पुणा International School
Shree Swaminarayan Gurukul, Zundal

Grade – 10 maths Specimen copy

Year 22-23

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CHAPTER NO. – 1

CHAPTER NAME – REAL NUMBERS

KEY POINTS TO REMEMBER –

- **Natural Numbers: Counting Numbers are called Natural Numbers are denoted by**

  N = {1, 2, 3, 4, 5, …….}

- **Whole Numbers : The collection of Natural Numbers along with zero is the collection of Whole Numbers and is denoted by W.**

  W = {0, 1, 2, 3, 4, …..}

- **Integers: The collection of Natural numbers, their negatives along with the number zero are called Integers. This collection is denoted by Z.**

  Z = {……..-3, -2, -1, 0, 1, 2, 3……}

- **Rational number: The numbers, which are obtained by dividing two integers, are called Rational numbers. Division by zero is not defined.**

  Q = { p/q: p and q are integers ,q ≠0}

- **Prime number: The number other than 1, with only factors namely 1 and the number itself, is a prime number.**

  { 2, 3, 5, 7, 11, 13, 17, 19,……..}

- **Co-prime number: If HCF of two numbers is 1, then the two numbers are called co-prime.**

Euclid's division lemma :

- For given positive integers 'a' and 'b' there exist unique whole numbers 'q' and 'r' satisfying the relation $a = b q + r$, $0 \leqslant r < b$.

  Theorem: If a and b are non-zero integers, the least positive integer which is

  expressible as a linear combination of a and b is the HCF of a and b,

  i.e. if d is the HCF of a and b, then these exist integers x_1 and y_1,

  such that $d = ax_1 + by_1$ and d is the smallest positive integer which is expressible in this form.

The HCF of a and b is denoted by HCF(a, b)

. Euclid's division algorithms :

- HCF of any two positive integers a and b. With a > b is obtained as follows:

Step 1 : Apply Euclid's division lemma to a and b to find q and r such that

$a = b\,q + r,\ 0 \leqslant r < b.$

b = Divisor

q = Quotient

r = Remainder

Step II: If r = 0, HCF (a ,b) = b if r ≠ 0, apply Euclid's lemma to b and r.

Step III: Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

- **The Fundamental Theorem of Arithmetic**

Every composite number can be expressed (factorized) as a product of primes and this factorization is unique, apart from the order in which the prime factors occur.

Ex : 24 = 2 X 2 X 2 X 3 = 3 X 2 X 2 X 2

The Fundamental Theorem of Arithmetic says that every composite number can be factorized as a product of primes.

- **HCF and LCM:(by prime factorization method)**

HCF: Product of the smallest power of each common prime factor in the numbers.

LCM: Product of the greatest power of each common prime factor in the numbers.

- **For any two positive integers a and b**

HCF (a x b) X LCM (a x b) = a x b

- **Revisiting Irrational Numbers**

Theorem 1.3: Let p be a prime number. If p divides a^2, then p divides a, Where a is a positive integer.

Theorem1.4: $\sqrt{2}$ is irrational.

- **Revisiting Rational Numbers and Their Decimal Expansions**

Theorem 1.5 : Let *x* be a rational number. Whose decimal expansion terminates then x can be expressed in the form $\frac{p}{q}$. Where p and q are co-prime, and prime factorization of q is of the form $2^m 5^n$, where m, n are non negative integers.

Theorem 1.6: Let $x = \frac{p}{q}$ $q \neq 0$ to be a rational number, such that the prime factorization of q is not of the form $2^m 5^n$, where m, n are non negative integers. Then *x* has a decimal expansion which terminates.

Theorem 1.7: : Let $x = \frac{p}{q}$, $q \neq 0$ to be a rational number, such that the prime factorization of q is of the form $2^m 5^n$, where m, n are non negative integers. Then *x* has a decimal expansion which is non-terminating repeating

Example: 1 Express 140 as a product of its prime factor

Solution: 140 = 2 X 2 X 5 X 7

= 2^2 X 5 X 7

Example: 2 SFind the HCF and LCM 91 and 26 by prime factorization.

Solution: 26 = 2 X 13

91 = 7 X 13

HCF = 13

LCM = 2 X 7 X 13 = 182

Example: 3 Find the HCF and LCM 12, 15 and 21 by prime factorization.

Solution: 12 = 2 x 2 x 3 = 2^2 x 3

15 = 3 x 5

21 = 3 x 7

HCF = 3

LCM = 2^2 x 3 x 5 x 7 = 420

Example: 4 **Given that HCF (306, 657) = 9, find LCM (306, 657).**

Solution:

$$\text{HCF}(306, 657) = 9$$

We know that, LCM × HCF = Product of two numbers

$$\therefore \text{LCM} \times \text{HCF} = 306 \times 657$$

$$\text{LCM} = \frac{306 \times 657}{\text{HCF}} = \frac{306 \times 657}{9}$$

$$\text{LCM} = 22338$$

Example: 5 **Check whether 6^n can end with the digit 0 for any natural number *n*.**

Solution: If any number ends with the digit 0, it should be divisible by 10 or in other words, it will also
be divisible by 2 and 5 as 10 = 2 x 5

Prime factorization of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorization of 6^n.

Hence, for any value of n, 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n.

Example: 6 **Explain why 7 x 11 x 13 + 13 and 7 x 6 x 5 x 4 x 3 x 2 x 1 + 5 are composite numbers.**

Solution: Numbers are of two types - prime and composite. Prime numbers can be divided by 1 and only itself, where as composite numbers have factors other than 1 and itself.

It can be observed that

7 x 11 x 13 + 13 = 13 x (7 x 11 + 1) = 13 x (77 + 1)

=13 x 78

=13 x 13 x 6

The given expression has 6 and 13 as its factors. Therefore, it is a composite number.

7 x 6 x 5 x 4 x 3 x 2 x 1 + 5 = 5 x (7 x 6 x 4 x 3 x 2 x 1 + 1)

=5 x (1008 + 1)

=5 x 1009

1009 can not be factorized further.
Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

Example: 7: **Prove that $\sqrt{5}$ is irrational.**

Answer : Let $\sqrt{5}$ is a rational number.

Therefore, we can find two integers a, b ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$

Let a and b have a common factor other than 1. Then we can divide them by the common factor, and assume that a and b are co-prime.

$$a = \sqrt{5}b$$
$$a^2 = 5b^2$$

Therefore, a^2 is divisible by 5 and it can be said that a is divisible by 5.

Let $a = 5k$, where k is an integer

$$(5k)^2 = 5b^2$$
$$b^2 = 5k^2$$

This means that b^2 is divisible by 5 and hence, b is divisible by 5.

This implies that a and b have 5 as a common factor.

And this is a contradiction to the fact that a and b are co-prime.

Hence, $\sqrt{5}$ cannot be expressed as $\frac{p}{q}$ or it can be said that $\sqrt{5}$ is irrational.

Example: 8 **Prove that $3+2\sqrt{5}$ is irrational.**

Answer :

Let $3+2\sqrt{5}$ is rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that

$$3+2\sqrt{5} = \frac{a}{b}$$
$$2\sqrt{5} = \frac{a}{b} - 3$$
$$\sqrt{5} = \frac{1}{2}\left(\frac{a}{b} - 3\right)$$

Since a and b are integers, $\frac{1}{2}\left(\frac{a}{b} - 3\right)$ will also be rational

And therefore, $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3+2\sqrt{5}$ is rational is false.

Therefore, $3+2\sqrt{5}$ is irrational.

Example: 9 Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:
Answer :

(i) $\frac{13}{3125}$

$3125 = 5^5$

The denominator is of the form 5^m.

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

$8 = 2^3$

The denominator is of the form 2^m.

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{64}{455}$

$455=5\times7\times13$

Since the denominator is not in the form $2^m \times 5^n$, and it also contains 7 and 13 as its factors, its decimal expansion will be non-terminating repeating.

(iv) $\frac{15}{1600}$

1600 = 26 × 52

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

Example: 10 Using Euclid's division algorithm find the HCF of 225 and 135.

Sol. On applying the division lemma to 225 and 135

We get

225 = 135 ×× 1 + 90

90 = 45 × 2 + 0

Hence HCF(225,135) = 45

Example: 11

Use Euclid's division algorithm to find the HCF of 196 and 38220

Sol. 196 and 38220

We have 38220 > 196,

So, we apply the division lemma to 38220 and 196 to obtain

38220 = 196 × 195 + 0

Since we get the remainder as zero, the process stops.

The divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

Example :12

Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where *q* is some integer.

Sol. Let a be any positive integer and $b = 6$. Then, by Euclid's algorithm,

$a = 6q + r$ for some integer $q \geq 0$, and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times(3q + 1) = 2k_1 + 1$, where k_1 is a positive integer

$6q + 3 = (6q + 2) + 1 = 2\,(3q + 1) + 1 = 2k_2 + 1$, where k_2 is an integer

$6q + 5 = (6q + 4) + 1 = 2\,(3q + 2) + 1 = 2k_3 + 1$, where k_3 is an integer

Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where k is an integer.

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2.

Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$, or $6q + 5$

Example: 13

Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where *q* is some integer.

Sol. Let a be any positive integer and $b = 6$. Then, by Euclid's algorithm,

$a = 6q + r$ for some integer $q \geq 0$, and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times (3q + 1) = 2k_1 + 1$, where k_1 is a positive integer

$6q + 3 = (6q + 2) + 1 = 2(3q + 1) + 1 = 2k_2 + 1$, where k_2 is an integer

$6q + 5 = (6q + 4) + 1 = 2(3q + 2) + 1 = 2k_3 + 1$, where k_3 is an integer

Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where k is an integer.

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2.

Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$, or $6q + 5$

WORK SHEET

1. Using prime factorization , find the HCF of
   (i) 405 and 2520
   (ii) 504 and 1188
   (iii) 960 and 1575
2. Using prime factorization, find the HCF and LCM of:
   (i) 36 and 84
   (ii) 23 and 31
   (iii) 96 and 404
   (iv) 144 and 198
   (v) 396 and 1080

In each case, verify that HCF X LCM = product of given number

3. Using prime factorization, find the HCF and LCM of:
   (i) 8, 9 and 25
   (ii) 12 ,15 and 21
   (iii) 17,23 and 29
   (iv) 24, 36 and 40
   (v) 30, 72 and 432
4. The HCF of two number is 23 and their LCM is 1449. If one of the number is 161, find the other.
5. The HCF of two number is 145 and their LCM is 2175. If one of the number is 725, find the other.
6. The HCF of two number is 18 and their product is 12960. Find their LCM.
7. State Euclid's division lemma.
8. State whether the given statement is true or false.
   (i) The sum of two rational is always rational.
   (ii) The product of two rational is always rational.

(iii) The sum of two irrational is always an irrational.
(iv) The product of two irrational is always an irrational.
(v) The sum of rational and an irrational is always irrational.
(vi) The product of rational and an irrational is always irrational.

CHAPTER NO. – 2

CHAPTER NAME – POLYNOMIALS

KEY POINTS TO REMEMBER –

- **Geometrical meaning of the Zeroes of the Polynomial.**
- **Zeroes and coefficients of a Polynomial.**
- **Division Algorithm for polynomial.**

1. **MONOMIALS:** Algebraic expression with one term is known as Monomial.
2. **BINOMIAL:** Algebraic expression with two terms is called Binomial.
3. **TRINOMIAL:** Algebraic expression with three terms is called Trinomial.
4. **POLYNOMIALS**: All above mentioned **algebraic expressions are called Polynomials.**
5. **LINEAR POLYNOMIAL:** Polynomial with degree 1 is called Linear polynomial.
6. **QUADRATIC POLYNOMIAL:** Polynomial with degree 2 is called Quadratic polynomial.
7. **CUBIC POLYNOMIAL:** Polynomial with degree 3 is called Cubic Polynomial.
8. **BIQUADRATIC POLYNOMIAL: :** Polynomial with degree 4 is called bi-quadratic Polynomial.

STANDARD FORM OF QUADRATIC POLYNOMIAL

- **A quadratic polynomial in x with real coefficients is the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.**
- **The zeros of a polynomial p(x) are precisely the x coordinates of the points where the graph of**
  y = p(x) intersect the x axis i.e. x = a is a zero of polynomial P (x) = 0.
- **A polynomial can have at most the same number of zeros as the degree of polynomial.**
- **For Quadratic polynomial : $ax^2 + bx + c, \; a \neq 0$**

  Sum of zeroes = $\frac{-b}{a}$ and product of zeroes = $\frac{c}{a}$
- **For cubic polynomials: $ax^3 + bx^2 + cx + d$, if $a, Q,$ are the zeroes of the polynomial.**

Then $a + Q + \; = \frac{-b}{a}$

$aQ + Q + \; a = \frac{c}{a}$

$ax\,Qx == \frac{d}{a}$

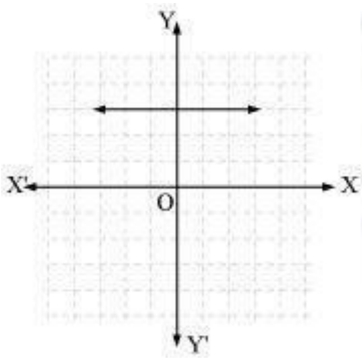
CHAPTER - 2

Polynomials

Question 1:

The graphs of y = p(x) are given in following figure, for some polynomials p(x). Find the number of zeroes of p(x), in each case.

(i)



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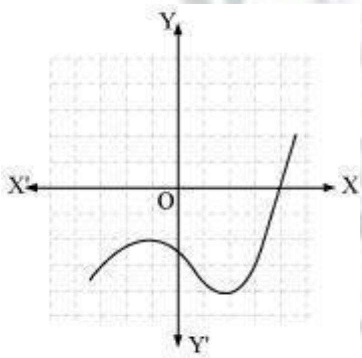

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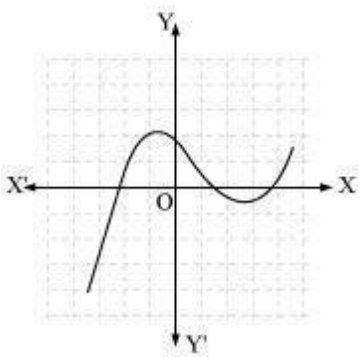

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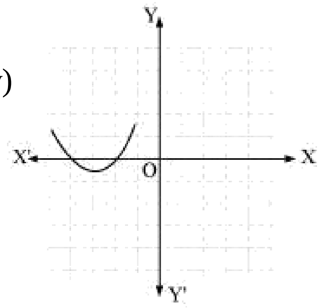

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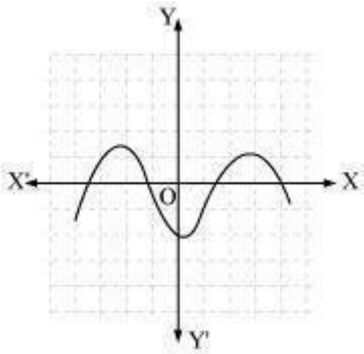

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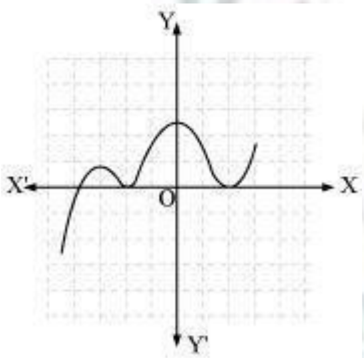

Answer:

The number of zeroes is 0 as the graph does not cut the x-axis at any point.

The number of zeroes is 1 as the graph intersects the x-axis at only 1 point.

The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

The number of zeroes is 2 as the graph intersects the x-axis at 2 points.

The number of zeroes is 4 as the graph intersects the x-axis at 4 points.

The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

Question : 2

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$(i)x^2-2x-8 \quad (ii)4s^2-4s+1 \quad (iii)6x^2-3-7x$

Answer:

(i) $x^2-2x-8=(x-4)(x+2)$

The value of x^2-2x-8 is zero when x − 4 = 0 or x + 2 = 0, i.e., when x = 4 or x = −2

Therefore, the zeroes of x^2-2x-8 are 4 and −2.

Sum of zeroes = $4-2=2=\frac{-(-2)}{1}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=4\times(-2)=-8=\frac{(-8)}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(ii) $4s^2-4s+1=(2s-1)^2$

The value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, i.e., $s=\frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are s = ½, ½

.

Sum of zeroes = ½ + ½ = 2/2 =1

Product of zeroes $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$

(iii) $6x^2 - 3 - 7x = 6x^2 - 7x - 3 = (3x+1)(2x-3)$

The value of $6x^2 - 3 - 7x$ is zero when $3x + 1 = 0$ or $2x - 3 = 0$, i.e., $x = \frac{-1}{3}$ or $x = \frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Sum of zeroes = $\frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes = $\frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Question :3

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4}, -1$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and β .

$$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$$

$$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$$

If $a = 4$, then $b = -1$, $c = -4$

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii) $\sqrt{2}, \frac{1}{3}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and .

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$

$$\alpha\beta = \frac{1}{3} = \frac{c}{a}$$

If $a = 3$, then $b = -3\sqrt{2}$, $c = 1$

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}\,x + 1$.

CHAP – 2

WORK SHEET

SUB: MATHS

Answer the questions

1) If α and β are the zeros of quadratic polynomial $x^2 + px + 2q$, find the value of $\alpha^2 + \beta^2$.
2) If a and b are the zeros of quadratic polynomial $x^2 + 2px + q$, find the value of 1/a + 1/b.
3) If α and β are the zeros of quadratic polynomial x2 + 3x - 4, find the value of α3 + β3.
4) Find the zeros of the polynomial f(x) = x3 - 12x2 + 47x - 60, if it is given that sum of its two zeros is 9.
5) Find the quadratic polynomial such that sum of its zeros is 10 and difference between zeros is 8.
6) Find a quadratic polynomial whose zeros are reciprocals of the zeros of the polynomial x2 + 7x + 12.
7) If two zeros of polynomial x3 + bx2 + cx + d are 3+√3 and 3-√3, find its third zero.
8) If α and β are the zeros of polynomial x2 - 6x + k, such that α2 + β2 = 20. Find the value of k.
9) If α and β are the zeros of quadratic polynomial x2 - 4x - 5, find the value of 1/α3 + 1/β3.

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CHAPTER 7

COORDINATES GEOMETRY

KEY POINTS TO REMEMBER –

- **DISTANCE FORMULA**
- **SECTION FORMULA**

1. **Distance Formula: The length of a line segment joining A and B is the distance between two points A (x_1, y_1) and B (x_2, y_2)**

THE DISTANCE FORMULA

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

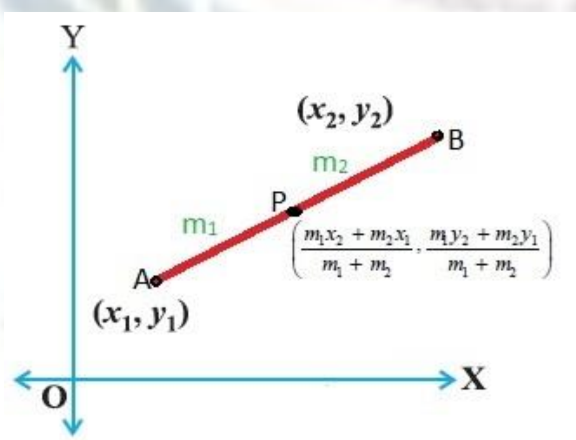
2. **The distance of a point P (x, y) from the origin (0, 0) is**

O (0, 0) ———————————————— **P (x, y)**

$$\mathbf{OP} = \sqrt{(x-0)^2 + (y-0)^2}$$

$$\mathbf{OP} = \sqrt{x^2 + y^2}$$

3. **SECTION FORMULA: The coordinate of the point P (x, y) which divides the line segment joining the points A (x_1, y_1) and B (x_2, y_2) internally in the ratio $m_1 : m_2$ are**



Section Formula

So, the coordinates of the point **P**(x, y) which divides the line segment joining the points **A**(x_1, y_1) and **B**(x_2, y_2), internally, in the ratio **m1: m2** are

$$\left(\frac{m_1x_2 + m_2x_1}{m_2 + m_1}, \frac{m_1y_2 + m_2y_1}{m_2 + m_1}\right)$$

This is known as the **section formula**.

4. MID-POINT FORMULA: The midpoint of the line segment joining the points A (x_1, y_1) and B (x_2, y_2) is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Chapter - 7

Coordinate Geometry
Exercise 7.1

1. Find the distance between the following pairs of points:

(i) **(2, 3), (4,1)**

(ii) **(–5, 7), (–1, 3)**

(iii) **(a, b), (–a, b)**

Ans. (i) Applying Distance Formula to find distance between points (2, 3) and (4,1), we get d =

$$\sqrt{(4-2)^2+(1-3)^2}=\sqrt{(2)^2+(-2)^2}=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}\text{ units}$$

(ii) Applying Distance Formula to find distance between points (–5, 7) and (–1, 3), we get d =

$$\sqrt{[-1-(-5)]^2+(3-7)^2}=\sqrt{(4)^2+(-4)^2}=\sqrt{16+16}=\sqrt{32}=4\sqrt{2}\text{ units}$$

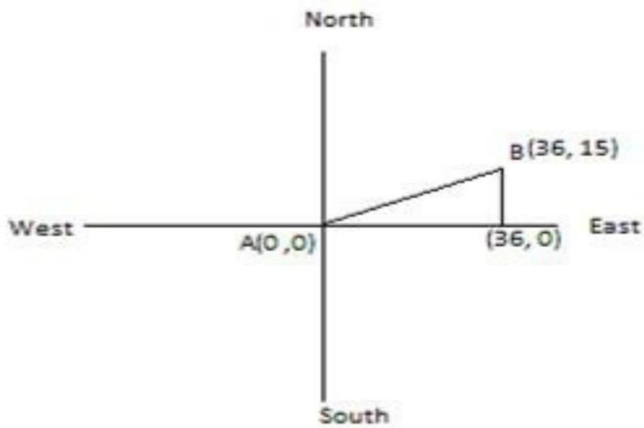
(iii) Applying Distance Formula to find distance between points (a, b) and (–a, –b), we get

d= $\sqrt{(-a-a)^2+(-b-b)^2}=\sqrt{(-2a)^2+(-2b)^2}=\sqrt{4a^2+4b^2}=\sqrt{4(a^2+b^2)}=2\sqrt{a^2+b^2}$

2. Find the distance between the points (0, 0) and (36, 15). Also, find the distance between towns A and B if town B is located at 36 km east and15 km north of town A.

Ans. Applying Distance Formula to find distance between points (0, 0) and (36, 15), we get d = $\sqrt{(36-0)^2+(15-0)^2} = \sqrt{(36)^2+(15)^2} = \sqrt{1296+225} = \sqrt{1521} = 39$ units

Town B is located at 36 km east and15 km north of town A. So, the location of town A and B can be shown as:



Clearly, the coordinates of point A are (0, 0) and coordinates of point B are (36, 15).

To find the distance between them, we use Distance formula:

d = $\sqrt{[36-0]^2+(15-0)^2} = \sqrt{(36)^2+(15)^2} = \sqrt{1296+225} = \sqrt{1521} = 39$ Km

3. Determine if the points (1, 5), (2, 3) and (–2, –11) are collinear.

Ans. Let A = (1, 5), B = (2, 3) and C = (–2, –11)

Using Distance Formula to find distance AB, BC and CA.

AB = $\sqrt{[2-1]^2+(3-5)^2} = \sqrt{(1)^2+(-2)^2} = \sqrt{1+4} = \sqrt{5}$

BC = $\sqrt{[-2-2]^2+(-11-3)^2} = \sqrt{(-4)^2+(-14)^2} = \sqrt{16+196} = \sqrt{212} = 2\sqrt{53}$

CA = $\sqrt{[-2-1]^2+(-11-5)^2} = \sqrt{(-3)^2+(-16)^2} = \sqrt{9+256} = \sqrt{265}$

Since AB + AC ≠ BC, BC + AC ≠ AB and AC ≠ BC.

Therefore, the points A, B and C are not collinear.

4. Check whether (5, –2), (6, 4) and (7, –2) are the vertices of an isosceles triangle.

Ans. Let A = (5, –2), B = (6, 4) and C = (7, –2)

Using Distance Formula to find distances AB, BC and CA.

AB = $\sqrt{[6-5]^2+[4-(-2)]^2} = \sqrt{(1)^2+(6)^2} = \sqrt{1+36} = \sqrt{37}$

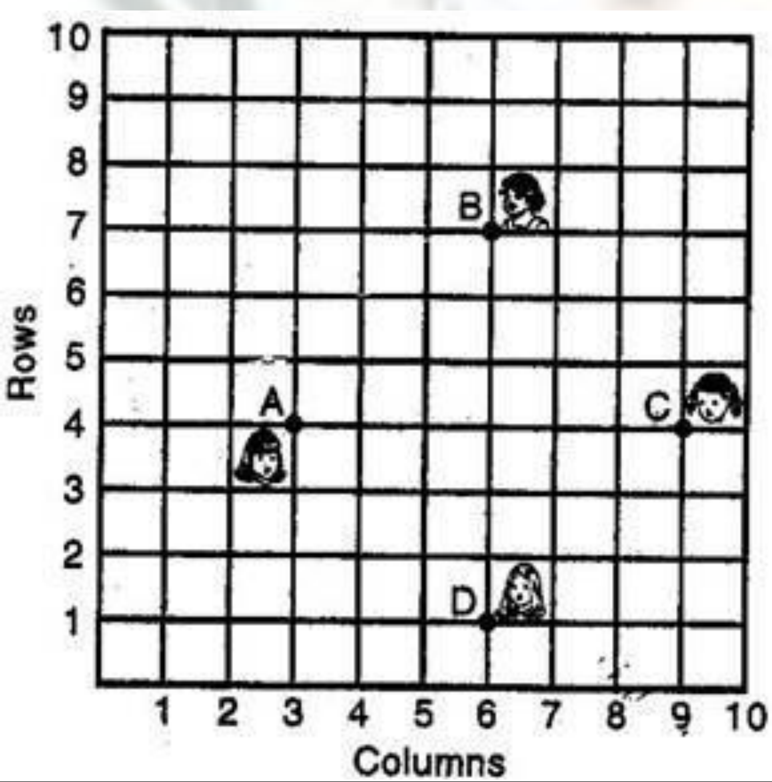
BC = $\sqrt{[7-6]^2+(-2-4)^2} = \sqrt{(1)^2+(-6)^2} = \sqrt{1+36} = \sqrt{37}$

CA = $\sqrt{[7-5]^2+[-2-(-2)]^2} = \sqrt{(2)^2+(0)^2} = \sqrt{4+0} = \sqrt{4} = 2$

Since AB = BC.

Therefore, A, B and C are vertices of an isosceles triangle.

5. In a classroom, 4 friends are seated at the points A (3, 4), B (6, 7), C (9, 4) and D (6, 1). Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli. "Don't you think ABCD is a square?"Chameli disagrees. Using distance formula, find which of them is correct.



Ans. We have A = (3, 4), B = (6, 7), C = (9, 4) and D = (6, 1)

Using Distance Formula to find distances AB, BC, CD and DA, we get AB =

$$\sqrt{[6-3]^2+[7-4]^2} = \sqrt{(3)^2+(3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

BC = $\sqrt{[9-6]^2+[4-7]^2} = \sqrt{(3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

CD = $\sqrt{[6-9]^2+[1-4]^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

DA = $\sqrt{[6-3]^2+[1-4]^2} = \sqrt{(3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Therefore, All the sides of ABCD are equal here. … (1)

Now, we will check the length of its diagonals.

AC = $\sqrt{[9-3]^2+[4-4]^2} = \sqrt{(6)^2+(0)^2} = \sqrt{36+0} = 6$

BD = $\sqrt{[6-6]^2+[1-7]^2} = \sqrt{(0)^2+(-6)^2} = \sqrt{0+36} = \sqrt{36} = 6$

So, Diagonals of ABCD are also equal. … (2)

From (1) and (2), we can definitely say that ABCD is a square.

Therefore, Champa is correct.

6. Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer.

(i) (–1, –2), (1, 0), (–1, 2), (–3, 0)

(ii) (–3, 5), (3, 1), (0, 3), (–1, –4)

(iii) (4, 5), (7, 6), (4, 3), (1, 2)

Ans. (i) Let A = (–1, –2), B = (1, 0), C= (–1, 2) and D = (–3, 0)

Using Distance Formula to find distances AB, BC, CD and DA, we get

AB = $\sqrt{[1-(-1)]^2+[0-(-2)]^2} = \sqrt{(2)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

BC = $\sqrt{[-1-1]^2+[2-0]^2} = \sqrt{(-2)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

CD = $\sqrt{[-3-(-1)]^2+[0-2]^2} = \sqrt{(-2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

DA = $\sqrt{[-3-(-1)]^2+[0-(-2)]^2} = \sqrt{(-2)^2+(2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

Therefore, all four sides of quadrilateral are equal. … (1)

Now, we will check the length of diagonals.

AC = $\sqrt{[-1-(-1)]^2+[2-(-2)]^2} = \sqrt{(0)^2+(4)^2} = \sqrt{0+16} = \sqrt{16} = 4$

BD = $\sqrt{[-3-1]^2+[0-0]^2} = \sqrt{(-4)^2+(0)^2} = \sqrt{16+0} = \sqrt{16} = 4$

Therefore, diagonals of quadrilateral ABCD are also equal. … (2)

From (1) and (2), we can say that ABCD is a square.

