Std: 11 Chapter **–3** Sub: MATHS

Trigonometric Functions

Exercise 3.1

1. Find the radian measures corresponding to the following degree measures:

(i) $25°$

(ii) $-47°30'$

(iii) $240°$

(iv) $520°$

Ans. (i) $25° = \left(25 \times \frac{\pi}{180}\right)^c = \left(\frac{5\pi}{36}\right)^c$

(ii) $-47°30' = -\left(47\frac{30}{60}\right)^° = -\left(\frac{95}{2}\right)^° = -\left(\frac{95}{2} \times \frac{\pi}{180}\right)^c = -\left(\frac{19\pi}{72}\right)^c$

(iii) $240° = \left(240 \times \frac{\pi}{180}\right)^c = \left(\frac{4\pi}{3}\right)^c$

(iv) $520° = \left(520 \times \frac{\pi}{180}\right)^c = \left(\frac{26\pi}{9}\right)^c$

2. Find the degree measures corresponding to the following radian measures $\left(\text{Use } \pi = \frac{22}{7}\right)$.

(i) $\frac{11}{16}$

(ii) -4

(iii) $\frac{5\pi}{3}$

(iv) $\frac{7\pi}{6}$

Ans. (i) $\left(\frac{11}{16}\right)^c = \left(\frac{11}{16}\times\frac{180}{\pi}\right)^\circ = \left(\frac{11}{16}\times\frac{180\times 7}{22}\right)^\circ = \left(\frac{315}{8}\right)^\circ = \left(39\frac{3}{8}\right)^\circ = \left(39\frac{3}{8}\times 60\right)^\circ$

= $39^\circ 22\frac{1}{2}' = 39^\circ 22'\frac{1}{2}\times 60 = 39^\circ 22' 30''$

(ii)

$(-4)^c = -\left(4\times\frac{180}{\pi}\right)^\circ = -\left(4\times\frac{180\times 7}{22}\right)^\circ = -\left(\frac{2520}{11}\right)^\circ = -\left(229\frac{1}{11}\right)^\circ = -229^\circ\left(\frac{1}{11}\times 60\right)^\circ$

$= -229^o 5'\left(\frac{5}{11}\times 60\right)'' = -229^o 5' 27''$

(iii) $\left(\frac{5\pi}{3}\right)^c = \left(\frac{5\pi}{3}\times\frac{180}{\pi}\right)^\circ = 300^\circ$

(iv) $\left(\frac{7\pi}{6}\right)^c = \left(\frac{7\pi}{6}\times\frac{180}{\pi}\right)^\circ = 210^\circ$

3. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Ans. Number of revolutions in 1 minute = 360

$\therefore$ Number of revolution in 60 seconds = 360

$\Rightarrow$ Number of revolutions in 1 second = $\frac{360}{60}$ = 6 revolutions

$\therefore$ Angle made by wheel in 6 revolutions = $360 \times 6 = 2160°$

$$\Rightarrow 2160° = \left(2160 \times \frac{\pi}{180}\right)^c = (12\pi)^c$$

4. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm $\left(\text{Use } \pi = \frac{22}{7}\right)$.

Ans. Here $r = 100$ cm and l = 22 cm

$$\because \theta^c = \frac{l}{r}$$

$$\therefore \theta^c = \frac{22}{100} = \left(\frac{11}{50}\right)^c$$

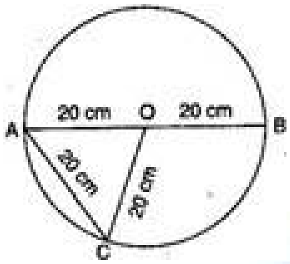
$$\Rightarrow \left(\frac{11}{50}\right)^c = \left(\frac{11}{50} \times \frac{180°}{\pi}\right) = \left(\frac{11}{50} \times \frac{180° \times 7}{22}\right)$$

$$= \left(\frac{63}{5}\right)^\circ = 12°\left(\frac{2}{5} \times 0\right)' = 12°36'$$

5. In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.

Ans. Given: Diameter AB = 40 cm, Radius OA = 20 cm and Chord AC = 20 cm

$\therefore \Delta AOC$ is an equilateral triangle.



