# PUNA INTERNATIONAL SCHOOL

# •CLASS – 9

# •SUBJECT - MATHS

**Semester - 2**

# SAMPLE NOTE-BOOK

# CHAPTER 10

## Circles

## Ex. 10.1

1. **Fill in the blanks:**

**(I)The centre of a circle lies in _______________ of the circle.**

**(II) A point, whose distance from the centre of a circle is greater than its radius lies in _______________ of the circle.**

**(III) The longest chord of a circle is a _______________ of the circle.**

**(IV) An arc is a _______________ when its ends are the ends of a diameter.**

**(V)Segment of a circle is the region between an arc and _______________ of the circle.**

**(VI) A circle divides the plane, on which it lies, in _______________ parts.**

**Ans. (I)** interior

(II) exterior

(III) Diameter

(IV) semi-circle

(V) the chord

(VI) three

**2. Write True or False:**

(I) **Line segment joining the centre to any point on the circle is a radius of the circle.**

(II) **A circle has only finite number of equal chords.**

(III) **If a circle is divided into three equal arcs each is a major arc.**

**(IV)A chord of a circle, which is twice as long as its radius is a diameter of the circle.**

**(V)Sector is the region between the chord and its corresponding arc.**

**(VI)A circle is a plane figure.**

**Ans. (i)** True

(II) False

(IV) False

(V) True

(V) False

(VI) True

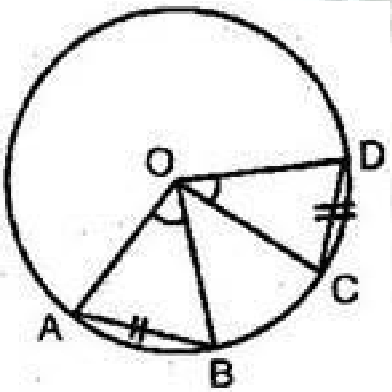
# CHAPTER 10

# Circles

## Ex. 10.2

**1. Recall that two circles are congruent if they have the same radii. Prove that equal chords of congruent circles subtend equal angles at their centres.**

**Ans. I Part**: Two circles are said to be congruent if and only if one of them can be superposed on the other so as to cover it exactly.




Let C (O, $r$ ) and C (O', $s$ ) be two circles. Let us imagine that the circle C (O', $s$ ) is superposed on C (O, $r$ ) so that O' coincide with O. Then it can easily be seen that C (O', $s$ ) will cover C (O, $r$ ) completely if and only if $r = s$.

Hence we can say that two circles are congruent, if and only if they have equal radii.

**3. Part: Given**: In a circle (O, $r$ ), AB and CD are two equal chords, subtend $\angle$ AOB and $\angle$ COB at the centre.

**To Prove**: $\angle$ AOB = $\angle$ COD

**Proof**: In $\Delta$AOB and $\Delta$COD,

AB = CD [Given]

AO = CO [Radii of the same circle]

BO = DO [Radii of the same circle]

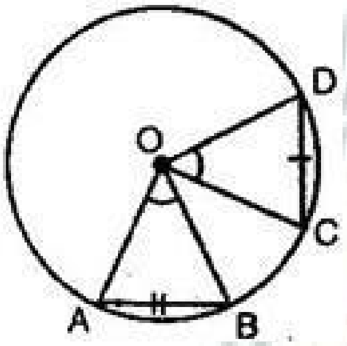
$\therefore \Delta AOB \cong \Delta COD$ [By SSS congruency]

$\Rightarrow \angle AOB = \angle COD$ [By C.P.C.T.]

Hence Proved.

**4. Prove that if chords of congruent circles subtend equal angles at their centres, then the chords are equal.**

**Ans. Given**: In a circle (O, $r$ ), AB and CD subtend two angles at the centre such that $\angle AOB = \angle COD$




**To Prove**: AB = CD

**Proof**: : In $\Delta AOB$ and $\Delta COD$,

AO = CO [Radii of the same circle]

BO = DO [Radii of the same circle]

$\angle AOB = \angle COD$ [Given]

$\therefore \Delta AOB \cong \Delta COD$ [By SAS congruency]

$\Rightarrow$ AB = CD [By CPCT]

Hence proved.

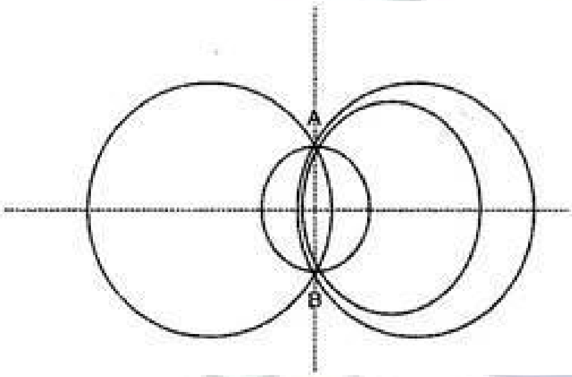
# CHAPTER 10

# Circles

## Ex. 10.3

1. **Draw different pairs of circles. How many points does each pair have in common? What is the maximum number of common points?**

**Ans.** From the figure, we observe that when different pairs of circles are drawn, each pair have two common points (say A and B). Maximum number of common points are two in each pair of circles.




Suppose two circles C (O, $r$ ) and C (O', $s$ ) intersect each other in three points, say A, B and C.

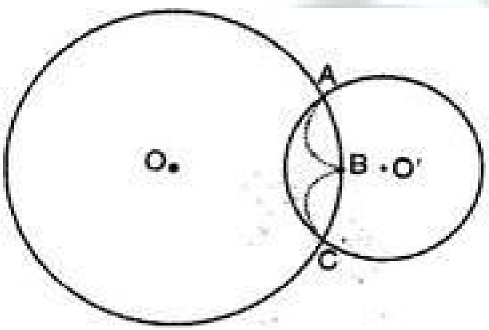
Then A, B and C are non-collinear points.

We know that:

There is one and only one circle passing through three non-collinear points.

Therefore, a unique circle passes through A, B and C.

$\Rightarrow$ O' coincides with O and S = r




A contradiction to the fact that C (O',S) ≠ C (O, r)

∴ Our supposition is wrong.

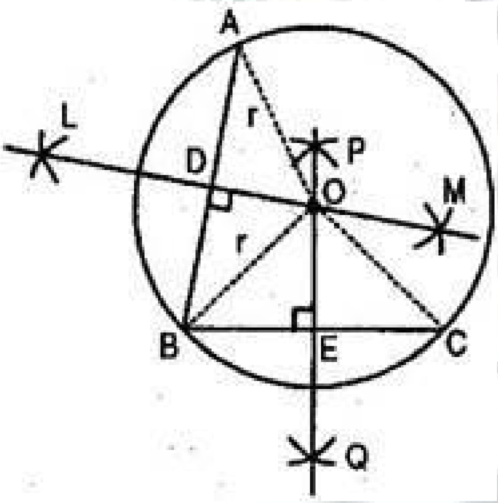
Hence two different circles cannot intersect each other at more than two points.

2. **Suppose you are given a circle. Give a construction to find its centre.**

   **Ans. Steps of construction**:

- Take any three points A, B and C on the circle.
- Join AB and BC.
- Draw perpendicular bisector say LM of AB.
- Draw perpendicular bisector PQ of BC.
- Let LM and PQ intersect at the point O.