(ii) Let A = (–3, 5), B= (3, 1), C= (0, 3) and D= (–1, –4)

Using Distance Formula to find distances AB, BC, CD and DA, we get

AB = $\sqrt{[3-(-3)]^2+[1-5]^2} = \sqrt{(6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

BC = $\sqrt{[0-3]^2+[3-1]^2} = \sqrt{(-3)^2+(2)^2} = \sqrt{9+4} = \sqrt{13}$

CD = $\sqrt{[-1-0]^2+[-4-3]^2} = \sqrt{(-1)^2+(-7)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$

DA = $\sqrt{[-1-(-3)]^2+[-4-5]^2} = \sqrt{(2)^2+(-9)^2} = \sqrt{4+81} = \sqrt{85}$

We cannot find any relation between the lengths of different sides.

Therefore, we cannot give any name to the quadrilateral ABCD

(iii) Let A = (4, 5), B= (7, 6), C= (4, 3) and D= (1, 2)

Using Distance Formula to find distances AB, BC, CD and DA, we get AB =

$\sqrt{[7-4]^2+[6-5]^2} = \sqrt{(3)^2+(1)^2} = \sqrt{9+1} = \sqrt{10}$

BC = $\sqrt{[4-7]^2+[3-6]^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

CD = $\sqrt{[1-4]^2+[2-3]^2} = \sqrt{(-3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

DA = $\sqrt{[1-4]^2+[2-5]^2} = \sqrt{(-3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Here opposite sides of quadrilateral ABCD are equal. … (1)

We can now find out the lengths of diagonals.

AC = $\sqrt{[4-4]^2+[3-5]^2} = \sqrt{(0)^2+(-2)^2} = \sqrt{0+4} = \sqrt{4} = 2$

BD = $\sqrt{[1-7]^2+[2-6]^2} = \sqrt{(-6)^2+(-4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

Here diagonals of ABCD are not equal. … (2)

From (1) and (2), we can say that ABCD is not a rectangle therefore it is a parallelogram.

---

7. Find the point on the x–axis which is equidistant from (2, –5) and (–2, 9).

Ans. Let the point be (x, 0) on x–axis which is equidistant from (2, –5) and (–2, 9). Using

Distance Formula and according to given conditions we have:

$$\sqrt{[x-2]^2+[0-(-5)]^2}=\sqrt{[x-(-2)]^2+[(0-9)]^2}$$

$\Rightarrow \sqrt{x^2+4-4x+25}=\sqrt{x^2+4+4x+81}$

Squaring both sides, we get

$\Rightarrow x^2+4-4x+25 = x^2+4+4x+81$

⇒ −4x + 29 = 4x + 85

⇒ 8x = −56

⇒ x = −7

Therefore, point on the x–axis which is equidistant from (2, –5) and (–2, 9) is (–7, 0)

---

8. Find the values of y for which the distance between the points P (2, –3) and Q (10, y) is 10units.

Ans. Using Distance formula, we have

$$10=\sqrt{(2-10)^2+(-3-y)^2}$$

$\Rightarrow 10=\sqrt{(-8)^2+9+y^2+6y}$

$\Rightarrow 10=\sqrt{64+9+y^2+6y}$

Squaring both sides, we get

100 =

$\Rightarrow y^2+6y-27=0$

Solving this Quadratic equation by factorization, we can write

$\Rightarrow y^2+9y-3y-27=0$

$\Rightarrow$ y (y + 9) – 3 (y + 9) = 0

$\Rightarrow$ (y + 9) (y − 3) = 0

$\Rightarrow$ y = 3, −9

9. If, Q (0, 1) is equidistant from P (5, –3) and R (x, 6), find the values of x. Also, find the distances QR and PR.

Ans. It is given that Q is equidistant from P and R. Using Distance Formula, we get PQ=RQ

$$\Rightarrow \sqrt{(0-5)^2 + [1-(-3)]^2} = \sqrt{(0-x)^2 + (1-6)^2}$$

$\Rightarrow \sqrt{(-5)^2 + (4)^2} = \sqrt{(-x)^2 + (-5)^2}$

$\Rightarrow \sqrt{25+16} = \sqrt{x^2+25}$

Squaring both sides, we get

$\Rightarrow$ 25+16= $x^2 + 25$

$\Rightarrow x^2 = 16$

$\Rightarrow$ x = 4, −4

Thus, Q is (4, 6) or (–4, 6).

Using Distance Formula to find QR, we get

Using value of x = 4 QR = $\sqrt{(4-0)^2+(6-1)^2} = \sqrt{16+25} = \sqrt{41}$

Using value of x = –4 QR = $\sqrt{(-4-0)^2+(6-1)^2} = \sqrt{16+25} = \sqrt{41}$

Therefore, QR = $\sqrt{41}$

Using Distance Formula to find PR, we get

Using value of x = 4 PR = $\sqrt{(4-5)^2+[6-(-3)]^2} = \sqrt{1+81} = \sqrt{82}$

Using value of x =–4 PR = $\sqrt{(-4-5)^2+[6-(-3)]^2} = \sqrt{81+81} = \sqrt{162} = 9\sqrt{2}$

Therefore, x = 4, –4

QR= $\sqrt{41}$,PR= $\sqrt{82}, 9\sqrt{2}$

10. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (–3, 4).

Ans. It is given that (x, y) is equidistant from (3, 6) and (–3, 4).

Using Distance formula, we can write

$$\sqrt{(x-3)^2+(y-6)^2} = \sqrt{[x-(-3)]^2+(y-4)^2}$$

$$\Rightarrow \sqrt{x^2+9-6x+y^2+36-12y} = \sqrt{x^2+9+6x+y^2+16-8y}$$

Squaring both sides, we get

$$\Rightarrow x^2+9-6x+y^2+36-12y = x^2+9+6x+y^2+16-8y$$

⇒ −6x − 12y + 45 = 6x − 8y + 25

⇒ 12x + 4y = 20

3x + y = 5

Exercise 7.2

1. **Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2:3.**

Ans. Let $x_1 = -1,\ x_2 = 4,\ y_1 = 7$ and $y_2 = -3,\ m_1 = 2\ and\ m_2 = 3$

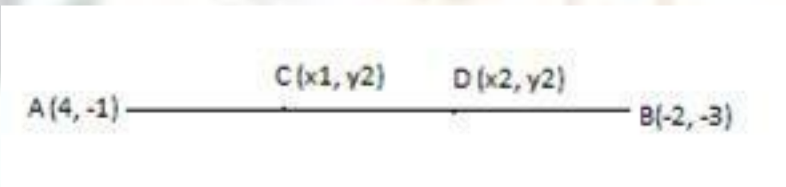
Using Section Formula to find coordinates of point which divides join of (–1, 7) and (4, –3) in the ratio 2:3, we get

$$x = \frac{m_1x_2 + m_2x_1}{m_1 + m_2} = \frac{2\times4 + 3\times(-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1$$

$$y = \frac{m_1y_2 + m_2y_1}{m_1 + m_2} = \frac{2\times(-3) + 3\times7}{2+3} = \frac{-6+21}{5} = \frac{15}{5} = 3$$

Therefore, the coordinates of point are (1, 3) which divides join of (–1, 7) and (4, –3) in the ratio 2:3.

2. **Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).**

Ans. 

We want to find coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).

We are given AC = CD = DB

We want to find coordinates of point C and D.

Let coordinates of point C be (x_1, y_1) and let coordinates of point D be (x_2, y_2).

Clearly, point C divides line segment AB in 1:2 and point D divides line segment AB in 2:1.

Using Section Formula to find coordinates of point C which divides join of (4, –1) and (–2, –3)

in the ratio 1:2, we get

$$x_1 = \frac{1\times(-2)+2\times4}{1+2} = \frac{-2+8}{3} = \frac{6}{3} = 2$$

$$y_1 = \frac{1\times(-3) + 2\times(-1)}{1+2} = \frac{-3-2}{3} = \frac{-5}{3}$$

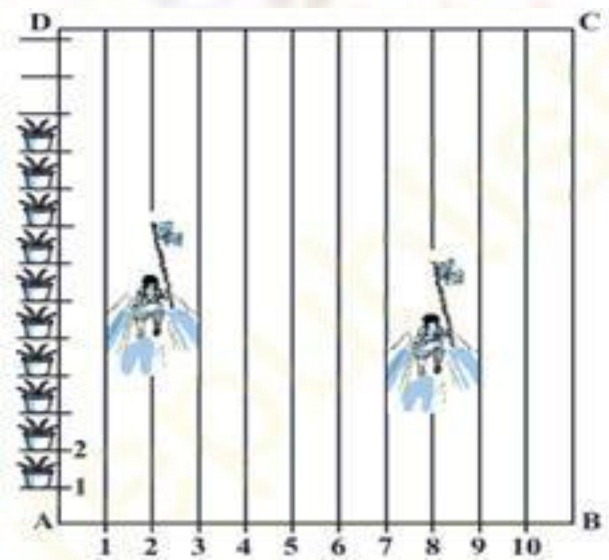
Using Section Formula to find coordinates of point D which divides join of (4, –1) and (–2, –3) in the ratio 2:1, we get

$$x_2 = \frac{2\times(-2) + 1\times4}{1+2} = \frac{-4+4}{3} = \frac{0}{3} = 0$$

$$y_2 = \frac{2\times(-3) + 1\times(-1)}{1+2} = \frac{-6-1}{3} = \frac{-7}{3}$$

Therefore, coordinates of point C are (2, -5/3) and coordinates of point D are (0, -7/3)

3. **To conduct sports day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD. Niharika runs 14th of the distance AD on the 2nd line and posts a green flag. Preet runs 15th of the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segments joining the two flags, where should she post her flag?**



Ans. Niharika runs 14^{th} of the distance AD on the 2^{nd} line and posts a green flag.

There are 100 flower pots. It means, she stops at 25th flower pot.

Therefore, the coordinates of point where she stops are (2 m, 25 m).

Preet runs 15th of the distance AD on the eighth line and posts a red flag. There are 100 flower pots. It means, she stops at 20th flower pot.

Therefore, the coordinates of point where she stops are (8, 20).

Using Distance Formula to find distance between points (2 m, 25 m) and (8 m, 20 m), we get

$$d = \sqrt{(2-8)^2 + (25-20)^2} = \sqrt{(-6)^2 + 5^2} = \sqrt{36+25} = \sqrt{61}m$$

Rashmi posts a blue flag exactly halfway the line segment joining the two flags. Using

section formula to find the coordinates of this point, we get

$$x = \frac{2+8}{2} = \frac{10}{2} = 5$$

$$y = \frac{25+20}{2} = \frac{45}{2}$$

Therefore, coordinates of point, where Rashmi posts her flag are $\left(5, \frac{45}{2}\right)$.

It means she posts her flag in 5th line after covering $\frac{45}{2}$ = 22.5 m of distance.

---

4. Find the ratio in which the line segment joining the points (–3, 10) and (6, –8) is divided by (–1, 6).

Ans. Let (–1, 6) divides line segment joining the points (–3, 10) and (6, –8) in k:1.

Using Section formula, we get

$$-1 = \frac{(-3)\times 1 + 6\times k}{k+1}$$

−k – 1 = (−3 + 6k)

−7k = −2

k = $\frac{2}{7}$

Therefore, the ratio is $\frac{2}{7}$:1 which is equivalent to 2:7.

Therefore, (–1, 6) divides line segment joining the points (–3, 10) and (6, –8) in 2:7.

---

5. Find the ratio in which the line segment joining A (1, –5) and B (–4, 5) is divided by the x–axis. Also find the coordinates of the point of division.

Ans. Let the coordinates of point of division be (x, 0) and suppose it divides line segment joining A (1, –5) and B (–4, 5) in k:1.

According to Section formula, we get

$$x = \frac{1\times1+(-4)\times k}{k+1} = \frac{1-4k}{k+1} \; and \; 0 = \frac{(-5)\times1+5k}{k+1} \quad \dots (1)$$

$$0 = \frac{(-5)\times1+5k}{k+1}$$

5 = 5k

k = 1

Putting value of k in (1), we get

$$x = \frac{1\times1+(-4)\times1}{1+1} = \frac{1-4}{2} = \frac{-3}{2}$$

Therefore, point ($\frac{-3}{2}$, 0) on x–axis divides line segment joining A (1, –5) and B (–4, 5) in 1:1.

6. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Ans. Let A = (1, 2), B = (4, y), C = (x, 6) and D = (3, 5)

We know that diagonals of parallelogram bisect each other. It means that coordinates of midpoint of diagonal AC would be same as coordinates of midpoint of diagonal BD. … (1)

Using Section formula, the coordinates of midpoint of AC are:

$$\frac{1+x}{2}, \frac{2+6}{2} = \frac{1+x}{2}, 4$$

Using Section formula, the coordinates of midpoint of BD are:

$$\frac{4+3}{2}, \frac{5+y}{2} = \frac{7}{2}, \frac{5+y}{2}$$

According to condition (1), we have

$$\frac{1+x}{2}=\frac{7}{2}$$

(1 + x) = 7

x = 6

Again, according to condition (1), we also have

$$4=\frac{5+y}{2}$$

8 = 5 + y

y = 3

Therefore, x = 6 and y = 3

7.Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, –3) and B is (1, 4).

Ans. We want to find coordinates of point A. AB is the diameter and coordinates of center are (2, –3) and, coordinates of point B are (1, 4).

Let coordinates of point A are (x, y). Using section formula, we get

$$2=\frac{x+1}{2}$$

4 = x + 1

x = 3

Using section formula, we get

$$-3=\frac{4+y}{2}$$

$-6 = 4 + y$

$y = -10$

Therefore, Coordinates of point A are (3, –10).

8. If A and B are (–2, –2) and (2, –4) respectively, find the coordinates of P such that AP = 3/7 AB and P lies on the line segment AB.

Ans. A = (–2, –2) and B = (2, –4)

It is given that AP= $\frac{3}{7}$ AB

PB=AB–AP=AB− $\frac{3}{7}$ AB= $\frac{4}{7}$ AB

So, we have AP: PB = 3: 4

Let coordinates of P be (x, y)

Using Section formula to find coordinates of P, we get

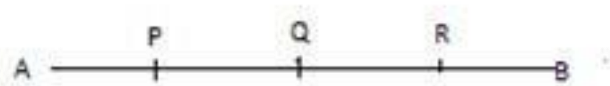
$$x = \frac{(-2)\times 4 + 2\times 3}{3+4} = \frac{6-8}{7} = \frac{-2}{7}$$

$$y = \frac{(-2)\times 4 + (-4)\times 3}{3+4} = \frac{-8-12}{7} = \frac{-20}{7}$$

Therefore, Coordinates of point P are $\left(\frac{-2}{7}, \frac{-20}{7}\right)$.

9. Find the coordinates of the points which divides the line segment joining A (–2, 2) and B (2, 8) into four equal parts.

Ans. A = (–2, 2) and B = (2, 8)



Let P, Q and R are the points which divide line segment AB into 4 equal parts.

Let coordinates of point $P = (x_1, y_1)$, $Q = (x_2, y_2)$ and $R = (x_3, y_3)$

We know AP = PQ = QR = RS.

It means, point P divides line segment AB in 1:3.

Using Section formula to find coordinates of point P, we get

$$x_1 = \frac{(-2)\times 3 + 2\times 1}{1+3} = \frac{-6+2}{4} = \frac{-4}{4} = -1$$

$$y_1 = \frac{2\times 3 + 8\times 1}{1+3} = \frac{6+8}{4} = \frac{14}{4} = \frac{7}{2}$$

Since, AP = PQ = QR = RS.It means, point Q is the mid-point of AB.

Using Section formula to find coordinates of point Q, we get

$$x_2 = \frac{(-2)\times 1 + 2\times 1}{1+1} = \frac{-2+2}{2} = \frac{0}{2} = 0$$

$$y_2 = \frac{2\times 1 + 8\times 1}{1+1} = \frac{2+8}{2} = \frac{10}{2} = 5$$

Because, AP = PQ = QR = RS.

It means, point R divides line segment AB in 3:1

Using Section formula to find coordinates of point P, we get

$$x_3 = \frac{(-2)\times 1 + 2\times 3}{1+3} = \frac{-2+6}{4} = \frac{4}{4} = 1$$

$$y_3 = \frac{2\times 1 + 8\times 3}{1+3} = \frac{2+24}{4} = \frac{26}{4} = \frac{13}{2}$$

Therefore, P = (–1, $\frac{7}{2}$), Q= (0, 5) and R = (1,$\frac{13}{2}$)

10. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (–1, 4) and (–2, –1) taken in order.

{Hint: Area of a rhombus = $\frac{1}{2}$ (product of its diagonals)}

Ans. Let A = (3, 0), B = (4, 5), C = (–1, 4) and D = (–2, –1)

Using Distance Formula to find length of diagonal AC, we get

$$AC = \sqrt{[3-(-1)]^2 + (0-4)^2} = \sqrt{4^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

Using Distance Formula to find length of diagonal BD, we get

$$BD = \sqrt{[4-(-2)]^2 + [5-(-1)]^2} = \sqrt{6^2 + 6^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

$\because$ Area of rhombus = ½ (product of its diagonals)

= $\frac{1}{2} \times AC \times BD = \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$ = 24 sq. units

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WORK - SHEET

CHAPTER – 7 STD -10th

COORDINATE GEOMETRY

SOLVE (EACH CARRY ONE MARK)

1. The number of coordinate axes in a plane is

   (a) 1 (b) 2 (c) 3 (d) 4

2. The coordinate of origin are

   (a) (0, 0) (b) (0, 1) (c) (1, 0) (d) (1, 1)

3. The angle between x- axis and y- axis is

   (a) 0° (b) 45° (c) 90° (d) 60°

4. The distance of the point (3, 4) from x- axis is

   (a) 3 (b) 1 (c) 7 (d) 4

5. Find the distance between the point (2, 3) and (4, 5)

   (a) 3 (b) $\sqrt{8}$ (c) 5 (d) 4

6. If A (x, 2), b (-3, -4) and C (7, -5) are collinear, than find the value of x

7. Find the point on x-axis which is equidistance from points (-1, 0) and (5, 0)

8. Determine if the points (1, 5), (2, 3) and (-2, -11) are collinear.

SOLVE (EACH CARRY TWO MARKS)

9. Check whether (5, –2), (6, 4) and (7, –2) are the vertices of an isosceles triangle.

10. Find the point on the x–axis which is equidistant from (2, –5) and (–2,9).

11. Find the values of y for which the distance between the points P (2, –3) and Q (10,y) Is 10 units

12. If, Q (0, 1) is equidistant from P (5, –3) and R (x, 6), find the values of x. Also, find the distances QR and PR.

13. Find a relation between x and y such that the point (x, y) is equidistant from the Point(3, 6) and (-3, 4).

SOLVE (EACH CARRY THREE MARKS)

14. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2:3.

15. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3)

16. Find the ratio in which the line segment joining the points (–3, 10) and (6, –8) is divided by (–1, 6).

पुणा International School
Shree Swaminarayan Gurukul, Zundal

Grade – 10
MATHS
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Year 22-23

INDEX

Notes
CHAPTER 05
ARITHMETIC PROGRESSIONS

1. **Sequence:** A set of numbers arranged in some definite order and formed according to some rules is called a sequence.
2. **Progression:** The sequence that follows a certain pattern is called progression.
3. **Arithmetic Progression:** A sequence in which the difference obtained by subtracting any term from its preceding term is constant throughout, is called an arithmetic sequence or arithmetic progression (A.P.).

The general form of an A.P. is **a, a + d, a + 2d, ... (a = first term, d = common difference).**
The terms of A.P. is denoted by **$a, a_2, a_3, ...a_n$**

1. **General Term:** If 'a' is the first term and 'd' is common difference in an A.P., then n^{th} term (general term) is given by $a_n = a + (n – 1) d$

Sum of n Terms of an A.P. : If 'a' is the first term and 'd' is the common difference of an A.P., then sum of first n terms is given by

$$S_n = \frac{n}{2}[\,2a + (n - 1)\,d\,]$$

If 'l' is the last term of a finite A.P. then the sum is given by

$$S_n = \frac{n}{2}[\,a + l\,]$$

(i) If a_n is given, then common difference $d = a_n - a_{n-1}$

(ii) If Sn is given, then n^{th} term is given by $a_n = S_n - S_{n-1}$

(iii) If a, b, c are in A.P., then $2b = a + c$.

(iv) **If a sequence has n terms, its r^{th} term from the end = $(n-r+1)^{th}$ term from the beginning.**

EX : 5.1

1(1). The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km. Is this situation make an arithmetic progression and why?

Sol.

Taxi fare for 1 km = Rs 15 = a_1

Taxi Fare for 2 kms

=Rs15+Rs8=Rs23=a2=Rs15+Rs8=Rs23=a2

Taxi fare for 3 km s

=Rs23+Rs8=Rs31=a3=Rs23+Rs8=Rs31=a3

Taxi fare for 4 kms

=RS31+Rs8=RS39=a4=RS31+Rs8=RS39=a4 and so on

a2−a1=Rs.23−Rs.15=Rs8a2−a1=Rs.23−Rs.15=Rs8

a3−a2=Rs31−Rs·23=Rs8a3−a2=Rs31−Rs·23=Rs8

a4−a3=Rs39−Rs31=Rs8a4−a3=Rs39−Rs31=Rs8

So, the arithmetic progression formed is:-

i.e.,a_{k+1}-a_k is the same every time.

So, this list of numbers form an arithmetic

Progression with the first term a = Rs 15 and the common difference d = Rs 8.

1(2). The amount of air present in a cylinder when a vacuum pump removes 1414 of the air remaining in the cylinder at a time. Does this situation make an arithmetic progression and why?

Sol.

Let the volume of the cylinder be 16 litres(a_1).
Air removed by pump = 14×16=4 litres14×16=4 litres
Air present after first removal = 16 - 4 = 12 litres(a_2)
Air again removed = 14×12=3litres14×12=3litres
Air present after second removal = 12 - 3 = 9 litres(a_3)
The amount of air present in the cyinder is the series
16,12,9.....
a_2 - a_1 = 12-16 = -4
a_3 - a_2 = 9-12 = -3
Since the difference is not same. This is not A.P.

1(3). The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs 50 for each subsequent metre. Is this situation make an arithmetic progression and why?

Sol.
Cost of digging the well after 1 metre of digging = Rs 150 = a_1
Cost digging the well after 2 metres of digging
= Rs 150 + Rs 50 = Rs 200 = a_2
Cost of digging the well after 3 metres of digging
= Rs 200 + Rs 50 = Rs 250 = a_3
Cost of digging the well after 4 metres of digging
= Rs 250 + Rs 50 = Rs 300 = a_4
and so on.
$a_2 - a_1$ = Rs 200 – Rs 150 = Rs 50
$a_3 - a_2$ = RS 250 – Rs 200 = Rs 50
$a_4 - a_3$ = Rs 300 – Rs 250 = Rs 50
i.e. $a_{k+1} - a$ is the same every time.
So this list of numbers forms an AP with the first term a = Rs 150
and the common difference d = Rs 50.

1(4). The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum. Is this situation make an arithmetic progression and why?

Sol.
Amount of money after 1 year =Rs10000(1+8100)=a1=Rs10000(1+8100)=a_1
Amount of money after 2 year =Rs10000(1+8100)2=a2=Rs10000(1+8100)^2 = a_2
Amount of money after 3 year =Rs10000(1+8100)3=a3=Rs10000(1+8100)^3 = a_3
Amount of money after 4 year =Rs10000(1+8100)4=a4=Rs10000(1+8100)^4 = a_4
a2−a1=Rs10000(1+8100)2−Rs10000(1+8100)
=Rs10000 (1+8100)(1+8100−1)
=10000(1+8100)(8100)
a_3-a_2
=10000(1+8100)3−10000(1+8100)^2
=10000(1+8100)(1+8100−1)=10000(1+8100)(1+8100−1)
=10000(1+8100)(8100)=10000(1+8100)(8100)
Since.a3−a2≠a2−a1a3−a2≠a2−a1.It does not form AP.

2(1). Write first four terms of the AP, when the first term a = 10 and the common difference d = 10.

Sol.
a = 10, d = 10
First term a = 10
Second term = 10 + d = 10 + 10 = 20
Third term = 20 + d = 20 + 10 = 30
Fourth term = 30 + d = 30 + 10 = 40
Hence, first four terms of the given AP are 10, 20, 30, 40

2(2). Write the first four terms of the AP, when the first term a = -2 and the common difference d = 0

Sol.
a = -2, d = 0
First term = a = -2
Second term = -2 + d = -2 + 0 = -2
Third term = -2 + d = -2 + 0 = -2
Fourth term = -2 + d = -2 + 0 = -2
Hence, first four terms of the given AP are -2, -2, -2, -2 respectively.

2(3). Write first four terms of the AP, when the first term a = 4 and the common difference d = -3

Sol.
a = 4, d = -3
First term = a = 4
Second term = 4 + d = 4 + (-3) = 1
Third term = 1 + d = 1 + (-3) = -2
Fourth term = −2+d=−2+(−3)=−5−2+d=−2+(−3)=−5
Hence, four first terms of the AP are 4, 1, -2, -5.

2(4). Write the first four terms of the AP, when the first term a = -1 and the common difference d = 1212

Sol.
a = -1, d=12d=12
First term = a = -1
Second term = -1 + d =−1+12=−12=−1+12=−12
Third term =−12+d=−12+12=0=−12+d=−12+12=0
Fourth term = 0 + d =0+12=12=0+12=12
Hence, the first four terms of the given AP are -1, −12,0,12−12,0,12.

2(5). Write first four terms of the AP, when the first term a = -1.25 and the common difference, d = –0.25

Sol.
Here, a_1 = a = -1.25, d = -0.25
First term, a = -1.25
Second term, a_2 = -1.25 + d = -1.25 + (-0.25) = -1.50
Third term = -1.50 + d = -1.50 + (-0.25) = -1.75
Fourth term = -1.75 + d = -1.75 + (-0.25) = -2.00
Hence, first four terms of the given AP are -1.25, -1.75, -2.00

3(1). For the AP 3, 1, -1, -3 ......, write the first term and the common difference.

Sol.
3, 1, -1, -3...
First term = a= 3,

Common difference (d) = Second term - first term = Third term - second term and so on
Therefore, Common difference (d) = 1 - 3 = -2

3(2). For the AP –5, –1, 3, 7,... write the first term and the common difference.

Sol.
Given A.P is: -5, - 1, 3, 7, ...
First term (a) = -5
Common difference (d) = -1 - (-5) = -1 + 5 = 4

3(3). For the AP $1/3$, $5/3$, $9/3$, $13/3$,..... write the first term and the common difference.

Sol.
$1/3$, $5/3$, $9/3$, $13/3$,.....
First term (a) = $1/3$
Common difference (d) = $5/3 - 1/3 = 4/3$

3(4). For the AP 0.6, 1.7, 2.8, 3.9 ...., write the first term and the common difference.

Sol.
0.6, 1.7, 2.8, 3.9...
First term = a= 0.6
Common difference (d) = Second term -First term
= Third term - Second term and so on
Therefore, Common difference (d) = 1.7−0.6=1.1

4(1). Is the given series 2, 4, 8, 16, ...form an AP? If It forms an AP, then find the common difference d and write the next three terms.

Sol.
If $a_{k+1} - a_k$ is same for different values of k, then the series is in the form of an AP.
here, we have $a_1 = 2$, $a_2 = 4$, $a_3 = 8$ and $a_4 = 16$
$a_4 - a_3 = 16 - 8 = 8$
$a_3 - a_2 = 8 - 4 = 4$
$a_2 - a_1 = 4 - 2 = 2$
Here, $a_{k+1} - a_k$ i.e. the common difference is not same for all values of k
Hence, the given series does not form an AP.

4(2). Is the given series: 2, $5/2$, 3, $7/2$,… form an AP? If It forms an AP, then find the common difference d and write the next three terms.

Sol.
As per the question:
$a_1 = 2$
$a_2 = 5/2$
$a_3 = 3$
$a_4 = 7/2$

now check the common difference (d)

$a_2 - a_1 = 5/2 - 2 = 1/2$

$a_3 - a_2 = 3 - 5/2 = 1/2$

$a_4 - a_3 = 7/2 - 3 = 1/2$

we can see that the common difference is the same everywhere, so the given series forms an AP.

now next three terms are: $a_5 = 7/2 + 1/2 = 4$

$a_6 = 4 + 1/2 = 9/2$

$a_7 = 9/2 + 1/2 = 5$

4(3). Is this –1.2, –3.2, –5.2, –7.2, …. an AP? If it forms an AP, find the common difference d and write three more terms.

Sol.

-1.2, -3.2, -5.2, -7.2, ...

$a^2 - a^1 = -3.2 - (-1.2) = -3.2 + 1.2 = -2.0$

$a^3 - a^2 = -5.2 - (-3.2) = -5.2 + 3.2 = -2.0$

$a^4 - a^3 = -7.2 - (-5.2) = -7.2 + 5.2 = -2.0$

i.e. $a_{k+1} - a_k$ is the same everytime, So, the given list of numbers form an AP with the common differenced d = -2.0

The next three terms are:

-7.2 + (-2.0) = -9.2

-9.2 + (-2.0) = -11.2

and -11.2 + (-2.0) = -13.2

4(4). Is this –10, –6, –2, 2, ... an AP? If it forms an AP, find the common difference d and write three more terms.

Sol.

-10, -6, -2, 2, ...

$a_2 - a_1 = -6 - (-10) = -6 + 10 = 4$

$a_3 - a_2 = -2 - (-6) = -2 + 6 = 4$

$a_4 - a_3 = 2 - (-2) = 2 + 2 = 4$

i.e. $a_{k+1} - a_k$ is the same every time.

So, the given lists of numbers form an AP with the common difference d = 4.

The next three terms are:

2 + 4 = 6, 6 + 4 = 10 and 10 + 4 = 14

4(7). Is the given series: 0, -4, -8, -12, ...forms an AP? If it forms an AP, then find the common difference d and write three more terms.

