Std: 11

Exercise 5.1

Chapter **– 5**

Sub: MATHS

Complex Numbers and

Express each of the complex numbers given in the exercises 1 to 10 in the form a+ib:

1. $(5i)\left(\frac{-3}{5}i\right)$

Ans. Here $(5i)\left(\frac{-3}{5}i\right) = -3i^2$

$= -3\times -1$

$=3$

2. $i^9 + i^{19}$

Ans. Here $i^9 + i^{19}$

$= (i^2)^4 i + (i^2)^9 i = (-1)^4 i + (-1)^9 i = i - i = 0$

3. i^{-39}

Ans. $\frac{1}{i^{39}} = \frac{1}{(i^2)^{19} i} = \frac{1}{(-1)^{19} i} = \frac{1}{-i}$

$\frac{-1}{i}\times\frac{i}{i} = \frac{-i}{i^2} = \frac{-i}{-1} = i$

4. $3(7+i7)+i(7+i7)$

Ans. Here

$$\frac{13}{3}+\frac{8}{3}i+\frac{4}{3}-i \quad \left(\frac{13}{3} \quad \frac{4}{3}\right) \quad \left(\frac{8}{3}-1\right)i \quad \frac{17}{3}+\frac{5}{3}i$$

$$3(7+i7)+ = \quad = \quad + \quad + \quad =$$

$$= 21+21i+7i+7i^2 = 21+28i-7 = 14+28i$$

---

5. $(1-i)-(-1+i6)$ 8. $(1-i)^4$

Ans. Here $(1-i)-(-1+i6)$ **Ans.** Here $\left[(1-i)^2\right]^2$ =

$$= \left[1+i^2-2i\right]^2 = (1-1-2i)^2 = (-2i)^2$$

$$1-i+1-i6 = 2-7i$$

---

6. $\left(\frac{1}{5}+\frac{2}{5}i\right)-\left(4+\frac{5}{2}i\right)$ = $4i^4 = -4$

Ans. Here $\left(\frac{1}{5}+\frac{2}{5}i\right)-\left(4+\frac{5}{2}i\right)$ 9. $\left(\frac{1}{3}+3i\right)^3$

$$= \frac{1}{5}+\frac{2}{5}i-4-\frac{5}{2}i \qquad \left(-\frac{1}{3}\right)^3$$

Ans. Here

$$= \left(\frac{1}{5}-4\right)+\left(\frac{2}{5}-\frac{5}{2}\right)i = \frac{-19}{5}-\frac{21}{10}i$$

$$= \left(\frac{1}{3}\right)^3+(3i)^3+3\times\left(\frac{1}{3}\right)^2\times 3i+3\times\frac{1}{3}\times(3i)^2$$

---

7. $\left[\left(\frac{1}{3}+\frac{7}{3}i\right)+\left(4+\frac{1}{3}i\right)\right]-\left[\frac{-4}{3}+i\right]$

$$\frac{1}{27}+27i^3+i+9i^2$$

Ans. Here $\left[\left(\frac{1}{3}+\frac{7}{3}i\right)+\left(4+\frac{1}{3}i\right)\right]-\left[\frac{-4}{3}+i\right]$

$$= \frac{1}{27}-27i+i-9$$

$$= \left[\left(\frac{1}{3}+4\right)+\left(\frac{7}{3}+\frac{1}{3}i\right)\right]-\left[\frac{-4}{3}+i\right]$$

$= \frac{1}{27}\left(-9-26i\right)$

$= \frac{-242}{27} - 26i$

10. $\left(-2-\frac{1}{3}i\right)^3$

Ans. Here $\left(-2-\frac{1}{3}i\right)^3 = -\left(2+\frac{1}{3}i\right)^3$

$= -\left[(2)^3 + \left(\frac{1}{3}i\right)^3 + 3\times(2)^2\times\frac{1}{3}i + 3\times 2\times\left(\frac{1}{3}i\right)^2\right]$

$= -\left[8 + \frac{1}{27}i^3 + 4i + \frac{2}{3}i^2\right]$

$= -\left[8 - \frac{1}{27}i + 4i - \frac{2}{3}\right] = \frac{-22}{3} - \frac{107}{27}i$

Find the multiplicative inverse of each of the complex numbers given in the exercises 11 to 13.

11. $4-3i$

Ans. Multiplicative Inverse of $4-3i$

$= \frac{1}{4-3i}\times\frac{4+3i}{4+3i}$

$= \frac{4+3i}{(4)^2-(3i)^2}$

$= \frac{4+3i}{16-9i^2}$

$= \frac{4+3i}{16+9} = \frac{1}{25}(4+3i)$

$= \frac{4}{25} + \frac{3}{25}i$

12. $\sqrt{5}+3i$

Ans. Multiplicative Inverse of $\sqrt{5}+3i$

$= \frac{1}{\sqrt{5}+3i}\times\frac{\sqrt{5}-3i}{\sqrt{5}-3i}$

$= \frac{\sqrt{5}-3i}{(\sqrt{5})^2-(3i)^2}$

$= \frac{\sqrt{5}-3i}{5+9} = \frac{1}{14}\left(\sqrt{5}-3i\right)$

$= \frac{\sqrt{5}}{14} - \frac{3}{14}i$

13. $-i$

Ans. Multiplicative Inverse of $-i$

$= \frac{1}{-i} \times \frac{i}{i}$

$= \frac{i}{-i^2}$

$= \frac{i}{-(-1)} = i$

14. Express the following expression in the form of $a + ib$: $\frac{(3+\sqrt{5}i)(3-\sqrt{5}i)}{(3+\sqrt{2}i)-(3-\sqrt{2}i)}$