Then O is the centre of the circle.




**Verification**:

O lies on the perpendicular bisector of AB.

∴ OA = OB ……….(i)

O lies on the perpendicular bisector of BC.

∴ OB = OC ……….(ii)

From eq. (i) and (ii), we observe that

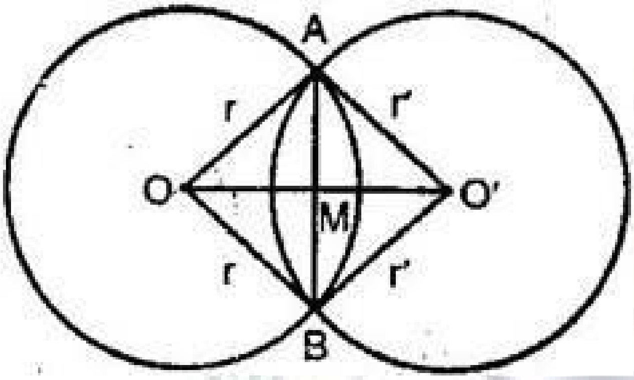
OA = OB = OC = $r$ (say)

Three non-collinear points A, B and C are at equal distance $(r)$ from the point O inside the circle.

Hence O is the centre of the circle.

3. **If two circles intersect at two points, prove that their centres lie on the perpendicular bisector of the common chord.**

**Ans. Given**: Let C (O, r) and C (O', r') be two circles intersecting at A and B. AB is the common chord.




**To prove**: OO' is the perpendicular bisector of the chord AB, i.e., AM = MB and $\angle$OMA = $\angle$OMB = $90^\circ$

**Construction**: Join OA, OB, O'A, O'B.

**Proof**: In triangles OAO' and OBO',

OA = OB [Each radius]

O'A = O'B [Each radius]

OO' = OO' [Common]

$\therefore \Delta$OAO' $\cong \Delta$OBO' [By SSS congruency]

$\Rightarrow \angle$ AOO' = $\angle$ BOO' [By CPCT]

$\Rightarrow \angle$ AOM = $\angle$ BOM (i)

Now in $\Delta$AOB, OA = OB

And $\angle$ AOB = $\angle$ OBA [Proved earlier]

Also $\angle$ AOM = $\angle$ BOM [From eq.(i)]

$\therefore$ Remaining $\angle$ AMO = $\angle$ BMO

Since $\angle$AMO + $\angle$BMO = $180^\circ$ [Linear pair]

$\Rightarrow$ 2$\angle$AMO= $180^\circ$

$\Rightarrow$ $\angle$ AMO = $\angle$ BMO = $90^\circ$

$\Rightarrow$ OM $\perp$ AB

$\Rightarrow$ OO' $\perp$ AB

Since OM $\perp$ AB

$\therefore$ M is the mid-point of AB, i.e., AM = BM Hence

OO' is the perpendicular bisector of AB.

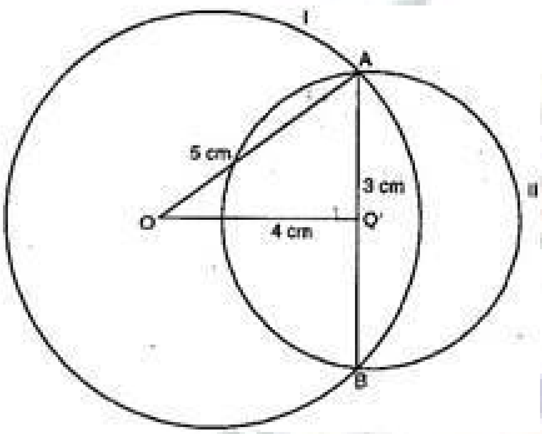
# CHAPTER 10

# Circles

## Ex. 10.4

**2. Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.**

**Ans.** Let two circles with centres O and O' intersect each other at points A and B. On joining A and B, AB is a common chord.




Radius OA = 5 cm, Radius O'A = 3 cm,

Distance between their centres OO' = 4 cm

In triangle AOO',

$5^2=4^2+3^2$

$\Rightarrow$ 25=16+9

$\Rightarrow$ 25=25

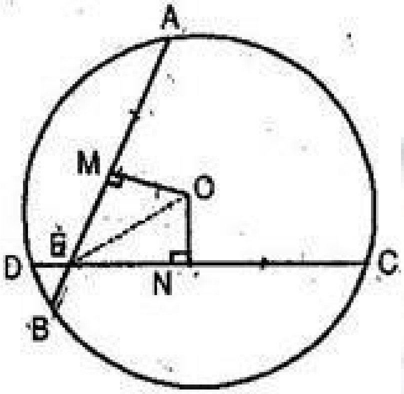
Hence AOO' is a right triangle, right angled at O'.

Since, perpendicular drawn from the centre of the circle bisects the chord. Hence O' is the mid-point of the chord AB. Also O' is the centre of the circle II. Therefore length of chord AB = Diameter of circle II

∴ Length of chord AB = 2 x 3 = 6 cm.

**4. If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.**

**Ans. Given**: Let AB and CD are two equal chords of a circle of centers O intersecting each other at point E within the circle.




**To prove**: (a) AE = CE

(b) BE = DE

**Construction**: Draw OM $\perp$ AB, ON $\perp$ CD. Also join OE.

**Proof**: In right triangles OME and ONE,

$\angle$ OME = $\angle$ ONE = $90^\circ$ [By construction]

OM=ON

[Equal chords are equidistance from the centre] OE =

OE [Common]

∴ $\Delta$OME $\cong$ $\Delta$ ONE [RHS rule of congruency]

∴ ME = NE [By C.P.C.T.] ……….(i)

Now, O is the centre of circle and OM $\perp$ AB

∴ AM= $\frac{1}{2}$AB

[Perpendicular from the centre bisects the chord] …..(ii)

Similarly, NC = $\frac{1}{2}$ CD ……….(iii)

But AB = CD [Given]

From eq. (ii) and (iii), AM
= NC ……….(iv)

Also MB = DN ……….(v)

Adding (i) and (iv), we get,

AM+ME=NC+NE

$\Rightarrow$ AE = CE **[Proved part (a)]**

Now AB = CD [Given]

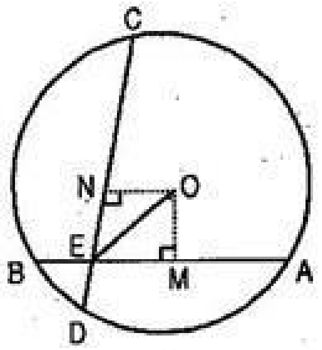
AE = CE [Proved]

$\Rightarrow$ AB–AE=CD–CE

$\Rightarrow$ BE = DE **[Proved part (b)]**

**5. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chord.**

**Ans. Given**: AB and CD be two equal chords of a circle with centre O intersecting each other with in the circle at point E. OE is joined.




**To prove**: $\angle$ OEM = $\angle$ OEN

**Construction**: Draw OM $\perp$ AB and

ON $\perp$ CD.