$\therefore \angle$ AOC = $60^\circ = \left(60^\circ \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{3}\right)^c$

Now, $\theta^c = \frac{l}{r} \Rightarrow \frac{\pi}{3} = \frac{l}{20}$

$\Rightarrow l = \frac{20\pi}{3}$ cm

6. If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Ans. Let r_1 and r_2 be radii of two circles in which arcs of same length l subtend angles $\theta_1 = 60^\circ$ and $\theta_2 = 75^\circ$ respectively.

$\therefore \theta_1 = \frac{l}{r_1} \Rightarrow \left(60 \times \frac{\pi}{180}\right)^c = \frac{l}{r_1}$

$\Rightarrow r_1 = \frac{3l}{\pi}$

And $\theta_2 = \frac{l}{r_2} \Rightarrow \left(75 \times \frac{\pi}{180}\right)^c = \frac{l}{r_2}$

$\Rightarrow r_2 = \frac{12l}{5\pi}$

$$\therefore \frac{r_1}{r_2} = \frac{3l/\pi}{12l/5\pi} = \frac{5}{4}$$

$$\Rightarrow r_1 : r_2 = 5 : 4$$

7. Find the angle in radians through which a pendulum swings if its length is 75 cm and the tip describes an arc of length:

(i) 10 cm

(ii) 15 cm

(iii) 21 cm

Ans. (i) Given: length of pendulum (r) = 75 cm and length of arc (l) = 10 cm

$$\therefore \theta^c = \frac{l}{r} = \frac{10}{75} = \left(\frac{2}{15}\right)^c$$

(ii) Given: length of pendulum (r) = 75 cm and length of arc (l) = 15 cm

$$\therefore \theta^c = \frac{l}{r} = \frac{15}{75} = \left(\frac{1}{5}\right)^c$$

(iii) Given: length of pendulum (r) = 75 cm and length of arc (l) = 21 cm

$$\therefore \theta^c = \frac{l}{r} = \frac{21}{75} = \left(\frac{7}{25}\right)^c$$

Exercise 3.2

1. Find the values of other trigonometric functions in exercises 1 to 5.

I. $\cos x = -\frac{1}{2}$, x **lies in third quadrant.**

Ans. Given: $\cos x = -\frac{1}{2}$

$\because \sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2 x + \left(-\frac{1}{2}\right)^2 = 1$

$\Rightarrow \sin^2 x = 1 - \frac{1}{4}$

$\Rightarrow \sin^2 x = \frac{3}{4}$

$\Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow \sin x = -\frac{\sqrt{3}}{2}$ [x lies in third quadrant]

Now, $\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{2}{\sqrt{3}}$

$\sec x = \frac{1}{\cos x} = -2$

$$\tan x = \frac{\sin x}{\cos x} = \frac{-\sqrt{3}/2}{-1/2} = \sqrt{3}$$

$$\cot x = \frac{\cos x}{\sin x} = \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

2. $\sin x = \frac{3}{5}$, x **lies in secondquadrant.**

Ans. Given: $\sin x = \frac{3}{5}$

$$\because \sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \left(\frac{3}{5}\right)^2 + \cos^2 x = 1$$

$$\Rightarrow \cos^2 x = 1 - \frac{9}{25}$$

$$\Rightarrow \cos^2 x = \frac{16}{25}$$

$$\Rightarrow \cos x = \pm\frac{4}{5}$$

$$\Rightarrow \cos x = -\frac{4}{5}$$ [x lies in second quadrant]