Sol.

Here: $a_2 - a_1 = -4 - 0 = -4$

$a_3 - a_2 = -8 + 4 = -4$

$a_4 - a_3 = -12 + 8 = -4$, since $a_{k+1} - a_k$ is same for all values of k

Hence, this is an AP.

The next three terms can be calculated as follows:

$a_5 = a + 4d = 0 + 4(-4) = -16$

$a_6 = a + 5d = 0 + 5(-4) = -20$

$a_7 = a + 6d = 0 + 6(-4) = -24$

Thus, the next three terms are: -16, -20 and -24

Exercise 5.2

1. Fill in the blanks in the following table, given that a is the first term, d is the common difference and

	a	d	n	a_n
i	7	3	8	...
ii	-18	...	10	0
iii	...	-3	18	-5
iv	-18.9	2.5	...	3.6
v	3.5	0	105	...

a_n is the n^{th} term of the AP:

Sol.

i. Given: a = 7, d = 3 and n = 8,
a_n = ?
We know that $a_n = a + (n – 1)d$
Thus, $a_8 = 7 + (8 – 1)3 = 7 + 21 = 28$

ii. Given: a = - 18, n = 10, a_n = 0, d = ?
We know that $a_n = a + (n – 1)d$
Thus, $0 = - 18 + (10 – 1)d$
$0 = -18 + 9d$
or, $9d = 18$
$d = 18/9 = 2$

iii. Given: d = - 3, n = 18, a_n = - 5, a = ?
We know that $a_n = a + (n – 1)d$
$-5 = a + (18 - 1) (-3)$
$-5 = a - 51$
or $a = -5 + 51 = 46$

iv. Given: a = -18.9, d = 2.5, a_n = 3.6, n = ?
We know that $a_n = a + (n – 1)d$
$3.6 = – 18.9 + (n – 1)2.5$
or, $2.5(n – 1) = 3.6 + 18.9 = 22.5$
$n – 1 = 22.5/2.5 = 9$
$n = 9 + 1 = 10$

v. Given: a = 3.5, d = 0, n = 105, a_n= ?
We know $a_n = a + (n - 1)d$
$a_{105} = 3.5 + (105 - 1)0$
$a_{105} = 3.5 + 0$
$a_{105} = 3.5$

a. Find the missing variable from a, d, n and a_n, where a is the first term, d is the common difference and a_n is the nth term of AP.

(i) a = 7, d = 3, n = 8

(ii) a = –18, n = 10, $a_n = 0$

(iii) d = –3, n = 18, $a_n = -5$

(iv) a = –18.9, d = 2.5, $a_n = 3.6$

(v) a = 3.5, d = 0, n = 105

Ans. (i) a = 7, d = 3, n = 8

We need to find a_n here.

Using formula $a_n = a + (n-1)d$

Putting values of a, d and n,

=7+(8−1)3

=7+(7)3=7+21=28

⇒ a = –18, n= 10, $a_n = 0$

We need to find d here.

Using formula $a_n = a + (n-1)d$

Putting values of a, a_n and n,
0 = –18 + (10 – 1) d

(ii) 0 = −18 + 9*d*

(iii) 18 = 9*d* ⇒ *d* = 2

(ii) d = –3, n = 18, $a_n = -5$

We need to find a here.

Using formula $a_n = a + (n-1)d$

Putting values of d, a_n and n,

–5 = a + (18 – 1) (–3)

(i) −5 = *a* + (17) (−3)

(ii) −5 = *a* – 51 ⇒ *a* = 46

(iii) a = –18.9, d = 2.5, $a_n = 3.6$

We need to find n here.

Using formula $a_n = a + (n-1)d$

Putting values of d, a_n and n,

3.6 = –18.9 + (n – 1) (2.5)

(i) $3.6 = -18.9 + 2.5n - 2.5$

(ii) $2.5n = 25 \Rightarrow n = 10$

(iii) a = 3.5, d = 0, n = 105

We need to find a_n here.

Using formula $a_n = a + (n-1)d$

Putting values of d, n and a,

$a_n = 3.5 + (105 - 1)(0)$

$\Rightarrow a_n = 3.5 + 104 \times 0$

$\Rightarrow a_n = 3.5 + 0 \Rightarrow a_n = 3.5$

2.Choose the correct choice in the following and justify:

(i) 30th term of the AP: 10, 7, 4... is

(A) 97

(B) 77

(C) –77

(D) –87

(ii) 11th term of the AP: −3, −12, 2... is

(A) 28

(B) 22

(C) –38

(D) $-48\frac{1}{2}$

Ans.(i) 10, 7, 4...

First term = a = 10, Common difference = d = 7 – 10 = 4 – 7 = –3

And n = 30{Because, we need to find 30^{th} term}

$a_n = a + (n - 1)d$

(ii) a_{30} =10+(30−1)(−3)=10–87=−77

Therefore, the answer is (C).

(i) −3, −½, 2...

First term = a = –3, Common difference = d = − $\frac{1}{2}$ − (−3) = 2 − (− $\frac{1}{2}$) = $\frac{5}{2}$

And n = 11 (Because, we need to find 11th term)

a_n =–3+(11–1) $\frac{5}{2}$ =–3+25=22

Therefore 11th term is 22 which means answer is (B).

(ii) In the following AP's find the missing terms:

(i) 2, __ , 26 (ii)

__, 13, __, 3

(i) 5, __, __, $9\frac{1}{2}$

(ii) –4. __, __, __, __, 6

(iii) __, 38, __, __, __, –22

Ans. (i) 2, __ , 26

We know that difference between consecutive terms is equal in any A.P. Let the

missing term be x.

$x - 2 = 26 - x$

(iii) $2x = 28 \Rightarrow x = 14$

Therefore, missing term is 14.

(ii)__, 13, __, 3

Let missing terms be x and y.

The sequence becomes x, 13, y, 3

We know that difference between consecutive terms is constant in any A.P. $y - 13 = 3 -$

y

(iv) $2y = 16 \Rightarrow y =$

8 And $13 - x = y -$

13

(v) $x + y = 26$

But, we have $y = 8$,

(v) $x + 8 = 26 \Rightarrow x = 18$

Therefore, missing terms are 18 and 8.

(i) 5, __, __, $9\frac{1}{2}$

Here, first term = a = 5 And, 4th term = $a_4 = 9\frac{1}{2}$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$a_4 = 5 + (4 - 1)\, d$

(ii) $\frac{19}{2} = 5 + 3d$

(iii) 19 = 2 (5 + 3d)

(iv) 19 = 10 + 6d

(v) 6d = 19 – 10

(iii) $6d = 9 \Rightarrow d = \frac{3}{2}$

Therefore, we get common difference = d = $\frac{3}{2}$

Second term = a + d = $5+\frac{3}{2}=\frac{13}{2}$

Third term = second term + d = $\frac{13}{2}+\frac{3}{2}=\frac{16}{2}=8$

Therefore, missing terms are $\frac{13}{2}$ and 8

(iv)–4. __, __, __, __, 6

Here, First term = a = –4 and 6th term = $a_6 = 6$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$a_6 = -4 + (6-1)d$

(iii) $6 = -4 + 5d$

(iv) $5d = 10 \Rightarrow d = 2$

Therefore, common difference = d = 2

Second term = first term + d = a + d = –4 + 2 = –2

Third term = second term + d = –2 + 2 = 0

Fourth term = third term + d = 0 + 2 = 2

Fifth term = fourth term + d = 2 + 2 = 4

Therefore, missing terms are –2, 0, 2 and 4.

(iv) __, 38, __, __, __, –22

We are given 2nd and 6th term.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, $a_2 = a + (2-1)d$ and $a_6 = a + (6-1)d$

iv) $38 = a + d$ and $-22 = a + 5d$

These are equations in two variables, we can solve them using any method.

Using equation ($38 = a + d$), we can say that $a = 38 - d$.

Putting value of a in equation ($-22 = a + 5d$),

$-22 = 38 - d + 5d$

(i) $4d = -60$

(ii) $d = -15$

Using this value of d and putting this in equation $38 = a + d$, $38 = a - 15 \Rightarrow a = 53$

Therefore, we get $a = 53$ and $d = -15$

First term = a = 53

Third term = second term + d = 38 – 15 = 23

Fourth term = third term + d = 23 – 15 = 8

Fifth term = fourth term + d = 8 – 15 = –7

Therefore, missing terms are 53, 23, 8 and –7.

4. Which term of the AP: 3, 8, 13, 18 ... is 78?

Ans. First term = a = 3, Common difference = d = 8 – 3 = 13 – 8 = 5 and $a_n = 78$

Using formula

a_n = 3 + (*n* − 1) 5,

78 = 3 + (n-1) 5

75 = 5*n* − 5

80 = 5*n*⇒ *n* = 16

It means 16th term of the given AP is equal to 78.

(v) Find the number of terms in each of the following APs:

(i) 7, 13, 19...., 205

(ii) 18, $15\frac{1}{2}$, 13..., −47

Ans. (i) 7, 13, 19 ..., 205

First term = a = 7, Common difference = d = 13 – 7 = 19 – 13 = 6 And

$a_n = 205$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, 205 = 7 +

(*n* − 1) 6 = 7 + 6*n* – 6

(i) 205 = 6*n* + 1

(ii) 204 = 6*n*⇒ *n* = 34

Therefore, there are 34 terms in the given arithmetic progression.

(ii) 18, $15\frac{1}{2}$, 13 ..., −47

First term = a =18, Common difference = d = $15\frac{1}{2}-18=\frac{31}{2}-18=\frac{31-36}{2}=\frac{-5}{2}$

And $a_n = -47$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$

$= 36 - \frac{5}{2}n + \frac{5}{2}$

(ii) $-94 = 36 - 5n + 5$

(iii) $5n = 135 \Rightarrow n = 27$

Therefore, there are 27 terms in the given arithmetic progression.

6. Check whether –150 is a term of the AP: 11, 8, 5, 2...

Ans. Let −150 is the n^{th} of AP 11, 8, 5, 2... which means that $a_n = -150$ Here,

First term = a = 11, Common difference = d = 8 – 11 = –3

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$-150 = 11 + (n-1)(-3)$

(iii) $-150 = 11 - 3n + 3$

(iv) $3n = 164 \Rightarrow n = \frac{164}{3}$

But, n cannot be in fraction.

Therefore, our supposition is wrong. −150 cannot be term in AP.

x **Find the 31st term of an AP whose 11th term is 38 and 16th term is 73. Ans.** Here

$a_{11} = 38 \, and \, a_{16} = 73$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

38 = *a* + (11 − 1) (*d*)And73 = *a* + (16 − 1) (*d*)

y 38 = *a* + 10*d* And 73 = *a* + 15*d*

These are equations consisting of two variables.

We have, 38 = *a* + 10*d*

x *a* = 38 − 10*d*

Let us put value of a in equation (73 = *a* + 15*d*),

73 = 38 − 10*d* + 15*d*

4035 = 5*d*

Therefore, Common difference = *d* = 7

Putting value of *d* in equation 38 = *a* + 10*d*,

38 = *a* + 70

40*a* = −32

Therefore, common difference = d = 7 and First term = a = –32

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

a_{31} = −32 + (31 − 1) (7)

= −32 + 210 = 178

Therefore, 31st term of AP is 178.

8. **An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.**

Ans. An AP consists of 50 terms and the 50th term is equal to 106 and $a_3 = 12$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, a_{50} = *a* + (50 − 1) *d* And a_3 = *a* + (3 − 1) *d*

= 106 = *a* + 49*d* And12 = *a* + 2*d*

12 = 106 − 49*d* + 2*d*

= 47*d* = 94 ⇒ *d* = 2

Putting value of *d* in the equation, *a* = 106 − 49*d*, *a* = 106 – 49 (2) = 106 – 98 = 8

Therefore, First term = *a* = 8 and Common difference = *d* = 2

To find 29th term, we use formula $a_n = a + (n-1)d$ which is used to find nth term of arithmetic progression,

a_{29} =8+(29−1)2=8+56=64

Therefore, 29th term of AP is equal to 64.

- **If the third and the ninth terms of an AP are 4 and –8 respectively, which term of this AP is zero?**

Ans. It is given that 3rd and 9th term of AP are 4 and –8 respectively.

It means $a_3 = 4 and\ a_9 = -8$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

4 = a + (3 – 1) d And, –8 = a + (9 – 1) d

⇒ 4 = *a* + 2*d* *a*nd −8 = *a* + 8*d*

These are equations in two variables.

Using equation 4 = *a* + 2*d*, we can say that *a* = 4 − 2*d*

Putting value of a in other equation $-8 = a + 8d$, $-8 = 4 - 2d$

$+ 8d$

(i) $-12 = 6d \Rightarrow d = -2$

Putting value of d in equation $-8 = a + 8d$,

$-8 = a + 8(-2)$

(iv) $-8 = a - 16 \Rightarrow a = 8$

Therefore, first term = $a = 8$ and Common Difference = $d = -2$ We

want to know which term is equal to zero.

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$0 = 8 + (n-1)(-2)$

$0 = 8 - 2n + 2$

$0 = 10 - 2n$

$2n = 10 \Rightarrow n = 5$

Therefore, 5^{th} term is equal to 0.

10. **The 17^{th} term of an AP exceeds its 10^{th} term by 7. Find the common difference.**

Ans. $a_{17} = a_{10} + 7 \ldots$ (1)

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a_{17} = a + 16d \ldots$ (2)

$a_{10} = a + 9d \ldots$ (3)

Putting (2) and (3) in equation (1),

- $+ 16d = a + 9d + 7$

$\Rightarrow 7d = 7$

$\Rightarrow d = 1$

11. Which term of the AP: 3, 15, 27, 39... will be 132 more than its 54^{th} term?

Ans. Lets first calculate 54th of the given AP.

First term = a = 3, Common difference = d = 15 – 3 = 12

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, a_{54} = a + (54 − 1) d = 3 + 53 (12) = 3 + 636 = 639

We want to find which term is 132 more than its 54th term.

Let us suppose it is nth term which is 132 more than 54th term.

$a_n = a_{54} + 132$

⇒ 3 + (n − 1) 12 = 639 + 132

⇒ 3 + 12n – 12 = 771

⇒ 12n – 9 = 771

⇒ 12n = 780

⇒ n = 65

Therefore, 65th term is 132 more than its 54th term.

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12. Two AP's have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms.

Ans. Let first term of 1st AP = a

Let first term of 2nd AP = a'

It is given that their common difference is same.

Let their common difference be d.

It is given that difference between their 100th terms is 100.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

a + (100 − 1) d – [a' + (100 − 1) d]

= a + 99d − a' − 99d = 100

⇒ $a - a'$ = 100… (1)

We want to find difference between their 1000th terms which means we want to calculate:

$a + (1000 - 1)\,d - [a' + (1000 - 1)\,d] =$

$a + 999d - a' - 999d = a - a'$

Putting equation (1) in the above equation,

$a + (1000 - 1)\,d - [a' + (1000 - 1)\,d]$
$a \;= a + 999d - a' + 999d = a - a' = 100$

Therefore, difference between their 1000th terms would be equal to 100.

13. How many three digit numbers are divisible by 7?

Ans. We have AP starting from 105 because it is the first three digit number divisible by 7.

AP will end at 994 because it is the last three digit number divisible by 7.

Therefore, we have AP of the form 105, 112, 119..., 994

Let 994 is the nth term of AP.

We need to find n here.

First term = a = 105, Common difference = d = 112 – 105 = 7

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

$994 = 105 + (n - 1)(7)$

$\Rightarrow 994 = 105 + 7n - 7$

$\Rightarrow 896 = 7n \Rightarrow n = 128$

It means 994 is the 128th term of AP.

Therefore, there are 128 terms in AP.

14. How many multiples of 4 lie between 10 and 250?

Ans. First multiple of 4 which lie between 10 and 250 is 12. The last

multiple of 4 which lie between 10 and 250 is 248. Therefore, AP is of the form 12, 16, 20... ,248

First term = a = 12, Common difference = d = 4

Using formula $a_n = a + (n - 1)\, d$, to find n^{th} term of arithmetic progression,

$248 = 12 + (n - 1)(4)$

$\Rightarrow 248 = 12 + 4n - 4$

$\Rightarrow 240 = 4n$

$\Rightarrow n = 60$

It means that 248 is the 60^{th} term of AP.

So, we can say that there are 60 multiples of 4 which lie between 10 and 250.

15. For what value of n, are the nth terms of two AP's: 63, 65, 67... and 3, 10, 17... equal? Ans. Lets

first consider AP 63, 65, 67...

First term = a = 63, Common difference = d = 65 – 63 = 2

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,
$a_n = 63 + (n - 1)(2)$… (1)

Now, consider second AP 3, 10, 17...

First term = a = 3, Common difference = d = 10 – 3 = 7

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,
$a_n = 3 + (n - 1)(7)$… (2)

According to the given condition:

(1) = (2)

$\Rightarrow 63 + (n - 1)(2) = 3 + (n - 1)(7)$

$\Rightarrow 63 + 2n - 2 = 3 + 7n - 7$
$\Rightarrow 65 = 5n \Rightarrow n = 13$

Therefore, 13^{th} terms of both the AP's are equal.

16. Determine the AP whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

Ans. Let first term of AP = a

Let common difference of AP = d

It is given that its 3^{rd} term is equal to 16.

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$16 = a + (3 - 1)(d)$

$\Rightarrow 16 = a + 2d$... (1)

It is also given that 7^{th} term exceeds 5^{th} term by12.

According to the given condition:

$a_7 = a_5 + 12$

$\Rightarrow a + (7 - 1)d = a + (5 - 1)d + 12$

$\Rightarrow 2d = 12 \Rightarrow d = 6$

Putting value of d in equation $16 = a + 2d$,

$16 = a + 2(6) \Rightarrow a = 4$

Therefore, first term = a = 4

And, common difference = d = 6

Therefore, AP is 4, 10, 16, 22...

17. Find the 20th term from the last term of the AP: 3, 8, 13... , 253.

Ans. We want to find 20th term from the last term of given AP.

So, let us write given AP in this way:253 ... 13, 8, 3

Here First term = a = 253, Common Difference = d = 8 – 13 = –5

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

a_{20} = 253 + (20 − 1) (−5)

⇒ a_{20} = 253 + 19 (−5) = 253 – 95 = 158

Therefore, the 20th term from the last term of given AP is 158.

18. The sum of the 4th and 8th terms of an AP is 24 and the sum of 6th and 10th terms is 44. Find the three terms of the AP.

Ans. The sum of 4th and 8th terms of an AP is 24 and sum of 6th and 10th terms is 44.

$a_4 + a_8 = 24$

and $a_6 + a_{10} = 44$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, => $a + (4 - 1) d + [a + (8 - 1) d] = 24$

And, $a + (6 - 1) d + [a + (10 - 1) d] = 44$

⇒ $a + 3d + a + 7d = 24$

And $a + 5d + a + 9d = 44$

⇒ $2a + 10d = 24$ And $2a + 14d = 44$

⇒ $a + 5d = 12$ And $a + 7d = 22$

These are equations in two variables.

Using equation, $a + 5d = 12$, we can say that $a = 12 - 5d$... (1)

Putting (1) in equation $a + 7d = 22$,

$12 - 5d + 7d = 22$

$\Rightarrow 12 + 2d = 22$

$\Rightarrow 2d = 10$

$\Rightarrow d = 5$

Putting value of d in equation $a = 12 - 5d$,

$a = 12 – 5 (5) = 12 – 25 = -13$

Therefore, first term = $a = -13$ and, Common difference = $d = 5$

Therefore, AP is –13, –8, –3, 2...

Its first three terms are –13, –8 and –3.

19. Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Ans. Subba Rao's starting salary = Rs 5000

It means, first term = a = 5000

He gets an increment of Rs 200 after every year.

Therefore, common difference = d = 200

His salary after 1 year = 5000 + 200 = Rs 5200

His salary after two years = 5200 + 200 = Rs 5400

Therefore, it is an AP of the form 5000, 5200, 5400, 5600... , 7000

We want to know in which year his income reaches Rs 7000.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression, 7000 = 5000 + (n − 1) (200)

⇒ 7000 = 5000 + 200n − 200

⇒ 7000 – 5000 + 200 = 200n

⇒ 2200 = 200n

⇒ n = 11

It means after 11 years, Subba Rao's income would be Rs 7000.

20. Ramkali saved Rs. 5 in the first week of a year and then increased her weekly savings by Rs. 1.75. If in the nth week, her weekly savings become Rs 20.75, find n.

Ans. Ramkali saved Rs. 5 in the first week of year. It means first term = a = 5

Ramkali increased her weekly savings by Rs 1.75.

Therefore, common difference = d = Rs 1.75

Money saved by Ramkali in the second week = a + d = 5 + 1.75 = Rs 6.75

Money saved by Ramkali in the third week = 6.75 + 1.75 = Rs 8.5

Therefore, it is an AP of the form:5, 6.75, 8.5 ... , 20.75

We want to know in which week her savings become 20.75.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression, 20.75 = 5 + (n − 1) (1.75)

⇒ 20.75 = 5 + 1.75n − 1.75

⇒ 17.5 = 1.75n

⇒ n = 10

It means in the 10^{th} week her savings become Rs 20.75

Chapter - 5

Arithmetic Progressions

Exercise 5.3

1. **Find the sum of the following AP's.**

(i) 2, 7, 12... to 10 terms

(ii) –37, –33, –29... to 12 terms (iii)

0.6, 1.7, 2.8... to 100 terms

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10} \ldots$ **to 11 terms**

Ans. (i) 2, 7, 12... to 10 terms

Here First term = a = 2, Common difference = d = 7 – 2 = 5 and n = 10

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP,

$$S_n = \frac{10}{2}\left[4 + (10-1)5\right] = 5(4+45) = 5 \times 49 = 245$$

(vi) –37, –33, –29... to 12 terms

Here First term = a = –37, Common difference = d = –33 – (–37) = 4 And n

= 12

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP

$$S_n = \frac{12}{2}\left[-74 + (12-1)4\right] = 6(-74+44) = 6 \times (-30) = -180$$

⇒ 0.6, 1.7, 2.8... to 100 terms

Here First term = a = 0.6, Common difference = d = 1.7 – 0.6 = 1.1 And n =

100

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP,

$$Sn = \frac{100}{2}\left[1.2 + (100-1)\,1.1\right] = 50\,(1.2 + 108.9) = 50 \times 110.1 = 5505$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10} \ldots$ to 11 terms

Here First term =a = $\frac{1}{15}$ Common difference = d = $\frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP,

$$S_n = \frac{11}{2}\left[\frac{2}{15} + (11-1)\frac{1}{60}\right] = \frac{11}{2}\left(\frac{2}{15} + \frac{1}{6}\right) = \frac{11}{2}\left(\frac{4+5}{30}\right) = \frac{11}{2} \times \frac{9}{30} = \frac{33}{20}$$

2. **Find the sums given below:**

(i) $7 + 10\frac{1}{2} + 14 + \ldots + 84$

(ii) 34+32+30+…+10

(i) –5 + (–8) + (–11) + ... + (–230)

Ans. (i) $7 + 10\frac{1}{2} + 14 + \ldots + 84$

Here First term = a = 7, Common difference = d = $\frac{21}{2} - 7 = \frac{21-14}{2} = \frac{7}{2} = 3.5$

And Last term = *l* = 84

We do not know how many terms are there in the given AP.

So, we need to find n first.

Using formula $a_n = a + (n-1)d$ to find n^{th} term of arithmetic progression,

$= 7 + (3.5)n - 3.5 = 84$

$= 3.5n = 84 + 3.5 - 7$

$= 3.5n = 80.5$

$n = 23$

Therefore, there are 23 terms in the given AP.

It means n = 23.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP,

$$S_{23} = \frac{23}{2}(7 + 84)$$

$$\Rightarrow S_{23} = \frac{23}{2} \times 91 = 1046.5$$

(ii) 34+32+30+...+10

Here First term = a = 34, Common difference = d = 32 – 34 = –2

And Last term = l = 10

We do not know how many terms are there in the given AP.

So, we need to find n first.

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$[34 + (n - 1)(-2)] = 10$

$\Rightarrow 34 - 2n + 2 = 10$

$-2n = -26 \Rightarrow n = 13$

Therefore, there are 13 terms in the given AP.

It means n = 13.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP,

$$S_{13} = \frac{13}{2}(34 + 10) = \frac{13}{2} \times 44 = 286$$

(iii) $-5 + (-8) + (-11) + ... + (-230)$

Here First term = a = –5, Common difference = d = –8 – (–5) = –8 + 5 = –3 And

Last term = $l = -230$

We do not know how many terms are there in the given AP.

So, we need to find n first.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$[-5 + (n - 1)(-3)] = -230$

$-5 - 3n + 3 = -230$

$-3n = -228 \Rightarrow n = 76$

Therefore, there are 76 terms in the given AP.

It means n = 76.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP,

$$S_{76} = \frac{76}{2}(-5-230) = 38 \times (-235) = -8930$$

3. In an AP

(iii) given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n.

(iv) given $a = 7$, $a_{13} = 35$, find d and S_{13}.

(v) given $a_{12} = 37$, $d = 3$, find a and S_{12}.

(vi) given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.

(vii) given $d = 5$, $S_9 = 75$, find a and a_9.

(viii) given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n.

(ix) given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d.

(x) given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a.

(xi) given $a = 3$, $n = 8$, $S = 192$, find d.

(xii) given $l = 28$, $S = 144$, and there are total of 9 terms. Find a.

Ans. (i) Given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$a_n = 5 + (n-1)(3)$

$50 = 5 + 3n - 3$

$48 = 3n \Rightarrow n = 16$

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP,

$$S_{16} = \frac{16}{2}[10+(16-1)3] = 8(10+45) = 8\times 55 = 440$$

Therefore, n = 16 and $S_n = 440$

(ii) Given a = 7, $a_{13} = 35$, find d and S_{13}.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

a_{13} = 7 + (13 − 1) (d)

35 = 7 + 12d

28 = 12d ⇒ d = $\frac{7}{3}$

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP,

$$S_{13} = \frac{13}{2}\left[14+(13-1)\frac{7}{3}\right] = \frac{13}{2}(14+28) = \frac{13}{2}\times 42 = 273$$

Therefore, d = $\frac{7}{3}$ and $S_{13} = 273$

(iii) Given $a_{12} = 37$, d = 3, find a and S_{12}.

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

a_{12} = a + (12 − 1) 3

37 = a + 33 ⇒ a = 4

Applying formula, $S_n = \frac{n}{2}[2a+(n-1)d]$ to find sum of n terms of AP,

$$S_{12} = \frac{12}{2}[8+(12-1)3] = 6(8+33) = 6\times 41 = 246$$

Therefore, a = 4 and S_{12} = 246

(iv) Given $a_3 = 15, S_{10} = 125$, find d and a_{10}.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$a_3 = a + (3 - 1)(d)$

$15 = a + 2d$

$a = 15 - 2d \ldots (1)$

Applying formula, $a_n = a + (n-1)d$ to find sum of n terms of AP,

$$S_{10} = \frac{10}{2}[2a + (10-1)d]$$

$125 = 5(2a + 9d) = 10a + 45d$

Putting (1) in the above equation,

$125 = 5[2(15 - 2d) + 9d] = 5(30 - 4d + 9d)$

$125 = 150 + 25d$

$125 - 150 = 25d$

$-25 = 25d \Rightarrow d = -1$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, $a_{10} = a + (10 - 1)d$

Putting value of d and equation (1) in the above equation, $a_{10} = 15 - 2d + 9d = 15 + 7d$

$= 15 + 7(-1) = 15 - 7 = 8$

Therefore, $d = -1$ and $a_{10} = 8$

(v) Given $d = 5$, $S_9 = 75, find\ a\ and\ a_9$.

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP,

$$S_9 = \frac{9}{2}\left[2a + (9-1)5\right]$$

$$\Rightarrow 75 = \frac{9}{2}\left[2a + 40\right]$$

$150 = 18a + 360$

$-210 = 18a$

$a = \frac{-35}{3}$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$a_9 = \frac{-35}{3} + (9-1)(5)$

$= \frac{-35}{3} + 40 = \frac{-35+120}{3} = \frac{85}{3}$

Therefore, $a = \frac{-35}{3}$ and $a_9 = \frac{85}{3}$

(vi) Given $a = 2$, $d = 8$, $S_n = 90, find\ n\ and\ a_n$.

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP,

$$90 = \frac{n}{2}\left[4 + (n-1)8\right]$$

$\Rightarrow 90 = \frac{n}{2}[4 + 8n - 8]$

$\Rightarrow 90 = \frac{n}{2}[8n - 4]$

$\Rightarrow 8n^2 - 4n - 180 = 0$

$\Rightarrow 2n^2 - n + 45 = 0$

$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$

$2n(n - 5) + 9(n - 5) = 0$

$(n - 5)(2n + 9) = 0$

$n = 5, -9/2$

We discard negative value of n because here n cannot be in negative or fraction.

The value of n must be a positive integer.

Therefore, $n = 5$

Using formula $a_n = a + (n - 1)d$,to find n^{th} term of arithmetic progression,
$a_5 = 2 + (5 - 1)(8) = 2 + 32 = 34$

Therefore, $n = 5$ and $a_n = 34$

(vii) Given $a = 8$, $a_n = 62, S_n = 210$, find n and d.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$62 = 8 + (n - 1)(d) = 8 + nd - d$

$62 = 8 + nd - d$

$nd - d = 54$

$nd = 54 + d$... (1)

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP,

$$210 = \frac{n}{2}[16 + (n-1)d] = \frac{n}{2}(16 + nd - d)$$

Putting equation (1) in the above equation,

$$210 = \frac{n}{2}[16 + 54 + d - d] = \frac{n}{2} \times 70$$

$\Rightarrow n = \frac{210 \times 2}{70} = 6 \Rightarrow n = 6$

Putting value of n in equation (1),

$6d = 54 + d \Rightarrow d = \frac{54}{5}$

Therefore, $n = 6$ and $d = \frac{54}{5}$

(viii) Given $a_n = 4, d = 2, S_n = -14$, find n and a.