Ans. Here $\frac{(3+\sqrt{5}i)(3-\sqrt{5}i)}{(3+\sqrt{2}i)-(3-\sqrt{2}i)}$

$= \frac{(3)^2 - (\sqrt{5}i)^2}{\sqrt{3} + \sqrt{2}i - \sqrt{3} + \sqrt{2}i}$

$= \frac{9 - 5i^2}{2\sqrt{2}i}$

$= \frac{9+5}{2\sqrt{2}i} = \frac{14}{2\sqrt{2}i}$

$= \frac{7}{\sqrt{2}i} = \frac{7}{\sqrt{2}i} \times \frac{i}{i} = \frac{7i}{\sqrt{2}i^2} = \frac{-7i}{\sqrt{2}} = \frac{-7\sqrt{2}i}{2}$

Exercise 5.2

Find the modulus and the argument of each of the complex numbers in exercises 1 to 2 1. $z = -1 - i\sqrt{3}$

Ans. Given: $z = r(\cos\theta + i\sin\theta) = -1 - i\sqrt{3}$

$\therefore r\cos\theta = -1$ and $|z| = 2$ and arg

$z = -\sqrt{3} + i$ **2.**

Ans. Given:

Squaring both sides and adding both the equations, we get

$r^2(\cos^2\theta + \sin^2\theta) = 1 + 3$

$\Rightarrow r^2 = 4$

$\Rightarrow r = 2$

and $\sin\theta = \frac{-\sqrt{3}}{2}$

$\therefore 2\cos\theta = -1$ and

$\Rightarrow \cos\theta = \frac{-1}{2}$

[θ lies in third quadrant]

$\therefore \theta = \left(-\pi + \frac{\pi}{3}\right) = \frac{-2\pi}{3}$

Therefore, $(z) = \frac{-2\pi}{}$

$z = r(\cos\theta$

$\therefore r\cos\theta = -\sqrt{3}$ and $r\sin\theta = 1$

Squaring both sides and adding both the equations, we get

$$r^2\left(\cos^2\theta+\sin^2\theta\right)=3+1$$

$$\Rightarrow r^2=4$$

$$\Rightarrow r=2$$

$$\therefore 2\cos\theta=-\sqrt{3} \quad \text{and } 2\sin\theta=1$$

$$\Rightarrow \cos\theta=\frac{-\sqrt{3}}{2} \quad \text{and } \sin\theta=\frac{1}{2}$$

[θ lies in second quadrant]

$$\therefore \theta=\left(\pi-\frac{\pi}{6}\right)=\frac{5\pi}{6}$$

Therefore, $|z|=2$ and arg $(z)=\frac{5\pi}{6}$

QUESTIONS 3 TO 8 NOT IN SYLLABUS

Exercise 5.3

Solve each of the following equations:

1. $x^2+3=0$

Ans. Given: $x^2+3=0$

$\Rightarrow x^2=-3$

$\Rightarrow x=\pm\sqrt{-3}$

$\Rightarrow x=\pm\sqrt{3}i$

2. $2x^2+x+1=0$

Ans. Given: $2x^2+x+1=0$ Comparing with

$ax^2+bx+c=0$,

$a=2, b=1$ and $c=1$

$\therefore x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\dfrac{-1\pm\sqrt{(1)^2-4\times2\times1}}{2\times2}$

$=\dfrac{-1\pm\sqrt{-7}}{4}$ $=\dfrac{-1\pm\sqrt{7}i}{4}$

Therefore, $x=\dfrac{-1+\sqrt{7}i}{4}$ and $x=\dfrac{-1-\sqrt{7}i}{4}$

3. $x^2+3x+9=0$

Ans. Given: $x^2+3x+9=0$

Comparing with $ax^2 + bx + c = 0$,

$a = 1, b = 3$ and $c = 9$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{3 \pm \sqrt{(3)^2 - 4 \times 1 \times 9}}{2 \times 1}$$

$$= \frac{-3 \pm \sqrt{-27}}{2} = \frac{-3 \pm \sqrt{27}i}{2}$$

Therefore, $x = \frac{-3 + \sqrt{27}i}{2}$ and $x = \frac{-3 - \sqrt{27}i}{2}$

4. $-x^2 + x - 2 = 0$

Ans. Given: $-x^2 + x - 2 = 0$ Comparing with $ax^2 + bx + c = 0$,

$a = -1, b = 1$ and $c = -2$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{(1)^2 - 4 \times (-1) \times (-2)}}{2 \times (-1)}$$