Now, $\operatorname{cosec} x = \frac{1}{\sin x} = \frac{5}{3}$

$$\sec x = \frac{1}{\cos x} = -\frac{5}{4}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{3/5}{-4/5} = \frac{-3}{4}$$

$$\cot x = \frac{\cos x}{\sin x} = \frac{-4/5}{3/5} = \frac{-4}{3}$$

---

3. $\cot x = \frac{3}{4}$, **lies in third quadrant.**

Ans. Given: $\cot x = \frac{3}{4}$

$\because \operatorname{cosec}^2\theta - \cot^2\theta = 1$

$\Rightarrow \operatorname{cosec}^2 x - \left(\frac{3}{4}\right)^2 = 1$

$\Rightarrow \operatorname{cosec}^2 x = 1 + \frac{9}{16}$

$\Rightarrow \operatorname{cosec}^2 x = \frac{25}{16}$

$\Rightarrow \operatorname{cosec} x = \pm\frac{5}{4}$

$\Rightarrow \operatorname{cosec} x = -\frac{5}{4}$ [x lies in third quadrant]

Now, $\sin x = \frac{1}{\operatorname{cosec} x} = \frac{-4}{5}$

$\cos x = \sqrt{1 \quad \sin^2 x} = \sqrt{1 \quad \frac{16}{25}} = \quad \frac{3}{5}$

$$\tan x = \frac{1}{\cot x} = \frac{4}{3}$$

$$\sec x = \frac{1}{\cos x} = \frac{-5}{3}$$

4. $\sec x = \frac{13}{5}$, x **lies in fourth quadrant.**

Ans. Given: $\sec x = \frac{13}{5}$

$$\because \sec^2 \theta - \tan^2 \theta = 1$$

$$\Rightarrow \left(\frac{13}{5}\right)^2 - \tan^2 x = 1$$

$$\Rightarrow \tan^2 x = \left(\frac{13}{5}\right)^2 - 1$$

$$\Rightarrow \tan^2 x = \frac{169}{25} - 1$$

$$\Rightarrow \tan^2 x = \frac{144}{25}$$

$$\Rightarrow \tan x = \pm \frac{12}{5}$$

$$\Rightarrow \tan x = \frac{-12}{5}$$ [x lies in fourth quadrant]

Now $\cot x = \frac{1}{\tan x} = \frac{-5}{12}$

$$\cos x = \frac{1}{\quad} = \frac{5}{\quad}$$

$$\sin x = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\frac{144}{169}} = -\frac{12}{13}$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{-13}{12}$$

5. $\tan x = \frac{-5}{12}, x$ **lies in second quadrant.**

Ans. Given: $\tan x = \frac{-5}{12}$

$$\therefore \cot x = \frac{1}{\tan x} = \frac{-12}{5}$$

$$\because \sec^2\theta - \tan^2\theta = 1$$

$$\Rightarrow \sec^2 x - \left(\frac{-5}{12}\right)^2 = 1$$

$$\Rightarrow \sec^2 x = 1 + \frac{25}{144}$$

$$\Rightarrow \sec^2 x = \frac{169}{144}$$

$$\Rightarrow \sec x = \pm\frac{13}{12}$$

$\Rightarrow \sec x = \frac{-13}{12}$ [x lies in second quadrant]

Now, $\cos x = \frac{1}{\sec x} = \frac{-12}{13}$

$$\sin x = \sqrt{1-\cos^2 x} = \sqrt{1-\frac{144}{169}} = \frac{5}{13}$$

Cosecx = 1/sinx =13/5

Find the values of the trigonometric functions in exercises 6 o 10.