**Proof**: In right angled triangles OME and ONE,

$\angle$ OME = $\angle$ ONE [Each $90^\circ$] [By construction]

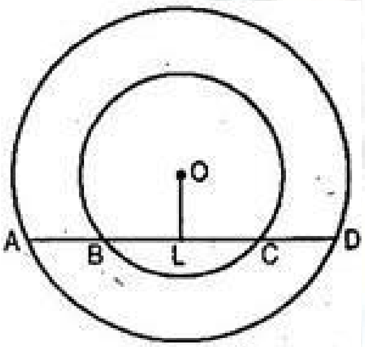
OM = ON [Equal chords are equidistant from the centre]

OE = OE [Common]

$\therefore$ $\Delta$OME $\cong$ $\Delta$ ONE [RHS rule of congruency]

$\therefore$ $\angle$ OEM = $\angle$ OEN [By C.P.C.T.]

**(ii) If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that AB = CD. (See figure)**




**Ans. Given:** Line $l$ intersects two concentric circles with centre O at points A, B, C and D.

**To prove**: AB = CD

**Construction**: Draw OL $\perp l$

**Proof**: AD is a chord of outer circle and OL $\perp$ AD.

$\therefore$ AL = LD ………(i) [Perpendicular drawn from the centre bisects the chord]

Now, BC is a chord of inner circle and

OL $\perp$ BC

$\therefore$ BL = LC …(ii) [Perpendicular drawn from the centre bisects the chord]

Subtracting (ii) from (i), we get,

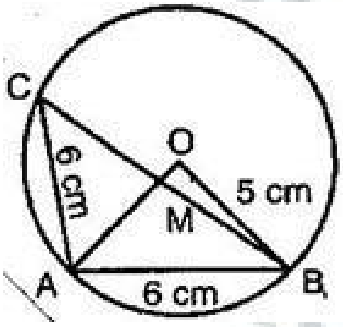
AL–BL=LD–LC

$\Rightarrow$ AB=CD

**5. Three girls Reshma, Salma and Mandip are standing on a circle of radius 5 m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6 m each, what is the distance between Reshma and Mandip?**

**Ans.** Let Reshma, Salma and Mandip takes the position C, A and B on the circle. Given: AB = AC

The centre lies on the bisector of $\angle$ BAC.




Let M be the point of intersection of BC and OA.

Again, since AB = AC and AM bisects

$\angle$ CAB.

$\therefore$ AM $\perp$ CB and M is the mid-point of CB.

Let OM = $x$, then MA = $5-x$

From right angled triangle OMB,

$OB^2 = OM^2 + MB^2$

$\Rightarrow$ $5^2 = x^2 + MB^2$ ……….(i)

Again, in right angled triangle AMB,

$AB^2 = AM^2 + MB^2$

$\Rightarrow \quad 6^2 = (5-x)^2 + MB^2$ ……….(ii)

Equating the value of $MB^2$ from eq. (i) and (ii),

$$5^2 - x^2 = 6^2 - (5-x)^2$$

$$\Rightarrow (5-x)^2 - x^2 = 6^2 - 5^2$$

$$\Rightarrow 25 - 10x + x^2 - x^2 = 36 - 25$$

$$\Rightarrow 10x = 25 - 11$$

$$\Rightarrow 10x = 14$$

$$\Rightarrow x = \frac{14}{10}$$

Hence, from eq. (i),

$MB^2 = 5^2 - x^2 = 5^2 - \left(\frac{14}{10}\right)^2$

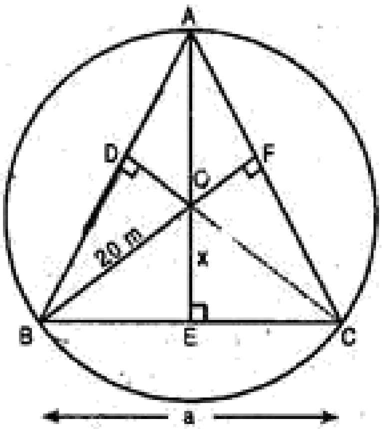
$= \left(5 + \frac{14}{10}\right)\left(5 - \frac{14}{10}\right) = \frac{64}{10} \times \frac{36}{10}$

$\Rightarrow MB = \frac{8 \times 6}{10} = 4.8$ cm

$\therefore$ BC = 2MB = 2 x 4.8 = 9.6 cm

**(i) A circular park of radius 20 m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.**

**Ans.** Let position of three boys Ankur, Syed and David are denoted by the points A, B and C respectively.




AB = BC = CA = $a$ [say]

Since equal sides of equilateral triangle are as equal chords and perpendicular distances of equal chords of a circle are equidistant from the centre.

$\therefore$ OD = OE = OF = $x$ cm [say]

Join OA, OB and OC.

$\Rightarrow$ Area of $\Delta$AOB

(i) Area of $\Delta$BOC = Area of

$\Delta$AOC And Area of $\Delta$ABC

(ii) Area of $\Delta$AOB + Area of $\Delta$BOC + Area of

$\Delta$AOC $\Rightarrow$ And Area of $\Delta$ABC = 3 x Area of $\Delta$BOC

$\Rightarrow \frac{\sqrt{3}}{4}a^2$ = 3 ( $\frac{1}{2}$ BC x OE)

$\Rightarrow \frac{\sqrt{3}}{4}a^2$ =3( $\frac{1}{2} \times a \times x$ )

$\Rightarrow \frac{a^2}{a} = 3 \times \frac{1}{2} \times \frac{4}{\sqrt{3}} \times x$

$\Rightarrow a = 2\sqrt{3}x$ ……….(i)

Now, CE $\perp$ BC

$\therefore$ BE = EC = $\frac{1}{2}$ BC [$\because$ Perpendicular drawn from the centre bisects the chord]

$\Rightarrow$ BE=EC= $\frac{1}{2}a$

$\Rightarrow$ BE = EC = $\frac{1}{2}\left(2\sqrt{3}x\right)$ [Using eq. (i)]

$\Rightarrow$ BE=EC= $\sqrt{3}x$

Now in right angled triangle BEO,

$OE^2 + BE^2 = OB^2$ [Using Pythagoras theorem]

$\Rightarrow x^2 + \left(\sqrt{3}x\right)^2 = (20)^2$

$\Rightarrow x^2 + 3x^2 = 400$

$\Rightarrow 4x^2 = 400$

$\Rightarrow x^2 = 100$

$\Rightarrow x$ = 10 m

And $a = 2\sqrt{3}x$ = $2\sqrt{3} \times 10$ = $20\sqrt{3}$ m

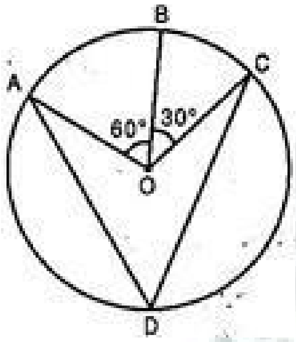
Thus distance between any two boys is $20\sqrt{3}$ m.