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$4 = a + (n - 1)(2) = a + 2n - 2$

$4 = a + 2n - 2$

$6 = a + 2n$

$a = 6 - 2n$... (1)

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP,

$$-14 = \frac{n}{2}[2a + (n-1)2] = \frac{n}{2}(2a + 2n - 2)$$

$\Rightarrow$ $-14 = \frac{n}{2}(2a + 2n - 2)$

Putting equation (1) in the above equation, we get −28 = *n*

[2 (6 − 2*n*) + 2*n* – 2]

x −28 = *n* (12 − 4*n* + 2*n* − 2)

x −28 = *n* (10 − 2*n*)

$\Rightarrow$ $2n^2 - 10n - 28 = 0$

$\Rightarrow$ $n^2 - 5n - 14 = 0$

$\Rightarrow$ $n^2 - 7n + 2n - 14 = 0$

y *n* (*n* − 7) + 2 (*n* − 7) = 0

y (*n* + 2) (*n* − 7) = 0

y *n* = −2, 7

Here, we cannot have negative value of *n*.

Therefore, we discard negative value of *n* which means *n* = 7.

Putting value of *n* in equation (1), we get

a = 6 − 2*n* = 6 – 2 (7) = 6 – 14 = −8

Therefore, *n* = 7 and *a* = −8

(ix) Given *a* = 3, *n* = 8, *S* = 192, find d.

Using formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$192 = \frac{8}{2} [6 + (8 - 1) d] = 4 (6 + 7d)$

x 192 = 24 + 28d

x 168 = 28d ⇒ d = 6

(x) Given l = 28, S = 144, and there are total of 9 terms. Find a.

Applying formula, $S_n = \frac{n}{2}[a + l]$, to find sum of n terms, we get

$144 = \frac{9}{2} [a + 28]$

41288 = 9 [a + 28]

4232 = a + 28⇒ a = 4

4. How many terms of the AP: 9, 17, 25, ... must be taken to give a sum of 636? Ans. First term = a = 9, Common difference = d = 17 – 9 = 8, S_n = 636

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$636 = \frac{n}{2}[18 + (n - 1) (8)]$

= 1272 = n (18 + 8n − 8)

⇒ $1272 = 18n + 8n^2 - 8n$

⇒ $8n^2 + 10n - 1272 = 0$

⇒ $4n^2 + 5n - 636 = 0$

Comparing equation $4n^2 + 5n - 636 = 0$ with general form $an^2 + bn + c = 0$, we get

$a = 4$, $b = 5$ and $c = -636$

Applying quadratic formula, $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and putting values of a, b and c, we get

$$n = \frac{-5 \pm \sqrt{5^2 - 4(4)(-636)}}{8}$$

$\Rightarrow n = \frac{-5 \pm \sqrt{25 + 10176}}{8}$

$\Rightarrow n = \frac{-5 \pm 101}{8}$

$\Rightarrow n = \frac{96}{8}, \frac{-106}{8} = 12, -\frac{106}{8}$

We discard negative value of *n* here because n cannot be in negative, n can only be a positive integer.

Therefore, $n = 12$

Therefore, 12 terms of the given sequence make sum equal to 636.

11. **The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.**

Ans. First term = a = 5, Last term = l = 45, $S_n = 400$

Applying formula, $S_n = \frac{n}{2}\left[a + l\right]$ to find sum of n terms of AP, we get

$$400 = \frac{n}{2}[5 + 45]$$

$\Rightarrow \frac{400}{50} = \frac{n}{2} \Rightarrow$ n = 16

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP and putting value of n, we get

$$400 = \frac{16}{2}[10 + (16-1)d]$$

= 400 = 8 (10 + 15*d*)

= 400 = 80 + 120*d*

= 320 = 120*d*

= $d = \frac{320}{120} = \frac{8}{3}$

12. **The first and the last terms of an AP are 17 and 350 respectively. If, the common difference is 9, how many terms are there and what is their sum?**

Ans. First term = a = 17, Last term = l = 350 and Common difference = d = 9

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we get 350 = 17 + (n − 1) (9)

⇒ 350 = 17 + 9n − 9

⇒ 342 = 9n ⇒ n = 38

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP and putting value of n, we get

$$S_{38} = \frac{38}{2}[34 + (38-1)9]$$

S_{38} = 19 (34 + 333) = 6973

Therefore, there are 38 terms and their sum is equal to 6973.

13. **Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.**

Ans. It is given that 22nd term is equal to 149 $\Rightarrow a_{22} = 149$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we get

$149 = a + (22 - 1)(7)$

$149 = a + 147 \Rightarrow a = 2$

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP and putting value of a, we get

$$S_{22} = \frac{22}{2}\left[4 + (22-1)7\right]$$

$S_{22} = 11(4 + 147)$

$S_{22} = 1661$

Therefore, sum of first 22 terms of AP is equal to 1661.

14. **Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.**

Ans. It is given that second and third term of AP are 14 and 18 respectively.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we get $14 = a + (2 - 1)d$

$\Rightarrow 14 = a + d$ ... (1)

And, $18 = a + (3 - 1)d$

$\Rightarrow 18 = a + 2d$ ... (2)

These are equations consisting of two variables.

Using equation (1), we get, $a = 14 - d$

Putting value of a in equation (2), we get

$18 = 14 - d + 2d$

$\Rightarrow d = 4$

Therefore, common difference $d = 4$

Putting value of d in equation (1), we get

$18 = a + 2\,(4)$

$\Rightarrow a = 10$

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{51} = \frac{51}{2}\left[20 + (51-1)d\right] = \frac{51}{2}(20 + 200) = \frac{51}{2} \times 220 = 51 \times 110 = 5610$$

Therefore, sum of first 51 terms of an AP is equal to 5610.

15. **If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.**

Ans. It is given that sum of first 7 terms of an AP is equal to 49 and sum of first 17 terms is equal to 289.

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP, we get

$$49 = \frac{7}{2}\left[2a + (7-1)d\right]$$ 49

$\Rightarrow 98 = 7\,(2a + 6d)$

$\Rightarrow 7 = a + 3d \Rightarrow a = 7 - 3d$ … (1)

And, $289 = \frac{17}{2}[2a + (17-1)d]$

578 = 17 (2a + 16d)

34 = 2a + 16d

17 = a + 8d

Putting equation (1) in the above equation, we get

17d = 7-3d + 8d

10 = 5d ⇒ d = 2

Putting value of d in equation (1), we get

$a = 7 - 3d = 7 - 3(2) = 7 - 6 = 1$

Again applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_n = \frac{n}{2}[2(1) + (n-1)2]$$

$$\Rightarrow S_n = \frac{n}{2}[2 + 2n - 2]$$

$$\Rightarrow S_n = \frac{n}{2} \times 2n \Rightarrow S_n = n^2$$

Therefore, sum of n terms of AP is equal to n^2.

16. **Show that $a_1, a_2 \ldots a_n$ form an AP where a_n is defined as below:**

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Ans. (i) We need to show that $a_1, a_2 \ldots a_n$ form an AP where $a_n = 3 + 4n$

Let us calculate values of $a_1, a_2, a_3 \ldots$ using $a_n = 3 + 4n$

$a_1 = 3 + 4(1) = 3+4=7$

$a_2 = 3 + 4(2) = 3 + 8 = 11$

$a_3 = 3 + 4(3) = 3+12=15$

$a_4 = 3 + 4(4) = 3 + 16 = 19$

So, the sequence is of the form 7, 11, 15, 19 ...

Let us check difference between consecutive terms of this sequence.

11–7=4,15–11=4,19–15=4

Therefore, the difference between consecutive terms is constant which means terms a_1, a_2 ... a_n form an AP.

We have sequence 7, 11, 15, 19 ...

First term = a = 7 and Common difference = d = 4

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_{15} = \frac{15}{2}[14 + (15-1)4] = \frac{15}{2}(14+56) = \frac{15}{2} \times 70 = 15 \times 35 = 525$$

Therefore, sum of first 15 terms of AP is equal to 525.

(ii) We need to show that $a_1, a_2 \ldots a_n$ form an AP where $a_n = 9 - 5n$

Let us calculate values of $a_1, a_2, a_3 \ldots$ using $a_n = 9 - 5n$

$a_1 = 9 - 5(1) = 9-5 = 4$ $a_2 = 9 - 5(2) = 9 - 10 = -1$

$a_3 = 9 - 5(3) = 9-15=-6$ $a_4 = 9 - 5(4) = 9 - 20 = -11$

So, the sequence is of the form 4, −1, −6, −11 ...

Let us check difference between consecutive terms of this sequence.

$-1 - (4) = -5, -6 - (-1)$

$\Rightarrow -6 + 1 = -5, -11 - (-6)$

$\Rightarrow -11+6=-5$

Therefore, the difference between consecutive terms is constant which means terms $a_1, a_2 ... a_n$ form an AP.

We have sequence 4, −1, −6, −11 ...

First term = a = 4 and Common difference = d = –5

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP , we get

$$S_{15} = \frac{15}{2}[8 + (15-1)(-5)] = \frac{15}{2}(8-70) = \frac{15}{2}\times(-62) = 15\times(-31) = -465$$

Therefore, sum of first 15 terms of AP is equal to –465.

11. If the sum of the first n terms of an AP is $(4n - n^2)$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

Ans. It is given that the sum of n terms of an AP is equal to $(4n - n^2)$

It means $S_n = 4n - n^2$

Let us calculate $S_1 and S_2$ using $S_n = 4n - n^2$

$S_1 = 4(1) - (1)^2$ =4–1=3

$S_2 = 4(2) - (2)^2$ =8–4=4

First term = a = = 3 … (1)

Let us find common difference now.

We can write any AP in the form of general terms like a , a + d, a + 2d ...

We have calculated that sum of first two terms is equal to 4 i.e. $S_2 = 4$

Therefore, we can say that a + (a + d) = 4

Putting value of a from equation (1), we get

2a + d = 4

2 (3) + d = 4

6 + d = 4

d = –2

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

Second term of AP = a_2 = a + (2 – 1) d = 3 + (2 – 1) (–2) = 3 – 2 = 1

Third term of AP = a_3 = a + (3 – 1) d = 3 + (3 – 1) (–2) = 3 – 4 = –1

Tenth term of AP = a_{10} = a + (10 – 1) d = 3 + (10 – 1) (–2) = 3 – 18 = –15

n^{th} term of AP = a_n = a + (n – 1) d = 3 + (n – 1) (–2) = 3 – 2n + 2 = 5 – 2n

12. Find the sum of the first 40 positive integers divisible by 6.

Ans. The first 40 positive integers divisible by 6 are 6, 12, 18, 24 ... 40 terms.

Therefore, we want to find sum of 40 terms of sequence of the form:

6, 12, 18, 24 ... 40 terms

Here, first term = a = 6 and Common difference = d = 12 – 6 = 6, n = 40

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{40} = \frac{40}{2}[12+(40-1)6]$$

= $20\ (12 + 39 \times 6)$

= 20 (12 + 234)

= $20 \times 246 = 4920$

13. Find the sum of the first 15 multiples of 8.

Ans. The first 15 multiples of 8 are 8,16, 24, 32 ... 15 terms

First term = a = 8 and Common difference = d = 16 – 8 = 8, n = 15

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{15} = \frac{15}{2}[16+(15-1)8] = \frac{15}{2}(16+14\times 8) = \frac{15}{2}(16+112) = \frac{15}{2}\times 128 = 15\times 64 = 960$$

14. Find the sum of the odd numbers between 0 and 50. Ans.

The odd numbers between 0 and 50 are 1, 3, 5, 7 ... 49

It is an arithmetic progression because the difference between consecutive terms is constant. First term = a = 1, Common difference = 3 – 1 = 2, Last term = *l* = 49

We do not know how many odd numbers are present between 0 and 50. Therefore, we need to find n first.

Using formula $a_n = a + (n - 1) d$, to find nth term of arithmetic progression, we get

$49 = 1 + (n - 1) 2$

$\Rightarrow 49 = 1 + 2n - 2$

$\Rightarrow 50 = 2n \Rightarrow n = 25$

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP, we get

$$S_{25} = \frac{25}{2}(1+49) = \frac{25}{2} \times 50 = 25 \times 25 = 625$$

15. **A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?**

Ans. Penalty for first day = Rs 200, Penalty for second day = Rs 250 Penalty

for third day = Rs 300

It is given that penalty for each succeeding day is Rs 50 more than the preceding day.

It makes it an arithmetic progression because the difference between consecutive terms is constant.

We want to know how much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

So, we have an AP of the form200, 250, 300, 350 ... 30 terms First

term = a = 200, Common difference = d = 50, n = 30

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$to find sum of n terms of AP, we get

$$S_n = \frac{30}{2}[400 + (30-1)50]$$

$\Rightarrow$ $S_n = 15\ (400\ +\ 29 \times 50)$

S_n = 15 (400 + 1450) = 27750

Therefore, penalty for 30 days is Rs. 27750.

16. **A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If, each prize is Rs 20 less than its preceding term, find the value of each of the prizes.**

Ans. It is given that sum of seven cash prizes is equal to Rs 700.

And, each prize is R.s 20 less than its preceding term.

Let value of first prize = Rs. a

Let value of second prize =Rs (a − 20)

Let value of third prize = Rs (a − 40)

So, we have sequence of the form:

a, a − 20, a − 40, a – 60 ...

It is an arithmetic progression because the difference between consecutive terms is constant. First term = a,

Common difference = d = (a – 20) – a = –20 n = 7 (Because there are total of seven prizes)

S_7 = Rs 700{given}

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$S_7 = \frac{7}{2}\left[2a + (7-1)(-20)\right]$

$\Rightarrow 700 = \frac{7}{2}\left[2a - 120\right]$

⇒ 200 = 2a – 120

⇒ 320 = 2a ⇒ a = 160

Therefore, value of first prize = Rs 160

Value of second prize = 160 – 20 = Rs 140

Value of third prize = 140 – 20 = Rs 120

Value of fourth prize = 120 – 20 = Rs 100

Value of fifth prize = 100 – 20 = Rs 80

Value of sixth prize = 80 – 20 = Rs 60

Value of seventh prize = 60 – 20 = Rs 40

17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g, a section of Class I will plant 1 tree, a section of class II will plant two trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Ans. There are three sections of each class and it is given that the number of trees planted by any class is equal to class number.

The number of trees planted by class I = number of sections $\times 1 = 3 \times 1 = 3$ The number of trees planted by class II = number of sections $\times 2 = 3 \times 2 = 6$

The number of trees planted by class III = number of sections $\times 3 = 3 \times 3 = 9$

Therefore, we have sequence of the form 3, 6, 9 ... 12 terms

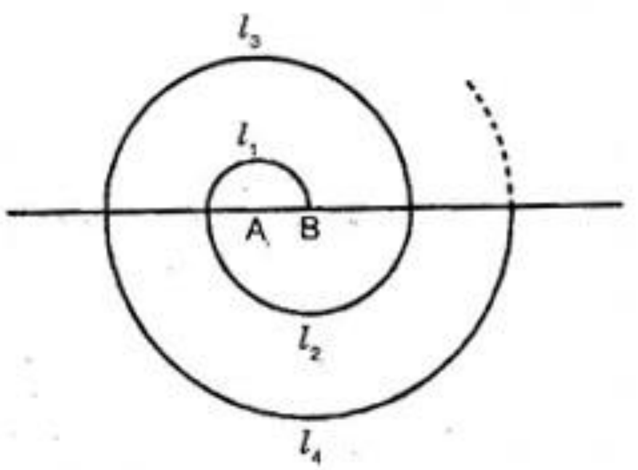
To find total number of trees planted by all the students, we need to find sum of the sequence 3, 6, 9, 12 ... 12 terms.

First term = a = 3, Common difference = d= 6 – 3 = 3 and n = 12

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP , we get

$$S_{12} = \frac{12}{2}\left[6+(12-1)3\right] = 6(6+33) = 6 \times 39 = 234$$

18. **A spiral is made up of successive semicircles, with centers alternatively at A and B, starting with center at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... What is the total length of such a spiral made up of thirteen consecutive semicircles.**



Ans. Length of semi–circle = $\frac{\text{Circumference of circle}}{2} = \frac{2\pi r}{2} = \pi r$

Length of semi-circle of radii 0.5 cm = π (0.5) cm

Length of semi-circle of radii 1.0 cm = π (1.0) cm

Length of semi-circle of radii 1.5 cm = π (1.5) cm

Therefore, we have sequence of the form:

π(0.5), π(1.0), π(1.5) ... 13 terms {There are total of thirteen semi–circles}.

To find total length of the spiral, we need to find sum of the sequence π(0.5), π(1.0), π(1.5) ... 13 terms

Total length of spiral = π(0.5) + π(1.0) + π(1.5) ... 13 terms

45. Total length of spiral = π (0.5 + 1.0 + 1.5) ... 13 terms … (1)

Sequence 0.5, 1.0, 1.5 ... 13 terms is an arithmetic progression. Let us

find the sum of this sequence.

First term = a = 0.5, Common difference = 1.0 – 0.5 = 0.5 and n = 13

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP, we get

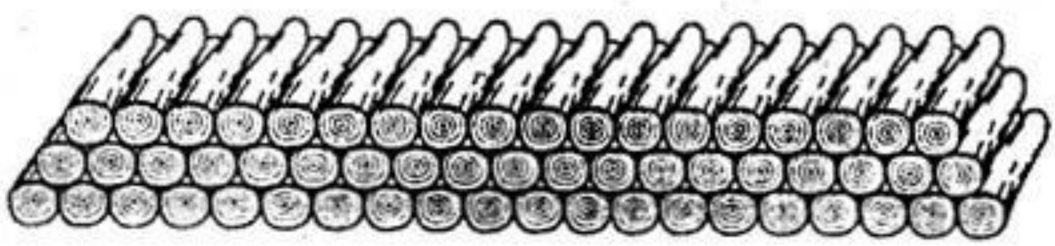
$$S_{13} = \frac{13}{2}\left[1 + (13-1)0.5\right] = 6.5(1+6) = 6.5 \times 7 = 45.5$$

Therefore, 0.5 + 1.0 + 1.5 + 2.0 ... 13 terms = 45.5

Putting this in equation (1), we get

Total length of spiral= π (0.5 + 1.5 + 2.0 + ... 13 terms) = π (45.5) = 143 cm

19. 200 logs are stacked in the following manner:20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?



Ans. The number of logs in the bottom row = 20

The number of logs in the next row = 19

The number of logs in the next to next row = 18

Therefore, we have sequence of the form 20, 19, 18 ...

First term = a = 20, Common difference = d = 19 – 20 = –1

We need to find that how many rows make total of 200 logs.

Applying formula, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$ to find sum of n terms of AP, we get

$200 = \frac{n}{2}\left[40+(n-1)(-1)\right]$

⇒ 400 = n (40 – n + 1)

⇒ 400 = $40n - n^2 + n$

⇒ $n^2 - 41n + 400 = 0$

It is a quadratic equation, we can factorize to solve the equation.

n^2 − 25n − 16n + 400 = 0

n (n − 25) – 16 (n − 25) = 0

(n − 25) (n − 16)

n = 25, 16

We discard n = 25 because we cannot have more than 20 rows in the sequence. The sequence is of the form: 20, 19, 18 ...

At most, we can have 20 or less number of rows.

Therefore, n = 16 which means 16 rows make total number of logs equal to 200.

We also need to find number of logs in the 16th row.

Applying formula, $S_n = \frac{n}{2}(a+l)$ to find sum of n terms of AP, we get

200 = 8 (20 + l)

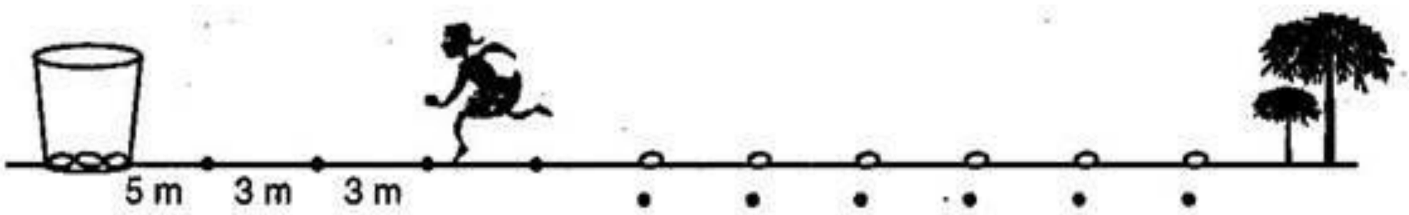
⇒ 200 = 160 + 8I

⇒ 40 = 8l ⇒ l = 5

Therefore, there are 5 logs in the top most row and there are total of 16 rows.

---

20. **In a potato race, a bucket is placed at the starting point, which is 5 meters from the first potato, and the other potatoes are placed 3 meters apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?**



Ans. The distance of first potato from the starting point = 5 meters

Therefore, the distance covered by competitor to pick up first potato and put it in bucket = 5×2 = 10 meters

The distance of Second potato from the starting point = 5 + 3 = 8 meters {All the potatoes are 3 meters apart from each other}

Therefore, the distance covered by competitor to pick up 2nd potato and put it in bucket = 8×2 = 16 meters

The distance of third potato from the starting point = 8 + 3 = 11 meters

Therefore, the distance covered by competitor to pick up 3rd potato and put it in bucket = 11×2 = 22 meters

Therefore, we have a sequence of the form 10, 16, 22 ... 10 terms (There are

ten terms because there are ten potatoes)

To calculate the total distance covered by the competitor, we need to find:

10 + 16 + 22 + ... 10 terms

First term = a = 10, Common difference = d = 16 – 10 = 6 n =

10{There are total of 10 terms in the sequence}

Applying formula, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ to find sum of n terms of AP, we get

$$S_{n10} = \frac{10}{2}\left[20 + (10-1)6\right] = 5(20 + 54) = 5 \times 74 = 370$$

Therefore, total distance covered by competitor is equal to 370 meters.

Chapter - 5

Arithmetic Progressions
Exercise 5.4

1. **Which term of the AP: 121, 117, 113, ….. is its first negative term? Ans.**

Given: 121, 117, 113, …….

Here a=121, d=117-121=-4

Now, $a_n = a+(n-1)d$

$= 121+(n-1)(-4)$

$= 121-4n+4 = 125-4n$

For the first negative term, $a_n < 0$

$\Rightarrow 125-4n<0$

$\Rightarrow 125<4n$

$\Rightarrow \frac{125}{4}<n$

$\Rightarrow 31\frac{1}{4}<n$

n is an integer and $n>31\frac{1}{4}$.

Hence, the first negative term is 32^{nd} term.

(vii) The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of sixteen terms of the AP.

Ans. Let the AP be $a-4d, a-3d, a-2d, a-d, a, a+d, a+2d, a+3d, \ldots$

Then, $a_3 = a-2d, a_7 = a+2d$

$\Rightarrow a_3 + a_7 = a-2d+a+2d = 6$

$\Rightarrow 2a = 6$

$\Rightarrow a = 3$ ……….(i)

Also $(a-2d)(a+2d) = 8$

$\Rightarrow a^2 - 4d^2 = 8$

$\Rightarrow 4d^2 = a^2 - 8$

$\Rightarrow 4d^2 = 3^2 - 8$

$\Rightarrow 4d^2 = 1$

$\Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \pm\frac{1}{2}$

Taking $d = \frac{1}{2}$,

$S_{16} = \frac{16}{2}\left[2\times(a-4d)+(16-1)d\right]$

= $8\left[2\times\left(3-4\times\frac{1}{2}\right)+15\times\frac{1}{2}\right]$

= $8\left[2+\frac{15}{2}\right]$ = $8\times\frac{19}{2}$ =76

Taking $d = \frac{-1}{2}$,

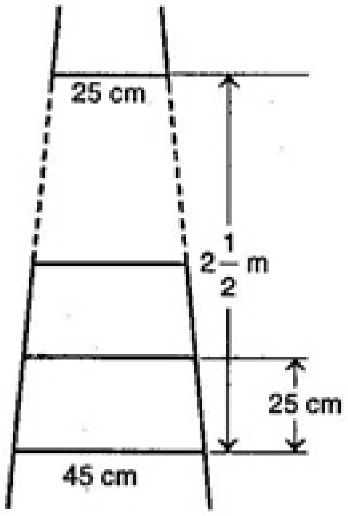
$$S_{16} = \frac{16}{2}\left[2\times(a-4d)+(16-1)d\right]$$

$$= 8\left[2\times\left(3-4\times\frac{-1}{2}\right)+15\times\frac{-1}{2}\right]$$

$= 8\left[\frac{20-15}{2}\right] = 8\times\frac{5}{2}$ =20

$\therefore S_{16} = 20$ and 76

⇒ A ladder has rungs 25 cm apart (see figure). The rungs decrease uniformly in length from 45 cm, at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?



Ans. Number of rungs $(n) = \dfrac{2\frac{1}{2}\times 100}{25}$ = 10

The length of the wood required for rungs = sum of 10 rungs

$= \frac{10}{2}[25+45] = 5\times 70$ = 350 cm

(iv) The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

Ans. Here $a = 1$ and $d = 1$

$\therefore S_{x-1} = \frac{x-1}{2}\left[2\times1+(x-1-1)\times1\right]$

= $\frac{x-1}{2}(2+x-2)$

$\frac{(x-1)x}{2}$ = $\frac{x^2-x}{2}$

$S_x = \frac{x}{2}\left[2\times1+(x-1)\times1\right]$

= $\frac{x}{2}(x+1)$ = $\frac{x^2+x}{2}$

$S_{49} = \frac{49}{2}\left[2\times1+(49-1)\times1\right]$

=$\frac{49}{2}(2+48)$ = 49 x 25

According to question,

$S_{x-1} = S_{49} - S_x$

$\Rightarrow \frac{x^2-x}{2}$ = $49\times25 - \frac{x^2+x}{2}$

$\Rightarrow \frac{x^2-x}{2}$ + $\frac{x^2+x}{2}$ = 49×25

$\Rightarrow \frac{x^2-x+x^2+x}{2} = 49\times25$

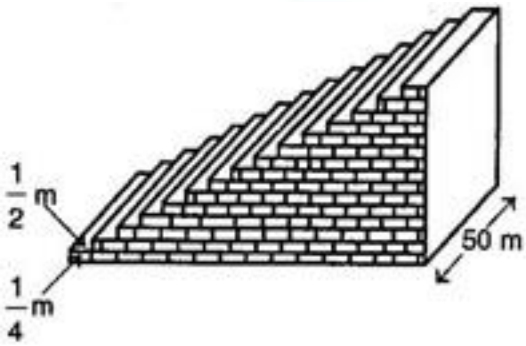
$\Rightarrow x^2$ = 49×25

$\Rightarrow x = \pm35$

Since, x is a counting number, so negative value will be neglected.

$\therefore x = 35$

(iii) A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.



Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see figure). Calculate the total volume of concrete required to build the terrace.

Ans. Volume of concrete required to build the first step, second step, third step, ……. (in m^2) are

$$\frac{1}{4}\times\frac{1}{2}\times 50, \left(2\times\frac{1}{4}\right)\times\frac{1}{2}\times 50, \left(3\times\frac{1}{4}\right)\times\frac{1}{2}\times 50, \ldots$$

$$\Rightarrow \frac{50}{8}, 2\times\frac{50}{8}, 3\times\frac{50}{8}, \ldots\ldots$$

$\therefore$ Total volume of concrete required = $\frac{50}{8}+2\times\frac{50}{8}+3\times\frac{50}{8}+\ldots\ldots$

$= \frac{50}{8}[1+2+3+\ldots\ldots]$

$= \frac{50}{8}\times\frac{15}{2}[2\times 1+(15-1)\times 1]\ [\because n=15]$

$= \frac{50}{8}\times\frac{15}{2}\times 16$

$= 750\ m^3$

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Grade – 10
MATHS
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CHAPTER 06
TRIANGLES

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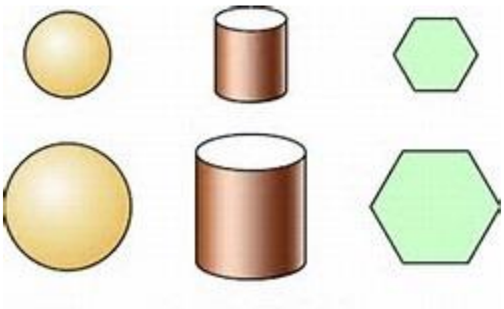
5. **Pythagoras Theorem**
6. **Miscellaneous Questions**

1. **Similar Figures:** Similar figures have the same shape (but not necessarily the same size). In geometry, two squares are similar, two equilateral triangles are similar, two circles are similar and two line segments are similar.

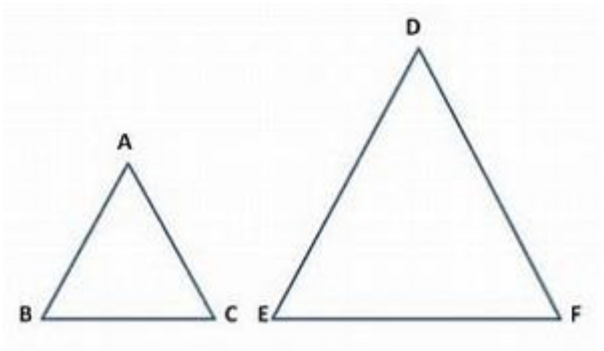


Similar Figures

2. **Similar Triangles**: Two triangles are said to be similar if their corresponding angles are equal and their corresponding sides are proportional.

