पुना International School

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MATH

GRADE VII

SPECIMEN COPY

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CHAPTER – 4 Simple Equations

KEY POINTS TO REMEMBER

What Equation Is?
An equation is a condition on a variable. The condition is that two expressions should have equal value. Note at least one of the two expressions must contain the variable.
An equation remains the same when the expressions on the left and on the right are interchanged. This property is often useful in solving equations.

Solving an Equation
For any balanced numerical equation, if we either:

- add the same number to both sides,
- or subtract the same number from both sides,
- or multiply by the same number to both sides,
- or divide by the same number both its sides, the balance is undisturbed.

More Equations
Transposing means moving to the other side. It has the same effect as adding the same number to (or subtracting the same number from) both sides of the equation.
When we transpose a number from one side of the equation to the other side, we change its sign.

Applications of Simple Equations to Practical Situations
We know how to convert statements in everyday language into simple equations. To solve the problems (or puzzles), we have to solve these equations by the usual method.

Equations involving only a linear polynomial are called simple equations.
e.g. $4x + 5 = 65, 10y - 20 = 50$

In an equation, there is always an equality sign.

A Simple Equation remains the same when the expression in the left and right are interchanged.

The value of a variable, which makes the equation a true statement is called the solution of a linear equation.
e.g. $5x – 12 = -2$ is a equation
L.H.S = $5x – 12 = 5 \times 2 – 12 = 10 – 12 = -2$
L.H.S = R.H.S

In the case of the balanced equation, we have

- add the same number to both the sides,
- subtract the same number from both the sides
- multiply both sides by the same number
- divide both sides by the same number, the balance remains undisturbed, ie. The value of the LHS remains equal to the value of the RHS.

Transposing means moving to the other side. Transposition of a number has the same effect as adding the same number to (or subtracting the same number from) both sides of the equation. When you transpose a number from one side of the equation to the other side, you change its sign. For example, transposing +3 from the LHS to the RHS in equation $x + 3 = 8$ gives $x = 8 – 3 = 5$.

EXERCISE 4.1

1. Complete the last column of the table.

S. No.	Equation	Value	Say, whether the equation is satisfied. (Yes/No)
(i)	$x + 3 = 0$	$x = 3$	
(ii)	$x + 3 = 0$	$x = 0$	
(iii)	$x + 3 = 0$	$x = -3$	
(iv)	$x – 7 = 1$	$x = 7$	
(v)	$x – 7 = 1$	$x = 8$	
(vi)	$5x = 25$	$x = 0$	
(vii)	$5x = 25$	$x = 5$	
(viii)	$5x = 25$	$x = -5$	
(ix)	$(m/3) = 2$	$m = – 6$	

(x)	(m/3) = 2	m = 0	
(xi)	(m/3) = 2	m = 6	

Solution:

(i) x + 3 = 0

LHS = x + 3

By substituting the value of x = 3

Then,

LHS = 3 + 3 = 6

By comparing LHS and RHS

LHS ≠ RHS

∴No, the equation is not satisfied.

(ii) x + 3 = 0

LHS = x + 3

By substituting the value of x = 0

Then,

LHS = 0 + 3 = 3

By comparing LHS and RHS

LHS ≠ RHS

∴No, the equation is not satisfied.

(iii) x + 3 = 0

LHS = x + 3

By substituting the value of x = – 3

Then,

LHS = – 3 + 3 = 0

By comparing LHS and RHS

LHS = RHS

∴Yes, the equation is satisfied

(iv) x – 7 = 1

LHS = x – 7

By substituting the value of x = 7

Then,

LHS = 7 – 7 = 0

By comparing LHS and RHS

LHS ≠ RHS

∴No, the equation is not satisfied

(v) x – 7 = 1

LHS = x – 7

By substituting the value of x = 8

Then,

LHS = 8 – 7 = 1

By comparing LHS and RHS

LHS = RHS

∴Yes, the equation is satisfied.

vi) 5x = 25

LHS = 5x

By substituting the value of x = 0

Then,

LHS = 5 × 0 = 0

By comparing LHS and RHS

LHS ≠ RHS

∴No, the equation is not satisfied.

(vii) 5x = 25

LHS = 5x

By substituting the value of x = 5

Then,

LHS = 5 × 5 = 25

By comparing LHS and RHS

LHS = RHS

∴Yes, the equation is satisfied.

(viii) 5x = 25

LHS = 5x

By substituting the value of x = -5

Then,

LHS = 5 × (-5) = – 25

By comparing LHS and RHS

LHS ≠ RHS

∴No, the equation is not satisfied.

(ix) m/3 = 2

LHS = m/3

By substituting the value of m = – 6

Then,

LHS = -6/3 = – 2

By comparing LHS and RHS

LHS ≠ RHS

∴No, the equation is not satisfied.

(x) m/3 = 2

LHS = m/3

By substituting the value of m = 0

Then,

LHS = 0/3 = 0

By comparing LHS and RHS

LHS ≠ RHS

∴No, the equation is not satisfied.

(xi) m/3 = 2

LHS = m/3

By substituting the value of m = 6

Then,

LHS = 6/3 = 2

By comparing LHS and RHS

LHS = RHS

∴Yes, the equation is satisfied.

S. No.	Equation	Value	Say, whether the equation is satisfied. (Yes/No)
(i)	x + 3 = 0	x = 3	No
(ii)	x + 3 = 0	x = 0	No
(iii)	x + 3 = 0	x = -3	Yes
(iv)	x – 7 = 1	x = 7	No
(v)	x – 7 = 1	x = 8	Yes

(vi)	5x = 25	x = 0	No
(vii)	5x = 25	x = 5	Yes
(viii)	5x = 25	x = -5	No
(ix)	(m/3) = 2	m = – 6	No
(x)	(m/3) = 2	m = 0	No
(xi)	(m/3) = 2	m = 6	Yes

2. Check whether the value given in the brackets is a solution to the given equation or not:

(a) n + 5 = 19 (n = 1)

Solution:

LHS = n + 5

By substituting the value of n = 1

Then,

LHS = n + 5

= 1 + 5

= 6

By comparing LHS and RHS

6 ≠ 19

LHS ≠ RHS

Hence, the value of n = 1 is not a solution to the given equation n + 5 = 19.

(b) 7n + 5 = 19 (n = – 2)

Solution:

LHS = 7n + 5

By substituting the value of n = -2

Then,

LHS = 7n + 5

= (7 × (-2)) + 5

= – 14 + 5

= – 9

By comparing LHS and RHS

-9 ≠ 19

LHS ≠ RHS

Hence, the value of n = -2 is not a solution to the given equation 7n + 5 = 19.

c) 7n + 5 = 19 (n = 2)

Solution:

LHS = 7n + 5

By substituting the value of n = 2

Then,

LHS = 7n + 5

= (7 × (2)) + 5

= 14 + 5

= 19

By comparing LHS and RHS

19 = 19

LHS = RHS

Hence, the value of n = 2 is a solution to the given equation 7n + 5 = 19.

(d) 4p – 3 = 13 (p = 1)

Solution:

LHS = 4p – 3

By substituting the value of p = 1

Then,

LHS = 4p – 3

= (4 × 1) – 3

= 4 – 3

= 1

By comparing LHS and RHS

1 ≠ 13

LHS ≠ RHS

Hence, the value of p = 1 is not a solution to the given equation 4p – 3 = 13.

(e) 4p – 3 = 13 (p = – 4)

Solution:

LHS = 4p – 3

By substituting the value of p = – 4

Then,

LHS = 4p – 3

= (4 × (-4)) – 3

= -16 – 3

= -19

By comparing LHS and RHS

-19 ≠ 13

LHS ≠ RHS

Hence, the value of p = -4 is not a solution to the given equation 4p – 3 = 13.

(f) 4p – 3 = 13 (p = 0)

Solution:

LHS = 4p – 3

By substituting the value of p = 0

Then,

LHS = 4p – 3

= (4 × 0) – 3

= 0 – 3

= -3

By comparing LHS and RHS

– 3 ≠ 13

LHS ≠ RHS

Hence, the value of p = 0 is not a solution to the given equation 4p – 3 = 13.

3. Solve the following equations by trial and error method:

(i) 5p + 2 = 17

Solution:

LHS = 5p + 2

By substituting the value of p = 0

Then,

LHS = 5p + 2

= (5 × 0) + 2

= 0 + 2

= 2

By comparing LHS and RHS

2 ≠ 17

LHS ≠ RHS

Hence, the value of p = 0 is not a solution to the given equation.

Let, p = 1

LHS = 5p + 2

= (5 × 1) + 2

= 5 + 2

= 7

By comparing LHS and RHS

7 ≠ 17

LHS ≠ RHS

Hence, the value of p = 1 is not a solution to the given equation.

Let, p = 2

LHS = 5p + 2

= (5 × 2) + 2

= 10 + 2

= 12

By comparing LHS and RHS

12 ≠ 17

LHS ≠ RHS

Hence, the value of p = 2 is not a solution to the given equation.

Let, p = 3

LHS = 5p + 2

= (5 × 3) + 2

= 15 + 2

= 17

By comparing LHS and RHS

17 = 17

LHS = RHS

Hence, the value of p = 3 is a solution to the given equation.

(ii) 3m – 14 = 4

Solution:

LHS = 3m – 14

By substituting the value of m = 3

Then,

LHS = 3m – 14

= (3 × 3) – 14

= 9 – 14

= – 5

By comparing LHS and RHS

-5 ≠ 4

LHS ≠ RHS

Hence, the value of m = 3 is not a solution to the given equation.

Let, m = 4

LHS = 3m – 14

= (3 × 4) – 14

= 12 – 14

= – 2

By comparing LHS and RHS

-2 ≠ 4

LHS ≠ RHS

Hence, the value of m = 4 is not a solution to the given equation.

Let, m = 5

LHS = 3m – 14

= (3 × 5) – 14

= 15 – 14

= 1

By comparing LHS and RHS

1 ≠ 4

LHS ≠ RHS

Hence, the value of m = 5 is not a solution to the given equation.

Let, m = 6

LHS = 3m – 14

= (3 × 6) – 14

= 18 – 14

= 4

By comparing LHS and RHS

4 = 4

LHS = RHS

Hence, the value of m = 6 is a solution to the given equation.

4. Write equations for the following statements:

(i) The sum of numbers x and 4 is 9.

Solution:

The above statement can be written in the equation form as,

= $x + 4 = 9$

(ii) 2 subtracted from y is 8.

Solution:

The above statement can be written in the equation form as,

= $y - 2 = 8$

(iii) Ten times a is 70.

Solution:

The above statement can be written in the equation form as,

= $10a = 70$

(iv) The number b divided by 5 gives 6.

Solution:

The above statement can be written in the equation form as,

= $(b/5) = 6$

(v) Three-fourth of t is 15.

Solution:

The above statement can be written in the equation form as,

= $\frac{3}{4}t = 15$

(vi) Seven times m plus 7 gets you 77.

Solution:

The above statement can be written in the equation form as,

Seven times m is 7m

= $7m + 7 = 77$

(vii) One-fourth of a number x minus 4 gives 4.