# CHAPTER 10

# Circles

## Ex. 10.5

**1. In figure, A, B, C are three points on a circle with centre O such that $\angle BOC = 30^o$, $\angle AOB = 60^o$. If D is a point on the circle other than the arc ABC, find $\angle ADC$**




**Ans.** $\angle$ AOC = $\angle$ AOB + $\angle$ BOC

$\Rightarrow \angle$ AOC = $60^\circ + 30^\circ = 90^\circ$

Now $\angle$ AOC = 2$\angle$ ADC

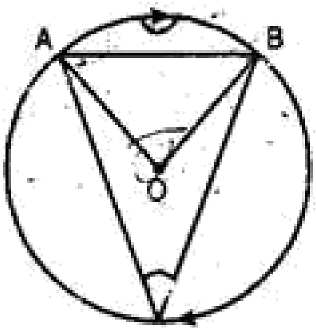
[$\because$ Angled subtended by an arc, at the centre of the circle is double the angle subtended by the same arc at any point in the remaining part of the circle]

$\Rightarrow \angle$ ADC = $\frac{1}{2}\angle$AOC

$\Rightarrow \angle$ ADC= $\frac{1}{2} \times 90^\circ = 45^\circ$

1. **A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord on a point on the minor arc and also at a point on the major arc.**

**Ans.** Let AB be the minor arc of circle.




∴ Chord AB = Radius OA = Radius OB

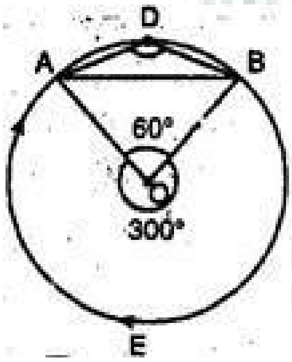
∴ $\Delta$ AOB is an equilateral triangle.

$\Rightarrow \angle$ AOB = $60^\circ$

Now $m\widehat{AB} + m\widehat{BA} = 360^\circ$

$\Rightarrow \angle$ AOB + $\angle$ BOA = $360^\circ$

$\Rightarrow 60^\circ + \angle$ BOA= $360^\circ$




$\Rightarrow \angle$ BOA = $360^\circ - 60^\circ = 300^\circ$

D is a point in the minor arc.

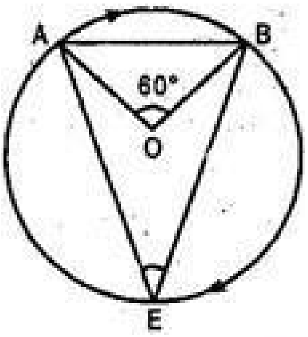
∴ $m\widehat{BA}$ =2$\angle$ BDA

$\Rightarrow \angle$ BOA = 2$\angle$ BDA

$\Rightarrow \angle$ BDA = $\frac{1}{2}\angle$ BOA = $\frac{1}{2} \times 300^\circ$

$\Rightarrow \angle$ BDA = $150^\circ$

Thus angle subtended by major arc, $\widehat{BA}$ at any point D in the minor arc is $150°$.




Let E be a point in the major arc $\widehat{BA}$.

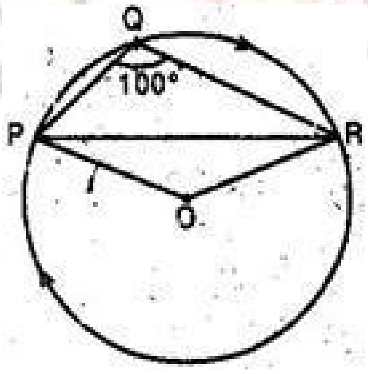
$\therefore m\widehat{AB}$ =2 $\angle$ AEB

$\Rightarrow \angle$ AOB = 2 $\angle$ AEB

$\Rightarrow \angle$ AEB = $\frac{1}{2}$ $\angle$ AOB

$\Rightarrow \angle$ AEB= $\frac{1}{2} \times 60^{\circ}$ = $30^{\circ}$

2. **In figure, $\angle$ PQR = $100°$, where P, Q, R are points on a circle with centre O. Find $\angle$ OPR.**




**Ans.** In the figure, Q is a point in the minor arc

$\widehat{PQR}$. $\therefore m\widehat{RP}$ =2 $\angle$ PQR

$\Rightarrow \angle$ ROP = 2 $\angle$ PQR

$\Rightarrow \angle$ ROP= $2 \times 100^\circ = 200^\circ$

Now $m\widehat{PR} + m\widehat{RP} = 360^\circ$

$\Rightarrow \angle$ POR + $\angle$ ROP = $360^\circ$

$\Rightarrow \angle$ POR+ $200^\circ = 360^\circ$

$\Rightarrow \angle$ POR = $360^\circ - 200^\circ = 160^\circ$ …..(i)

Now $\Delta$OPR is an isosceles triangle.

$\therefore$ OP = OR [radii of the circle]

$\Rightarrow \angle$ OPR = $\angle$ ORP [angles opposite to equal sides are equal] …..(ii)

Now in isosceles triangle OPR,

$\angle$ OPR + $\angle$ ORP + $\angle$ POR = $180^\circ$

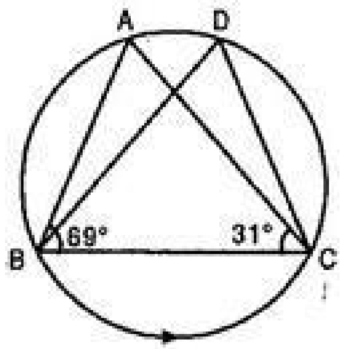
$\Rightarrow \angle$ OPR+ $\angle$ ORP+ $160^\circ = 180^\circ$

$\Rightarrow 2\angle$ OPR = $180^\circ - 160^\circ$ [Using (i) & (ii)]

$\Rightarrow 2\angle$ OPR= $20^\circ$

$\Rightarrow \angle$ OPR = $10^\circ$

**4. In figure, $\angle$ ABC = $69^\circ$, $\angle$ ACB = $31^\circ$, find $\angle$ BDC.**



**Ans.** In triangle ABC,

$\angle$ BAC + $\angle$ ABC + $\angle$ ACB = $180°$

$\Rightarrow \angle$ BAC + $69° + 31° = 180°$

$\Rightarrow \angle$ BAC = $180° - 69° - 31°$

$\Rightarrow \angle$ BAC = $80°$ …….(i)

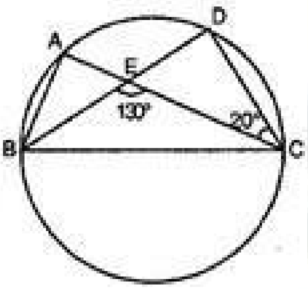
Since, A and D are the points in the same segment of the circle.