$$= \frac{-1 \pm \sqrt{-7}}{-2} = \frac{-1 \pm \sqrt{7}i}{-2}$$

Therefore, $x = \frac{-1 + \sqrt{7}i}{-2}$ and $x = \frac{-1 - \sqrt{7}i}{-2}$

5. $x^2+3x+5=0$

Ans. Given: $x^2+3x+5=0$

Comparing with $ax^2+bx+c=0$,

$a=1, b=3$ and $c=5$

$$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-3\pm\sqrt{(3)^2-4\times1\times5}}{2\times1}$$

$$=\frac{-3\pm\sqrt{-11}}{2}=\frac{-3\pm\sqrt{11}i}{2}$$

Therefore, $x=\frac{-3+\sqrt{11}i}{2}$ and $x=\frac{-3-\sqrt{11}i}{2}$

6. $x^2-x+2=0$

Ans. Given: $x^2-x+2=0$ Comparing with

$ax^2+bx+c=0$,

$a=1, b=-1$ and $c=2$

$$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-1)\pm\sqrt{(-1)^2-4\times1\times2}}{2\times1}$$

$$=\frac{1\pm\sqrt{-7}}{2}=\frac{1\pm\sqrt{7}i}{2}$$

Therefore, $x=\frac{1+\sqrt{7}i}{2}$ and $x=\frac{1-\sqrt{7}i}{2}$

7. $\sqrt{2}x^2+x+\sqrt{2}=0$

Ans. Given: $\sqrt{2}x^2+x+\sqrt{2}=0$

Comparing with $ax^2+bx+c=0$,

$a=\sqrt{2}, b=1$ and $c=\sqrt{2}$

$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$=\frac{-1\pm\sqrt{(1)^2-4\times\sqrt{2}\times\sqrt{2}}}{2\times\sqrt{2}}$

$=\frac{-1\pm\sqrt{-7}}{2\sqrt{2}}=\frac{-1\pm\sqrt{7}i}{2\sqrt{2}}$

Therefore, $x=\frac{-1+\sqrt{7}i}{2\sqrt{2}}$ and $x=\frac{-1-\sqrt{7}i}{2\sqrt{2}}$

8. $\sqrt{3}x^2-\sqrt{2}x+3\sqrt{3}=0$

Ans. Given: $\sqrt{3}x^2-\sqrt{2}x+3\sqrt{3}=0$

Comparing with $ax^2+bx+c=0$,

$a=\sqrt{3}, b=-\sqrt{2}$ and $c=3\sqrt{3}$

$\therefore x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \dfrac{-(-\sqrt{2}) \pm \sqrt{(-\sqrt{2})^2 - 4 \times \sqrt{3} \times 3\sqrt{3}}}{2 \times \sqrt{3}}$

$= \dfrac{\sqrt{2} \pm \sqrt{-34}}{2\sqrt{3}} = \dfrac{\sqrt{2} \pm \sqrt{34}i}{2\sqrt{3}}$

Therefore, $x = \dfrac{\sqrt{2} + \sqrt{34}i}{2\sqrt{3}}$ and $x = \dfrac{\sqrt{2} - \sqrt{34}i}{2\sqrt{3}}$

9 $x^2 + x + \dfrac{1}{\sqrt{2}} = 0$

Ans. Given: $x^2 + x + \dfrac{1}{\sqrt{2}} = 0$

Comparing with $ax^2 + bx + c = 0$,

$a = 1, b = 1$ and $c = \dfrac{1}{\sqrt{2}}$

$\therefore x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-1 \pm \sqrt{(1)^2 - 4 \times 1 \times \dfrac{1}{\sqrt{2}}}}{2 \times 1}$

$= \dfrac{-1 \pm \sqrt{1 - 2\sqrt{2}}}{2} = \dfrac{-1 \pm \sqrt{2\sqrt{2} - 1}i}{2}$

Therefore, $x = \dfrac{-1 + \sqrt{2\sqrt{2} - 1}i}{2}$ and $x = \dfrac{-1 - \sqrt{2\sqrt{2} - 1}i}{2}$

10. $x^2 + \dfrac{x}{\sqrt{2}} + 1 = 0$

Ans. Given: $x^2 + \dfrac{x}{\sqrt{2}} + 1 = 0$

Comparing with $ax^2 + bx + c = 0$,

$a = 1, b = \frac{1}{\sqrt{2}}$ and $c = 1$

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{\frac{-1}{\sqrt{2}} \pm \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 - 4 \times 1 \times 1}}{2 \times 1}$$

$$= \frac{-\frac{1}{\sqrt{2}} \pm \sqrt{\frac{1}{2} - 4}}{2} = \frac{-\frac{1}{\sqrt{2}} \pm \sqrt{\frac{-7}{2}}}{2}$$

$$= \frac{-1 \pm \sqrt{7}i}{2\sqrt{2}}$$

Therefore, $x = \frac{-1 + \sqrt{7}i}{2\sqrt{2}}$ and $x = \frac{-1 - \sqrt{7}i}{2\sqrt{2}}$