Solution:

The above statement can be written in the equation form as,

One-fourth of a number x is x/4

= $x/4 - 4 = 4$

(viii) If you take away 6 from 6 times y, you get 60.

Solution:

The above statement can be written in the equation form as,

6 times of y is 6y

= $6y - 6 = 60$

(ix) If you add 3 to one-third of z, you get 30.

Solution:

The above statement can be written in the equation form as,

One-third of z is z/3

= 3 + z/3 = 30

5. Write the following equations in statement forms:

(i) p + 4 = 15

Solution:

The sum of numbers p and 4 is 15.

(ii) m – 7 = 3

Solution:

7 subtracted from m is 3.

(iii) 2m = 7

Solution:

Twice of number m is 7.

(iv) m/5 = 3

Solution:

The number m divided by 5 gives 3.

(v) (3m)/5 = 6

Solution:

Three-fifth of m is 6.

(vi) 3p + 4 = 25

Solution:

Three times p plus 4 gives you 25.

(vii) 4p – 2 = 18

Solution:

Four times p minus 2 gives you 18.

(viii) p/2 + 2 = 8

Solution:

If you add half of a number p to 2, you get 8.

6. Set up an equation in the following cases:

(i) Irfan says that he has 7 marbles more than five times the marbles Parmit has. Irfan has 37 marbles. (Take m to be the number of Parmit's marbles.)

Solution:

From the question it is given that,

Number of Parmit's marbles = m

Then,

Irfan has 7 marbles more than five times the marbles Parmit has

= 5 × Number of Parmit's marbles + 7 = Total number of marbles Irfan having

= (5 × m) + 7 = 37

= 5m + 7 = 37

(ii) Laxmi's father is 49 years old. He is 4 years older than three times Laxmi's age. (Take Laxmi's age to be y years.)

Solution:

From the question it is given that,

Let Laxmi's age to be = y years old

Then,

Lakshmi's father is 4 years older than three times of her age

= 3 × Laxmi's age + 4 = Age of Lakshmi's father

= (3 × y) + 4 = 49

= 3y + 4 = 49

(iii) The teacher tells the class that the highest marks obtained by a student in her class is twice the lowest marks plus 7. The highest score is 87. (Take the lowest score to be l.)

Solution:

From the question it is given that,

Highest score in the class = 87

Let lowest score be l

= 2 × Lowest score + 7 = Highest score in the class

= (2 × l) + 7 = 87

= 2l + 7 = 87

(iv) In an isosceles triangle, the vertex angle is twice either base angle. (Let the base angle be b in degrees. Remember that the sum of angles of a triangle is 180 degrees).

Solution:

From the question it is given that,

We know that, the sum of angles of a triangle is 180^{o}

Let base angle be b

Then,

Vertex angle = 2 × base angle = 2b

$= b + b + 2b = 180^{o}$

$= 4b = 180^{o}$

EXERCISE 4.2

1. Give first the step you will use to separate the variable and then solve the equation:

(a) *x* – 1 = 0

Solution:

We have to add 1 to both the side of given equation,

Then we get,

= x – 1 + 1 = 0 + 1

= x = 1

(b) *x* + 1 = 0

Solution:

We have to subtract 1 to both the side of given equation,

Then we get,

= x + 1 – 1 = 0 – 1

= x = – 1

(c) *x* – 1 = 5

Solution:

We have to add 1 to both the side of given equation,

Then we get,

= x – 1 + 1 = 5 + 1

= x = 6

(d) *x* + 6 = 2

Solution:

We have to subtract 6 to both the side of given equation,

Then we get,

= x + 6 – 6 = 2 – 6

= x = – 4

(e) *y* – 4 = – 7

Solution:

We have to add 4 to both the side of given equation,

Then we get,

= y – 4 + 4 = – 7 + 4

= y = – 3

(f) *y* – 4 = 4

Solution:

We have to add 4 to both the side of given equation,

Then we get,

= y – 4 + 4 = 4 + 4

= y = 8

(g) *y* + 4 = 4

Solution:

We have to subtract 4 to both the side of given equation,

Then we get,

= y + 4 – 4 = 4 – 4

= y = 0

(h) *y* + 4 = – 4

Solution:

We have to subtract 4 to both the side of given equation,

Then we get,

= y + 4 – 4 = – 4 – 4

= y = – 8

2. Give first the step you will use to separate the variable and then solve the equation:

(a) 3l = 42

Solution:

Now we have to divide both sides of the equation by 3,

Then we get,

= 3l/3 = 42/3

= l = 14

(b) b/2 = 6

Solution:

Now we have to multiply both sides of the equation by 2,

Then we get,

= b/2 × 2= 6 × 2

= b = 12

(c) p/7 = 4

Solution:

Now we have to multiply both sides of the equation by 7,

Then we get,

= p/7 × 7= 4 × 7

= p = 28

(d) 4x = 25

Solution:

Now we have to divide both sides of the equation by 4,

Then we get,

= 4x/4 = 25/4

= x = 25/4

(e) 8y = 36

Solution:

Now we have to divide both sides of the equation by 8,

Then we get,

= 8y/8 = 36/8

= x = 9/2

(f) (z/3) = (5/4)

Solution:

Now we have to multiply both sides of the equation by 3,

Then we get,

= (z/3) × 3 = (5/4) × 3

= x = 15/4

(g) (a/5) = (7/15)

Solution:

Now we have to multiply both sides of the equation by 5,

Then we get,

= (a/5) × 5 = (7/15) × 5

= a = 7/3

(h) 20t = – 10

Solution:

Now we have to divide both sides of the equation by 20,

Then we get,

= 20t/20 = -10/20

= x = – ½

3. Give the steps you will use to separate the variable and then solve the equation:

(a) 3n – 2 = 46

Solution:

First we have to add 2 to the both sides of the equation,

Then, we get,

= 3n – 2 + 2 = 46 + 2

= 3n = 48

Now,

We have to divide both sides of the equation by 3,

Then, we get,

= 3n/3 = 48/3

= n = 16

(b) 5m + 7 = 17

Solution:

First we have to subtract 7 to the both sides of the equation,

Then, we get,

= 5m + 7 – 7 = 17 – 7

= 5m = 10

Now,

We have to divide both sides of the equation by 5,

Then, we get,

= 5m/5 = 10/5

= m = 2

(c) 20p/3 = 40

Solution:

First we have to multiply both sides of the equation by 3,

Then, we get,

= (20p/3) × 3 = 40 × 3

= 20p = 120

Now,

We have to divide both sides of the equation by 20,

Then, we get,

= 20p/20 = 120/20

= p = 6

(d) 3p/10 = 6

Solution:

First we have to multiply both sides of the equation by 10,

Then, we get,

= (3p/10) × 10 = 6 × 10

= 3p = 60

Now,

We have to divide both sides of the equation by 3,

Then, we get,

= 3p/3 = 60/3

= p = 20

4. Solve the following equations:

(a) 10p = 100

Solution:

Now,

We have to divide both sides of the equation by 10,

Then, we get,

= 10p/10 = 100/10

= p = 10

(b) 10p + 10 = 100

Solution:

First we have to subtract 10 to the both sides of the equation,

Then, we get,

= 10p + 10 – 10 = 100 – 10

= 10p = 90

Now,

We have to divide both sides of the equation by 10,

Then, we get,

= 10p/10 = 90/10

= p = 9

(c) p/4 = 5

Solution:

Now,

We have to multiply both sides of the equation by 4,

Then, we get,

= p/4 × 4 = 5 × 4

= p = 20

(d) – p/3 = 5

Solution:

Now,

We have to multiply both sides of the equation by – 3,

Then, we get,

= – p/3 × (- 3) = 5 × (- 3)

= p = – 15

(e) 3p/4 = 6

Solution:

First we have to multiply both sides of the equation by 4,

Then, we get,

= (3p/4) × (4) = 6 × 4

= 3p = 24

Now,

We have to divide both sides of the equation by 3,

Then, we get,

= 3p/3 = 24/3

= p = 8

(f) 3s = – 9

Solution:

Now,

We have to divide both sides of the equation by 3,

Then, we get,

= 3s/3 = -9/3

= s = -3

(g) 3s + 12 = 0

Solution:

First we have to subtract 12 to the both sides of the equation,

Then, we get,

= 3s + 12 – 12 = 0 – 12

= 3s = -12

Now,

We have to divide both sides of the equation by 3,

Then, we get,

= 3s/3 = -12/3

= s = – 4

(h) 3s = 0

Solution:

Now,

We have to divide both sides of the equation by 3,

Then, we get,

= 3s/3 = 0/3

= s = 0

(i) 2q = 6

Solution:

Now,

We have to divide both sides of the equation by 2,

Then, we get,

= 2q/2 = 6/2

= q = 3

(j) 2q – 6 = 0

Solution:

First we have to add 6 to the both sides of the equation,

Then, we get,

= 2q – 6 + 6 = 0 + 6

= 2q = 6

Now,

We have to divide both sides of the equation by 2,

Then, we get,

= 2q/2 = 6/2

= q = 3

(k) 2q + 6 = 0

Solution:

First we have to subtract 6 to the both sides of the equation, Then, we get,

= 2q + 6 – 6 = 0 – 6

= 2q = – 6

Now,

We have to divide both sides of the equation by 2,

Then, we get,

= 2q/2 = – 6/2

= q = – 3

(l) 2q + 6 = 12

Solution:

First we have to subtract 6 to the both sides of the equation,

Then, we get,

= 2q + 6 – 6 = 12 – 6

= 2q = 6

Now,

We have to divide both sides of the equation by 2,

Then, we get,

= 2q/2 = 6/2

= q = 3

EXERCISE : 4.3

1. Solve the following equations:

(a) 2y + (5/2) = (37/2)

Solution:

By transposing (5/2) from LHS to RHS it becomes -5/2

Then,

= 2y = (37/2) – (5/2)

= 2y = (37-5)/2

= 2y = 32/2

Now,

Divide both side by 2,

= 2y/2 = (32/2)/2

= y = (32/2) × (1/2)

= y = 32/4

= y = 8

(b) 5t + 28 = 10

Solution:

By transposing 28 from LHS to RHS it becomes -28

Then,

= 5t = 10 – 28

= 5t = – 18

Now,

Divide both side by 5,

= 5t/5= -18/5

t = -18/5

(c) (a/5) + 3 = 2

Solution:

By transposing 3 from LHS to RHS it becomes -3

Then,

= a/5 = 2 – 3

= a/5 = – 1

Now,

Multiply both side by 5,

= (a/5) × 5= -1 × 5

= a = -5

(d) (q/4) + 7 = 5

Solution:

By transposing 7 from LHS to RHS it becomes -7

Then,

= q/4 = 5 – 7

= q/4 = – 2

Now,

Multiply both side by 4,

= (q/4) × 4= -2 × 4

= a = -8

(e) (5/2) x = -5

Solution:

First we have to multiply both the side by 2,

= (5x/2) × 2 = – 5 × 2

= 5x = – 10

Now,

We have to divide both the side by 5,

Then we get,

= 5x/5 = -10/5

= x = -2

(f) (5/2) x = 25/4

Solution:

First we have to multiply both the side by 2,

= (5x/2) × 2 = (25/4) × 2

= 5x = (25/2)