$\therefore \angle$ BDC = $\angle$ BAC

[Angles subtended by the same arc at any points in the alternate segment of a circle are equal]

$\Rightarrow \angle$ BDC = $80°$ [Using (i)]

**5.In figure, A, B, C, D are four points on a circle. AC and BD intersect at a point E such that $\angle$ BEC = $130°$ and $\angle$ ECD = $20°$. Find $\angle$ BAC.**




**Ans.** Given: $\angle$ BEC = $130°$ and $\angle$ ECD = $20°$

$\angle$ DEC = $180° - \angle$ BEC = $180° - 130° = 50°$ [Linear pair]

Now in $\Delta$DEC,

$\angle$ DEC + $\angle$ DCE + $\angle$ EDC = $180°$ [Angle sum property]

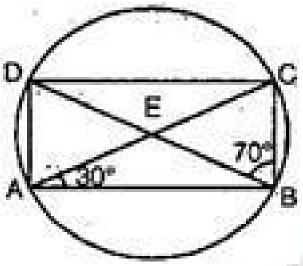
$\Rightarrow 50° + 20° + \angle$ EDC = $180°$

$\Rightarrow \angle$ EDC = $110°$

$\Rightarrow \angle BAC = \angle EDC = 110^\circ$ [Angles in same segment]

6. **ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. $\angle DBC = 70^\circ$, $\angle BAC$ is $30^\circ$ find $\angle BCD$. Further if AB = BC, find $\angle ECD$.**

**Ans.** For chord CD




$\angle CBD = \angle CAD$ (Angles in same segment)

$\angle CAD = 70^\circ$ [Given: CBD= ]

$\angle BAD = \angle BAC + \angle CAD$

$= 30^\circ + 70^\circ = 100^\circ$

$\angle BCD + \angle BAD = 180^\circ$ (Opposite angles of a cyclic quadrilateral are supplementary]

$\angle BCD + 100 = 180^\circ$

$\angle BCD = 80^\circ$

$In\ \Delta ABC$

AB = BC (given)

$\therefore \angle BCA = \angle CAB$ (Angles opposite to equal sides of a triangle are equal)

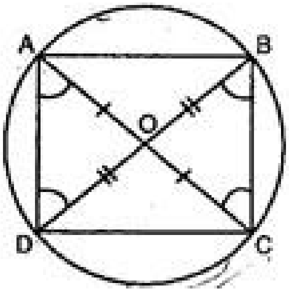
$\angle BCA = 30^\circ$

We have $\Rightarrow \angle BCA + \angle ACD = 80^\circ$

$\Rightarrow \angle ECD = 50^\circ$

7. **If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.**

**Ans.** Let ABCD a cyclic quadrilateral having diagonals as BD and AC intersecting each other at point O.




$\angle BAD = \frac{1}{2}\angle BOD = \frac{180°}{2} = 90°$

(Consider BD as a chord)

$\angle BCD + \angle BAD = 180°$ (Opposite angles of a cyclic quadrilateral are supplementary)

$\angle BCD = 180° - 90° = 90°$

$\angle ADC = \frac{1}{2}\angle AOC = \frac{180°}{2} = 90°$

(Considering AC as a chord)

$\angle ADC + \angle ABC = 180°$ (Opposite angles of a cyclic quadrilateral are supplementry)

$90° + \angle ABC = 180°$

$\angle ABC = 90°$

Here, each interior angle of cyclic quadrilateral is of $90°$ and its diagonals are equal (given).
Hence it is a rectangle.

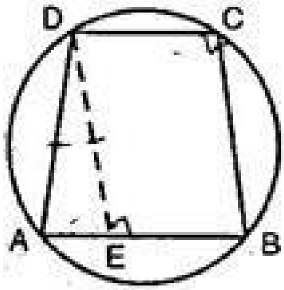
8. **If the non-parallel sides of a trapezium are equal, prove that it is cyclic.**

**Ans. Given**: A trapezium ABCD in which AB $\parallel$ CD and AD = BC.

**To prove**: The points A, B, C, D are cyclic.

**Construction**: Draw DE $\parallel$ CB.

**Proof**: Since DE $\parallel$ CB and EB $\parallel$ DC.




$\therefore$ EBCD is a parallelogram.

$\therefore$ DE = CB and $\angle$DEB = $\angle$DCB

Now AD = BC and DA = DE

$\Rightarrow \angle$DAE = $\angle$DEB

But $\angle$DEA + $\angle$DEB = $180^\circ$

$\Rightarrow \angle$DAE + $\angle$DCB = $180^\circ$

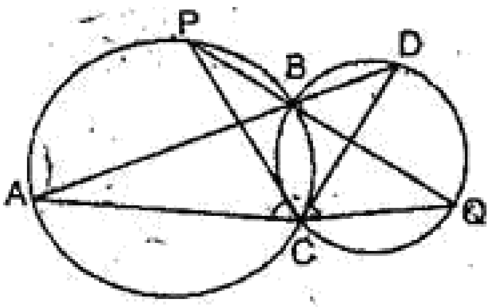
[$\because \angle$DEA = $\angle$DAE and $\angle$DEB = $\angle$DCB]

$\Rightarrow \angle$DAB + $\angle$DCB =

$\Rightarrow \angle$A+ $\angle$C= $180^\circ$

Hence, ABCD is a cyclic trapezium.

9. **Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D, P, Q respectively (see figure). Prove that $\angle$ACP = $\angle$QCD.**




**Ans.** In triangles ACD and QCP,

$\angle A = \angle P$ and $\angle Q = \angle D$ [Angles in same segment]

$\therefore \angle ACD = \angle QCP$ [Third angles of two triangles] ……….(i)

Subtracting $\angle PCD$ from both the sides of eq. (i), we get,

$\angle ACD - \angle PCD = \angle QCP - \angle PCD$

$\Rightarrow \angle ACP = \angle QCD$

Hence proved.

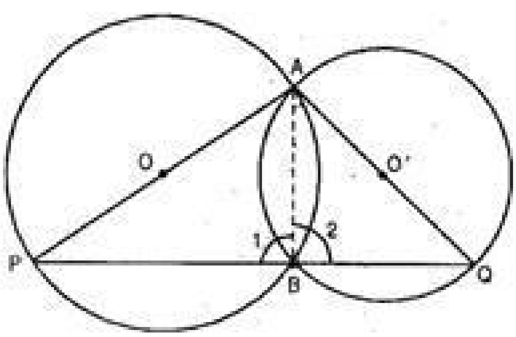
**10. If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.**

**Ans. Given**: Two circles intersect each other at points A and B. AP and AQ be their respective diameters.

**To prove**: Point B lies on the third side PQ.

**Construction**: Join A and B.

**Proof**: AP is a diameter.



$\therefore \angle 1 = 90^\circ$

[Angle in semicircle]

Also AQ is a diameter.

$\therefore \angle 2 =$

[Angle in semicircle]

$\angle 1 + \angle 2 =$

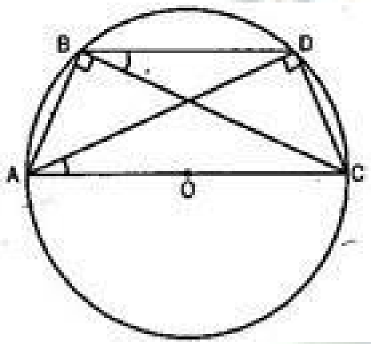
$\Rightarrow \angle PBQ =$

$\Rightarrow$ PBQ is a straight line.

Thus point B. i.e. point of intersection of these circles lies on the third side i.e., on PQ.

**7. ABC and ADC are two right triangles with common hypotenuse AC. Prove that $\angle$ CAD = $\angle$ CBD.**

**Ans.** We have ABC and ADC two right triangles, right angled at B and D respectively.




$\Rightarrow \angle$ ABC = ADC [Each $90^\circ$]

If we draw a circle with AC (the common hypotenuse) as diameter, this circle will definitely passes through of an arc AC, Because B and D are the points in the alternate segment of an arc AC.