6. $\sin 765°$

Ans. Here $\sin 765° = \sin\left(2\times 360° + 45°\right) = \sin 45° = \frac{1}{\sqrt{2}}$

7. $\operatorname{cosec}\left(-1410\right)°$

Ans. Here $\operatorname{cosec}\left(-1410\right)° = \operatorname{cosec}\left(-4\times 360° + 30°\right) = \operatorname{cosec}30° = 2$

8. $\tan\frac{19\pi}{3}$

Ans. Here $\tan\frac{19\pi}{3} = \tan\frac{19}{3}\times 180° = \tan 1140° = \tan\left(3\times 360° + 60°\right) = \tan 60° = \sqrt{3}$

9. $\sin\left(\frac{-11\pi}{3}\right)$

Ans. Here

$$\sin\left(\frac{-11\pi}{3}\right) = \sin\left(\frac{-11\times 180°}{3}\right) = \sin\left(-660°\right) = \sin\left(-2\times 360° + 60°\right) = \sin 60° = \frac{\sqrt{3}}{2}$$

10. $\cot\left(\frac{-15\pi}{4}\right)$

Ans. Here

$$\cot\left(\frac{-15\pi}{4}\right) = \cot\left(\frac{-15\times 180°}{4}\right) = \cot\left(-675°\right) = \cot\left(-2\times 360° + 45°\right) = \cot 45° = 1$$

Exercise 3.3

Prove that:

1. $\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$

Ans. Taking L.H.S.

$=\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$

$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2=\frac{1}{4}+\frac{1}{4}-1$

$=\frac{1+1-4}{4}=\frac{-2}{4}=\frac{-1}{2}$ R.H.S.

2. $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

Ans. Taking L.H.S

$=2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$=2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\left(\pi+\frac{\pi}{6}\right)\cos^2\frac{\pi}{3}$

$=2\sin^2\frac{\pi}{6}-\operatorname{cosec}^2\frac{\pi}{6}\cos^2\frac{\pi}{3}$

Exercise 3.3

Prove that:

1. $\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}=-\frac{1}{2}$

Ans. Taking L.H.S.

$=\sin^2\frac{\pi}{6}+\cos^2\frac{\pi}{3}-\tan^2\frac{\pi}{4}$

$=\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-(1)^2=\frac{1}{4}+\frac{1}{4}-1$

$=\frac{1+1-4}{4}=\frac{-2}{4}=\frac{-1}{2}$ R.H.S.

2. $2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}=\frac{3}{2}$

Ans. Taking L.H.S

$=2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$=2\sin^2\frac{\pi}{6}+\operatorname{cosec}^2\left(\pi+\frac{\pi}{6}\right)\cos^2\frac{\pi}{3}$

$=2\sin^2\frac{\pi}{6}-\operatorname{cosec}^2\frac{\pi}{6}\cos^2\frac{\pi}{3}$

$= 2\times\left(\frac{1}{2}\right)^2+(-2)^2\times\left(\frac{1}{2}\right)^2$

$=2\times\frac{1}{4}+4\times\frac{1}{4}=\frac{6}{4}=\frac{3}{2}$

3. $\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}=6$

Ans. Taking L.H.S

$=\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{5\pi}{6}+3\tan^2\frac{\pi}{6}$

$=\cot^2\frac{\pi}{6}+\operatorname{cosec}\left(\pi-\frac{\pi}{6}\right)+3\tan^2\frac{\pi}{6}$

$=\cot^2\frac{\pi}{6}+\operatorname{cosec}\frac{\pi}{6}+3\tan^2\frac{\pi}{6}$

$=(\sqrt{3})^2+2+3\times\left(\frac{1}{\sqrt{3}}\right)^2$

$=3+2+3\times\frac{1}{3}=5+1=6$ = R.H.S.

4. $2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}=10$

Ans. L.H.S. $=2\sin^2\frac{3\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

$=2\sin^2\left(\pi-\frac{\pi}{4}\right)+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

$=2\sin^2\frac{\pi}{4}+2\cos^2\frac{\pi}{4}+2\sec^2\frac{\pi}{3}$

$= \quad 2\times\left(\frac{1}{\sqrt{2}}\right)^2+2\times\left(\frac{1}{\sqrt{2}}\right)^2+2\times(2)^2$

$=2\times\frac{1}{2}+2\times\frac{1}{2}+2\times 4=1+1+8=10$ = R.H.S.