Now,

We have to divide both the side by 5,

Then we get,

= 5x/5 = (25/2)/5

= x = (25/2) × (1/5)

= x = (5/2)

g) 7m + (19/2) = 13

Solution:

By transposing (19/2) from LHS to RHS it becomes -19/2

Then,

= 7m = 13 – (19/2)

= 7m = (26 – 19)/2

= 7m = 7/2

Now,

Divide both side by 7,

= 7m/7 = (7/2)/7

= m = (7/2) × (1/7)

= m = ½

(h) 6z + 10 = – 2

Solution:

By transposing 10 from LHS to RHS it becomes – 10

Then,

= 6z = -2 – 10

= 6z = – 12

Now,

Divide both side by 6,

= 6z/6 = -12/6

= m = – 2

(i) (3/2) l = 2/3

Solution:

First we have to multiply both the side by 2,

= (3l/2) × 2 = (2/3) × 2

= 3l = (4/3)

Now,

We have to divide both the side by 3,

Then we get,

= 3l/3 = (4/3)/3

= l = (4/3) × (1/3)

= x = (4/9)

(j) (2b/3) – 5 = 3

Solution:

By transposing -5 from LHS to RHS it becomes 5

Then,

= 2b/3 = 3 + 5

= 2b/3 = 8

Now,

Multiply both side by 3,

= (2b/3) × 3= 8 × 3

= 2b = 24

And,

Divide both side by 2,

= 2b/2 = 24/2

= b = 12

2. Solve the following equations:

(a) 2(x + 4) = 12

Solution:

Let us divide both the side by 2,

= (2(x + 4))/2 = 12/2

= x + 4 = 6

By transposing 4 from LHS to RHS it becomes -4

= x = 6 – 4

= x = 2

(b) 3(n – 5) = 21

Solution:

Let us divide both the side by 3,

= (3(n – 5))/3 = 21/3

= n – 5 = 7

By transposing -5 from LHS to RHS it becomes 5

= n = 7 + 5

= n = 12

(c) 3(n – 5) = – 21

Solution:

Let us divide both the side by 3,

= (3(n – 5))/3 = – 21/3

= n – 5 = -7

By transposing -5 from LHS to RHS it becomes 5

= n = – 7 + 5

= n = – 2

(d) – 4(2 + x) = 8

Solution:

Let us divide both the side by -4,

= (-4(2 + x))/ (-4) = 8/ (-4)

= 2 + x = -2

By transposing 2 from LHS to RHS it becomes – 2

= x = -2 – 2

= x = – 4

(e) 4(2 – x) = 8

Solution:

Let us divide both the side by 4,

= (4(2 – x))/ 4 = 8/ 4

= 2 – x = 2

By transposing 2 from LHS to RHS it becomes – 2

= – x = 2 – 2

= – x = 0

= x = 0

3. Solve the following equations:

(a) 4 = 5(p – 2)

Solution:

Let us divide both the side by 5,

= 4/5 = (5(p – 2))/5

= 4/5 = p -2

By transposing – 2 from RHS to LHS it becomes 2

= (4/5) + 2 = p

= (4 + 10)/ 5 = p

= p = 14/5

(b) – 4 = 5(p – 2)

Solution:

Let us divide both the side by 5,

= – 4/5 = (5(p – 2))/5

= – 4/5 = p -2

By transposing – 2 from RHS to LHS it becomes 2

= – (4/5) + 2 = p

= (- 4 + 10)/ 5 = p

= p = 6/5

(c) 16 = 4 + 3(t + 2)

Solution:

By transposing 4 from RHS to LHS it becomes – 4

= 16 – 4 = 3(t + 2)

= 12 = 3(t + 2)

Let us divide both the side by 3,

= 12/3 = (3(t + 2))/ 3

= 4 = t + 2

By transposing 2 from RHS to LHS it becomes – 2

= 4 – 2 = t

= t = 2

(d) 4 + 5(p – 1) =34

Solution:

By transposing 4 from LHS to RHS it becomes – 4

= 5(p – 1) = 34 – 4

= 5(p – 1) = 30

Let us divide both the side by 5,

= (5(p – 1))/ 5 = 30/5

= p – 1 = 6

By transposing – 1 from RHS to LHS it becomes 1

= p = 6 + 1

= p = 7

(e) 0 = 16 + 4(m – 6)

Solution:

By transposing 16 from RHS to LHS it becomes – 16

= 0 – 16 = 4(m – 6)

= – 16 = 4(m – 6)

Let us divide both the side by 4,

= – 16/4 = (4(m – 6))/ 4

= – 4 = m – 6

By transposing – 6 from RHS to LHS it becomes 6

= – 4 + 6 = m

= m = 2

4. (a) Construct 3 equations starting with x = 2

Solution:

First equation is,

Multiply both side by 6

= 6x = 12 … [equation 1]

Second equation is,

Subtracting 4 from both side,

= 6x – 4 = 12 -4

= 6x – 4 = 8 … [equation 2]

Third equation is,

Divide both side by 6

= (6x/6) – (4/6) = (8/6)

= x – (4/6) = (8/6) … [equation 3]

(b) Construct 3 equations starting with x = – 2

Solution:

First equation is,

Multiply both side by 5

= 5x = -10 … [equation 1]

Second equation is,

Subtracting 3 from both side,

= 5x – 3 = – 10 – 3

= 5x – 3 = – 13 … [equation 2]

Third equation is,

Dividing both sides by 2

= (5x/2) – (3/2) = (-13/2)

EXERCISE : 4.4

1. Set up equations and solve them to find the unknown numbers in the following cases:

(a) Add 4 to eight times a number; you get 60.

Solution:

Let us assume the required number be x

Eight times a number = 8x

The given above statement can be written in the equation form as,

= 8x + 4 = 60

By transposing 4 from LHS to RHS it becomes – 4

= 8x = 60 – 4

= 8x = 56

Divide both side by 8,

Then we get,

= (8x/8) = 56/8

= x = 7

(b) One-fifth of a number minus 4 gives 3.

Solution:

Let us assume the required number be x

One-fifth of a number = (1/5) x = x/5

The given above statement can be written in the equation form as,

= (x/5) – 4 = 3

By transposing – 4 from LHS to RHS it becomes 4

= x/5 = 3 + 4

= x/5 = 7

Multiply both side by 5,

Then we get,

= (x/5) × 5 = 7 × 5

= x = 35

(c) If I take three-fourths of a number and add 3 to it, I get 21.

Solution:

Let us assume the required number be x

Three-fourths of a number = (3/4) x

The given above statement can be written in the equation form as,

= (3/4) x + 3 = 21

By transposing 3 from LHS to RHS it becomes – 3

= (3/4) x = 21 – 3

= (3/4) x = 18

Multiply both side by 4,

Then we get,

= (3x/4) × 4 = 18 × 4

= 3x = 72

Then,

Divide both side by 3,

= (3x/3) = 72/3

= x = 24

(d) When I subtracted 11 from twice a number, the result was 15.

Solution:

Let us assume the required number be x

Twice a number = 2x

The given above statement can be written in the equation form as,

= 2x –11 = 15

By transposing -11 from LHS to RHS it becomes 11

= 2x = 15 + 11

= 2x = 26

Then,

Divide both side by 2,

= (2x/2) = 26/2

= x = 13

(e) Munna subtracts thrice the number of notebooks he has from 50, he finds the result to be 8.

Solution:

Let us assume the required number be x

Thrice the number = 3x

The given above statement can be written in the equation form as,

= 50 – 3x = 8

By transposing 50 from LHS to RHS it becomes – 50

= – 3x = 8 – 50

= -3x = – 42

Then,

Divide both side by -3,

= (-3x/-3) = – 42/-3

= x = 14

(f) Ibenhal thinks of a number. If she adds 19 to it and divides the sum by 5, she will get 8.

Solution:

Let us assume the required number be x

The given above statement can be written in the equation form as,

= (x + 19)/5 = 8

Multiply both side by 5,

= ((x + 19)/5) × 5 = 8 × 5

= x + 19 = 40

Then,

By transposing 19 from LHS to RHS it becomes – 19

= x = 40 – 19

= x = 21

(g) Anwar thinks of a number. If he takes away 7 from 5/2 of the number, the result is 23.

Solution:

Let us assume the required number be x

5/2 of the number = (5/2) x

The given above statement can be written in the equation form as,

= (5/2) x – 7 = 23

By transposing -7 from LHS to RHS it becomes 7

= (5/2) x = 23 + 7

= (5/2) x = 30

Multiply both side by 2,

= ((5/2) x) × 2 = 30 × 2

= 5x = 60

Then,

Divide both the side by 5

= 5x/5 = 60/5

= x = 12

2. Solve the following:

(a) The teacher tells the class that the highest marks obtained by a student in her class is twice the lowest marks plus 7. The highest score is 87. What is the lowest score?

Solution:

Let us assume the lowest score be x

From the question it is given that,

The highest score is = 87

Highest marks obtained by a student in her class is twice the lowest marks plus 7= 2x + 7

5/2 of the number = (5/2) x

The given above statement can be written in the equation form as,

Then,

= 2x + 7 = Highest score

= 2x + 7 = 87

By transposing 7 from LHS to RHS it becomes -7

= 2x = 87 – 7

= 2x = 80

Now,

Divide both the side by 2

= 2x/2 = 80/2

= x = 40

Hence, the lowest score is 40

(b) In an isosceles triangle, the base angles are equal. The vertex angle is 40°.

What are the base angles of the triangle? (Remember, the sum of three angles of a triangle is 180°).

Solution:

From the question it is given that,

We know that, the sum of angles of a triangle is 180^o

Let base angle be b

Then,

= b + b + 40^o = 180^o

= 2b + 40 = 180^o

By transposing 40 from LHS to RHS it becomes -40

= 2b = 180 – 40

= 2b = 140

Now,

Divide both the side by 2

= 2b/2 = 140/2

= b = 70^o

Hence, 70^o is the base angle of an isosceles triangle.

(c) Sachin scored twice as many runs as Rahul. Together, their runs fell two short of a double century. How many runs did each one score?

Solution:

Let us assume Rahul's score be x

Then,

Sachin scored twice as many runs as Rahul is 2x

Together, their runs fell two short of a double century,

= Rahul's score + Sachin's score = 200 – 2

= x + 2x = 198

= 3x = 198

Divide both the side by 3,

= 3x/3 = 198/3

= x = 66

So, Rahul's score is 66

And Sachin's score is 2x = 2 × 66 = 132

3. Solve the following:

(i) Irfan says that he has 7 marbles more than five times the marbles Parmit has.

Irfan has 37 marbles. How many marbles does Parmit have?

Solution:

Let us assume number of Parmit's marbles = m

From the question it is given that,

Then,

Irfan has 7 marbles more than five times the marbles Parmit has

= 5 × Number of Parmit's marbles + 7 = Total number of marbles Irfan having

= (5 × m) + 7 = 37

= 5m + 7 = 37

By transposing 7 from LHS to RHS it becomes -7

= 5m = 37 – 7

= 5m = 30

Divide both the side by 5

= 5m/5 = 30/5

= m = 6

So, Permit has 6 marbles

(ii) Laxmi's father is 49 years old. He is 4 years older than three times Laxmi's age.

