geq 3,$

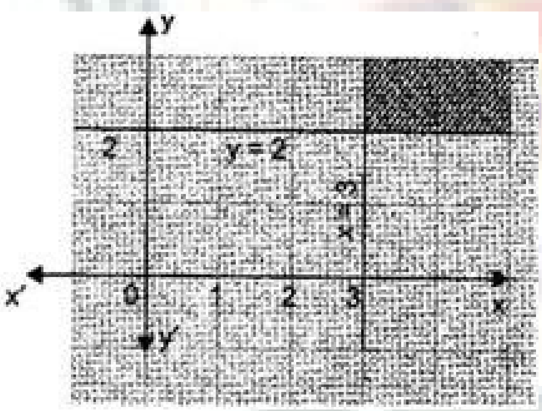
$0 > 3$ which is false.

∴ Half plane of $x \geq 3$ is away from origin.

Again Putting (0, 0) in $y \geq 2,$

$0 \geq 2$ which is false.

∴ Half plane of $y \geq 2$ is away from origin.

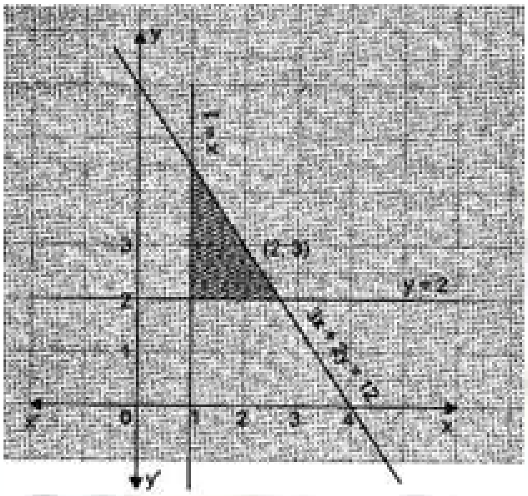


2. $3x + 2y \leq 12, x \geq 1, y \geq 2$

Ans. Given: $3x + 2y \leq 12, x \geq 1, y \geq 2$

Table of values satisfying the equation $3x + 2y = 12$

x	0	4
y	6	0



Putting (0, 0) in $3x+2y\le 12$

$3\times 0+2\times 0\le 12$

$\Rightarrow 0\le 12$ which is true.

$\therefore$ Half plane of $3x+2y\le 12$ is towards the origin.

Also, putting (0,0) in $x\ge 1$

$\Rightarrow 0\ge 1$ which is false.

$\therefore$ Half plane of $x\ge 1$ is away from origin.

Again, Again Putting (0,0) in $y\ge 2,$

$\Rightarrow 0\ge 2$ which is false.

$\therefore$ Half plane of $y\ge 2$ is away from origin.

3. $2x+y\ge 6, 3x+4y\le 12$

Ans.

$$2x+y=6$$

Given: $2x+y\geq 6, 3x+4y\leq 12$

Table of values satisfying the equation

$$2x+y=6$$