5. Find the value of:

(i) $\sin 75^\circ$

(ii) $\tan 15^\circ$

Ans. (i) $\sin 75^\circ=\sin\left(45^\circ+30^\circ\right)=\sin 45^\circ\cos 30^\circ+\cos 45^\circ\sin 30^\circ$

$= \quad \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{2}}\times\frac{1}{2}=\frac{\sqrt{3}}{2\sqrt{2}}+\frac{1}{2\sqrt{2}}=\frac{\sqrt{3}+1}{2\sqrt{2}}$

(ii) $\tan 15^\circ=\tan\left(45^\circ-30^\circ\right)=\frac{\tan 45^\circ-\tan 30^\circ}{1+\tan 45^\circ\tan 30^\circ}$

$\left[\because \tan\left(x-y\right)=\frac{\tan x-\tan y}{1+\tan x\tan y}\right]$

$= \frac{1-\frac{1}{\sqrt{3}}}{1+1\times\frac{1}{\sqrt{3}}}=\frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \quad \frac{3+1-2\sqrt{3}}{3-1}=\frac{4-2\sqrt{3}}{2}=2-\sqrt{3}$

Prove the following:

6. $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$

Ans taking L.H S

$=\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$

$=\cos\left[\frac{\pi}{4}-x+\frac{\pi}{4}-y\right]$

$\left[\because \cos(x+y)=\cos x\cos y-\sin x\sin y\right]$

$=\cos\left[\frac{\pi}{2}-(x+y)\right]=\sin(x+y)$ = R.H.S.

7. $\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}=\left[\dfrac{1+\tan x}{1-\tan x}\right]^2$

Ans. Taking L.H.S

$=\dfrac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)}$

[Using $\tan(x+y)=\dfrac{\tan x+\tan y}{1-\tan x\tan y}$, $\tan(x-y)=\dfrac{\tan x-\tan y}{1+\tan x\tan y}$]

$=\dfrac{\dfrac{\tan\frac{\pi}{4}+\tan x}{1-\tan\frac{\pi}{4}\tan x}}{\dfrac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}}=\dfrac{\dfrac{1+\tan x}{1-\tan x}}{\dfrac{1-\tan x}{1+\tan x}}$

$= \dfrac{(1+\tan x)^2}{(1-\tan x)^2}$

= R. H. S

8. $\dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\dfrac{\pi}{2}+x\right)} = \cot^2 x$

Ans. Taking L.H.S

$= \dfrac{\cos(\pi+x)\cos(-x)}{\sin(\pi-x)\cos\left(\dfrac{\pi}{2}+x\right)}$

$= \dfrac{-\cos x.\cos x}{\sin x(-\sin x)} = \dfrac{-\cos^2 x}{-\sin^2 x} = \cot^2 x$ = R.H.S.

9. $\cos\left(\dfrac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\dfrac{3\pi}{2}-x\right)+\cot(2\pi+x)\right] = 1$

Ans. Taking L.H.S

$= \cos\left(\dfrac{3\pi}{2}+x\right)\cos(2\pi+x)\left[\cot\left(\dfrac{3\pi}{2}-x\right)+\cot(2\pi+x)\right]$

$= \sin x.\cos x(\tan x+\cot x)$

$= \sin x.\cos x\left(\dfrac{\sin x}{\cos x}+\dfrac{\cos x}{\sin x}\right)$

$= \sin x.\cos x\left(\dfrac{\sin^2 x+\cos^2 x}{\sin x\cos x}\right)$

= 1 = R.H.S.

10. $\sin(n+1)x\sin(n+2)x+\cos(n+1)x\cos(n+2)x = \cos x$

Ans. Taking L.H.S.

$= \sin(n+1)x\sin(n+2)x + \cos(n+1)x\cos(n+2)x$

$= \cos[(n+1)x-(n+2)x]$

$= \cos[nx+x-nx-2x]$

$= \cos(-x) = \cos x$ = R.H.S.