What is Laxmi's age?

Solution:

Let Laxmi's age to be = y years old

From the question it is given that,

Lakshmi's father is 4 years older than three times of her age

= 3 × Laxmi's age + 4 = Age of Lakshmi's father

= (3 × y) + 4 = 49

= 3y + 4 = 49

By transposing 4 from LHS to RHS it becomes -4

= 3y = 49 – 4

= 3y = 45

Divide both the side by 3

= 3y/3 = 45/3

= y = 15

So, Lakshmi’s age is 15 years.

(iii) People of Sundargram planted trees in the village garden. Some of the trees were fruit trees. The number of non-fruit trees were two more than three times the number of fruit trees. What was the number of fruit trees planted if the number of non-fruit trees planted was 77?

Solution:

Let the number of fruit tress be f.

From the question it is given that,

3 × number of fruit trees + 2 = number of non-fruit trees

= $3f + 2 = 77$

By transposing 2 from LHS to RHS it becomes -2

=$3f = 77 – 2$

= $3f = 75$

Divide both the side by 3

= $3f/3 = 75/3$

= $f = 25$

So, number of fruit tree was 25.

4. Solve the following riddle:

I am a number,

Tell my identity!

Take me seven times over

And add a fifty!

To reach a triple century

You still need forty!

Solution:

Let us assume the number be x.

Take me seven times over and add a fifty = $7x + 50$

To reach a triple century you still need forty = $(7x + 50) + 40 = 300$

= $7x + 50 + 40 = 300$

= $7x + 90 = 300$

By transposing 90 from LHS to RHS it becomes -90

= $7x = 300 – 90$

= $7x = 210$

Divide both side by 7

= $7x/7 = 210/7$

= $x = 30$

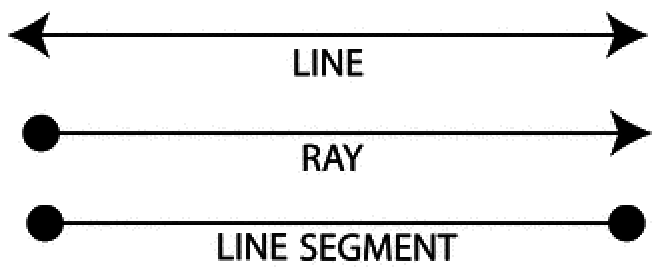
Hence the number is 30.

CHAPTER –5 Lines and Angles

KEY POINTS TO REMEMBER

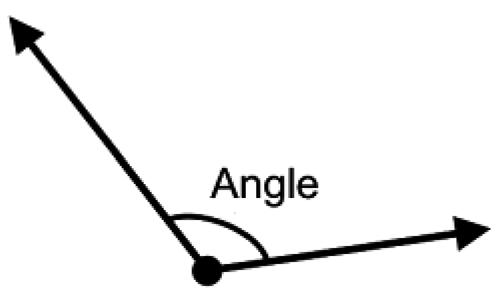
Line, line segment and ray

- If we take a point and draw a straight path that extends endlessly on both the sides, then the straight path is called as a **line**.
- A **ray** is a part of a line with one endpoint.
- A **line segment** is a part of a line with two endpoints.



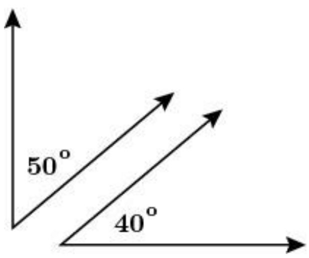
Angles

- An **angle** is formed when two rays originate from the same end point.
- The rays making an angle are called the **arms** of the angle.
- The end point is called the **vertex** of the angle.



Complementary Angles

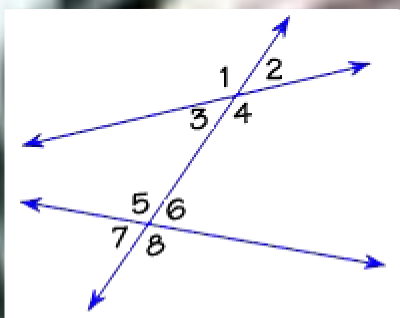
- Two angles whose sum is 90° are called complementary angles.
  Example: $50^\circ + 40^\circ = 90^\circ$
  $\therefore$ 50° and 40° angles are complementary angles.



Parallel Lines and a Transversal

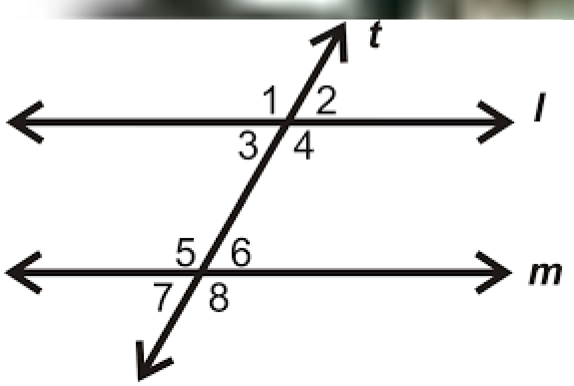
Transversal intersecting two lines

- Transversal is a line that intersects two or more lines at different points.



- **Corresponding Angles:**
  (i) ∠1 and ∠5 (ii) ∠2 and ∠6
  (iii) ∠3 and ∠7 (iv) ∠4 and ∠8
- **Alternate Interior Angles:**
  (i) ∠3 and ∠6 (ii) ∠4 and ∠5
- **Alternate Exterior Angles:**
  (i) ∠1 and ∠8 (ii) ∠2 and ∠7
- **Interior angles on the same side of the transversal:**
  (i) ∠3 and ∠5 (ii) ∠4 and ∠6

Transversal of Parallel Lines



- If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.
  (i) ∠1=∠5 (ii) ∠2=∠6
  (iii) ∠3=∠7 (iv) ∠4=∠8
- If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.
  (i) ∠3=∠6 (ii) ∠4=∠5
- If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.
  (i) $∠3+∠5=180^0$ (ii) $∠4+∠6=180^0$

Checking if two or more lines are parallel

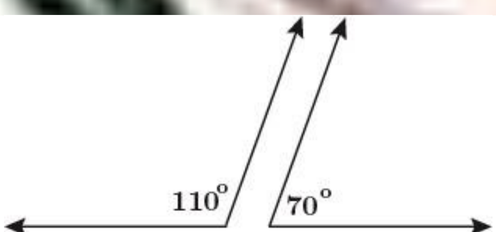
- There are three conditions to check whether the two lines are parallel. They are:
  (i) If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel to each other.
  (ii) If a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel.
  (iii) If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.

To know more about Parallel Lines and a Transversal, visit here.

Intersecting Lines and Pairs of Angles

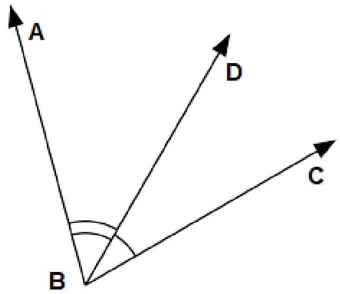
Supplementary angles

- Two angles whose sum is 180^0 are called supplementary angles.
  Example: $110^0+70^0=90^0$
  ∴ 110^0 and 70^0 angles are supplementary angles.



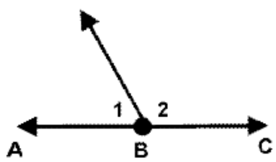
Adjacent Angles

- Two angles are adjacent, if they have
  (i) A common vertex
  (ii) A common arm
  (iii) Their non-common arms on different sides of the common arm.



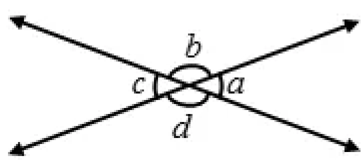
Linear Pair

- Linear pair of angles are adjacent angles whose sum is equal to 180∘.



Vertically Opposite Angles

- Vertically opposite angles are formed when two straight lines intersect each other at a common point.
- Vertically opposite angles are equal.

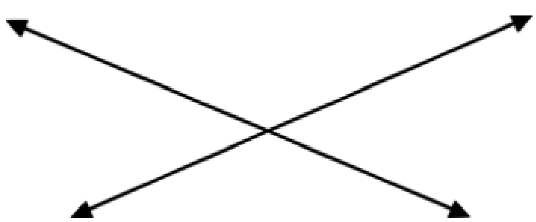


- Here, the following pairs of angles are vertically opposite angles.
  (i) **a** and **c**
  (ii) **b** and **d**

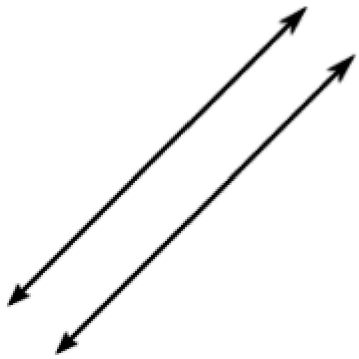
To know more about Adjacent Angle, Linear Pair and Vertically Opposite Angles, visit here.

Intersecting and Non-Intersecting lines

- Intersecting lines are lines which intersect at a common point called the point of intersection.

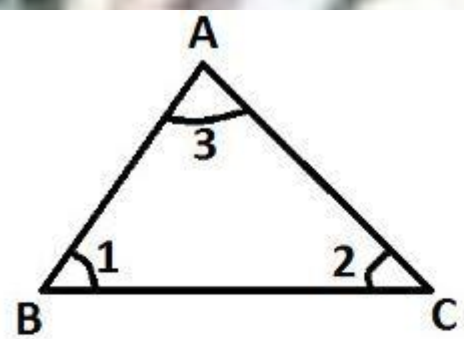


- Parallel lines are lines which do not intersect at any point. Parallel lines are also known as non-intersecting lines.



Sum of Interior Angles in a Triangle

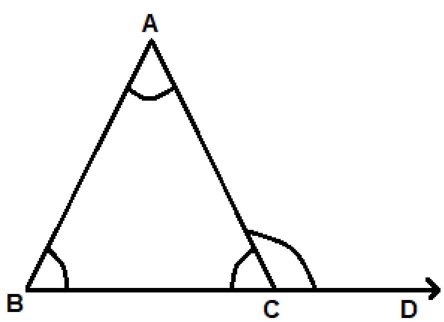
- Angle sum property of a triangle: Sum of all interior angles of a triangle is 180^0.



- In △ABC, ∠1+∠2+∠3=180^0

The exterior angle of a triangle = Sum of opposite internal angles

- If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.

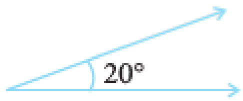


In △ABC, ∠CAB+∠ABC=∠ACD.

EXERCISE : 5.1

1. Find the complement of each of the following angles:

(i)



Solution:-

Two angles are said to be complementary if the sum of their measures is 90^{o}.

The given angle is 20^{o}

Let the measure of its complement be x^{o}.

Then,

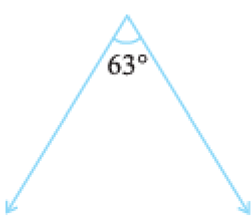
$= x + 20^{o} = 90^{o}$

$= x = 90^{o} – 20^{o}$

$= x = 70^{o}$

Hence, the complement of the given angle measures 70^{o}.