3. Equiangular Triangles: Two triangles are equiangular if their corresponding angles are equal. The ratio of any two corresponding sides in two equiangular triangles is always the same.

4. **Criteria for Similarity**:
in and ΔABC and ΔDEF



(i) **AAA Similarity**: $\Delta ABC \sim \Delta DEF$ when $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

(ii) **SAS Similarity:** $\Delta ABC \sim \Delta DEF$, when $\angle A = \angle D$ or $\angle B = \angle E$ or $\angle C = \angle F$ and

AB = DE, AC = DF, or BC = EF, AB = DE

(iii) **SSS Similarity**: ΔABC~ΔDEF, when AB = DE, AC = DF, BC = EF

4. The proof of the following theorems can be asked in the examination:

(i) **Basic Proportionality Theorem**: If a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, the other two sides are divided in the same ratio.

(ii) **Converse of Basic Proportionality Theorem**: If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.

(iii) If one angle of a triangle is equal to one angle of other triangle and the sides including these angles are proportional ,the triangles are similar.

(iv) If a perpendicular drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.

(v) **Area Theorem**: The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

(iii) **Pythagoras Theorem**: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

(iv) **Converse of Pythagoras Theorem**: In a triangle, if the square of one side is equal to the sum of the squares of the other two sides then the angle opposite to the first side is a right angle.

Exercise 6.1

1. Fill in the blanks using the correct word given in brackets:

(i) All circles are _______________. (congruent, similar) (ii) All

squares are _______________. (similar, congruent)

(iii) All _______________ triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are _______________ and (b) their corresponding sides are _______________. (equal, proportional)

Ans. (i) similar

(ii) similar

(iii) equilateral

(iv) equal, proportional

2. Give two different examples of pair of:

(i) similar figures

(ii) non-similar figures

Ans. (i) Two different examples of a pair of similar figures are:

(a) Any two rectangles

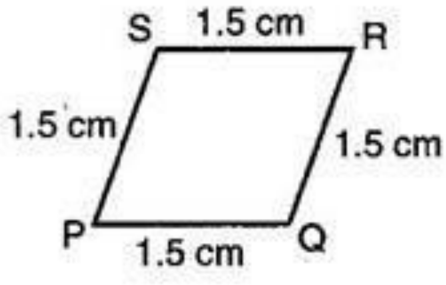
(b) Any two squares

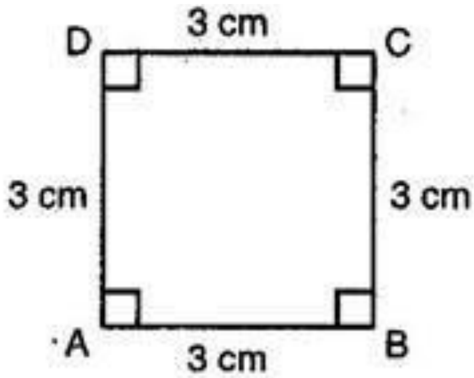
(ii) Two different examples of a pair of non-similar figures are:

(a) A scalene and an equilateral triangle

(b) An equilateral triangle and a right angled triangle

3. State whether the following quadrilaterals are similar or not:



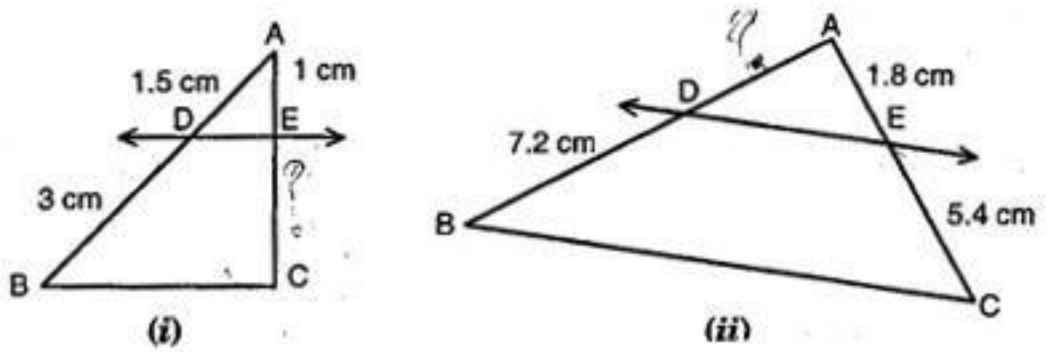


Ans. On looking at the given figures of the quadrilaterals, we can say that they are not similar because their angles are not equal.

Chapter - 6

Triangles - Exercise 6.2

1. In figure (i) and (ii), DE $\parallel$ BC. Find EC in (i) and AD in (ii).



Ans. (i) Since DE $\parallel$ BC,

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{1.5}{3} = \frac{1}{EC}$$

$$\Rightarrow EC = \frac{3}{1.5}$$

$$\Rightarrow EC = 2 \text{ cm}$$

(ii) Since DE $\parallel$ BC,

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4}$$

$\Rightarrow$ AD = $\frac{1.8 \times 7.2}{5.4}$

$\Rightarrow$ EC = 2.4 cm

2. E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF $\parallel$ QR:

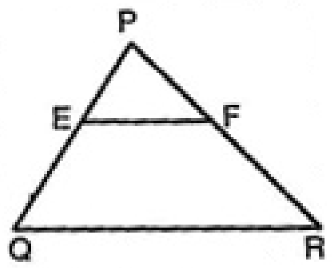
(v) PE = 3.9 cm, EQ = 4 cm, PF = 3.6 cm and FR = 2.4 cm

(vi) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

(vii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm Ans.

(i) Given: PE = 3.9 cm, EQ = 4 cm, PF = 3.6 cm and FR = 2.4 cm

Now, $\frac{PE}{EQ} = \frac{3.9}{4}$ = 0.97 cm



And $\frac{PF}{FR} = \frac{3.6}{2.4}$ = 1.2 cm

$\because \frac{PE}{EQ} \neq \frac{PF}{FR}$

Therefore, EF does not divide the sides PQ and PR of ΔPQR in the same ratio.

$\therefore$ EF is not parallel to QR.

(ii) Given: PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

Now, $\frac{PE}{EQ} = \frac{4}{4.5}$ = $\frac{8}{9}$ cm

And $\frac{PF}{FR} = \frac{8}{9}$ cm

$\because \frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF divides the sides PQ and PR of ΔPQR in the same ratio.

$\therefore$ EF is parallel to QR.

(iii) Given: PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm $\Rightarrow$

EQ = PQ – PE = 1.28 – 0.18 = 1.10 cm

And ER = PR – PF = 2.56 – 0.36 = 2.20 cm

Now, $\frac{PE}{EQ} = \frac{0.18}{1.10} = \frac{18}{110}$ = $\frac{9}{55}$ cm

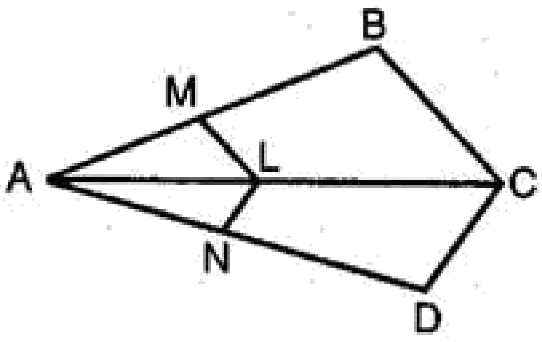
And $\frac{PF}{FR} = \frac{0.36}{2.20} = \frac{36}{220}$ = $\frac{9}{55}$cm

$\because \frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF divides the sides PQ and PR of ΔPQR in the same ratio.

$\therefore$ EF is parallel to QR.

3. In figure, if LM $\parallel$ CB and LN $\parallel$ CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



Ans. In ΔABC, LM $\parallel$ CB

$\therefore \frac{AM}{AB} = \frac{AL}{AC}$ [Basic Proportionality theorem] ……….(i)

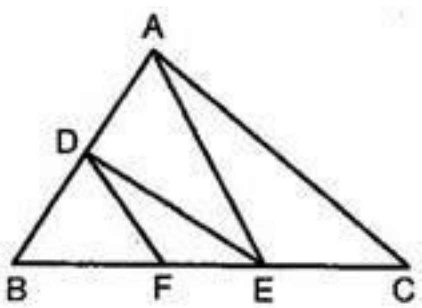
And in ΔACD, LN $\parallel$ CD

$\therefore \frac{AL}{AC} = \frac{AN}{AD}$ [Basic Proportionality theorem] ……….(ii)

From eq. (i) and (ii), we have

$$\frac{AM}{AB} = \frac{AN}{AD}$$

4. In the given figure, DE $\parallel$ AC and DF $\parallel$ AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.



Ans. In ΔBCA, DE $\parallel$ AC

$\therefore \frac{BE}{EC} = \frac{BD}{DA}$ [Basic Proportionality theorem] ……….(i)

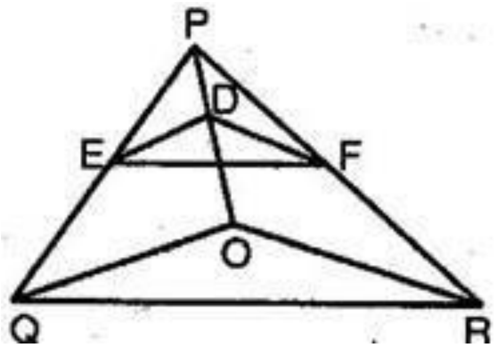
And in ΔBEA, DF $\parallel$ AE

$\therefore \frac{BE}{FE} = \frac{BD}{DA}$ [Basic Proportionality theorem] ……….(ii)

From eq. (i) and (ii), we have

$$\frac{BF}{FE} = \frac{BE}{EC}$$

5. In the given figure, DE $\parallel$ OQ and DF $\parallel$ OR. Show that EF $\parallel$ QR.



Ans. In ΔPQO, DE $\parallel$ OQ

$\therefore \frac{PE}{EQ} = \frac{PD}{DO}$ [Basic Proportionality theorem] ……….(i)

And in ΔPOR, DF $\parallel$ OR

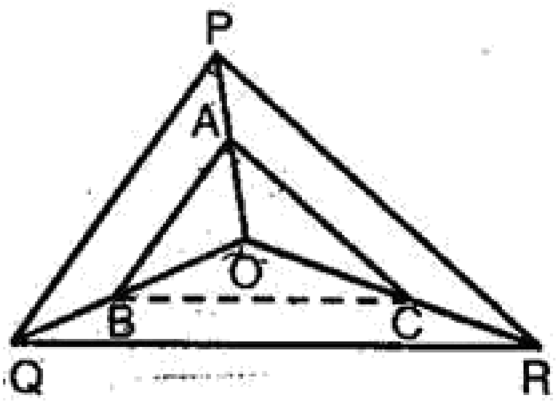
$\therefore \frac{PD}{DO} = \frac{PF}{FR}$ [Basic Proportionality theorem] ……….(ii)

From eq. (i) and (ii), we have

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$\therefore$ EF $\parallel$ QR [By the converse of BPT]

(v) In the given figure, A, B, and C are points on OP, OQ and OR respectively such that AB $\parallel$ PQ and AC $\parallel$ PR. Show that BC $\parallel$ QR.



Ans. Given: O is any point in ΔPQR, in which AB $\parallel$ PQ and AC $\parallel$ PR.

To prove: BC $\parallel$ QR

Construction: Join BC.

Proof: In ΔOPQ, AB $\parallel$ PQ

$\therefore \frac{OA}{AP} = \frac{OB}{BQ}$ [Basic Proportionality theorem] ……….(i)

And in ΔOPR, AC $\parallel$ PR

$\therefore \frac{OA}{AP} = \frac{OC}{CR}$ [Basic Proportionality theorem] ……….(ii)

From eq. (i) and (ii), we have

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

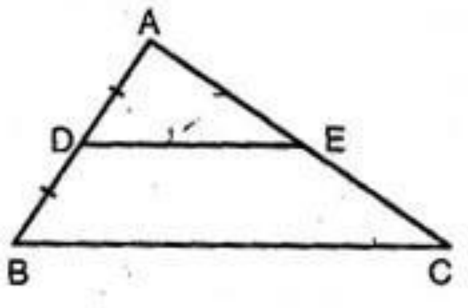
$\because$ In ΔOQR, B and C are points dividing the sides OQ and OR in the same ratio. $\therefore$ By the converse of Basic Proportionality theorem,

$\Rightarrow$ BC $\parallel$ QR

3. Using Theorem 6.1, prove that a line drawn through the midpoint of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Ans. Given: A triangle ABC, in which D is the midpoint of side AB and the

line DE is drawn parallel to BC, meeting AC at E.



To prove: AE = EC

Proof: Since DE $\parallel$ BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [Basic Proportionality theorem] ……….(i)

But AD = DB [Given]

$\Rightarrow \frac{AD}{DB} = 1$

$\Rightarrow \frac{AE}{EC} = 1$ [From eq. (i)]

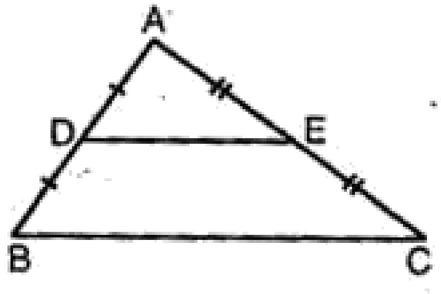
$\Rightarrow$ AE=EC

Hence, E is the midpoint of the third side AC.

(c) Using Theorem 6.2, prove that the line joining the midpoints of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Ans. Given: A triangle ABC, in which D and E are the midpoints of

sides AB and AC respectively.



To Prove: DE $\parallel$ BC

Proof: Since D and E are the midpoints of AB and AC

respectively.

$\therefore$ AD = DB and AE = EC

Now, AD = DB

$\Rightarrow \frac{AD}{DB} = 1$ and AE = EC

$\Rightarrow \frac{AE}{EC} = 1$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ = 1

$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$

Thus, in triangle ABC, D and E are points dividing the sides AB and AC in the same ratio. Therefore,

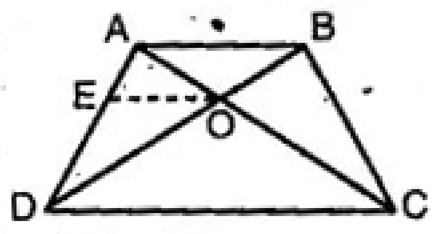
by the converse of Basic Proportionality theorem, we have

DE $\parallel$ BC

(iii) ABCD is a trapezium in which AB $\parallel$ DC and its diagonals intersect each other at the point O.

Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Ans. Given: A trapezium ABCD, in which AB $\parallel$ DC and its diagonals AC and BD intersect each other at O.



To Prove: $\frac{AO}{BO} = \frac{CO}{DO}$

Construction: Through O, draw OE $\parallel$ AB, i.e. OE $\parallel$ DC.

Proof: In ΔADC, we have OE $\parallel$ DC

$\therefore \frac{AE}{ED} = \frac{AO}{CO}$ [By Basic Proportionality theorem]……….(i)

Again, in ΔABD, we have OE $\parallel$ AB[Construction]

$\therefore \frac{ED}{AE} = \frac{DO}{BO}$ [By Basic Proportionality theorem]

$\Rightarrow \frac{AE}{ED} = \frac{BO}{DO}$ ……….(ii)

From eq. (i) and (ii), we get

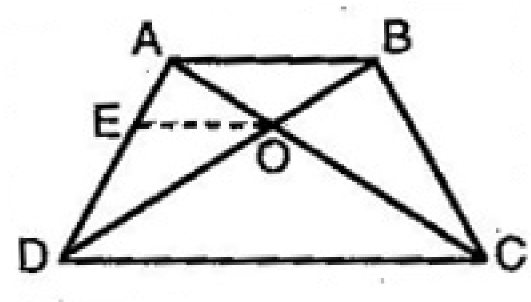
$$\frac{AO}{CO} = \frac{BO}{DO}$$

$$\Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$$

(c) The diagonals of a quadrilateral ABCD intersect each other at the point O such that Show that $\frac{AO}{BO} = \frac{CO}{DO}$. **ABCD is a trapezium.**

Ans. Given: A quadrilateral ABCD, in which its diagonals AC and

BD intersect each other at O such that $\frac{AO}{BO} = \frac{CO}{DO}$, i.e.



$\frac{AO}{CO} = \frac{BO}{DO}$.

To Prove: Quadrilateral ABCD is a trapezium.

Construction: Through O, draw OE $\parallel$ AB meeting AD at E.

Proof: In ΔADB, we have OE $\parallel$ AB [By construction]

$\therefore \frac{DE}{EA} = \frac{OD}{BO}$ [By Basic Proportionality theorem]

$\Rightarrow \frac{EA}{DE} = \frac{BO}{DO}$

$\Rightarrow \frac{EA}{DE} = \frac{BO}{DO} = \frac{AO}{CO}$

$\left[\because \frac{AO}{CO} = \frac{BO}{DO}\right]$

$\Rightarrow \frac{EA}{DE} = \frac{AO}{CO}$

Thus in ΔADC, E and O are points dividing the sides AD and AC in the same ratio. Therefore by the converse of Basic Proportionality theorem, we have

EO $\parallel$ DC

But EO $\parallel$ AB[By construction]

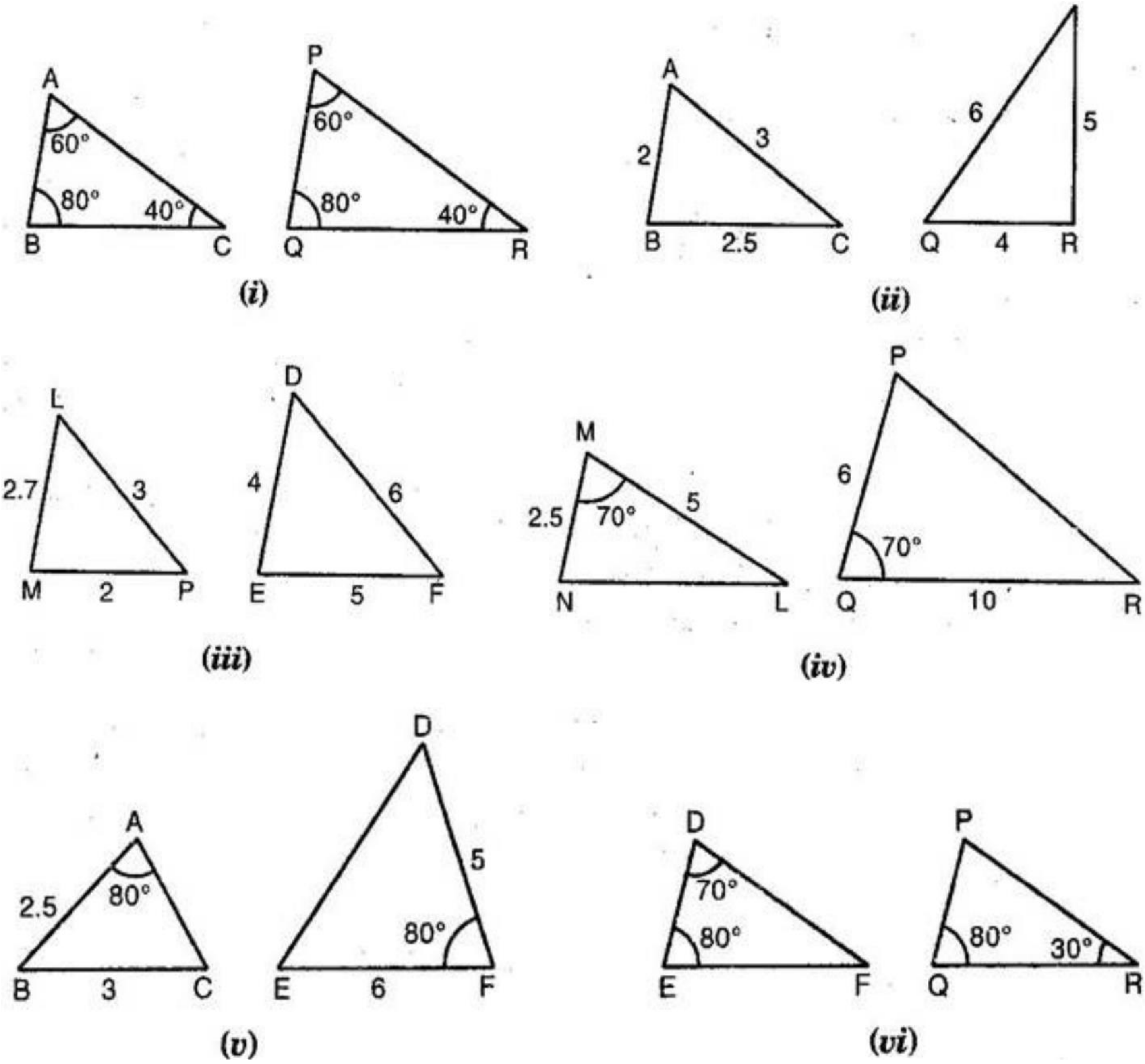
$\therefore$ AB $\parallel$ D

$\therefore$ Quadrilateral ABCD is a trapezium

Chapter - 6

Triangles - Exercise 6.3

3. State which pairs of triangles in the given figure, are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:



Ans. (i) In Δs ABC and PQR, we observe that,

$\angle A = \angle P = 60^\circ$, $\angle B = \angle Q = 80^\circ$ and $\angle C = \angle R = 40^\circ$

$\therefore$ By AAA criterion of similarity, $\Delta ABC \sim \Delta PQR$

(viii) In Δs ABC and PQR, we observe that,

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ} = \frac{1}{2}$$

$\therefore$ By SSS criterion of similarity, $\Delta ABC \sim \Delta PQR$

(vi) In Δs LMP and DEF, we observe that the ratio of the sides of these triangles is not equal. Therefore, these two triangles are not similar.

(vii) In Δs MNL and QPR, we observe that, $\angle M = \angle Q = 70^\circ$

But, $\frac{MN}{PQ} \neq \frac{ML}{QR}$

$\therefore$ These two triangles are not similar as they do not satisfy SAS criterion of similarity.

4. In Δs ABC and FDE, we have, $\angle A = \angle F = 80^\circ$

But, $\frac{AB}{DE} \neq \frac{AC}{DF}$ [$\because$ AC is not given]

$\therefore$ These two triangles are not similar as they do not satisfy SAS criterion of similarity.

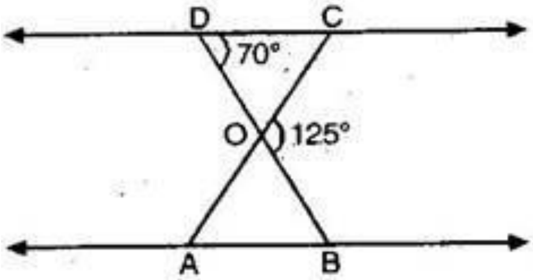
(d) In Δs DEF and PQR, we have, $\angle D = \angle P = 70^\circ$

[$\because \angle P = 180^\circ - 80^\circ - 30^\circ = 70^\circ$]

And $\angle E = \angle Q = 80^\circ$

$\therefore$ By AAA criterion of similarity, $\Delta DEF \sim \Delta PQR$

(iv) In figure, Δ ODC $\sim \Delta$ OBA, $\angle$ BOC = $125°$ and $\angle$ CDO = $70°$. Find $\angle$ DOC, $\angle$ DCO and $\angle$ OAB.



Ans. Since BD is a line and OC is a ray on it.

$\therefore \angle DOC + \angle BOC = 180°$

$\Rightarrow \angle DOC + 125° = 180°$

$\Rightarrow \angle DOC = 55°$

In Δ CDO, we have $\angle CDO + \angle DOC + \angle DCO = 180°$

$\Rightarrow 70° + 55° + \angle DCO = 180°$

$\Rightarrow \angle DCO = 55°$

It is given that Δ ODC $\sim \Delta$ OBA

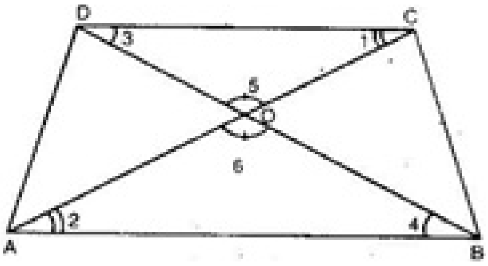
$\therefore \angle OBA = \angle ODC, \angle OAB = \angle OCD$

$\Rightarrow \angle OBA = 70°, \angle OAB = 55°$

Hence $\angle$ DOC = $55°$, $\angle$ DCO = $55°$ and $\angle$ OAB = $55°$

(d) Diagonals AC and BD of a trapezium ABCD with AB || CD intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Ans. Given: ABCD is a trapezium in which AB $\parallel$ DC.



To Prove: $\frac{OA}{OC} = \frac{OB}{OD}$

Proof: In Δs OAB and OCD, we have,

$\angle 5 = \angle 6$ [Vertically opposite angles]

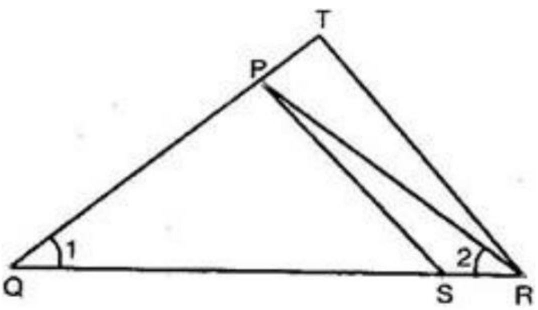
$\angle 1 = \angle 2$ [Alternate angles]

And $\angle 3 = \angle 4$ [Alternate angles]

$\therefore$ By AAA criterion of similarity, $\Delta OAB \sim \Delta ODC$

Hence, $\frac{OA}{OC} = \frac{OB}{OD}$

4. In figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\Delta PQS \sim \Delta TQR$.



Ans. We have, $\frac{QR}{QS} = \frac{QT}{PR}$

$\Rightarrow \frac{QT}{QR} = \frac{PR}{QS}$ ……….(1)

Also, $\angle 1 = \angle 2$ [Given]

$\therefore$ PR = PQ ……….(2) [$\because$ Sides opposite to equal $\angle$s are equal]

From eq.(1) and (2), we get

$$\frac{QT}{QR} = \frac{PR}{QS} \Rightarrow \frac{PQ}{QT} = \frac{QS}{QR}$$

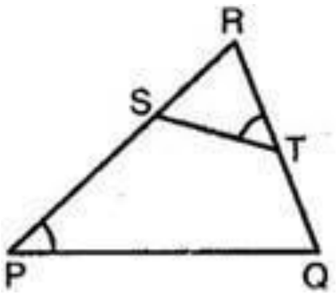
In Δs PQS and TQR, we have,

$\frac{PQ}{QT} = \frac{QS}{QR}$ and $\angle$PQS = $\angle$TQR = $\angle$Q

$\therefore$ By SAS criterion of similarity, ΔPQS $\sim \Delta$TQR

5. S and T are points on sides PR and QR of a ΔPQR such that $\angle$P = $\angle$RTS. Show that ΔRPQ $\sim \Delta$ RTS.

Ans. In Δs RPQ and RTS, we have



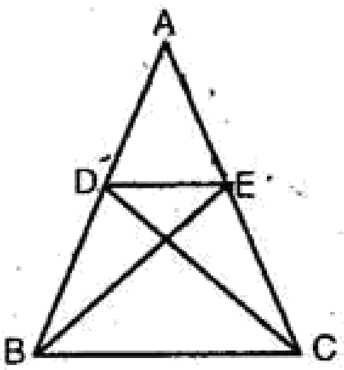
$\angle$RPQ = $\angle$RTS [Given]

$\angle$PRQ = $\angle$TRS [Common]

$\therefore$ By AA-criterion of similarity,

ΔRPQ $\sim \Delta$RTS

6. In the given figure, if ΔABE $\cong \Delta$ ACD, show that ΔADE $\sim \Delta$ABC.



Ans. It is given that $\Delta ABE \cong \Delta ACD$

$\therefore$ AB = AC and AE = AD

$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$

$\Rightarrow \frac{AB}{AC} = \frac{AD}{AE}$ ……….(1)

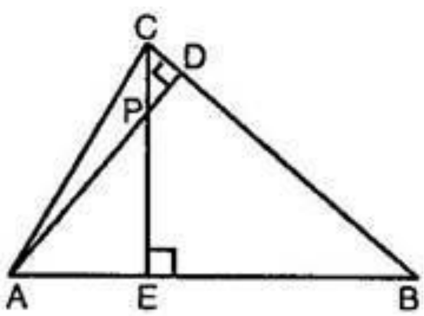
$\therefore$ In Δs ADE and ABC, we have,

$\frac{AB}{AC} = \frac{AD}{AE}$ [from eq.(1)]

And $\angle$BAC = $\angle$DAE [Common]

Thus, by SAS criterion of similarity, $\Delta ADE \sim \Delta ABC$

7. In figure, altitude AD and CE of a ΔABC intersect each other at the point P. Show that:



(i) $\Delta AEP \sim \Delta CDP$

(ii) Δ**ABD** $\sim \Delta$ **CBE**

(iii) Δ**AEP** $\sim \Delta$ **ADB**

(iv) Δ**PDC** $\sim \Delta$ **BEC**

Ans. (i) In Δs AEP and CDP, we have,

$\angle$AEP = $\angle$CDP = 90° [$\because$ CE $\perp$AB, AD $\perp$BC]

And $\angle$APE = $\angle$CPD[Vertically opposite]