11 $cos\left(\frac{3\pi}{4}+x\right) - cos\left(\frac{3\pi}{4}-x\right) = -\sqrt{2}sinx$

Ans. Taking L.H.S

$= cos\left(\frac{3\pi}{4}+x\right) - cos\left(\frac{3\pi}{4}-x\right)$

$= -2\sin\frac{3\pi}{4}\sin x$ $= -2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$

$= -2\sin\frac{\pi}{4}\sin x$ $= -2\times\frac{1}{\sqrt{2}}\sin x$

$= -\sqrt{2}\sin x$ = R.H.S.

12. $\sin^2 6x - \sin^2 4x = \sin 2x\sin 10x$ **Ans.**

L.H.S. = $\sin^2 6x - \sin^2 4x$

$= \sin(6x+4x).\sin(6x-4x)$

$[\because \sin^2 x - \sin^2 y - \sin(x+y)\sin(x-y)]$

$= \sin 10x\sin 4x$ = R.H.S.

13. $\cos^2 2x - \cos^2 6x = \sin 4x\sin 8x$

Ans. L.H.S. = $\cos^2 2x - \cos^2 6x$

= $\sin(2x+6x).\sin(6x-2x)$

$\left[\because \cos^2 y - \cos^2 x = \sin(x+y)\sin(x-y)\right]$

= $\sin 8x \sin 4x$ = R.H.S.

14. $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$

Ans. L.H.S. = $\sin 2x + 2\sin 4x + \sin 6x$

= $[\sin 4x + \sin 2x] + [\sin 6x + \sin 4x]$

=$2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + 2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)$

=2sin3x.cosx+2sin5xcosx

= $2\cos x[\sin 3x + \sin 5x]$

=$2\cos x\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$

= $2\cos x[2\sin 4x.\cos x]$

= $4\cos^2 x \sin 4x$ = R.H.S.

15. $\cot 4x(\sin 5x + \sin 3x) = \cot x(\sin 5x - \sin 3x)$ **Ans.**

L.H.S. = $\cot 4x(\sin 5x + \sin 3x)$

= $\frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$

$=\frac{\cos 4x}{\sin 4x}[2\sin 4x\cos x]$ $= 2\cos 4x\cos x$

R.H.S. = cot x(sin5x - sin3x)

$=\frac{cosx}{sinx}\left[2cos(\frac{5x+3x}{2})sin(\frac{5x-3x}{2})\right]$

$=\frac{cosx}{sinx}[2cos4xsinx]$

$= 2\cos 4x\cos x$

∴ L.H.S. = R.H.S.

16. $\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}=-\frac{\sin 2x}{\cos 10x}$

Ans. L.H.S. $=\frac{\cos 9x-\cos 5x}{\sin 17x-\sin 3x}$

$$=\frac{-2sin(\frac{9x+5x}{2})sin(\frac{9x-5x}{2})}{2cos(\frac{17x+3x}{2})sin(\frac{17x-3x}{2})}$$

$=\frac{-2\sin 7x\sin 2x}{2\cos 10x\sin 7x}=-\frac{\sin 2x}{\cos 10x}$ = R.H.S.

17. $\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}=\tan 4x$

Ans. L.H.S. $=\frac{\sin 5x+\sin 3x}{\cos 5x+\cos 3x}$

$$=\frac{2sin(\frac{5x+3x}{2})cos(\frac{5x-3x}{2})}{2cos(\frac{5x+3x}{2})cos(\frac{5x-3x}{2})}$$

$=\frac{2\sin 4x}{2\cos 4x}=\tan 4x$ = R.H.S.

18. $\frac{\sin x-\sin y}{\cos x+\cos y}=\tan\left(\frac{x-y}{2}\right)$

Ans. L.H.S. $=\frac{\sin x-\sin y}{\cos x+\cos y}$

$=\frac{2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$

$=\frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}=\tan\left(\frac{x-y}{2}\right)$ = R.H.S.