(ii)



Solution:-

Two angles are said to be complementary if the sum of their measures is 90^{o}.

The given angle is 63^{o}

Let the measure of its complement be x^{o}.

Then,

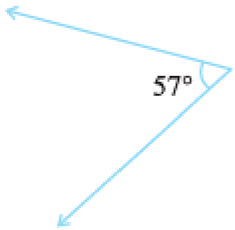
$= x + 63^{o} = 90^{o}$

$= x = 90^{o} – 63^{o}$

$= x = 27^{o}$

Hence, the complement of the given angle measures 27^{o}.

(iii)



Solution:-

Two angles are said to be complementary if the sum of their measures is 90°.

The given angle is 57°

Let the measure of its complement be x°.

Then,

$= x + 57^\circ = 90^\circ$

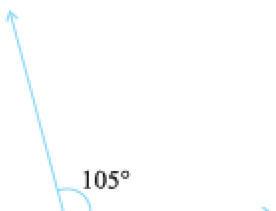
$= x = 90^\circ – 57^\circ$

$= x = 33^\circ$

Hence, the complement of the given angle measures 33°.

2. Find the supplement of each of the following angles:

(i)



Solution:-

Two angles are said to be supplementary if the sum of their measures is 180°.

The given angle is 105°

Let the measure of its supplement be x°.

Then,

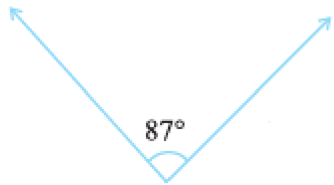
$= x + 105^\circ = 180^\circ$

$= x = 180^\circ – 105^\circ$

$= x = 75^\circ$

Hence, the supplement of the given angle measures 75°.

(ii)



Solution:-

Two angles are said to be supplementary if the sum of their measures is 180°.

The given angle is 87°

Let the measure of its supplement be x°.

Then,

= x + 87° = 180°

= x = 180° – 87°

= x = 93°

Hence, the supplement of the given angle measures 93°.

(iii)

Solution:-

Two angles are said to be supplementary if the sum of their measures is 180°.

The given angle is 154°

Let the measure of its supplement be x°.

Then,

= x + 154° = 180°

= x = 180° – 154°

= x = 26°

Hence, the supplement of the given angle measures 93°.

3. Identify which of the following pairs of angles are complementary and which are supplementary.

(i) 65°, 115°

Solution:-

We have to find the sum of given angles to identify whether the angles are complementary or supplementary.

Then,

= 65° + 115°

= 180°

If the sum of two angle measures is 180°, then the two angles are said to be supplementary.

∴These angles are supplementary angles.

(ii) 63°, 27°

Solution:-

We have to find the sum of given angles to identify whether the angles are complementary or supplementary.

Then,

= 63° + 27°

= 90°

If the sum of two angle measures is 90°, then the two angles are said to be complementary.

∴These angles are complementary angles.

(iii) 112°, 68°

Solution:-

We have to find the sum of given angles to identify whether the angles are complementary or supplementary.

Then,

= 112° + 68°

= 180°

If the sum of two angle measures is 180°, then the two angles are said to be supplementary.

∴These angles are supplementary angles.

(iv) 130°, 50°

Solution:-

We have to find the sum of given angles to identify whether the angles are complementary or supplementary.

Then,

= 130° + 50°

= 180°

If the sum of two angle measures is 180°, then the two angles are said to be supplementary.

∴These angles are supplementary angles.

(v) 45°, 45°

Solution:-

We have to find the sum of given angles to identify whether the angles are complementary or supplementary.

Then,

= 45° + 45°

= 90°

If the sum of two angle measures is 90°, then the two angles are said to be complementary.

∴These angles are complementary angles.

(vi) 80°, 10°

Solution:-

We have to find the sum of given angles to identify whether the angles are complementary or supplementary.

Then,

$= 80^\circ + 10^\circ$

$= 90^\circ$

If the sum of two angle measures is 90°, then the two angles are said to be complementary.

∴These angles are complementary angles.

4. Find the angles which is equal to its complement.

Solution:-

Let the measure of the required angle be x°.

We know that, sum of measures of complementary angle pair is 90°.

Then,

$= x + x = 90^\circ$

$= 2x = 90^\circ$

$= x = 90/2$

$= x = 45^\circ$

Hence, the required angle measures is 45°.

5. Find the angles which is equal to its supplement.

Solution:-

Let the measure of the required angle be x°.

We know that, sum of measures of supplementary angle pair is 180°.

Then,

$= x + x = 180^\circ$

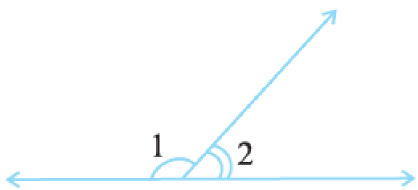
$= 2x = 180^\circ$

$= x = 180/2$

$= x = 90^\circ$

Hence, the required angle measures is 90°.

6. In the given figure, ∠1 and ∠2 are supplementary angles. If ∠1 is decreased, what changes should take place in ∠2 so that both angles still remain supplementary.



Solution:-

From the question, it is given that,

∠1 and ∠2 are supplementary angles.

If ∠1 is decreased, then ∠2 must be increased by the same value. Hence, this angle pair remains supplementary.

7. Can two angles be supplementary if both of them are:

(i). Acute?

Solution:-

No. If two angles are acute, means less than 90°, the two angles cannot be supplementary. Because, their sum will be always less than 90°.

(ii). Obtuse?

Solution:-

No. If two angles are obtuse, means more than 90°, the two angles cannot be supplementary. Because, their sum will be always more than 180°.

(iii). Right?

Solution:-

Yes. If two angles are right, means both measures 90°, then two angles can form a supplementary pair.

$\therefore 90^{\circ} + 90^{\circ} = 180$

8. An angle is greater than 45°. Is its complementary angle greater than 45° or equal to 45° or less than 45°?

Solution:-

Let us assume the complementary angles be p and q,

We know that, sum of measures of complementary angle pair is 90°.

Then,

$= p + q = 90^{\circ}$

It is given in the question that $p > 45^{\circ}$

Adding q on both the sides,

$= p + q > 45^{\circ} + q$

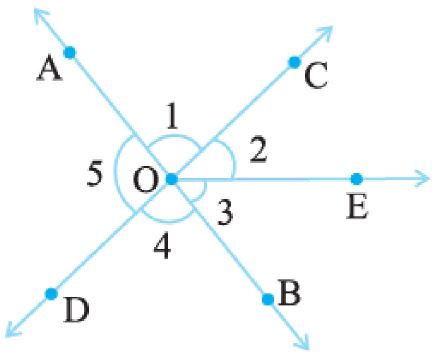
$= 90^{\circ} > 45^{\circ} + q$

$= 90^{\circ} – 45^{\circ} > q$

$= q < 45^{\circ}$

Hence, its complementary angle is less than 45°.

9. In the adjoining figure:



(i) Is ∠1 adjacent to ∠2?

Solution:-

By observing the figure we came to conclude that,

Yes, as ∠1 and ∠2 having a common vertex i.e. O and a common arm OC.

Their non-common arms OA and OE are on both the side of common arm.

(ii) Is ∠AOC adjacent to ∠AOE?

Solution:-

By observing the figure, we came to conclude that,

No, since they are having a common vertex O and common arm OA.

But, they have no non-common arms on both the side of the common arm.

(iii) Do ∠COE and ∠EOD form a linear pair?

Solution:-

By observing the figure, we came to conclude that,

Yes, as ∠COE and ∠EOD having a common vertex i.e. O and a common arm OE.

Their non-common arms OC and OD are on both the side of common arm.

(iv) Are ∠BOD and ∠DOA supplementary?

Solution:-

By observing the figure, we came to conclude that,

Yes, as ∠BOD and ∠DOA having a common vertex i.e. O and a common arm OE.

Their non-common arms OA and OB are opposite to each other.

(v) Is ∠1 vertically opposite to ∠4?

Solution:-

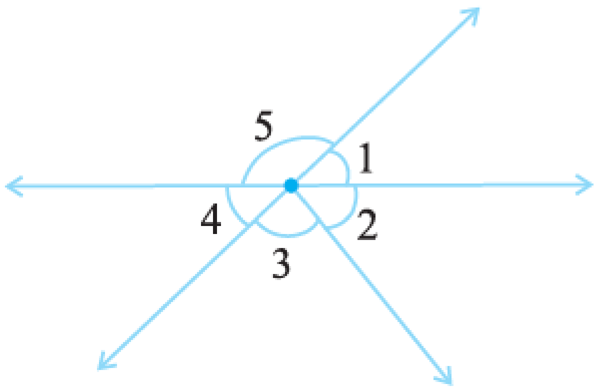
Yes, ∠1 and ∠2 are formed by the intersection of two straight lines AB and CD.

(vi) What is the vertically opposite angle of ∠5?

Solution:-

∠COB is the vertically opposite angle of ∠5. Because these two angles are formed by the intersection of two straight lines AB and CD.

10. Indicate which pairs of angles are:



(i) Vertically opposite angles.

Solution:-

By observing the figure we can say that,

∠1 and ∠4, ∠5 and ∠2 + ∠3 are vertically opposite angles. Because these two angles are formed by the intersection of two straight lines.

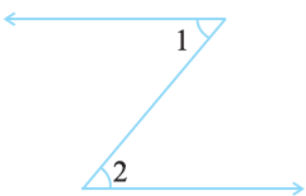
(ii) Linear pairs.

Solution:-

By observing the figure we can say that,

∠1 and ∠5, ∠5 and ∠4 as these are having a common vertex and also having non common arms opposite to each other.

11. In the following figure, is ∠1 adjacent to ∠2? Give reasons.

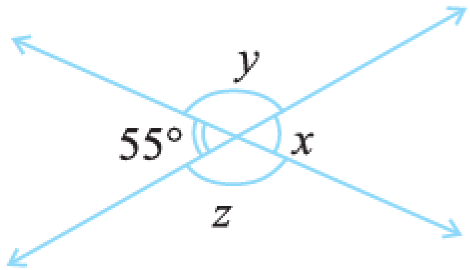


Solution:-

∠1 and ∠2 are not adjacent angles. Because, they are not lie on the same vertex.

12. Find the values of the angles x, y, and z in each of the following:

(i)



Solution:-

∠x = 55°, because vertically opposite angles.

∠x + ∠y = 180° … [∵ linear pair]

= 55° + ∠y = 180°

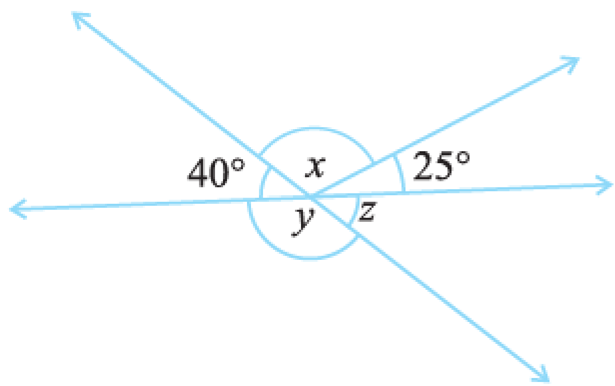
= ∠y = 180° – 55°

= ∠y = 125°

Then, ∠y = ∠z … [∵ vertically opposite angles]

∴ ∠z = 125°

(ii)



Solution:-

∠z = 40°, because vertically opposite angles.