$\therefore$ By AA-criterion of similarity, ΔAEP $\sim \Delta$CDP

(ii) In Δs ABD and CBE, we have,

$\angle$ADB = $\angle$CEB = 90°

And $\angle$ABD = $\angle$CBE[Common]

$\therefore$ By AA-criterion of similarity, ΔABD $\sim \Delta$CBE

(iii) In Δs AEP and ADB, we have,

$\angle$AEP = $\angle$ADB = 90° [$\because$ AD $\perp$BC, CE $\perp$AB]

And $\angle$PAE = $\angle$DAB[Common]

$\therefore$ By AA-criterion of similarity, ΔAEP $\sim \Delta$ADB

(iv) In Δs PDC and BEC, we have,

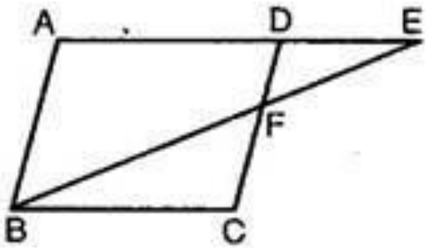
$\angle$PDC = $\angle$BEC = 90° [$\because$ CE $\perp$AB, AD $\perp$BC]

And $\angle$PCD = $\angle$BEC[Common]

$\therefore$ By AA-criterion of similarity, ΔPDC $\sim \Delta$BEC

8. E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that Δ**ABE** $\sim \Delta$ **CFB.**

Ans. In Δs ABE and CFB, we have,



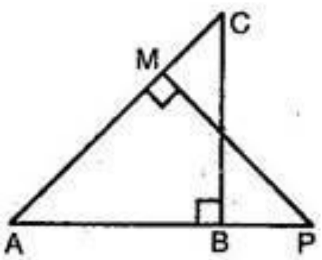
$\angle$AEB = $\angle$CBF[Alt. $\angle$s]

$\angle$A = $\angle$C [opp. $\angle$s of a $\parallel$gm]

$\therefore$ By AA-criterion of similarity, we have

$\Delta ABE \sim \Delta CFB$

9. In the given figure, ABC and AMP are two right triangles, right angles at B and M respectively. Prove that:



(i) Δ**ABC** $\sim \Delta$ **AMP**

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Ans. (i) In Δs ABC and AMP, we have,

$\angle$ABC = $\angle$AMP = 90° [Given]

$\angle$BAC = $\angle$MAP [Common angles]

$\therefore$ By AA-criterion of similarity, we have

$\Delta ABC \sim \Delta AMP$

(ii) We have ΔABC ~ ΔAMP [As prove above]

$$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP}$$

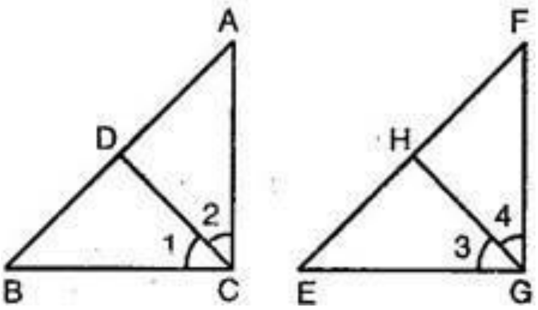
10. CD and GH are respectively the bisectors of $\angle$ACB and $\angle$EGF such that D and H lie on sides AB and FE at ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) Δ**DCB** ~ Δ **HE**

(iii) Δ**DCA** ~ Δ **HGF**

Ans. We have, ΔABC ~ ΔFEG



$\Rightarrow \angle$A = $\angle$F………(1)

And $\angle$C = $\angle$G

$\Rightarrow \frac{1}{2}\angle C = \frac{1}{2}\angle G$

$\Rightarrow \angle$1 = $\angle$3 and $\angle$2 = $\angle$4 ……….(2)

[$\because$ CD and GH are bisectors of $\angle$C and $\angle$G

respectively]

$\therefore$ In Δs DCA and HGF, we have

$\angle$A = $\angle$F[From eq.(1)]

$\angle 2 = \angle 4$[From eq.(2)]

$\therefore$ By AA-criterion of similarity, we have

$\Delta DCA \sim \Delta HGF$

Which proves the (iii) part

We have, $\Delta DCA \sim \Delta HGF$

$$\Rightarrow \frac{AG}{FG} = \frac{CD}{GH}$$

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

Which proves the (i) part

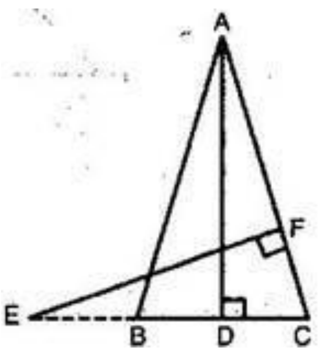
In Δs DCA and HGF, we have

$\angle 1 = \angle 3$[From eq.(2)]

$\angle B = \angle E$[$\because \Delta DCB \sim \Delta HE$]

Which proves the (ii) part

11. In the given figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD $\perp$ BC and EF $\perp$ AC, prove that $\Delta ABD \sim \Delta ECF$.



Ans. Here ΔABC is isosceles with AB = AC

$\therefore \angle B = \angle C$

In Δs ABD and ECF, we have

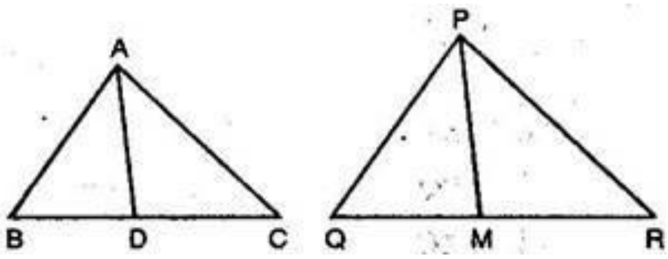
$\angle$ABD = $\angle$ECF[$\because$ $\angle$B = $\angle$C]

$\angle$ABD = $\angle$ECF = 90° [$\because$ AD $\perp$ BC and EF $\perp$ AC]

$\therefore$ By AA-criterion of similarity, we have

ΔABD $\sim$ ΔECF

12. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of a ΔPQR (see figure). Show that ΔABC $\sim$ ΔPQR.



Ans. Given: AD is the median of Δ ABC and PM is the median of Δ PQR such that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

To prove: Δ ABC $\sim$ Δ PQR

Proof: BD = $\frac{1}{2}$BC [Given]

And QM = $\frac{1}{2}$QR [Given]

Also $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ [Given]

$$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}$$

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

$\therefore \Delta ABD \sim \Delta PQM$[By SSS-criterion of similarity]

$\Rightarrow \angle B = \angle Q$[Similar triangles have corresponding angles equal]

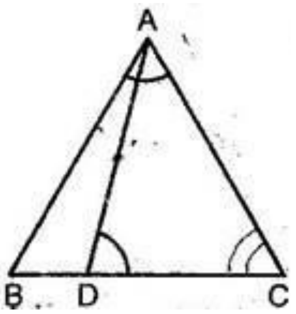
And $\frac{AB}{PQ} = \frac{BC}{QR}$[Given]

$\therefore$ By SAS-criterion of similarity, we have

$\Delta ABC \sim \Delta PQR$

13. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB.CD$.

ANS. In triangles ABC and DAC,



$\angle ADC = \angle BAC$ [Given]

and $\angle C = \angle C$[Common]

$\therefore$ By AA-similarity criterion,

$\Delta ABC \sim \Delta DAC$

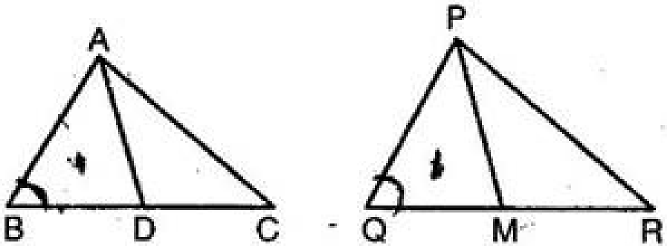
$\Rightarrow \frac{AB}{DA} = \frac{BC}{AC} = \frac{AC}{DC}$

$\Rightarrow \frac{CB}{CA} = \frac{CA}{CD}$

$\Rightarrow CA^2$ = CB.CD

14. Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC $\sim \Delta$ PQR.

ANS. Given: AD is the median of Δ ABC and PM is the median of Δ PQR such that

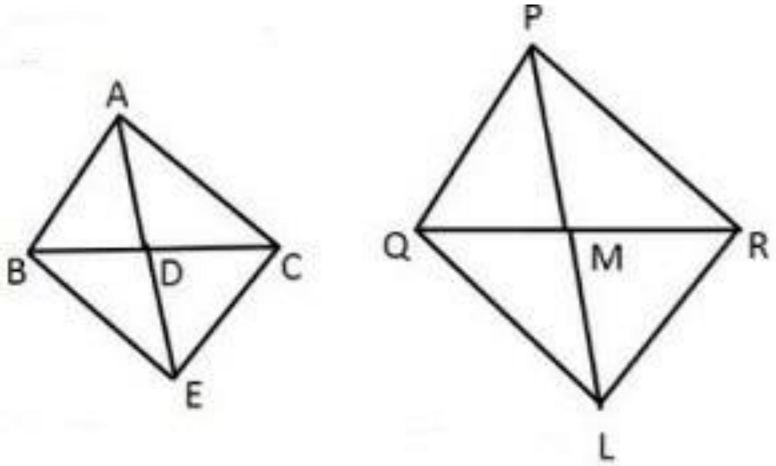


$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad \ldots\ldots\ldots(1)$$

To prove: Δ ABC $\sim \Delta$ PQR

Proof:

Let us extend AD to point D such that AD = DE and PM up to point L such that PM = ML



Join B to E. C to E,and Q to L, and R to L

We know that medians is the bisector of opposite side

Hence

BD=DC

Also, AD = DE (by construction)

Hence in quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

AC=BE

AB = EC (opposite sides of ||gm are equal) .... (2)

Similarly, we can prove that PQLR is a parallelogram

PR=QL

PQ = LR opposite sides of ||gm are equal) .... (3)

Given that

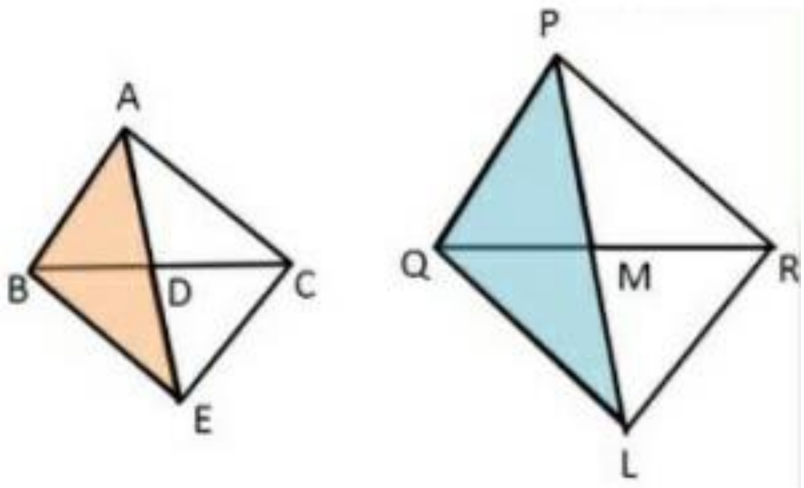
$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{AD}{PM}$ [from (2) (3)]

$$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$$

$$\frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL} \ [as\ AD = DE, AE = AD + DE = 2AD$$
$$PM = ML. PL = PM + ML = 2PM$$

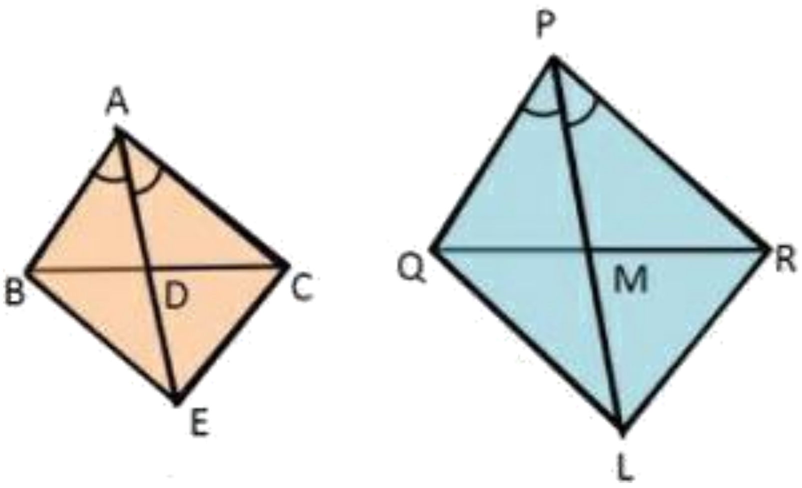
$\triangle ABE \sim \triangle PQL$ (By SSS Similiarity Criteria)



We know that corresponding angles of similar triangles are equal.

(4)

Similarly, we can prove that $\triangle A\ EC \sim \triangle P\ LR$.



We know that corresponding angles of similar triangles are equal.

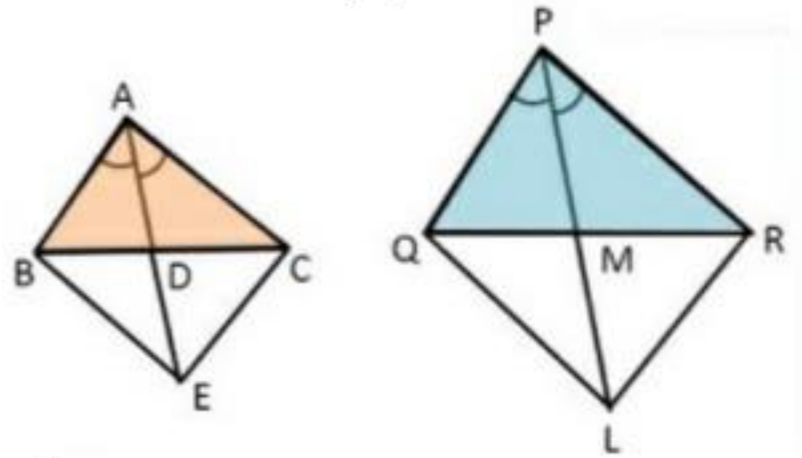
$\angle CAE = \angle RPL$ (5)

Adding (4) and (5),

$\angle BAE + \angle CAE = \angle QPL + \angle RPL$

$\angle CAB = \angle RPQ$

$In\ \triangle ABC\ and\ \triangle PQR,$



$\frac{AB}{PQ} = \frac{AC}{PR}$

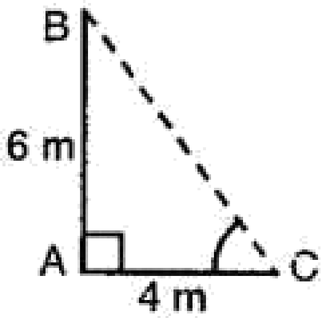
$\angle CAB = \angle RPQ$

$\triangle ABC \sim \triangle PQR$

Hence proved

15. A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Ans. Let AB the vertical pole and AC be its shadow. Also, let DE be the vertical tower and DF be its shadow. Joined BC and EF.



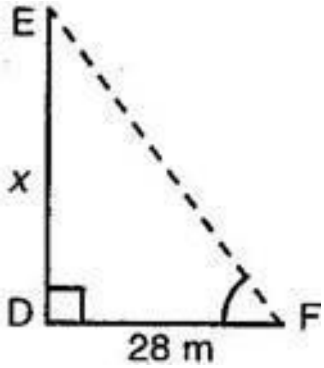
Let DE = x meters

Here, AB = 6 m, AC = 4 m and DF = 28 m

In the triangles ABC and DEF,

$\angle$A= $\angle$D= 90°

And $\angle$C = $\angle$F[Each is the angular elevation of the sun]



$\therefore$ By AA-similarity criterion,

ΔABC $\sim \Delta$DEF

$$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF}$$

$$\Rightarrow \frac{6}{x} = \frac{4}{28}$$

$$\Rightarrow \frac{6}{x} = \frac{1}{7}$$

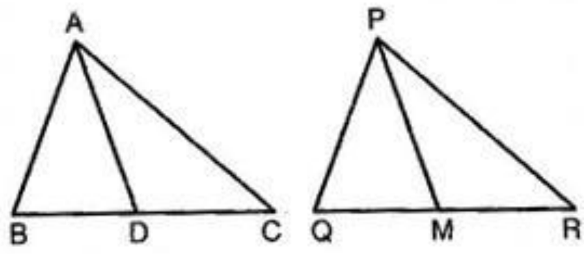
$\Rightarrow x$ = 42 m

Hence, the height of the tower is 42 meters.

16. If AD and PM are medians of triangles ABC and PQR respectively, where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Ans. Given: AD and PM are the medians of triangles

ABC and PQR respectively, where



$\Delta ABC \sim \Delta PQR$

To prove: $\frac{AB}{PQ} = \frac{AD}{PM}$

Proof: In triangles ABD and PQM,

$\angle B = \angle Q$ [Given]

And $\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$ [$\because$ AD and PM are the medians of BC and QR respectively]

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM}$

$\therefore$ By SAS-criterion of similarity,

$\Delta ABD \sim \Delta PQM$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PM}$$

Chapter - 6

Triangles - Exercise 6.4

4. Let ΔABC $\sim \Delta$ DEF and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.

Ans. We have, $\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \frac{BC^2}{EF^2}$

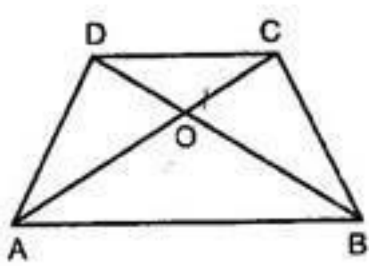
$$\Rightarrow \frac{64}{121} = \frac{BC^2}{(15.4)^2}$$

$$\Rightarrow \frac{8}{11} = \frac{BC}{15.4}$$

$\Rightarrow$ BC = $\left(\frac{8}{11} \times 15.4\right)$ cm = 11.2 cm

(ix) Diagonals of a trapezium ABCD with AB $\parallel$ DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.

Ans. In Δs AOB and COD, we have,



$\angle$AOB = $\angle$COD[Vertically opposite angles]

$\angle$OAB = $\angle$OCD[Alternate angles]

By AA-criterion of similarity,

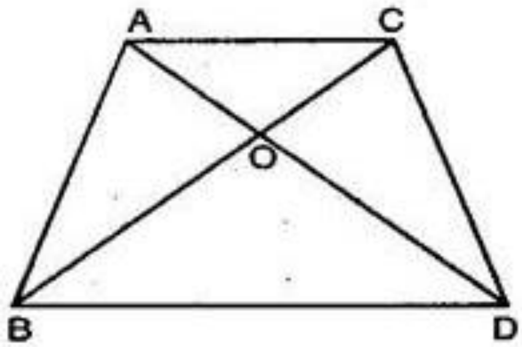
$\therefore \Delta_{AOB} \sim \Delta_{COD}$

$$\therefore \frac{\text{Area}(\Delta AOB)}{\text{Area}(\Delta COD)} = \frac{AB^2}{DC^2}$$

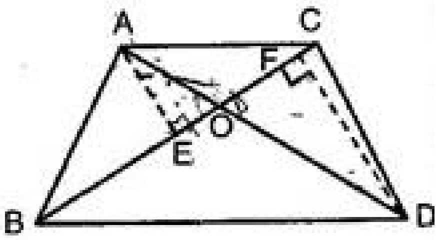
$$\Rightarrow \frac{\text{Area}(\Delta AOB)}{\text{Area}(\Delta COD)} = \frac{(2DC)^2}{DC^2} = \frac{4}{1}$$

Hence, Area (ΔAOB) : Area (ΔCOD) = 4 : 1

(viii) In the given figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$.



Ans. Given: Two Δs ABC and DBC which stand on the same base but on the opposite sides of BC.



To Prove: $\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta DBC)} = \frac{AO}{DO}$

Construction: Draw AE $\perp$ BC and DF $\perp$ BC.

Proof: In Δs AOE and DOF, we have, $\angle$AEO = $\angle$DFO = 90°

and $\angle$AOE = $\angle$DOF[Vertically opposite)

$\therefore$ ΔAOE $\sim$ ΔDOF[By AA-criterion]

$\therefore \frac{AE}{DF} = \frac{AO}{OD}$ ……….(i)

Now, $\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DBC)} = \frac{\frac{1}{2}\times BC\times AE}{\frac{1}{2}\times BC\times DF}$

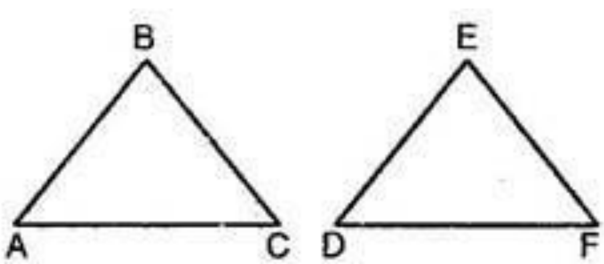
$\Rightarrow \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DBC)} = \frac{AE}{DF}$

$\Rightarrow \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DBC)} = \frac{AO}{OD}$ [using eq. (i)]

5. If the areas of two similar triangles are equal, prove that they are congruent.

Ans. Given: Two Δs ABC and DEF such that ΔABC $\sim$ ΔDEF

And Area(ΔABC) = Area (ΔDEF)



To Prove: ΔABC $\cong$ ΔDEF

Proof: ΔABC $\sim$ ΔDEF

$\therefore$ $\angle$A= $\angle$D, $\angle$B= $\angle$E, $\angle$C= $\angle$F

And $\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$

To establish $\Delta ABC \cong \Delta DEF$, it is sufficient to prove that, AB = DE, BC = EF and AC = DF Now,

Area(ΔABC) = Area (ΔDEF)

$\therefore \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)}=1$

$\Rightarrow \frac{AB^2}{DE^2}=\frac{BC^2}{EF^2}=\frac{AC^2}{DF^2}=1$

$\Rightarrow \frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}=1$

$\Rightarrow$ AB=DE,BC=EF,AC=DF

Hence, $\Delta ABC \cong \Delta DEF$

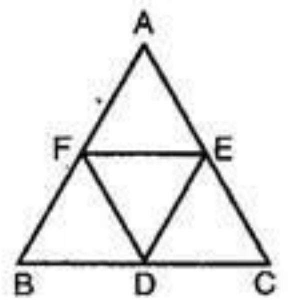
(e) D, E and F are respectively the midpoints of sides AB, BC and CA of ΔABC. Find the ratio of the areas of ΔDEF and ΔABC.

Ans. Since D and E are the midpoints of the sides BC and CA of ΔABC respectively.

$\therefore$ DE $\parallel$ BA $\Rightarrow$ DE $\parallel$ FA ……….(i)

Since D and F are the midpoints of the sides BC and AB of ΔABC respectively.

$\therefore$ DF $\parallel$ CA $\Rightarrow$ DE $\parallel$ AE ……….(ii)



From (i) and (ii), we can say that AFDE is a parallelogram.

Similarly, BDEF is a parallelogram.

Now, in Δs DEF and ABC, we have

$\angle$FDE = $\angle$A[opposite angles of $\|$gm AFDE]

And $\angle$DEF = $\angle$B[opposite angles of $\|$gm BDEF]

$\therefore$ By AA-criterion of similarity, we have ΔDEF ~ ΔABC

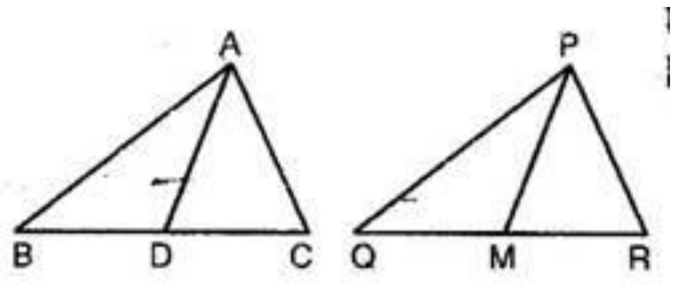
$$\Rightarrow \frac{\text{Area }(\Delta DEF)}{\text{Area }(\Delta ABC)} = \frac{DE^2}{AB^2} = \frac{\left(\frac{1}{2}AB\right)^2}{AB^2} \frac{1}{4}$$

[$\because$ DE= $\frac{1}{2}$AB]

Hence, Area (ΔDEF): Area (ΔABC) = 1 : 4

(v) Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Ans. Given: ΔABC ~ ΔPQR, AD and PM are the medians of Δs ABC and PQR respectively.



To Prove: $\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta PQR)} = \frac{AD^2}{PM^2}$

Proof: Since ΔABC ~ ΔPQR

$\therefore \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta PQR)} = \frac{AB^2}{PQ^2}$ ……….(1)

But, $\frac{AB}{PQ} = \frac{AD}{PM}$ ……….(2)

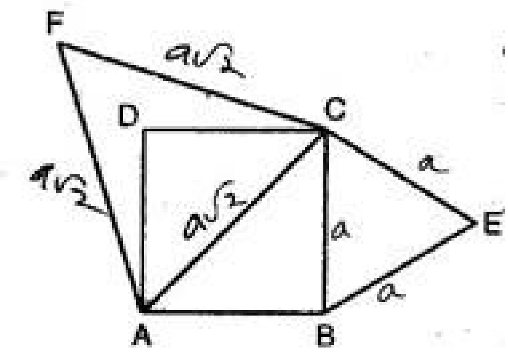
$\therefore$ From eq. (1) and (2), we have,

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{AD^2}{PM^2}$$

(e) Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of the diagonals.

Ans. Given: A square ABCD,

Equilateral Δs BCE and ACF have been drawn on side BC and the diagonal AC respectively.



To Prove: Area (ΔBCE) = $\frac{1}{2}$ Area (ΔACF)

Proof: ΔBCE ~ ΔACF

[Being equilateral so similar by AAA criterion of

similarity]

$$\Rightarrow \frac{\text{Area}(\Delta BCE)}{\text{Area}(\Delta ACF)} = \frac{BC^2}{AC^2}$$

$$\Rightarrow \frac{\text{Area}(\Delta BCE)}{\text{Area}(\Delta ACF)} = \frac{BC^2}{\left(\sqrt{2}BC\right)^2}$$

[$\because$ Diagonal = $\sqrt{2}$ side $\Rightarrow$ AC = $\sqrt{2}$ BC]

$$\Rightarrow \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta BDE)} = \frac{4}{1}$$

$$\Rightarrow \frac{\text{Area }(\Delta BDE)}{\text{Area }(\Delta ACF)} = \frac{1}{2}$$

Tick the correct answer and justify:

6. ABC and BDE are two equilateral triangles such that D is the midpoint of BC. The ratio of the areas of triangles ABC and BDE is:

2: 1

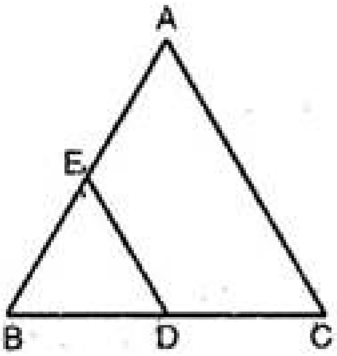
1: 2

4: 1

1: 4

Ans. (C) Since ΔABC and ΔBDE are equilateral, they are equiangular and hence,

$\Delta ABC \sim \Delta BDE$



$$\Rightarrow \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta BDE)} = \frac{BC^2}{BD^2}$$

$$\Rightarrow \frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta BDE)} = \frac{(2BD)^2}{BD^2}$$

[$\because$ D is the midpoint of BC]

$\therefore$ (C) is the correct answer.

(v)Sides of two similar triangles are in the ratio 4: 9. Areas of these triangles are in the ratio:

2: 3

4: 9

81: 16

16: 81

Ans. (D) Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides. Therefore,

Ratio of areas = $\frac{(4)^2}{(9)^2} = \frac{16}{81}$

$\therefore$ (D) is the correct answer.

पुOना International School

Shree Swaminarayan Gurukul, Zundal

Grade – 10 MATHS Specimen copy Year 22-23

INDEX

CHAPTER NO. – 8

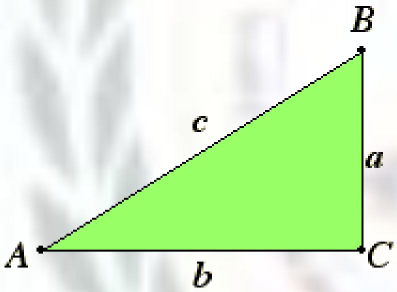
CHAPTER NAME – iNTRODUCTION TO TRIGONOMETRY

KEY POINTS TO REMEMBER –

Notes

- **Trigonometry** literally means measurement of sides and angles of a triangle.
- **Positive and Negative angles**: Angles in anti-clockwise direction are taken as positive angles and angles in clockwise direction are taken as negative angles.
- **Trigonometric Ratios of an acute angle of a right angled triangle**:

1. In a right triangle ABC, right-angled at B,



Solving right triangles

We can use the Pythagorean Theorem and properties of sines, cosines, and tangents to solve the triangle, that is, to find unknown parts in terms of known parts.

- Pythagorean Theorem: $a^2 + b^2 = c^2$.

- Sines: $\sin A = a/c$, $\sin B = b/c$.