Now we have $\overparen{CD}$ subtending $\angle$ CBD and $\angle$ CAD in the same segment.

$\therefore \angle CAD = \angle CBD$

Hence proved.

**12. Prove that a cyclic parallelogram is a rectangle**.

**Ans**. Given: A cyclic parallelogram ABCD.

**To prove**: ABCD is a rectangle.

**Proof**: $\angle B = \angle D$ [Opposite angles of a parallelogram are equal]

But, $\angle B + \angle D = 180^\circ$ [Sum of opposite angles of a parallelogram]

$\Rightarrow \quad 2\angle B = 180^\circ$

$\Rightarrow \quad \angle B = 90^\circ$ and $\angle D = 90^\circ$

Similarly,

$\angle A = 90^\circ$ and $\angle C = 90^\circ$

Hence, ABCD is a rectangle.

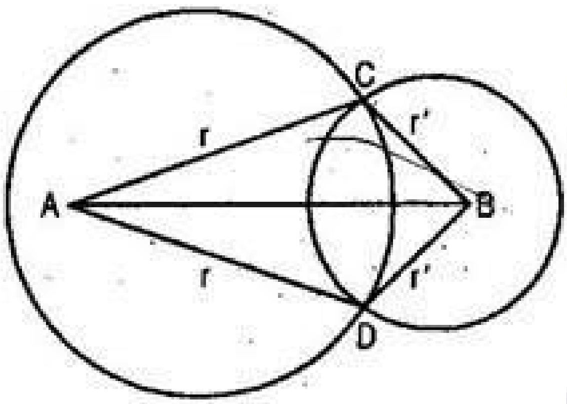
# CHAPTER 10

# Circles

## Ex. 10.6

1. **Prove that the line of centres of two intersecting circles subtends equal angles at the two points of intersection.**

**Ans.** Let two circles with respective centers A and B intersect each other at points C and D.




We have to prove $\angle$ ACB = $\angle$ ADB

Proof: In triangles ABC and ABD,

AC=AD= $r$

BC=BD= $r'$

AB = AB [Common]

$\therefore \Delta_{ABC} \cong \Delta_{ABD}$

[SSS rule of congruency]

$\Rightarrow \angle$ ACB = $\angle$ ADB [By C.P.C.T.]

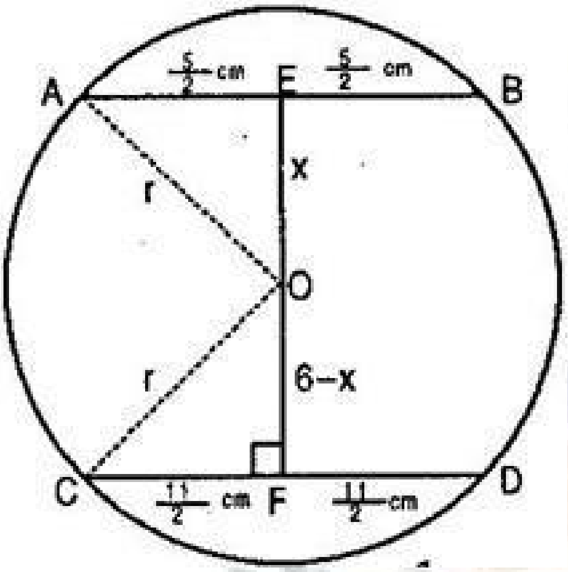
2. **Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD**

**is 6 cm, find he radius of the circle.**

**Ans.** Let O be the centre of the circle. Join OA and OC.

Since perpendicular from the centre of the circle to the chord bisects the chord. $\therefore$ AE = EB = $\frac{1}{2} \times AB = \frac{1}{2} \times 5 = \frac{5}{2}$ cm

And CF = FD = $\frac{1}{2} \times CD = \frac{1}{2} \times 11 = \frac{11}{2}$ cm




Let OE = $x$

$\therefore$ OF = 6 - x

Let radius of the circle be $r$.

In right angled triangle AEO,

$AO^2 = AE^2 + OE^2$

[Using Pythagoras theorem]

$$\Rightarrow r^2 = \left(\frac{5}{2}\right)^2 + x^2 \text{ ....(i)}$$

Again In right angled triangle CFO,

$OC^2 = CF^2 + OF^2$

[Using Pythagoras theorem]

$$\Rightarrow r^2 = \left(\frac{11}{2}\right)^2 + (6-x)^2 \text{....(ii)}$$

Equating eq. (i) and (ii),

$$\left(\frac{5}{2}\right)^2 + x^2 = \left(\frac{11}{2}\right)^2 + (6-x)^2$$

$$\Rightarrow \frac{25}{4} + x^2 = \frac{121}{4} + 36 + x^2 - 12x$$

$$\Rightarrow 12x = \frac{121}{4} - \frac{25}{4} + 36$$

$$\Rightarrow 12x = \frac{96}{4} + 36$$

$$\Rightarrow 12x = 24 + 36$$

$$\Rightarrow 12x = 60$$

$$\Rightarrow x = 5$$

Now from eq. (i),

$$r^2 = \frac{25}{4} + x^2$$

$$\Rightarrow r^2 = \frac{25}{4} + 5^2$$

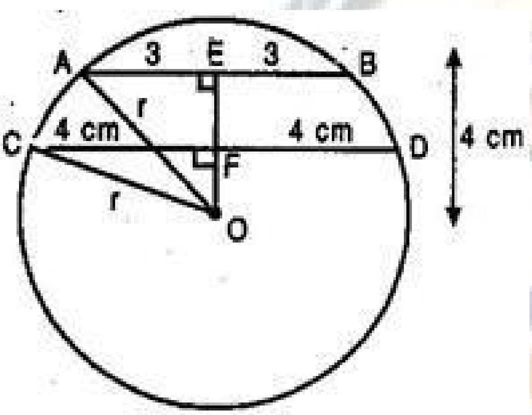
$$\Rightarrow r^2 = \frac{125}{4}$$

$\Rightarrow r = \frac{5\sqrt{5}}{2}$ cm

Hence radius of the circle is $\frac{5\sqrt{5}}{2}$ cm.

3. **The lengths of two parallel chords of a circle are 6 cm and 8 cm. If the smaller chord is at a distance of 4 cm from the centre, what is the distance of the other chord form the centre?**

**Ans.** Let AB = 6 cm and CD = 8 cm are the chords of circle with centre O. Join OA and OC.




Since perpendicular from the centre of the circle to the chord bisects the chord.

$\therefore$ AE = EB = $\frac{1}{2}$ AB = $\frac{1}{2}$ x 6 = 3 cm

And CF = FD = $\frac{1}{2}$ CD = $\frac{1}{2}$ x 8 = 4 cm

Perpendicular distance of chord AB from the centre O is OE.

$\therefore$ OE = 4 cm

Now in right angled triangle AOE,

$OA^2 = AE^2 + OE^2$ [Using Pythagoras theorem]

$\Rightarrow r^2 = 3^2 + 4^2$

$\Rightarrow r^2 = 9+16=25$

$\Rightarrow r = 5$ cm

Perpendicular distance of chord CD from the center O is OF.