∠y + ∠z = 180° … [∵ linear pair]

= ∠y + 40° = 180°

= ∠y = 180° – 40°

= ∠y = 140°

Then, 40 + ∠x + 25 = 180° … [∵angles on straight line]

65 + ∠x = 180°

∠x = 180° – 65

∴ ∠x = 115°

13. Fill in the blanks:

(i) If two angles are complementary, then the sum of their measures is _______.

Solution:-

If two angles are complementary, then the sum of their measures is 90^{o}.

(ii) If two angles are supplementary, then the sum of their measures is ______.

Solution:-

If two angles are supplementary, then the sum of their measures is 180^{o}.

(iii) Two angles forming a linear pair are _______________.

Solution:-

Two angles forming a linear pair are Supplementary.

(iv) If two adjacent angles are supplementary, they form a ___________.

Solution:-

If two adjacent angles are supplementary, they form a linear pair.

(v) If two lines intersect at a point, then the vertically opposite angles are always _____________.

Solution:-

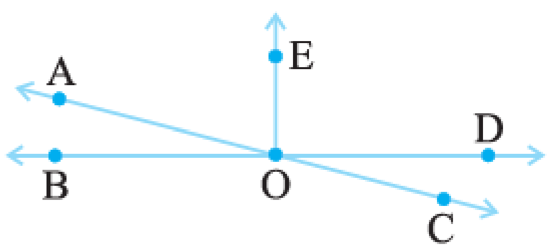
If two lines intersect at a point, then the vertically opposite angles are always equal.

(vi) If two lines intersect at a point, and if one pair of vertically opposite angles are acute angles, then the other pair of vertically opposite angles are __________.

Solution:-

If two lines intersect at a point, and if one pair of vertically opposite angles are acute angles, then the other pair of vertically opposite angles are Obtuse angles.

14. In the adjoining figure, name the following pairs of angles.



(i) Obtuse vertically opposite angles

Solution:-

∠AOD and ∠BOC are obtuse vertically opposite angles in the given figure.

(ii) Adjacent complementary angles

Solution:-

∠EOA and ∠AOB are adjacent complementary angles in the given figure.

(iii) Equal supplementary angles

Solution:-

∠EOB and EOD are the equal supplementary angles in the given figure.

(iv) Unequal supplementary angles

Solution:-

∠EOA and ∠EOC are the unequal supplementary angles in the given figure.

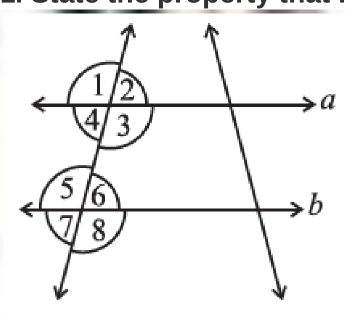
(v) Adjacent angles that do not form a linear pair

Solution:-

∠AOB and ∠AOE, ∠AOE and ∠EOD, ∠EOD and ∠COD are the adjacent angles that do not form a linear pair in the given figure.

EXERCISE : 5.2

1. State the property that is used in each of the following statements?



i) If a ∥ b, then ∠1 = ∠5.

Solution:-

Corresponding angles property is used in the above statement.

(ii) If ∠4 = ∠6, then a ∥ b.

Solution:-

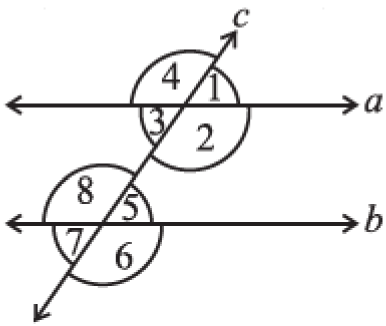
Alternate interior angles property is used in the above statement.

(iii) If ∠4 + ∠5 = 180°, then a ∥ b.

Solution:-

Interior angles on the same side of transversal are supplementary.

2. In the adjoining figure, identify



(i) The pairs of corresponding angles.

Solution:-

By observing the figure, the pairs of corresponding angle are,

∠1 and ∠5, ∠4 and ∠8, ∠2 and ∠6, ∠3 and ∠7

(ii) The pairs of alternate interior angles.

Solution:-

By observing the figure, the pairs of alternate interior angle are,

∠2 and ∠8, ∠3 and ∠5

(iii) The pairs of interior angles on the same side of the transversal.

Solution:-

By observing the figure, the pairs of interior angles on the same side of the transversal are ∠2 and ∠5, ∠3 and ∠8

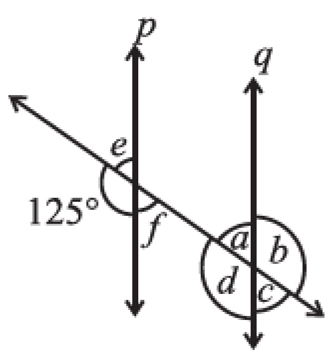
(iv) The vertically opposite angles.

Solution:-

By observing the figure, the vertically opposite angles are,

∠1 and ∠3, ∠5 and ∠7, ∠2 and ∠4, ∠6 and ∠8

3. In the adjoining figure, p ∥ q. Find the unknown angles.



Solution:-

By observing the figure,

∠d = ∠125° … [∵ corresponding angles]

We know that, Linear pair is the sum of adjacent angles is 180°

Then,

= ∠e + 125° = 180° … [Linear pair]

= ∠e = 180° – 125°

= ∠e = 55°

From the rule of vertically opposite angles,

∠f = ∠e = 55°

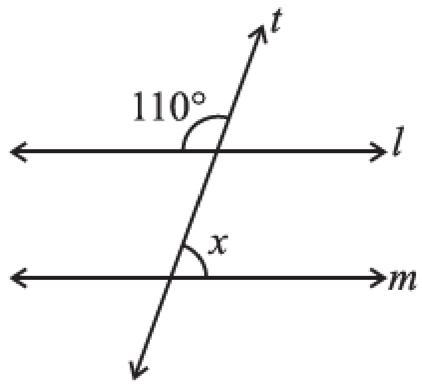
∠b = ∠d = 125°

By the property of corresponding angles,

∠c = ∠f = 55°

∠a = ∠e = 55°

4. Find the value of x in each of the following figures if l ∥ m.



(i)

Solution:-

Let us assume other angle on the line m be ∠y,

Then,

By the property of corresponding angles,

∠y = 110°

We know that Linear pair is the sum of adjacent angles is 180°

Then,

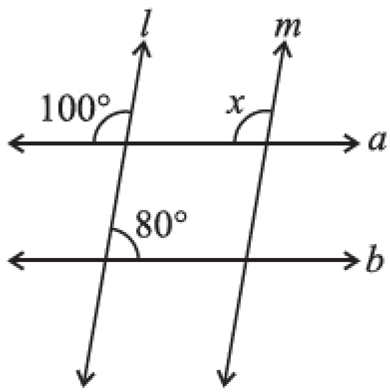
= ∠x + ∠y = 180°

= ∠x + 110° = 180°

= ∠x = 180° – 110°

= ∠x = 70°

(ii)

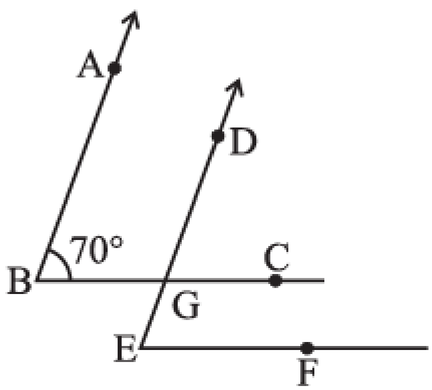


Solution:-

By the property of corresponding angles,

$\angle x = 100^{o}$

5. In the given figure, the arms of two angles are parallel.



If $\angle ABC = 70^{o}$, then find

(i) $\angle DGC$

(ii) $\angle DEF$

Solution:-

(i) Let us consider that AB ∥ DG

BC is the transversal line intersecting AB and DG

By the property of corresponding angles,

$\angle DGC = \angle ABC$

Then,

$\angle DGC = 70^{o}$

(ii) Let us consider that BC ∥ EF

DE is the transversal line intersecting BC and EF

By the property of corresponding angles,

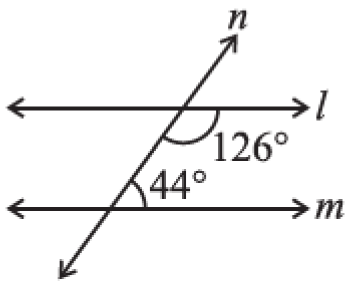
$\angle DEF = \angle DGC$

Then,

∠DEF = 70°

6. In the given figures below, decide whether l is parallel to m.

(i)



Solution:-

Let us consider the two lines l and m,

n is the transversal line intersecting l and m.

We know that the sum of interior angles on the same side of transversal is 180°.

Then,

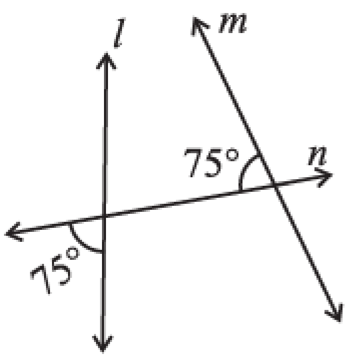
= $126^{\circ} + 44^{\circ}$

= 170°

But, the sum of interior angles on the same side of transversal is not equal to 180°.

So, line l is not parallel to line m.

(ii)



Solution:-

Let us assume ∠x be the vertically opposite angle formed due to the intersection of the straight line l and transversal n,

Then, ∠x = 75°

Let us consider the two lines l and m,

n is the transversal line intersecting l and m.

We know that the sum of interior angles on the same side of transversal is 180°.

Then,

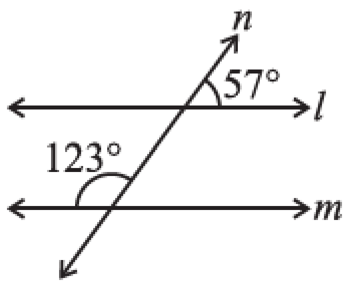
= $75^\circ + 75^\circ$

= 150°

But, the sum of interior angles on the same side of transversal is not equal to 180°.

So, line l is not parallel to line m.

(iii)



Solution:-

Let us assume ∠x be the vertically opposite angle formed due to the intersection of the Straight line l and transversal line n,

Let us consider the two lines l and m,

n is the transversal line intersecting l and m.

We know that the sum of interior angles on the same side of transversal is 180°.

Then,

= $123^\circ + \angle x$

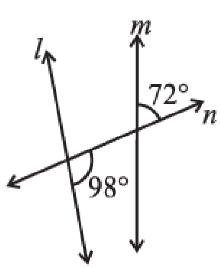
= $123^\circ + 57^\circ$

= 180°

∴The sum of interior angles on the same side of transversal is equal to 180°.