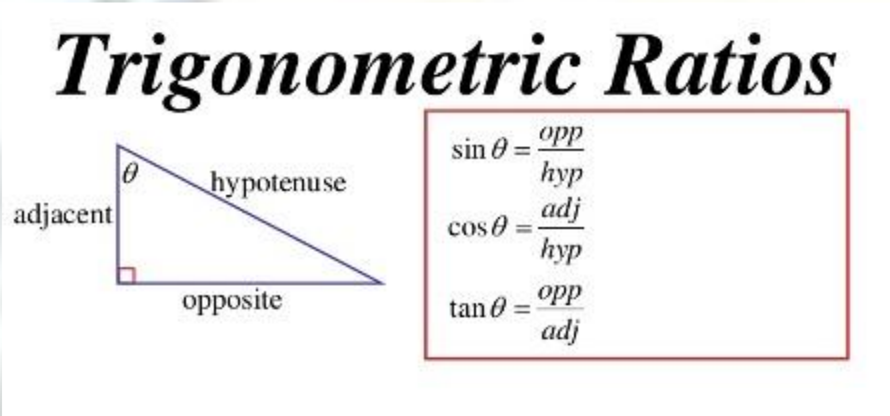
$$\sin A = \frac{\text{side opposite to angle A}}{\text{hypotenuse}}$$

- Cosines: $\cos A = b/c$, $\cos B = a/c$.

$\cos A = \frac{\text{side adjacent to angle A}}{hypotenuse}$

- Tangents: tan $A = a/b$, tan $B = b/a$.

$\tan A = \frac{\text{side opposite to angle A}}{side\ adjacent\ to\ angle\ A}$



- Cosec A = $\frac{\text{hypotenuse}}{side\ 0pposite\ to\ angle\ A}$
- secA = $\frac{\text{hypotenuse}}{side\ adjacent\ to\ angle\ A}$
- cot A= $\frac{\text{side opposite to angle A}}{side\ adjacent\ to\ angle\ A}$

Opposite of Sin: Cosecant

Opposite of Cos: Secant

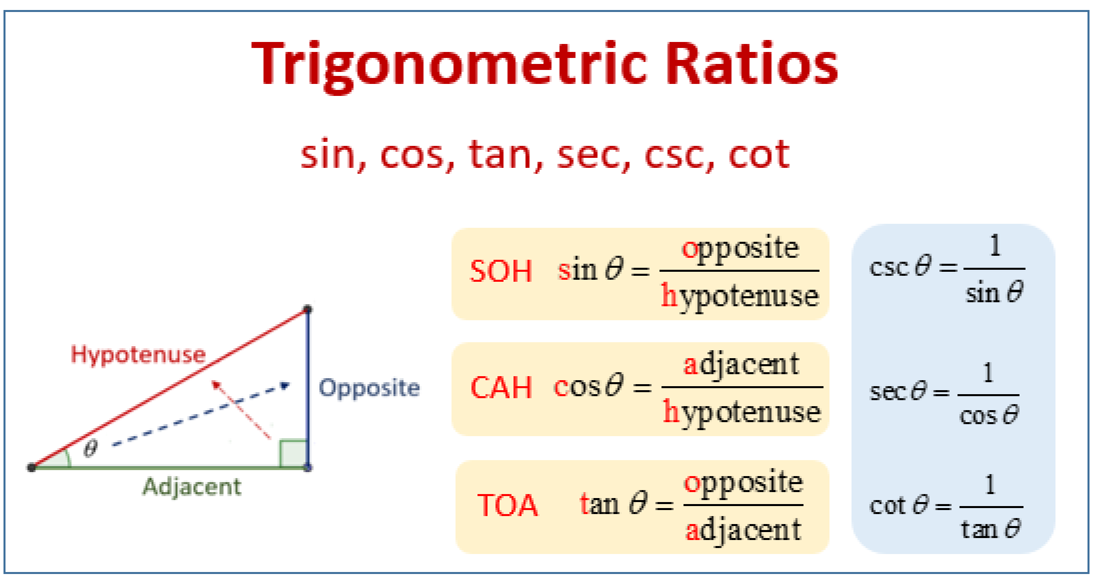
Opposite of Tan: Cotangent

Opposite of Cosecant: Sin

Opposite of Cotangent: Tan

Opposite of Secant: Cosecant

Sin ϴ (sin ϴ)	$\frac{\text{Perpendicular}}{\text{Hypotenuse}}$	$\frac{y}{r}$
Cosine ϴ (cos ϴ)	$\frac{\text{Base}}{\text{Hypotenuse}}$	$\frac{x}{r}$
Tangent ϴ (tan ϴ)	$\frac{\text{Perpendicular}}{\text{Base}}$	$\frac{y}{x}$
Cosecant ϴ (cossec ϴ)	$\frac{\text{Hypotenuse}}{\text{Perpendicular}}$	$\frac{r}{y}$
Secant ϴ (sec ϴ)	$\frac{\text{Hypotenuse}}{\text{Base}}$	$\frac{r}{x}$
Cotangent ϴ (cot ϴ)	$\frac{\text{Base}}{\text{Perpendicular}}$	$\frac{x}{y}$



- if one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of the angle can be easily determined.

(a) Find the sides of the right triangle in terms of k.

(b) Use Pythagoras Theorem and find the third side of the right triangle.

(c) Use definitions of t-ratios and substitute the values of sides.

(d) k is cancelled from numerator and denominator and the value of t-ratio is obtained.

- **Trigonometric Ratios of some specified angles**:

The values of trigonometric ratios for angles 0°, 30°, 45°, 60° and 90°

- The value of sin A or cos A never exceeds 1, whereas the value of sec A or cosec A is always greater than or equal to 1.

ratio \ angle θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
$\text{cosec}\,\theta$	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
$\cot\theta$	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

- **Trigonometric Ratios of Complementary Angles**:

sin(90° – A) = cos A, cos(90° – A) = sinA;

tan (90° – A) = cot A, cot (90° – A) = tan A;

sec (90° – A) = cosec A,   cosec (90° – A) = sec A.

- **Trigonometric Identities**:

1. $\sin^2 A + \cos^2 A = 1$
2. $\sec^2 A - \tan^2 A = 1$ for 0° ≤ A < 90°,
3. $\text{Cosec}^2 A - \cot^2 A = 1$ for 0° < A ≤ 90°

CHAPTER 8

INTRODUCTION TO TRIGONOMETRY

(Ex. 8.1)

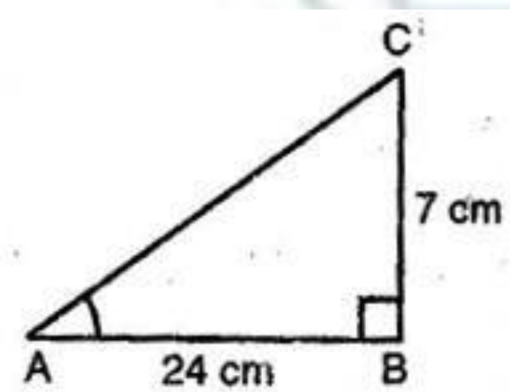
1. In ΔABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine:

(i) $\sin A \cos A$

(ii) $\sin C \cos C$

Ans. Let us draw a right angled triangle ABC, right angled at B. Using

Pythagoras theorem,



Let AC = 24k and BC = 7k

Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

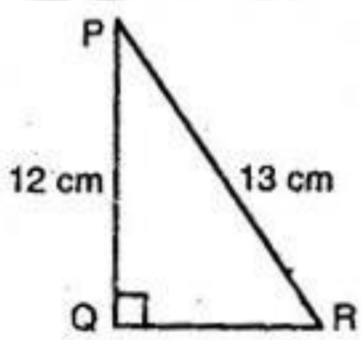
= $(24)^2 + (7)^2$ =576+49=625

$\Rightarrow$ AC = 25 cm

(i) $\sin A = \frac{P}{H} = \frac{BC}{AC} = \frac{7}{25}$, $\cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{24}{25}$

(ii) $\sin C = \frac{P}{H} = \frac{AB}{AC} = \frac{24}{25}$, $\cos C = \frac{B}{H} = \frac{BC}{AC} = \frac{7}{25}$

2. In adjoining figure, find $\tan P - \cot R$**:**



Ans. In triangle PQR, Using Pythagoras theorem,

$PR^2 = PQ^2 + QR^2$

$\Rightarrow (13)^2 = (12)^2 + QR^2$

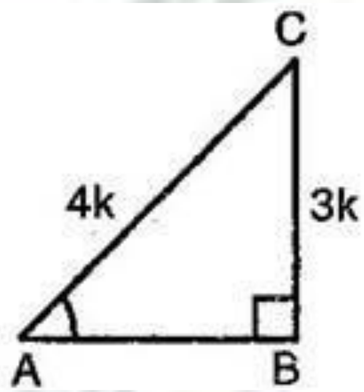
$\Rightarrow QR^2$ =169–144=25

$\Rightarrow$ QR = 5 cm

$\therefore \tan P - \cot R = \frac{P}{B} - \frac{B}{P} = \frac{QR}{PQ} - \frac{QR}{PQ} = \frac{5}{13} - \frac{5}{13} = 0$

3. If $\sin A = \frac{3}{4}$, **calculate** $\cos A$ **and** $\tan A$.

Ans. Given: A triangle ABC in which $\angle B = 90^\circ$



Then, Using Pythagoras theorem,

$AB = \sqrt{(AC)^2 - (BC)^2} = \sqrt{(4k)^2 - (3k)^2}$

$= \sqrt{16k^2 - 9k^2} = k\sqrt{7}$

$\therefore \cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{k\sqrt{7}}{4k} = \frac{\sqrt{7}}{4}$

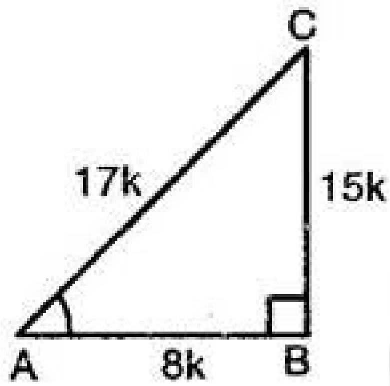
$\tan A = \frac{P}{B} = \frac{BC}{AB} = \frac{3k}{k\sqrt{7}} = \frac{3}{\sqrt{7}}$

4. Given $15 \cot A = 8$, **find** $\sin A$ **and** $\sec A$

Ans. Given: A triangle ABC in which $\angle B = 90^\circ$

$15 \cot A = 8$

$\Rightarrow \cot A = \frac{8}{15}$



Let AB = $8k$ and BC = $15k$

Then using Pythagoras theorem,

AC = $\sqrt{(AB)^2 + (BC)^2}$

= $\sqrt{(8k)^2 + (15k)^2}$

= $\sqrt{64k^2 + 225k^2}$

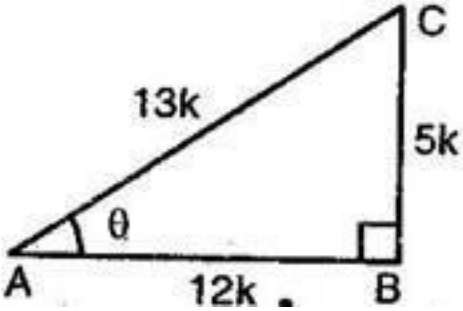
= $\sqrt{289k^2}$ = $17k$

$\therefore \sin A = \frac{P}{H} = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$

$\sec A = \frac{H}{B} = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$

5. Given $\sec\theta = \frac{13}{12}$, **calculate all other trigonometric ratios.**

Ans. Consider a triangle ABC in which $\angle$ A = θ and $\angle$ B = 90°



Let AB = $12k$ and BC = $5k$

Then, using Pythagoras theorem,

BC = $\sqrt{(AC)^2 - (AB)^2}$

= $\sqrt{(13k)^2 - (12k)^2}$

= $\sqrt{169k^2 - 144k^2}$

= $\sqrt{25k^2}$ = $5k$

$$\therefore \sin\theta = \frac{P}{H} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos\theta = \frac{B}{H} = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

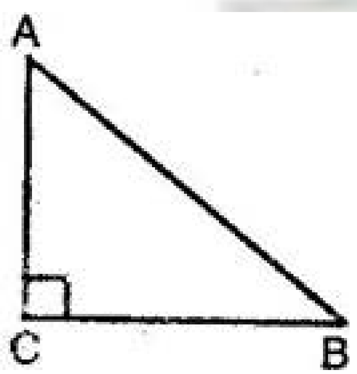
$$\tan\theta = \frac{P}{B} = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$$

$$\cot\theta = \frac{B}{P} = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$$

$$\cos ec\theta = \frac{H}{P} = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$$

6. If $\angle$ And $\angle$B are acute angles such that $\cos A = \cos B$, then show that $\angle$A = $\angle$B.

Ans. In right triangle ABC,



$\cos A = \dfrac{AC}{AB}$ $\quad$ $\cos B = \dfrac{BC}{AB}$

But $\cos A = \cos B$ [Given]

$\Rightarrow \frac{AC}{AB} = \frac{BC}{AB}$

$\Rightarrow$ AC=BC

$\Rightarrow \angle$A = $\angle$B [Angles opposite to equal sides are equal]

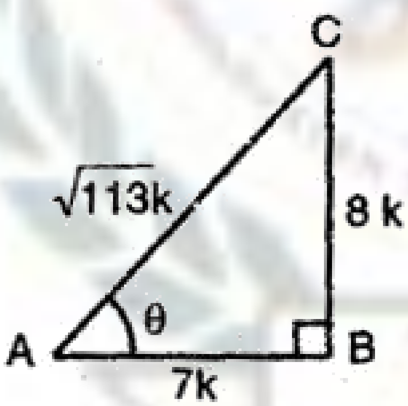
---

7. If $\cot\theta = \frac{7}{8}$, evaluate:

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\cot^2\theta$

Ans. Consider a triangle ABC in which $\angle$A = θ and $\angle$B = 90°



Let AB = $7k$ and BC = $8k$

Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2 + (AB)^2}$

$= \sqrt{(8k)^2 + (7k)^2}$

$= \sqrt{64k^2 + 49k^2}$

$= \sqrt{113k^2}$ $= \sqrt{113}k$

$$\therefore \sin\theta = \frac{BC}{AC} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{AB}{AC} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

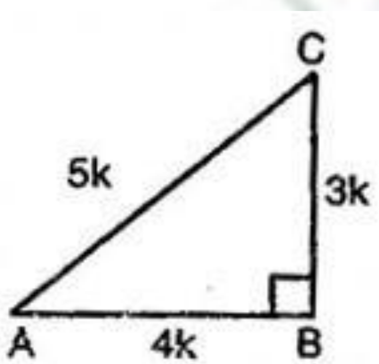
(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$

$$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}} = \frac{113-64}{113-49} = \frac{49}{64}$$

(ii) $\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta} = \frac{49/113}{64/113} = \frac{49}{64}$

8. If $3\cot A = 4,$ **check whether** $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ **or not.**

Ans. Consider a triangle ABC in which $\angle$ B = 90°.



And $3\cot A = 4$

$\Rightarrow \cot A = \frac{4}{3}$

Let AB = $4k$ and BC = $3k$.

Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2 + (AB)^2}$

= $\sqrt{(3k)^2 + (4k)^2}$

= $\sqrt{16k^2 + 9k^2}$

= $\sqrt{25k^2}$ = $5k$

Sin A = BC / AC = 3k / 5k = 3/5

$\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$

And tan A = BC / AB =3k / 4k = 3/4

Now, L.H.S. $\frac{1-\tan^2 A}{1+\tan^2 A}$ = $\frac{1-\frac{9}{16}}{1+\frac{9}{16}}$

= $\frac{16-9}{16+9}$= $\frac{7}{25}$

R.H.S. $\cos^2 A - \sin^2 A$ = $\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

= $\frac{16}{25} - \frac{9}{25}$ = $\frac{7}{25}$

$\because$ L.H.S. = R.H.S.

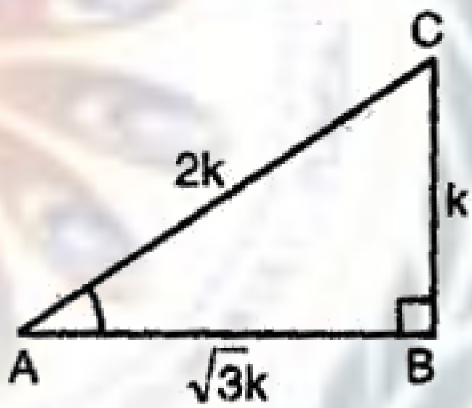
$\therefore \frac{1-\tan^2 A}{1+\tan^2 A}$ = $\cos^2 A - \sin^2 A$

---

9. In ΔABC right angles at B, if $\tan A = \frac{1}{\sqrt{3}}$, **find value of:**

(i) $\sin A \cos C + \cos A \sin C$

(ii) cos A cos C – sin A sin C



Ans. Consider a triangle ABC in which $\angle$B = 90°.
Let BC = k and AB = $\sqrt{3}k$

Then, using Pythagoras theorem,

AC = $\sqrt{(BC)^2 + (AB)^2}$

= $\sqrt{(k)^2 + (\sqrt{3}k)^2}$

= $\sqrt{k^2 + 3k^2}$ = $\sqrt{4k^2}$ = $2k$

$\therefore \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

For $\angle$C, Base = BC, Perpendicular = AB and Hypotenuse = AC

$$\therefore \sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

(i) $\sin A \cos C + \cos A \sin C = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

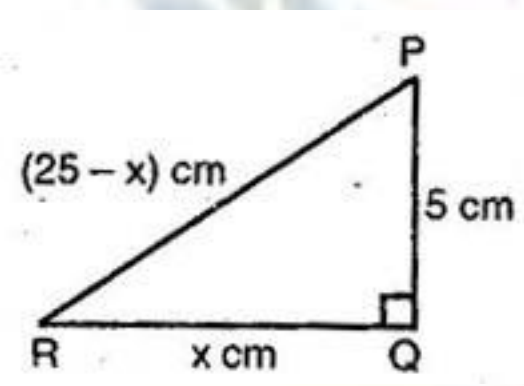
$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

(ii) $\cos A \cos C - \sin A \sin C =$

$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

10. In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of $\sin P, \cos P$ and $\tan P$.

Ans. In ΔPQR, right angled at Q.



PR + QR = 25 cm and PQ = 5 cm

Let QR = x cm, then PR = $(25-x)$ cm

Using Pythagoras theorem,

$$RP^2 = RQ^2 + QP^2$$

$$\Rightarrow (25-x)^2 = (x)^2 + (5)^2$$

$$\Rightarrow 625 - 50x + x^2 = x^2 + 25$$

$$\Rightarrow -50x = -600$$

$$\Rightarrow x = 12$$

$\therefore$ RQ = 12 cm and RP = 25 – 12 = 13 cm

$$\therefore \sin P = \frac{\text{RQ}}{\text{RP}} = \frac{12}{13}$$

$$\cos P = \frac{\text{PQ}}{\text{RP}} = \frac{5}{13}$$

$$\tan P = \frac{\text{RQ}}{\text{PQ}} = \frac{12}{5}$$

11. State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1.

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) $\cos A$ is the abbreviation used for the cosecant of angle A.

(iv) $\cot A$ **is the product of** $\cot$ **and A.**

(v) $\sin\theta = \frac{4}{3}$ **for some angle** θ.

Ans. (i) False because sides of a right triangle may have any length, so $\tan A$ may have any value.

(ii) True as $\sec A$ is always greater than 1.

(iii) False as $\cos A$ is the abbreviation of cosine A.

(iv) False as $\cot A$ is not the product of 'cot' and A. 'cot' is separated from A has no meaning.

(v) False as $\sin\theta$ cannot be > 1

(Ex. 8.2)

1. Evaluate:

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Ans. (i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$= \dfrac{\sqrt{3}}{2} \times \dfrac{\sqrt{3}}{2} + \dfrac{1}{2} \times \dfrac{1}{2}$

$= \dfrac{3}{4} + \dfrac{1}{4} = \dfrac{4}{4} = 1$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$= 2(1)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2 - \left(\dfrac{\sqrt{3}}{2}\right)^2$

$= 2 + \dfrac{3}{4} - \dfrac{3}{4}$

= 2

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$= \dfrac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \dfrac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$

$= \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2+2\sqrt{3}} = \dfrac{\sqrt{3}}{\sqrt{2} \times 2\left(\sqrt{3}+1\right)}$

$$= \frac{\sqrt{3}}{\sqrt{2}\times 2\left(\sqrt{3}+1\right)}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{\sqrt{3}\left(\sqrt{3}-1\right)}{\sqrt{2}\times 2\left(3-1\right)}$$ [Since $(a+b)(a-b)=a^2-b^2$]

$$= \frac{\sqrt{3}\left(\sqrt{3}-1\right)}{4\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) $\dfrac{\sin 30^\circ+\tan 45^\circ-\cos ec60^\circ}{\sec 30^\circ+\cos 60^\circ+\cot 45^\circ}$

$$= \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}} = \frac{3\sqrt{3}-4}{3\sqrt{3}+4}$$

$$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4}\times\frac{3\sqrt{3}-4}{3\sqrt{3}-4}$$

$$= \frac{27+16-24\sqrt{3}}{27-16}$$ [Since $(a+b)(a-b)=a^2-b^2$]

$$= \frac{43-24\sqrt{3}}{11}$$

(v) $\dfrac{5\cos^2 60^\circ+4\sec^2 30^\circ-\tan^2 45^\circ}{\sin^2 30^\circ+\cos^2 30^\circ}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5 \times \frac{1}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{4}{4}}$$

$$= \frac{15+64-12}{12} = \frac{67}{12}$$

---

2. Choose the correct option and justify:

(i) $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$ =

(A) $\sin 60^\circ$

(B) $\cos 60^\circ$

(C) $\tan 60^\circ$

(D) $\sin 30^\circ$

(ii) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ}$ =

(A) $\tan 90^\circ$

(B) 1

(C) $\sin 45^\circ$

(D) 0

(iii) $\sin 2A = 2\sin A$ **is true when A =**

(A) 0°

(B) 30°

(C) 45°

(D) 60°

(iv) $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$ =

(A) $\cos 60^\circ$

(B) $\sin 60^\circ$

(C) $\tan 60^\circ$

(D). None of these

Ans. (i) (A) $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$

$= \frac{2 \times 1/\sqrt{3}}{1+\left(1/\sqrt{3}\right)^2}$

$= \frac{2}{\sqrt{3}} \times \frac{3}{3+1} = \frac{\sqrt{3}}{2} = \sin 60^\circ$

(ii) (D) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

(iii). (A) Since A = 0, then

$\sin 2A = \sin 0^\circ = 0$ and

$2\sin A = 2\sin 0^\circ$ = $2 \times 0 = 0$

$\therefore \sin 2A = \sin A$ when A = 0

- **(iv).** $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$

= $\frac{2\times\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$ = $\frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$

= $\frac{2}{\sqrt{3}}\times\frac{3}{3-1}$ = $\sqrt{3} = \tan 60^\circ$

---

3. If $\tan(A+B) = \sqrt{3}$ **and** $\tan(A-B) = \frac{1}{\sqrt{3}};\ 0^\circ < A+B \le 90^\circ;\ A > B,$ **find A and B.**

Ans. $\tan(A+B) = \sqrt{3}$

$\Rightarrow \tan(A+B) = \tan 60^\circ$

$\Rightarrow$ A + B = 60° ……….(i)

Also, $\tan(A-B) = \frac{1}{\sqrt{3}}$

$\Rightarrow$ A – B = 30° ……….(ii)

On adding eq. (i) and (ii), we get,

2A= $90^\circ \Rightarrow$ A= 45°

On Subtracting eq. (i) and eq. (ii), we get

2B= $30^\circ \Rightarrow$ B= 15°

---

7. State whether the following are true or false. Justify your answer.

(i) $\sin(A+B) = \sin A + \sin B$

(ii)The value of $\sin\theta$ **increases as** θ **increases.**

(iii)The value of $\cos\theta$ **increases as** θ **increases.**

(iv) $\sin\theta = \cos\theta$ **for all values of** $\theta.$

(v) $\cot A$ **is not defined for** $A = 0^\circ.$

Ans. (i) False, because, let A = 60° and B = 30°

Then, $\sin(A+B) = \sin(60^\circ + 30^\circ) = \sin 90^\circ$ = 1

And $\sin A + \sin B = \sin 60^\circ + \sin 30^\circ = \frac{\sqrt{3}}{2} + \frac{1}{2}$ = $\frac{\sqrt{3}+1}{2}$

$\therefore \sin(A+B) \neq \sin A + \sin B$

Ans
\
(ii)True, because it is clear from the table below

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1

Therefore, it is clear, the value of $\sin\theta$ increases as θ increases.

(vi) False, because

θ	0°	30°	45°	60°	90°
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

It is clear, the value of $\cos\theta$ decreases as θ increases

(vi) False as it is only true for $\theta = 45^\circ$.

$$\Rightarrow \sin 45^\circ = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

(vii) True, because $\tan 0^\circ = 0$ and $\cot 0^\circ = \frac{1}{\tan 0^\circ}$

i.e. $\frac{1}{0}$ undefined.

Ex – 8.3 (deleted)

Ex. 8.4

1. **Express the trigonometric ratios** $\sin A, \sec A$ **and** $\tan A$ **in terms of** $\cot A$

Ans. For $\sin A,$

By using identity $\operatorname{cosec}^2 A - \cot^2 A = 1$

$$\Rightarrow \operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\Rightarrow \frac{1}{\sin^2 A} = 1 + \cot^2 A$$

$$\Rightarrow \sin^2 A = \frac{1}{1 + \cot^2 A}$$

$$\Rightarrow \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

For $\sec A,$

By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \sec^2 A = 1 + \tan^2 A$$

$$\sec^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$

For $\tan A,$

$$\tan A = \frac{1}{\cot A}$$

(iii) Write the other trigonometric ratios of A in terms of $\sec A$ Ans.

For $\sin A,$

By using identity, $\sin^2 A + \cos^2 A = 1$

$$\Rightarrow \sin^2 A = 1 - \cos^2 A$$

$$\Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A} = \frac{\sec^2 A - 1}{\sec^2 A}$$

$$\Rightarrow \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

For $\cos A,$

$$\cos A = \frac{1}{\sec A}$$

For $\tan A,$

By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \tan^2 A = \sec^2 A - 1$$

$$\Rightarrow \tan A = \sqrt{\sec^2 A - 1}$$

For $\cos ecA,$

$$\cos ecA = \frac{1}{\sin A} \quad \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

For $\cot A$,

$$\cot A = \frac{1}{\tan A}$$

$$\Rightarrow \cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$

5. Evaluate:

(i) $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

Ans. (i) $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

= $\frac{\sin^2 63° + \sin^2 (90° - 63°)}{\cos^2 (90° - 73°) + \cos^2 73°}$

= $\frac{\sin^2 63° + \cos^2 63°}{\sin^2 73° + \cos^2 73°}$

$\left[\because \sin(90° - \theta) = \cos\theta, \cos(90° - \theta) = \sin\theta\right]$

= $\frac{1}{1} = 1 \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

= $\sin 25°.\cos(90° - 25°) + \cos 25°.\sin(90° - 25°)$

= $\sin 25°.\sin 25° + \cos 25°.\cos 25°$

= $\sin^2 25° + \cos^2 25°$ = 1

$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

---

4. Choose the correct option. Justify your choice:

(i) $9\sec^2 A - 9\tan^2 A$ =

(A) 1

(B) 9

(C) 8

(D) 0

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$ =

(A) 0

(B) 1

(C) 2

(D) none of these

(iii) $(\sec A+\tan A)(1-\sin A)$ =

(A) $\sec A$

(B) $\sin A$

(C) $\operatorname{cosec} A$ **(D).** $\cos A$

(iv) $\dfrac{1+\tan^2 A}{1+\cot^2 A}$ =

(A) $\sec^2 A$

(B) -1

(C) $\cot^2 A$

(D) none of these

Ans. (i) (B) $9\sec^2 A - 9\tan^2 A$

= $9\left(\sec^2 A - \tan^2 A\right)$

= $9\times 1 = 9$ [Since $\sec^2\theta - \tan^2\theta = 1$]

(ii) (C) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$

= $\left(1+\dfrac{\sin\theta}{\cos\theta}+\dfrac{1}{\cos\theta}\right)\left(1+\dfrac{\cos\theta}{\sin\theta}-\dfrac{1}{\sin\theta}\right)$

= $\left(\dfrac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\dfrac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$

= $\dfrac{(\cos\theta+\sin\theta)^2-(1)^2}{\cos\theta.\sin\theta}$

= $\dfrac{\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$

= $\dfrac{1+2\cos\theta\sin\theta-1}{\cos\theta.\sin\theta}$

$\left[\because \sin^2\theta+\cos^2\theta=1\right]$

$\dfrac{2\cos\theta\sin\theta}{\cos\theta.\sin\theta}$

= = 2

(iii)(D) $(\sec A+\tan A)(1-\sin A)$

= $\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$

= $\left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$

= $\frac{1-\sin^2 A}{\cos A}$ [Since $(a+b)(a-b)=a^2-b^2$]

= $\frac{\cos^2 A}{\cos A}$

= $\cos A \left[\because 1-\sin^2 A=\cos^2 A\right]$

(iv)(D) $\frac{1+\tan^2 A}{1+\cot^2 A}$ = $\frac{\sec^2 A-\tan^2 A+\tan^2 A}{\operatorname{cosec}^2 A-\cot^2 A+\cot^2 A}$

= $\frac{\sec^2 A}{\operatorname{cosec}^2 A}$ = $\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$

= $\frac{\sin^2 A}{\cos^2 A}$ = $\tan^2 A$

5. **Prove the following identities, where the angles involved are acute angles for which the expressions are defined:**

(i) $(\operatorname{cosec}\theta-\cot\theta)^2=\frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\mathrm{cosec}\,\theta$

(iv) $\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$

(v) $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\mathrm{cosec}A+\cot A$**, using the identity** $\mathrm{cosec}^2 A=1+\cot^2 A$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}}=\sec A+\tan A$

(vii) $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}=\tan\theta$

(viii) $(\sin A+\mathrm{cosec}A)^2+(\cos A+\sec A)^2 = 7+\tan^2 A+\cot^2 A$

(ix) $(\mathrm{cosec}A-\sin A)(\sec A-\cos A) = \frac{1}{\tan A+\cot A}$

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

Ans. (i) L.H.S. $(\mathrm{cosec}\theta-\cot\theta)^2$

$= \cos ec^2\theta + \cot^2\theta - 2\cos ec\theta \cot\theta$ [Since $(a - b)^2 = a^2 + b^2 - 2ab$]

$= \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} - 2 \times \frac{1}{\sin\theta} \cdot \frac{\cos\theta}{\sin\theta}$

=

$= \frac{1 + \cos^2\theta - 2\cos\theta}{\sin^2\theta}$

$= \frac{(1 - \cos\theta)^2}{\sin^2\theta}$ $\left[\because a^2 + b^2 - 2ab = (a - b)^2\right]$

$= \frac{(1 - \cos\theta)(1 - \cos\theta)}{1 - \cos^2\theta}$

$= \frac{(1 - \cos\theta)(1 - \cos\theta)}{(1 + \cos\theta)(1 - \cos\theta)}$

(viii) $\frac{1 - \cos\theta}{1 + \cos\theta}$ = R.H.S.