Now in right angled triangle OFC,

$OC^2 = CF^2 + OF^2$ [Using Pythagoras theorem]

$\Rightarrow r^2 = 4^2 + (OF)^2$

$\Rightarrow 5^2 = 4^2 + (OF)^2$

$\Rightarrow 25 = 16 + (OF)^2$

$\Rightarrow OF^2=9$

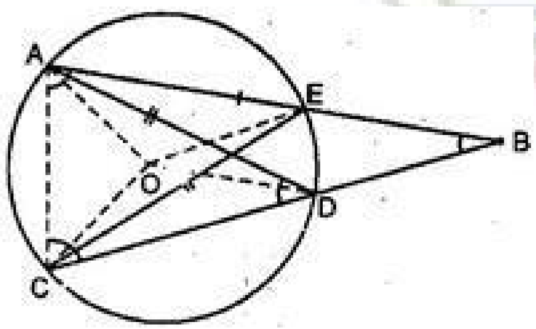
$\Rightarrow OF = 3cm$

Hence distance of other chord from the centre is 3 cm.

4. **Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect chords AD and CE with the circle. Prove that $\angle$ ABC is equal to half the difference of the angles subtended by the chords AC and DE at the centre.**

**Ans.** Vertex B of $\angle$ ABC is located outside the circle with centre O.

Side AB intersects chord CE at point E and side BC intersects chord AD at point D with the circle.




We have to prove that

$\angle ABC = \frac{1}{2}[\angle AOC - \angle DOE]$

Join OA, OC, OE and OD.

Now $\angle AOC = 2\angle AEC$

[Angle subtended by an arc at the centre of the circle is twice the angle subtended by the same arc at any point in the alternate segment of the circle]

$\Rightarrow \frac{1}{2}\angle AOC = \angle AEC$ ...(i)

Similarly $\frac{1}{2}\angle DOE = \angle DCE$ ....(ii)

Subtracting eq. (ii) from eq. (i),

$\frac{1}{2}[\angle AOC - \angle DOE] = \angle AEC - \angle DCE$ ....(iii)

Now $\angle AEC = \angle ADC$

[Angles in same segment in circle] ....(iv)

Also $\angle DCE = \angle DAE$

[Angles in same segment in circle] ....(v)

Using eq. (iv) and (v) in eq. (iii),

$\frac{1}{2}[\angle AOC - \angle DOE]$

$= \angle DAE + \angle ABD - \angle DAE$

$\Rightarrow \frac{1}{2}[\angle AOC - \angle DOE] = \angle ABD$

Or $\frac{1}{2}$ [∠AOC – ∠DOE] = ∠ ABC

Hence proved.

5. **Prove that the circle drawn with any drawn with any side of a rhombus as a diameter, passes through the point of intersection of its diagonals.**

**Ans.** Let ABCD be a rhombus in which diagonals AC and BD intersect each other at point O.

As we know that diagonals of a rhombus bisect and perpendicular to each other.

$\therefore \angle AOB = 90^\circ$

And if we draw a circle with side AB as diameter, it will definitely **pass through point O** (the point intersection of diagonals) because then $\angle AOB = 90^\circ$ will be the angle in a semi-circle.

6. **ABCD is a parallelogram. The circle through A, B and C intersect CD (produced it necessary) at E. Prove that AE = AD.**

**Ans.** In figure (a),

ABCD is a parallelogram.

$\Rightarrow \angle 1 = \angle 3$ ....(i)

ABCE is a cyclic quadrilateral.

$\therefore \angle 1 + \angle 6 = 180^\circ$ ....(ii)

And $\angle 5 + \angle 6 = 180^\circ$ ....(iii)

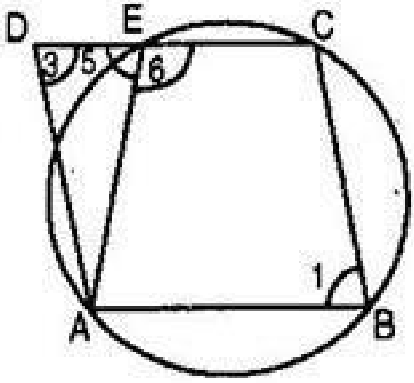
[Linear pair]

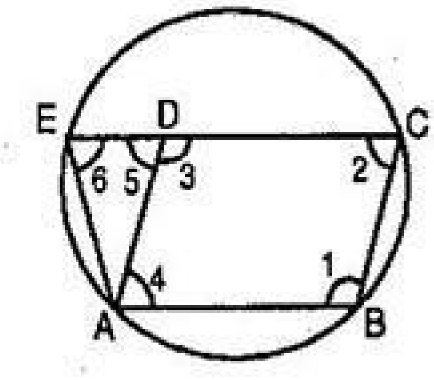
From eq. (ii) and (iii), $\angle 1 = \angle 5$ ....(iv)

Now, from eq. (i) and (iv),

$\angle 3 = \angle 5$

$\Rightarrow$ AE = AD [Sides opposite to equal angles are equal]




(a) (b)

In figure (b),

ABCD is a parallelogram.

$\therefore \angle 1 = \angle 3$ and $\angle 2 = \angle 4$

Also AB $\parallel$ CD and BC meets them.

$\therefore \angle 1 + \angle 2 = 180^\circ$ ....(i) [Sum of interior angles]

And AD $\parallel$ BC and EC meets them.

$\therefore \angle 5 = \angle 2$....(ii) [Corresponding angles]

Since ABCE is a cyclic quadrilateral.

$\therefore \angle 1 + \angle 6 = 180^\circ$ ....(iii)

From eq. (i) and (iii),

$\angle 1 + \angle 2 = \angle 1 + \angle 6$

$\Rightarrow \angle 2 = \angle 6$

But from eq. (ii), $\angle 2 = \angle 5$

$\therefore \angle 5 = \angle 6$

Now in triangle AED,

$\angle 5 = \angle 6$

$\Rightarrow$ AE = AD [Sides opposite to equal angles]

Hence in both the cases, AE = AD

7. **AC and BD are chords of a circle which bisect each other. Prove that:**

**(i) AC and BD are diameters.**

**(ii) ABCD is a rectangle.**

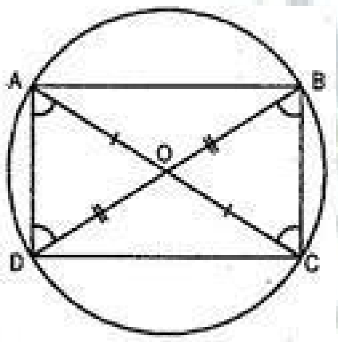
**Ans. Given**: AC and BD of a circle bisect each other at O. Then

OA = OC and OB = OD

**To prove**: (i) AC and BD are the diameters. In other words, O is the centre of the circle.

(ii) ABCD is a rectangle.

**Proof**: (i) In triangles AOD and BOC,




AO = OC [given]

$\angle$ AOD = $\angle$ BOC [Vertically opp.]

OD = OB [given]

$\therefore \Delta AOD \cong \Delta COB$ [SAS congruency]

$\Rightarrow$ AD = CB [By C.P.C.T.]