So, line l is parallel to line m.

(iv)



Solution:-

Let us assume ∠x be the angle formed due to the intersection of the Straight line l and transversal line n, We know that Linear pair is the sum of adjacent angles is equal to 180°.

$= \angle x + 98^\circ = 180^\circ$

$= \angle x = 180^\circ – 98^\circ$

$= \angle x = 82^\circ$

Now, we consider $\angle x$ and 72° are the corresponding angles.

For l and m to be parallel to each other, corresponding angles should be equal.

But, in the given figure corresponding angles measures 82° and 72° respectively.

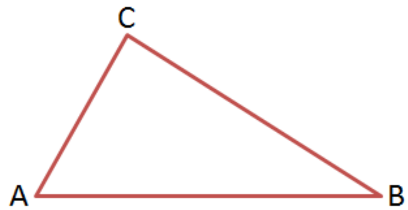
∴Line l is not parallel to line m.

CHAPTER – 6 The Triangles and its Properties

KEY POINTS TO REMEMBER

Triangle

- A triangle is a closed curve made of three line segments.

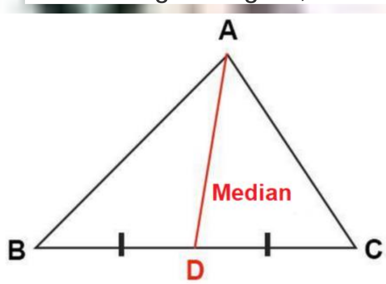


- It has three:Sides:
  (i) Sides: $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$
  (ii) Angles: ∠BAC, ∠ACB and ∠CBA
  (iii) Vertices: A, B and C

Important Lines in a Triangle

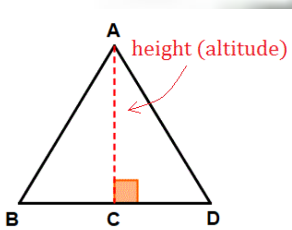
Median

- Median is the line that connects a vertex of a triangle to the mid-point of the opposite side.
- In the given figure, $\overline{AD}$ is the median, joining the vertex A to the midpoint of $\overline{BC}$.



Altitude

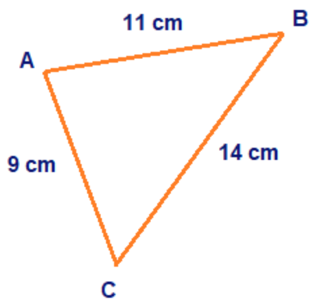
- An altitude is a line segment through a vertex of the triangle and perpendicular to a line containing the opposite side.



Sides Also Have Constraints

Sum of the lengths of two sides of a triangle

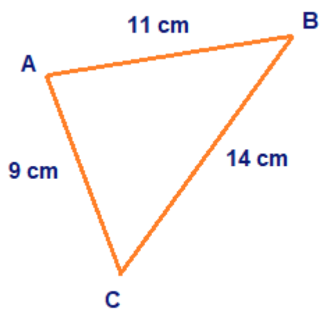
- The sum of the lengths of any two sides of a triangle is greater than the third side.



- In the above triangle,
  9+11=20 >14
  11+14=25 >9
  9+14=23 >11

Difference between lengths of two sides of a triangle

- The difference between lengths of any two sides is smaller than the length of the third side.

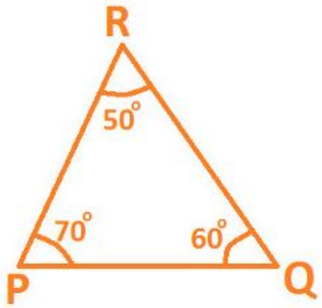


- In the above triangle,
  11 – 9 = 2 < 14
  14 – 11 = 3 < 9
  14 – 9 = 5 < 11
- Using the concept of sum of two sides and difference of two sides, it is possible to determine the range of lengths that the third side can take.

Triangle Properties

Angle sum property of a triangle

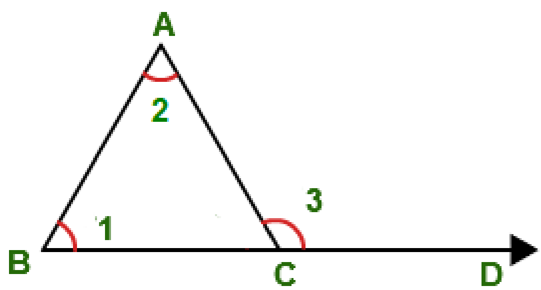
- The total measure of the three angles of a triangle is 180°.



- In △PQR,
  ∠RPQ+∠PQR+∠QRP
  $=70^{\circ} + 60^{\circ} + 50^{\circ} = 180^{\circ}$

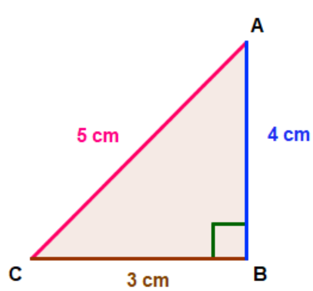
Exterior angle of a triangle and its property

- An exterior angle of a triangle is equal to the sum of its interior opposite angles.



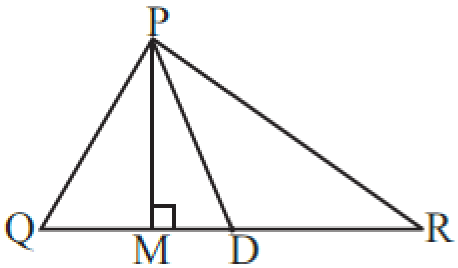
Pythagoras Theorem

- The side opposite to the right angle in a right-angled triangle is called the hypotenuse.
- The other two sides are known as legs of the right-angled triangle.
- In a right-angled triangle, square of hypotenuse is equal to the sum of squares of legs.



EXERCISE :6.1

1. In Δ PQR, D is the mid-point of $\overline{QR}$.



(i) $\overline{PM}$ is ___.

Solution:-

Altitude

An altitude has one end point at a vertex of the triangle and other on the line containing the opposite side.

(ii) PD is ___.

Solution:-

Median

A median connects a vertex of a triangle to the mid-point of the opposite side.

(iii) Is QM = MR?

Solution:-

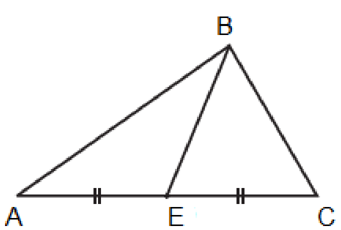
No, QM ≠ MR because, D is the mid-point of QR.

2. Draw rough sketches for the following:

(a) In ΔABC, BE is a median.

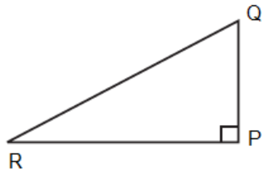
Solution:-

A median connects a vertex of a triangle to the mid-point of the opposite side.



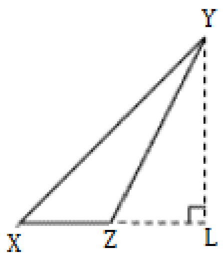
(b) In ΔPQR, PQ and PR are altitudes of the triangle.

Solution:-



(c) In ΔXYZ, YL is an altitude in the exterior of the triangle.

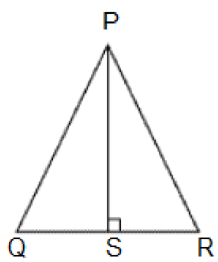
Solution:-



In the figure we may observe that for ΔXYZ, YL is an altitude drawn exteriorly to side XZ which is extended up to point L.

3. Verify by drawing a diagram if the median and altitude of an isosceles triangle can be same.

Solution:-

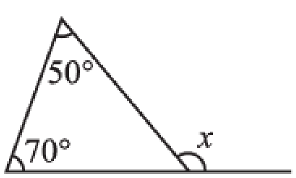


Draw a Line segment PS ⊥ BC. It is an altitude for this triangle. Here we observe that length of QS and SR is also same. So PS is also a median of this triangle.

EXERCISE :6.2

1. Find the value of the unknown exterior angle x in the following diagram:

(i)



Solution:-

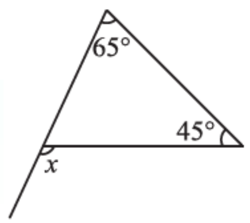
We Know That,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

= $x = 50^{o} + 70^{o}$

= $x = 120^{o}$

(ii)



Solution:-

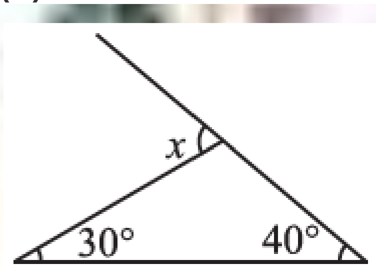
We Know That,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

= $x = 65^{o} + 45^{o}$

= $x = 110^{o}$

(iii)



Solution:-

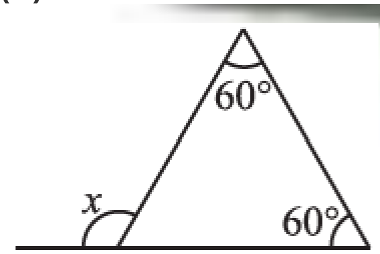
We Know That,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

= $x = 30^{o} + 40^{o}$

= $x = 70^{o}$

(iv)



Solution:-

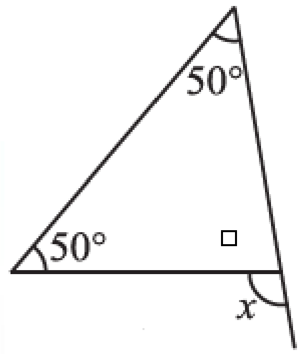
We Know That,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

$= x = 60^\circ + 60^\circ$

$= x = 120^\circ$

(v)



Solution:-

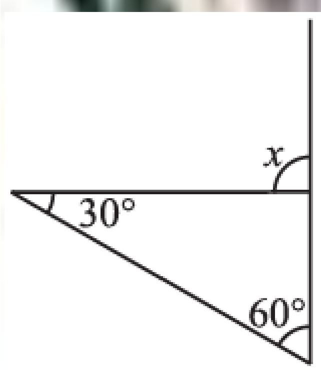
We Know That,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

$= x = 50^\circ + 50^\circ$

$= x = 100^\circ$

(vi)



Solution:-

We Know That,

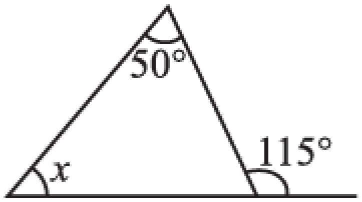
An exterior angle of a triangle is equal to the sum of its interior opposite angles.