= L.H.S. $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$

$= \frac{\cos^2\theta + 1 + \sin^2\theta + 2\sin A}{(1 + \sin A)\cos A}$

$= \frac{\cos^2\theta + \sin^2\theta + 1 + 2\sin A}{(1 + \sin A)\cos A}$

$= \frac{1 + 1 + 2\sin A}{(1 + \sin A)\cos A}$ $\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$= \dfrac{2+2\sin A}{(1+\sin A)\cos A} = \dfrac{2(1+\sin A)}{(1+\sin A)\cos A}$

$= \dfrac{2}{\cos A}$

= 2 sec A = RHS

(iii) L.H.S.

$= \dfrac{\dfrac{\sin\theta}{\cos\theta}}{1-\dfrac{\cos\theta}{\sin\theta}} + \dfrac{\dfrac{\cos\theta}{\sin\theta}}{1-\dfrac{\sin\theta}{\cos\theta}}$

$= \dfrac{\dfrac{\sin\theta}{\cos}}{\dfrac{\sin\theta-\cos\theta}{\sin\theta}} + \dfrac{\dfrac{\cos\theta}{\sin}}{\dfrac{\cos\theta-\sin\theta}{\cos\theta}}$

$= \dfrac{\sin\theta}{\cos\theta}\times\dfrac{\sin\theta}{\sin\theta-\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\times\dfrac{\cos\theta}{\cos\theta-\sin\theta}$

$= \dfrac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \dfrac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$= \dfrac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \dfrac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$= \dfrac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$= \dfrac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$\left[\because a^3-b^3=(a-b)(a^2+b^2+ab)\right]$

$= \dfrac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$ $\left[\because \sin^2\theta+\cos^2\theta=1\right]$

$= \dfrac{1}{\sin\theta\cos\theta} + \dfrac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta}+1 = 1+\dfrac{1}{\sin\theta\cos\theta}$

$= 1+\sec\theta\, \cos ec\theta$

(iv) $\dfrac{1+\sec A}{\sec A} = \dfrac{1+\dfrac{1}{\cos A}}{\dfrac{1}{\cos A}}$

$= \dfrac{\dfrac{\cos A+1}{\cos A}}{\dfrac{1}{\cos A}} = \dfrac{\cos A+1}{\cos A} \times \dfrac{\cos A}{1}$

$= 1+\cos A \times \dfrac{1-\cos A}{1-\cos A}$

$= \dfrac{1-\cos^2 A}{1-\cos A}$ [Since $(a+b)(a-b)=a^2-b^2$]

$= \dfrac{\sin^2 A}{1-\cos A}$ = R.H.S.

(v) L.H.S. $\dfrac{\cos A-\sin A+1}{\cos A+\sin A-1}$

Dividing all terms by $\sin A,$

$= \dfrac{\cot A-1+cosec\ A}{\cot A+1-cosec\ A} = \dfrac{\cot A+cosec\ A-1}{\cot A-cosec\ A+1}$

$$= \frac{\cot A + cosec\ A - (\ cosec^2 A - cot^2 A)}{\cot A + 1 - cosec\ A}$$

$$= \frac{(\cot A + cosec\ A\ ) - (cosec\ A + \cot A)\ (\text{cosec}\ A - cot\ A)}{\cot A + 1 - cosec\ A}$$

$$= \frac{(\cot A + cosec\ A\ )\ [1 - (cosec\ A - \cot A)]}{\cot A + 1 - cosec\ A}$$

$$= \frac{(\cot A + cosec\ A)\ (\ 1 - cosec\ A + \cot A\}}{(\cot A + 1 - cosec\ A)}$$

= cot A + cosec A

= R.H.S

(vi) L.H.S.

$$= \sqrt{\frac{1+\sin A}{1-\sin A}} \times \sqrt{\frac{1+\sin A}{1+\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}} \left[\because (a+b)(a-b) = a^2 - b^2\right]$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} \left[\because 1-\sin^2\theta = \cos^2\theta\right]$$

$$= \frac{1+\sin A}{\cos A} = \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

= $\sec A + \tan A$ = R.H.S.

(vii) L.H.S. $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

= $\dfrac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta-1\right)}$

= $\dfrac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left[2\left(1-\sin^2\theta\right)-1\right]}$

$\left[\because 1-\sin^2\theta=\cos^2\theta\right]$

= $\dfrac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2-2\sin^2\theta-1\right)}$

= $\dfrac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(1-2\sin^2\theta\right)}$ = $\dfrac{\sin\theta}{\cos\theta}$

= $\tan\theta$ = R.H.S

(viii) L.H.S. $\left(\sin A+\operatorname{cosec} A\right)^2+\left(\cos A+\sec A\right)^2$

= $\left(\sin A+\dfrac{1}{\sin A}\right)^2+\left(\cos A+\dfrac{1}{\cos A}\right)^2$

= $\sin^2 A+\dfrac{1}{\sin^2 A}+2.\sin A.\dfrac{1}{\sin A}+\cos^2 A+\dfrac{1}{\cos^2 A}+2.\cos A.\dfrac{1}{\cos A}$

= $2+2+\sin^2 A+\cos^2 A+\dfrac{1}{\sin^2 A}+\dfrac{1}{\cos^2 A}$

$= 4+1+\frac{1}{\sin^2 A}+\frac{1}{\cos^2 A}$

$= 5+\operatorname{cosec}^2 A+\sec^2 A$

$= 5+1+\cot^2 A+1+\tan^2 A$

$\left[\because \operatorname{cosec}^2\theta = 1+\cot^2\theta, \sec^2\theta = 1+\tan^2\theta\right]$

$= 7+\tan^2 A+\cot^2 A$

= R.H.S.

(ix) L.H.S. $(\operatorname{cosec} A-\sin A)(\sec A-\cos A)$

$= \left(\frac{1}{\sin A}-\sin A\right)\left(\frac{1}{\cos A}-\cos A\right)$

$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)$

$= \frac{\cos^2 A}{\sin A}\times\frac{\sin^2 A}{\cos A} = \sin A.\cos A$

=

Dividing all the terms by $\sin A.\cos A$,

$= \frac{\frac{\sin A.\cos A}{\sin A.\cos A}}{\frac{\sin^2 A}{\sin A.\cos A}+\frac{\cos^2 A}{\sin A.\cos A}}$

$= \frac{1}{\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}}$

$= \dfrac{1}{\tan A + \cot A}$ = R.H.S.

(x) L.H.S. $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right) = \dfrac{\sec^2 A}{\cos ec^2 A}$

$\left[\because 1+\tan^2\theta = \sec^2\theta, 1+\cot^2\theta = \cos ec^2\theta\right]$

$= \dfrac{1}{\cos^2 A} \times \dfrac{\sin^2 A}{1} = \tan^2 A$ = R.H.S.

Now, Middle side = $\left(\dfrac{1-\tan A}{1-\cot A}\right)^2 = \left(\dfrac{1-\tan A}{1-\dfrac{1}{\tan A}}\right)^2$

$= \left(\dfrac{1-\tan A}{\dfrac{\tan A - 1}{\tan A}}\right)^2$

$= \left(\dfrac{1-tan\ A}{\dfrac{-(1-tanA)}{tanA}}\right)^2$

$= (- \tan A)^2$

$= \tan^2 A$ -= R.H.S

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Grade – 10 MATHS Specimen copy Year 22-23

INDEX

Chapter – 15 Probability.

KEY POINTS TO REMEMBER –

- **(I) Probability**
- **(II) Miscellaneous Questions**

1. **The Theoretical probability of the occurrence of an event E written as P(E) is**

$$P(E) = \frac{\text{Number of outcomes favourable of } E}{\text{Number of all outcomes of the experiment}}$$

2. **Experiment:** An activity which ends in some well defined outcomes is called an experiment.
3. **Trial:** Performing an experiment once is called a trial.
4. **Event:** The possible outcomes of a trial are called an Event.
5. **Sure event**: An event whose occurrence is certain is called a sure event.
6. The sum of the probability of all the elementary events of an experiment is 1.
7. The probability of a sure event is 1 and probability of an impossible event is 0.
8. If E is an event, in general, it is true that

$$P(\text{not } E) = 1 - P(E)$$

$$P(\text{not } E) + P(E) = 1$$

∴ Probability of an event E + Probability of the event 'not E' = 1.

Event E and Event not E are called complementary events.

9. From the definition of the probability, the numerator is always less than or equal to the denominator therefore $0 \le P(E) \le 1$

Chapter - 15
Probability

Exercise 15.1

1. Complete the statements:

(i) **Probability of event E + Probability of event "not E" = _______________**

(ii) **The probability of an event that cannot happen is _______________. Such an event is called _______________.**

(iii) **The probability of an event that is certain to happen is _______________. Such an event is called _______________.**

(iv) **The sum of the probabilities of all the elementary events of an experiment is _______________.**

(v) **The probability of an event is greater than or equal to _______________ and less than or equal to _______________.**

Ans. (i) 1

(ii) 0, impossible event

(iii) 1, sure or certain event

(iv) 1

(v) 0, 1

2. Which of the following experiments have equally likely outcomes? Explain.

(i) A driver attempts to start a car. The car starts or does not start.

(ii) A player attempts to shoot a basketball. She/he shoots or misses the shot.

(iii) A trial is made to answer a true-false question. The answer is right or wrong.

(iv) A baby is born. It is a boy or a girl.

Ans. (i) In the experiment, "A driver attempts to start a car. The car starts or does not start", we are not justified to assume that each outcome is as likely to occur as the other. Thus, the experiment has no equally likely outcomes.

(ii) In the experiment, "A player attempts to shoot a basketball. She/he shoots or misses the shot", the outcome depends upon many factors e.g. quality of player. Thus, the experiment has no equally likely outcomes.

(iii) In the experiment, "A trial is made to answer a true-false question. The answer is right or wrong." We know, in advance, that the result can lead to one of the two possible ways – either right or wrong. We can reasonably assume that each outcome, right or wrong, is likely to occur as the other. Thus, the outcomes right or wrong are equally likely.

(iv) In the experiment, "A baby is born, It is a boy or a girl, we know, in advance that there are only two possible outcomes – either a boy or a girl. We are justified to assume that each outcome, boy or girl, is likely to occur as the other. Thus, the outcomes boy or girl are equally likely.

---

3. Why is tossing a coin considered to be a fair way of deciding which team should get the ball at the beginning of a football game?

Ans. The tossing of a coin is considered to be a fair way of deciding which team should get the ball at the beginning of a football game as we know that the tossing of the coin only land in one of two possible ways – either head up or tail up. It can reasonably be assumed that each outcome, head or tail, is as likely to occur as the other, i.e., the outcomes head and tail are equally likely. So the result of the tossing of a coin is completely unpredictable.

---

4. Which of the following cannot be the probability of an event:

(A) 2/ 3

(B) -1.5

(C) 15%

(D) 0.7

Ans.

(A) Since the probability of an event E is a number P(E) such that

$0 \leq P(E) \leq 1$ $0 \leq 2/3 \leq 1$ therefore 2/3 can be probability of Event

(B) Since the probability of an event E is a number P(E) such that

$0 \leq P(E) \leq 1$

$\therefore -1.5$ cannot be the probability of an event.

(c) Since the probability of an event E is a number P(E) such that

$0\% \leq P(E) \leq 100\%$

$\therefore$ $0\% \leq 15\% \leq 100\%$, therefore 15% can be probability of Event

(B) (D) Since the probability of an event E is a number P(E) such that

$0 \leq P(E) \leq 1$ $0 \leq 0.7 \leq 1$ therefore 0.7 can be probability of Event

5. If P(E) = 0.05, what is the probability of 'not E'? Ans.

Since P(E) + P (not E) = 1

$\therefore$ P (not E) = 1 – P(E) = 1 – 0.05 = 0.95

6. A bag contains lemon flavoured candles only. Malini takes out one candy without looking into the bag. What is the probability that she takes out:

(i) an orange flavoured candy?

(ii) a lemon flavoured candy?

Ans. (i) Consider the event related to the experiment of taking out of an orange flavoured candy from a bag containing only lemon flavoured candies. Since no outcome gives an orange flavoured candy, therefore, it is an impossible event. So its probability is 0.

(ii) Consider the event of taking a lemon flavoured candy out of a bag containing only lemon flavoured candies. This event is a certain event. So its probability is 1.

7. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

Ans. Let E be the event of having the same birthday

$\Rightarrow$ P(E) = 0.992

$\Rightarrow$ But $P(E) + P(\bar{E}) = 1$

$\therefore P(\bar{E}) = 1 - P(E) = 1 - 0.992 = 0.008$

8. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is:

(i) red?

(ii) not red?

Ans. There are 3 + 5 = 8 balls in a bag. Out of these 8 balls, one can be chosen in 8 ways

. ∴ Total number of elementary events = 8

(i) Since the bag contains 3 red balls, therefore, one red ball can be drawn in 3 ways. ∴

Favourable number of elementary events = 3

Hence P (getting a red ball) = $\frac{3}{8}$

(ii) Since the bag contains 5 black balls along with 3 red balls, therefore one black (not red) ball can be drawn in 5 ways.

∴ Favourable number of elementary events = 5

Hence P (getting a black ball) = $\frac{5}{8}$

---

9. of the box at random. What is the probability that the marble taken out will be:
A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out

(i) red? (ii) white? (iii) not green?

Ans. Total number of marbles in the box = 5 + 8 + 4 = 17

∴ Total number of elementary events = 17

(i) There are 5 red marbles in the box.

∴ Favourable number of elementary events = 5

∴ P (getting a red marble) = $\frac{5}{17}$

(ii) There are 8 white marbles in the box.

∴ Favourable number of elementary events = 8

∴ P (getting a white marble) = $\frac{8}{17}$

(iii) There are 5 + 8 = 13 marbles in the box, which are not green.

∴ Favourable number of elementary events = 13

∴ P (not getting a green marble) = $\frac{13}{17}$

---

10. A piggy bank contains hundred 50 p coins, fifty Re. 1 coins, twenty Rs. 2coins and ten Rs. 5 coins. If it is equally likely that of the coins will fall out when the bank is turned upside down, what is the probability that the coin:

(i) will be a 50 p coin?

(ii) will not be a Rs.5 coin?

Ans. Total number of coins in a piggy bank = 100 + 50 + 20 + 10 = 180

∴ Total number of elementary events = 180

(i) There are one hundred 50 coins in the piggy bank.
∴ Favourable number of elementary events = 100

∴ P (falling out of a 50 p coin) = $\frac{100}{180} = \frac{5}{9}$

(ii) There are 100 + 50 + 20 = 170 coins other than Rs. 5 coin. ∴

Favourable number of elementary events = 170

∴ P (falling out of a coin other than Rs. 5 coin) = $\frac{170}{180} = \frac{17}{18}$

---

11. Gopi buys a fish from a shop for his aquarium. The shopkeeper takes out one fish at random from a tank containing 5 male fishes and 8 female fishes (see figure). What is the probability that the fish taken out is a male fish?

Ans. Total number of fish in the tank = 5 + 8 = 13

$\therefore$ Total number of elementary events = 13

There are 5 male fishes in the tank.

$\therefore$ Favourable number of elementary events = 5

Hence, P (taking out a male fish) = $\frac{5}{13}$

12. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see figure) and these are equally likely outcomes. What is the probability that it will point at:

(i) 8?

(ii) an odd number?

(iii) a number greater than 2?

(iv) a number less than 9?

Ans. Out of 8 numbers, an arrow can point any of the numbers in 8 ways.

$\therefore$ Total number of possible outcomes = 8

(i) Favourable number of outcomes = 1

Hence, P (arrow points at 8) = $\frac{1}{8}$

(ii) Favourable number of outcomes = 4

Hence, P (arrow points at an odd number) = $\frac{4}{8} = \frac{1}{2}$

(iii) Favourable number of outcomes = 6

Hence, P (arrow points at a number > 2) = $\frac{6}{8} = \frac{3}{4}$

(iv) Favourable number of outcomes = 8

Hence, P (arrow points at a number < 9) = $\frac{8}{8}$=1

13. A dice is thrown once. Find the probability of getting:

(i) a prime number.

(ii) a number lying between 2 and 6.

(iii) an odd number.

Ans. Total number of Possible outcomes of throwing a dice = 6

(i) On a dice, the prime numbers are 2, 3 and 5.

Therefore, favourable outcomes = 3

Hence P (getting a prime number) = $\frac{3}{6} = \frac{1}{2}$

(ii) On a dice, the number lying between 2 and 6 are 3, 4, 5.

Therefore, favourable outcomes = 3

Hence P (getting a number lying between 2 and 6) = $\frac{3}{6} = \frac{1}{2}$

(iii) On a dice, the odd numbers are 1, 3 and 5.

Therefore, favourable outcomes = 3

Hence P (getting an odd number) = $\frac{3}{6} = \frac{1}{2}$

---

14. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting:

(i) a king of red colour

(ii) a face card

(iii) a red face card

(iv) the jack of hearts

(v) a spade

(vi) the queen of diamonds.

Ans. Total number of possible outcomes = 52

(i) There are two suits of red cards, i.e., diamond and heart. Each suit contains one king.

$\therefore$ Favourable outcomes = 2

Hence, P (a king of red colour) = $\frac{2}{52} = \frac{1}{26}$

(ii) There are 12 face cards in a pack

. $\therefore$ Favourable outcomes = 12

Hence, P (a face card) = $\frac{12}{52} = \frac{3}{13}$

(iii) There are two suits of red cards, i.e., diamond and heart. Each suit contains 3 face cards

. $\therefore$ Favourable outcomes = 2×3 = 6

Hence, P (a red face card) = $\frac{6}{52} = \frac{3}{26}$

(iv) There are only one jack of heart. $\therefore$

Favourable outcome = 1

Hence, P (the jack of hearts) = $\frac{1}{52}$

(v) There are 13 cards of spade.

$\therefore$ Favourable outcomes = 13

Hence, P (a spade) =

(vi) There is only one queen of diamonds. $\therefore$

Favourable outcome = 1

Hence, P (the queen of diamonds) = $\frac{1}{52}$

---

15. Five cards – then ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.

(i) What is the probability that the card is the queen?

(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

Ans. Total number of possible outcomes = 5

(i) There is only one queen.

$\therefore$ Favourable outcome = 1

Hence, P (the queen) = $\frac{1}{5}$

(ii) In this situation, total number of favourable outcomes = 4

(a) Favourable outcome = 1

Hence, P (an ace) = $\frac{1}{4}$

(b) There is no card as queen.

$\therefore$ Favourable outcome = 0

Hence, P (the queen) = $\frac{0}{4} = 0$

16. 12 defective pens are accidently mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

Ans. Total number of possible outcomes = 132 + 12 = 144

Number of favourable outcomes = 132

Hence, P (getting a good pen) = $\frac{132}{144} = \frac{11}{12}$

---

17. (i) A lot of 20 bulbs contains 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?

(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

Ans. (i) Total number of possible outcomes = 20

Number of favourable outcomes = 4

Hence P (getting a defective bulb) = $\frac{4}{20} = \frac{1}{5}$

(ii) Now total number of possible outcomes = 20 – 1 = 19

Number of favouroable outcomes = 19 – 4 = 15

Hence P (getting a non-defective bulb) = $\frac{15}{19}$

18. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.

Ans. Total number of possible outcomes = 90

Number of two-digit numbers from 1

to 90 are 90 – 9 = 81

$\therefore$ Favourable outcomes = 81

Hence, P (getting a disc bearing a two-digit number) = $\frac{81}{90} = \frac{9}{10}$

(ii) From 1 to 90, the perfect squares are 1, 4, 9, 16, 25, 36, 49, 64 and 81. $\therefore$

Favourable outcomes = 9

Hence P (getting a perfect square) = $\frac{9}{90} = \frac{1}{10}$

(iii) The numbers divisible by 5 from 1 to 90 are 18

. ∴ Favourable outcomes = 18

Hence P (getting a number divisible by 5) = $\frac{18}{90} = \frac{1}{5}$

19. A child has a die whose six faces show the letters as given below:

A B C D E A

The die is thrown once. What is the probability of getting:

(i) A?

(ii) D?

Ans. Total number of possible outcomes = 6

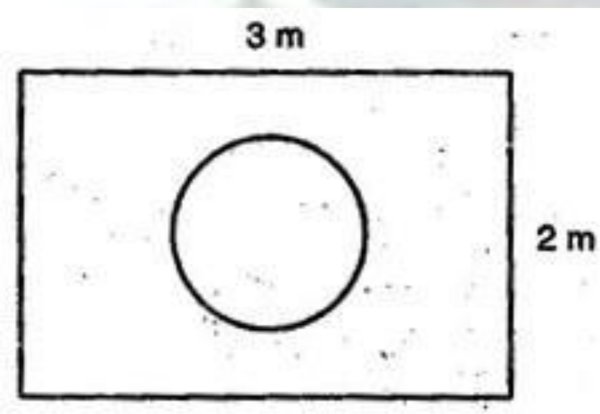
(i) Number of favourable outcomes = 2

Hence P (getting a letter A) = 2/6 = 1/3

(ii) Number of favourable outcomes = 1

Hence P (getting a letter D) = 1/6.

20. Suppose you drop a die at random on the rectangular region shown in the figure given on the next page. What is the probability that it will land inside the circle with diameter 1 m?



Ans. Total area of the given figure (rectangle) = $3 \times 2 = 6\ m^2$

And Area of circle = πr^2 = $\pi\left(\frac{1}{2}\right)^2$ = $\frac{\pi}{4}$ m^2

Hence, P (die to land inside the circle) = $\frac{\pi/4}{6} = \frac{\pi}{24}$

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21. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that:

(i) she will buy it?

(ii) she will not buy it?

Ans. Total number of possible outcomes = 144

Number of non-defective pens = 144 – 20 = 124

$\therefore$ Number of favourable outcomes = 124

Hence P (she will buy) = P (a non-defective pen) = 124 / 144

(ii) Number of favourable outcomes = 20

Hence P (she will not buy) = P (a defective pen) = $\frac{20}{144} = \frac{5}{36}$

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22. Refer to example 13.

(i) Complete the following table:

Event: Sum of 2 dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$						$\frac{5}{36}$				$\frac{1}{36}$

(ii) A student argues that 'there are 11 possible outcomes 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. Therefore each of them has a probability $\frac{1}{11}$. Do you agree with this argument? Justify your answer.

Ans. Total possible outcomes of throwing two dice are:

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

$\therefore$ Total number of favourable outcomes = 36

(i) Favourable outcomes of getting the sum as 3 = 2

Hence P (getting the sum as 3) = $\frac{2}{36} = \frac{1}{18}$

Favourable outcomes of getting the sum as 4 = 3

Hence P (getting the sum as 4) = $\frac{3}{36} = \frac{1}{12}$

Favourable outcomes of getting the sum as 5 = 4

Hence P (getting the sum as 5) = $\frac{4}{36} = \frac{1}{9}$

Favourable outcomes of getting the sum as 6 = 5

Hence P (getting the sum as 6) = $\frac{5}{36}$

Favourable outcomes of getting the sum as 7 = 6

Hence P (getting the sum as 7) = $\frac{6}{36}=\frac{1}{6}$

Favourable outcomes of getting the sum as 9 = 4

Hence P (getting the sum as 9) = $\frac{4}{36}=\frac{1}{9}$

Favourable outcomes of getting the sum as 10 = 3

Hence P (getting the sum as 10) = $\frac{3}{36}=\frac{1}{12}$

Favourable outcomes of getting the sum as 11 = 2

Hence P (getting the sum as 11) = $\frac{2}{36}=\frac{1}{18}$

Event: Sum of 2 dice	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

(ii) I do not agree with the argument given here. Justification has already been given in part

23. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result, i.e., three heads or three tails and loses otherwise. Calculate the probability that Hanif will lose the game.

Ans. The outcomes associated with the experiment in which a coin is tossed thrice:

HHH, HHT, HTH, THH, TTH, HTT, THT, TTT

Therefore, Total number of possible outcomes = 8

Number of favourable outcomes = 6

Hence required probability = $\frac{6}{8} = \frac{3}{4}$

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24. A die is thrown twice. What is the probability that:

(i) 5 will not come up either time?

(ii) 5 will come up at least once?

Ans. (i) The outcomes associated with the experiment in which a dice is thrown is twice:

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

Therefore, Total number of possible outcomes = 36

Now consider the following events:

A = first throe shows 5 and B = second throw shows 5

Therefore, the number of favourable outcomes = 6 in each case.

$\therefore$ P(A) = $\frac{6}{36}$ and P(B) = $\frac{6}{36}$

$\Rightarrow P(\overline{A}) = 1 - \frac{6}{36} = \frac{30}{36} = \frac{5}{6}$ and $P(\overline{B}) = \frac{5}{6}$

∴ Required probability = $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

(ii) Let S be the sample space associated with the random experiment of throwing a die twice.

Then, n(S) = 36

∴ $A \cap B$ = first and second throw shoe 5, i.e. getting 5 in each throw.

We have, A = (5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)

And B = (1, 5) (2, 5) (3, 5) (4, 5) (5, 5) (6, 5)

∴ $P(A) = \frac{6}{36}$, $P(B) = \frac{6}{36}$ and $P(A \cap B) = \frac{1}{36}$

∴ Required probability = Probability that at least one of the two throws shows 5

$= P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}$

25. Which of the following arguments are correct and which are not correct? Give reasons for your answer:

(i) If two coins are tossed simultaneously there are three possible outcomes – two heads, two tails or one of each. Therefore, for each of these outcomes, the probability is 1 / 3

(ii) If a die is thrown, there are two possible outcomes – an odd number or an even number.

Therefore, the probability of getting an odd number is $\frac{1}{2}$.

Ans. (i) Incorrect: We can classify the outcomes like this but they are not then, 'equally likely'. Reason is that 'one of each' can result in two ways – from a head on first coin and tail on the second coin or from a tail on the first coin and head on the second coin. This makes it twice as likely as two heads (or two tails).

Correct: The two outcomes considered in the question are equally likely

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WORK - SHEET

CHAPTER – 15 **Std -10th**

PROBABILITY

1. Complete the statements:

(I) Probability of event E + Probability of event "not E" = _______________

(II) The probability of an event that cannot happen is _______________. Such an event is called _______________.

(III) The probability of an event that is certain to happen is _______________. Such an event is called _______________.

(IV) The sum of the probabilities of all the elementary events of an experiment is _____________.

(V) The probability of an event is greater than or equal to _______________ and less than or equal to _______________.

2. Which of the following cannot be the probability of an event:

(A) = $\frac{2}{3}$ (B) = -1.5 (C) = 0.7 (D) = 15%

3. If P(E) = 0.05, what is the probability of 'not E'

* SOLVE (EACH CARRY TWO MARKS)

4. Find the probability of getting a head when a coin tossed once. Also find the probability of getting a tail.

5. It is given that in a group of 3 students, the probability of 2 students not having the same birthday is 0.992. What is the probability that the 2 students have the same birthday?

6. A bag contains 3 red balls and 5 black balls. A ball is drawn at random from the bag. What is the probability that the ball drawn is:

(I) red? (II) not red?

7.A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out the box at random. What is the probability that the marble taken out will be:

(I) red ? (ii) white ?

8. A coin is tossed once. What is the probability of getting a head ?

9. A die is thrown once. What is the probability that it show

(i) a 3 (ii) a 5 (iii) a number greater than 4?

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* SOLVE (EACH CARRY THREE MARKS)

10. A bag contain a red ball, a blue ball and a yellow ball, all the balls being of the same size. Rritika takes out a ball from the bag without looking into it. What is the probability that she take out the

(i) yellow ball (ii) red ball (iii) blue ball

11. One card is drawn from a well – shuffled deck of 52 cards. Calculate the probability that the card will

(i) be an ace (ii) not be an ace (iii) black king

12. A box contain 3 blue, 2 white and 4 red marbles. If a marble is drawn at random from the box, What is the Probability that it will be

(i) whit? (ii) blue ? (iii) red ?

13. In a lottery there are 10 prizes and 25 blanks. What is the probability of getting a prize?

14. In a family of 3 children, find the probability of having at least one boy.

15. Two different dice are rolled simultaneously. Find the probability that

(i) the sum of the numbers appeared is less than 7 ?

(ii) the product of the number appeared is less than 18.

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