Similarly $\Delta_{AOB} \cong \Delta_{COD}$

$\Rightarrow$ AB=CD

$\Rightarrow \widehat{AB} \cong \widehat{CD}$ [Arcs opposite to equal chords]

$\Rightarrow \widehat{AB} + \widehat{BC} \cong \widehat{CD} + \widehat{BC}$

$\Rightarrow \widehat{ABC} \cong \widehat{BCD}$

$\Rightarrow$ AC = BD [Chords opposites to equal arcs]

$\therefore$ AC and BD are the diameters as only diameters can bisects each other as the chords of the circle.

(ii) AC is the diameter. [Proved in (i)]

$\therefore \angle B = \angle D = 90^\circ$ ....(i) [Angle in semi-circle]

Similarly BD is the diameter.

$\therefore \angle A = \angle C = 90^\circ$ ...(ii) [Angle in semi-circle]

Now diameters AC = BD

$\Rightarrow \widehat{AC} \cong \widehat{BD}$ [Arcs opposite to equal chords]

$\Rightarrow \widehat{AC} - \widehat{DC} \cong \widehat{BD} - \widehat{DC}$

$\Rightarrow \widehat{AD} \cong \widehat{BC}$

$\Rightarrow$ AD = BC [Chords corresponding to the equal arcs] ....(iii)

Similarly AB = DC ....(iv)

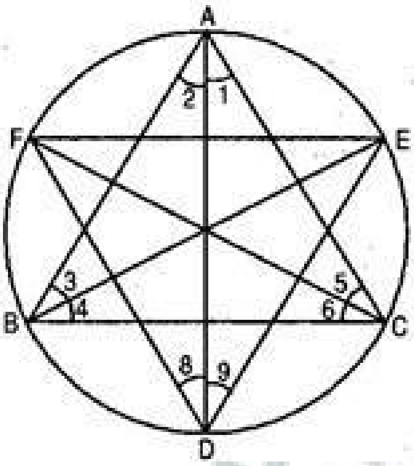
From eq. (i), (ii), (iii) and (iv), we observe that each angle of the quadrilateral is $90^\circ$ and opposite sides are equal.

Hence ABCD is a rectangle.

**8. Bisectors of angles A, B and C of a triangle ABC intersect its circum-circle at D, E and F respectively. Prove that angles of the triangle are** $\left(90^\circ - \frac{A}{2}\right), \left(90^\circ - \frac{B}{2}\right)$ **and** $\left(90^\circ - \frac{C}{2}\right)$ **respectively.**

**Ans.** According to question, AD is bisector of $\angle$ A.

$\therefore \angle 1 = \angle 2 = \frac{A}{2}$




And BE is the bisector of $\angle$ B.

$\therefore \angle 3 = \angle 4 = \frac{B}{2}$

Also CF is the bisector of $\angle$ C.

$\therefore \angle 5 = \angle 6 = \frac{C}{2}$

Since the angles in the same segment of a circle are equal.

$\therefore \angle 9 = \angle 3$ [angles subtended by $\overset{\frown}{AE}$ ] ....(i)

And $\angle 8 = \angle 5$ [angles subtended by $\overset{\frown}{FA}$ ] ....(ii)

Adding both equations,

$\angle 9 + \angle 8 = \angle 3 + \angle 5$

$\Rightarrow \angle D = \frac{B}{2} + \frac{C}{2}$

Similarly $\angle E = \frac{A}{2} + \frac{C}{2}$

And $\angle F = \frac{A}{2} + \frac{B}{2}$

In triangle DEF,

$\angle D + \angle E + \angle F = 180^\circ$ [Angle sum property]

$\Rightarrow \angle D = 180^\circ - (\angle E + \angle F)$

$\Rightarrow \angle D = 180^\circ - \left(\frac{A}{2} + \frac{C}{2} + \frac{A}{2} + \frac{B}{2}\right)$

$\Rightarrow \angle D = 180^\circ - \left(\frac{A}{2} + \frac{B}{2} + \frac{C}{2}\right) - \frac{A}{2}$

$\Rightarrow \angle D = 180^\circ - 90^\circ - \frac{A}{2}$ [$\because \angle A + \angle B + \angle C = 180^\circ$ ]

$\Rightarrow \angle D = 90^\circ - \frac{A}{2}$

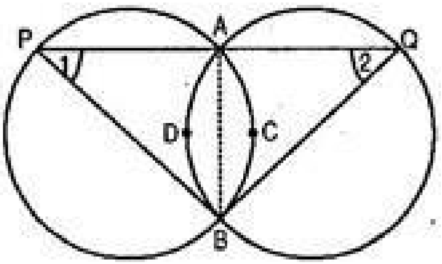
Similarly, we can prove that

$\angle E = 90^\circ - \frac{B}{2}$ and $\angle F = 90^\circ - \frac{C}{2}$

**9, Two congruent circles intersect each other at points A and B. Through A any line segment PAQ is drawn so that P, Q lie on the two circles. Prove that BP = BQ.**

**Ans. Given**: Two equal circles intersect in A and B.

A straight line through A meets the circles in P and Q.




**To prove**: BP = BQ

**Construction**: Join A and B.

**Proof:** AB is a common chord and the circles are equal.

$\therefore$ Arc about the common chord are equal, i.e.,

$\overparen{ACB} = \overparen{ADB}$

Since equal arcs of two equal circles subtend equal angles at any point on the remaining part of the circle, then we have,

$\angle 1 = \angle 2$

In triangle PBQ,

$\angle 1 = \angle 2$ [proved]

$\therefore$ Sides opposite to equal angles of a triangle are equal.

Then we have, BP = BQ

**3. In any triangle ABC, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circum circle of the triangle ABC.**

**Ans. Given**: ABC is a triangle and a circle passes through its vertices.

Angle bisector of $\angle$ A and the perpendicular bisector (say $l$) of its opposite side BC intersect each other at a point P.

**To prove**: Circum-circle of triangle ABC also passes through point P.

**Proof**: Since any point on the perpendicular bisector is equidistant from the end points of the corresponding side,

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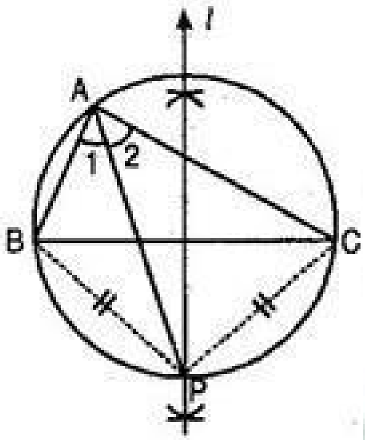


$\therefore$ BP = PC ....(i)

Also we have $\angle 1 = \angle 2$ [$\because$ AP is the bisector of $\angle$ A (given)] ....(ii)

From eq. (i) and (ii) we observe that equal line segments are subtending equal angles in the same segment i.e., at point A of circum-circle of $\Delta$ABC. Therefore BP and PC acts as chords of

Circum-circle of $\Delta$ABC and the corresponding congruent arcs $\overparen{BP}$ and $\overparen{PC}$ acts as parts of circum-circle. Hence point P lies on the circum-circle. In other words, points A, B, P and C are concyclic (proved).