19. $\frac{\sin x+\sin 3x}{\cos x+\cos 3x}=\tan 2x$

Ans. L.H.S. $=\frac{\sin x+\sin 3x}{\cos x+\cos 3x}$

$=\frac{2\sin\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}{2\cos\left(\frac{x+3x}{2}\right)\cos\left(\frac{x-3x}{2}\right)}$

$=\frac{\sin 2x}{\cos 2x}=\tan 2x$ = R.H.S.

20. $\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}-2\sin x$

Ans. L.H.S. $=\frac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}$

$$=\frac{-(\sin 3x-\sin x)}{-(\cos^2 x-\sin^2 x)}$$

$$=\frac{2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right)}{\cos 2x}$$

$$=\frac{2\cos 2x\sin x}{\cos 2x}=2\sin x$$ = R.H.S.

21. $\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}=\cot 3x$

Ans. L.H.S. $=\frac{\cos 4x+\cos 3x+\cos 2x}{\sin 4x+\sin 3x+\sin 2x}$

$$=\frac{(\cos 4x+\cos 2x)+\cos 3x}{(\sin 4x+\sin 2x)+\sin 3x}$$

$$=\frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right)+\sin 3x}$$

$$=\frac{2\cos 3x\cos x+\cos 3x}{2\sin 3x\cos x+\sin 3x}$$

$$=\frac{\cos 3x(2\cos x+1)}{\sin 3x(2\cos x+1)}=\cot 3x$$ = R.H.S.

22. $\cot x\cot 2x-\cot 2x\cot 3x-\cot 3x\cot x=1$

Ans. We know that $\cot 3x=\cot(2x+x)$

$$\Rightarrow \cot 3x = \frac{\cot 2x \cot x - 1}{\cot 2x + \cot x}$$

$$\Rightarrow \cot 3x(\cot 2x + \cot x) = \cot 2x \cot x - 1$$

$$\Rightarrow \cot 3x \cot 2x + \cot 3x \cot x = \cot 2x \cot x - 1$$

$$\Rightarrow \cot 3x \cot 2x + \cot 3x \cot x - \cot 2x \cot x + 1 = 0$$

$$\Rightarrow \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$$

23. $\tan 4x = \dfrac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$

Ans. L.H.S. $= \tan 4x = \dfrac{2\tan 2x}{1-\tan^2 2x}$

$$= \frac{2.\dfrac{2\tan x}{1-\tan^2 x}}{1-\left(\dfrac{2\tan x}{1-\tan^2 x}\right)^2}$$

$$= \frac{\dfrac{4\tan x}{1-\tan^2 x}}{\dfrac{(1-\tan^2 x)^2 - 4\tan^2 x}{(1-\tan^2 x)^2}}$$

$$= \frac{4\tan x}{1-\tan^2 x} \times \frac{(1-\tan^2 x)^2}{1+\tan^4 x - 2\tan^2 x - 4\tan^2 x}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$$ = R.H.S.

24. $\cos 4x = 1 \quad 8\sin^2 x \cos^2 x$

Ans. L.H.S. $= \cos 4x = 1 - 2\sin^2 2x$

$= 1 - 2(2\sin x \cos x)^2$

$= 1 - 2(4\sin^2 x \cos^2 x)$

$= 1 - 8\sin^2 x \cos^2 x$ = R.H.S.

25. $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$ **Ans.**

L.H.S. $= \cos 6x = 2\cos^2 3x - 1$

$= 2\left[4\cos^3 x - 3\cos x\right]^2 - 1$

$= 2\left[16\cos^6 x + 9\cos^2 x - 24\cos^4 x\right] - 1$

$= 32\cos^6 x + 18\cos^2 x - 48\cos^4 x - 1$

$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$ = R.H.S.

EX 3.4 [NOT IN SYLLUBUS]