$= x = 30^\circ + 60^\circ$

$= x = 90^\circ$

2. Find the value of the unknown interior angle x in the following figures:

(i)



Solution:-

We Know That,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

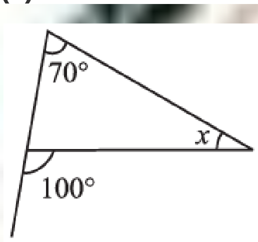
= $x + 50^\circ = 115^\circ$

By transposing 50° from LHS to RHS it becomes – 50°

= $x = 115^\circ – 50^\circ$

= $x = 65^\circ$

(ii)



Solution:-

We Know That,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

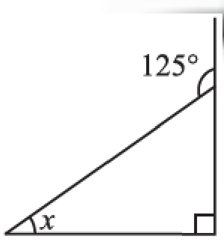
= $70^\circ + x = 100^\circ$

By transposing 70° from LHS to RHS it becomes – 70°

= $x = 100^\circ – 70^\circ$

= $x = 30^\circ$

(iii)



Solution:-

We Know That,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

The given triangle is a right angled triangle. So the angle opposite to the x is 90°.

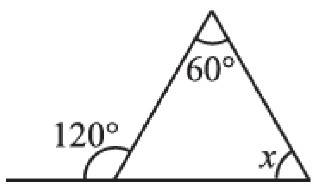
$= x + 90^\circ = 125^\circ$

By transposing 90° from LHS to RHS it becomes $– 90^\circ$

$= x = 125^\circ – 90^\circ$

$= x = 35^\circ$

(iv)



Solution:-

We Know That,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

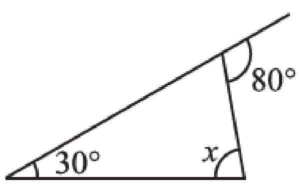
$= x + 60^\circ = 120^\circ$

By transposing 60° from LHS to RHS it becomes $– 60^\circ$

$= x = 120^\circ – 60^\circ$

$= x = 60^\circ$

(v)



Solution:-

We Know That,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

The given triangle is a right angled triangle. So the angle opposite to the x is 90°.

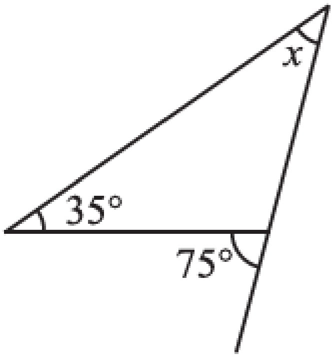
$= x + 30^\circ = 80^\circ$

By transposing 30° from LHS to RHS it becomes $– 30^\circ$

$= x = 80^\circ – 30^\circ$

$= x = 50^\circ$

(vi)



Solution:-

We Know That,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

The given triangle is a right angled triangle. So the angle opposite to the x is 90^{o}.

= x + 35^{o} = 75^{o}

By transposing 35^{o} from LHS to RHS it becomes – 35^{o}

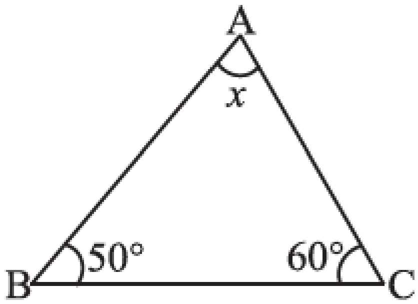
= x = 75^{o} – 35^{o}

= x = 40^{o}

EXERCISE :6.3

1. Find the value of the unknown x in the following diagrams:

(i)



Solution:-

We know that,

The sum of all the interior angles of a triangle is 180^{o}.

Then,

= ∠BAC + ∠ABC + ∠BCA = 180^{o}

= x + 50^{o} + 60^{o} = 180^{o}

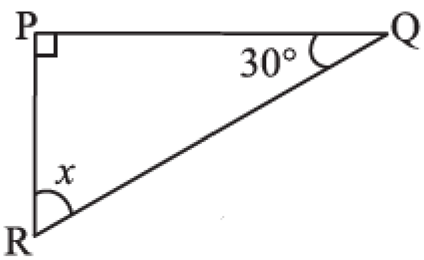
= x + 110^{o} = 180^{o}

By transposing 110° from LHS to RHS it becomes -110°

$= x = 180^\circ - 110^\circ$

$= x = 70^\circ$

(ii)



Solution:-

We know that,

The sum of all the interior angles of a triangle is 180°.

The given triangle is a right angled triangle. So the ∠QPR is 90°.

Then,

$= \angle QPR + \angle PQR + \angle PRQ = 180^\circ$

$= 90^\circ + 30^\circ + x = 180^\circ$

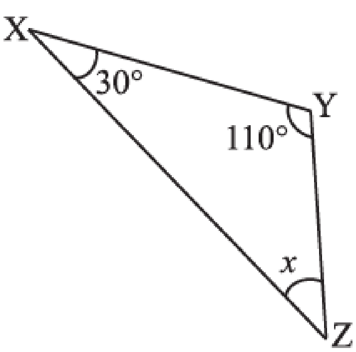
$= 120^\circ + x = 180^\circ$

By transposing 110° from LHS to RHS it becomes -110°

$= x = 180^\circ - 120^\circ$

$= x = 60^\circ$

(iii)



Solution:-

We know that,

The sum of all the interior angles of a triangle is 180°.

Then,

$= \angle XYZ + \angle YXZ + \angle XZY = 180^\circ$

$= 110^\circ + 30^\circ + x = 180^\circ$

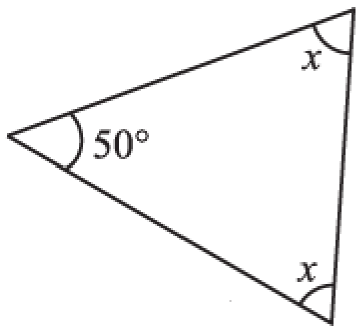
$= 140^\circ + x = 180^\circ$

By transposing 140° from LHS to RHS it becomes – 140°

$= x = 180^\circ – 140^\circ$

$= x = 40^\circ$

(iv)



Solution:-

We know that,

The sum of all the interior angles of a triangle is 180°.

Then,

$= 50^\circ + x + x = 180^\circ$

$= 50^\circ + 2x = 180^\circ$

By transposing 50° from LHS to RHS it becomes – 50°

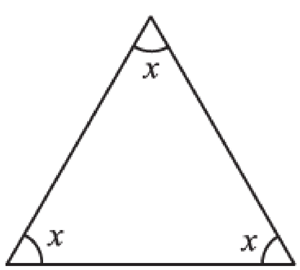
$= 2x = 180^\circ – 50^\circ$

$= 2x = 130^\circ$

$= x = 130^\circ/2$

$= x = 65^\circ$

(v)



Solution:-

We know that,

The sum of all the interior angles of a triangle is 180°.

Then,

$= x + x + x = 180^\circ$

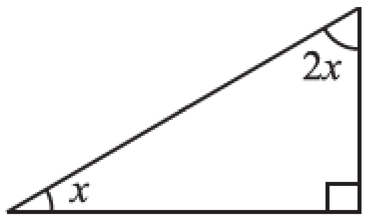
= 3x = 180°

= x = 180°/3

= x = 60°

∴The given triangle is an equiangular triangle.

(vi)



Solution:-

We know that,

The sum of all the interior angles of a triangle is 180°.

Then,

= 90° + 2x + x = 180°

= 90° + 3x = 180°

By transposing 90° from LHS to RHS it becomes – 90°

= 3x = 180° – 90°

= 3x = 90°

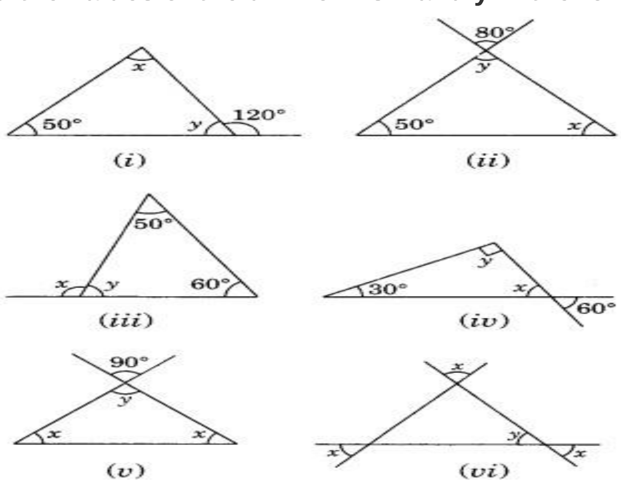
= x = 90°/3

= x = 30°

Then,

= 2x = 2 × 30° = 60°

2. Find the values of the unknowns x and y in the following diagrams:



i)

Solution:-

We Know That,

An exterior angle of a triangle is equal to the sum of its interior opposite angles.

Then,

$= 50^\circ + x = 120^\circ$

By transposing 50° from LHS to RHS it becomes -50°

$= x = 120^\circ - 50^\circ$

$= x = 70^\circ$

We also know that,

The sum of all the interior angles of a triangle is 180°.

Then,

$= 50^\circ + x + y = 180^\circ$

$= 50^\circ + 70^\circ + y = 180^\circ$

$= 120^\circ + y = 180^\circ$

By transposing 120° from LHS to RHS it becomes -120°

$= y = 180^\circ - 120^\circ$

$= y = 60^\circ$

(ii)

Solution:-

From the rule of vertically opposite angles,

$= y = 80^\circ$

Then,

We know that,

The sum of all the interior angles of a triangle is 180°.

Then,

$= 50^\circ + 80^\circ + x = 180^\circ$

$= 130^\circ + x = 180^\circ$

By transposing 130° from LHS to RHS it becomes -130°

$= x = 180^\circ - 130^\circ$

$= x = 50^\circ$

(iii)

Solution:-

We know that,

The sum of all the interior angles of a triangle is 180°.

Then,

= $50^o + 60^o + y = 180^o$

= $110^o + y = 180^o$

By transposing 110^o from LHS to RHS it becomes – 110^o

= $y = 180^o – 110^o$

= $y = 70^o$

Now,

From the rule of linear pair,

= $x + y = 180^o$

= $x + 70^o = 180^o$

By transposing 70^o from LHS to RHS it becomes – 70^o

= $x = 180^o – 70$

= $x = 110^o$

(iv)

Solution:-

From the rule of vertically opposite angles,

= $x = 60^o$

Then,

We know that,

The sum of all the interior angles of a triangle is 180^o.

Then,

= $30^o + x + y = 180^o$

= $30^o + 60^o + y = 180^o$

= $90^o + y = 180^o$

By transposing 90^o from LHS to RHS it becomes – 90^o

= $y = 180^o – 90^o$

= $y = 90^o$

(v)

Solution:-

From the rule of vertically opposite angles,

= $y = 90^o$

Then,

We know that,

The sum of all the interior angles of a triangle is 180^o.

Then,

= $x + x + y = 180^o$

= $2x + 90^o = 180^o$

By transposing 90^o from LHS to RHS it becomes – 90^o

= $2x = 180^o – 90^o$

= $2x = 90^o$

= $x = 90^o/2$

= $x = 45^o$

(vi)

Solution:-

From the rule of vertically opposite angles,

= $x = y$

Then,

We know that,

The sum of all the interior angles of a triangle is 180^o.

Then,

= $x + x + x = 180^o$

= $3x = 180^o$

= $x = 180^o/3$

= $x = 60^o$

EXERCISE :6.4

1. Is it possible to have a triangle with the following sides?

(i) 2 cm, 3 cm, 5 cm

Solution:-

Clearly, we have:

(2 + 3) = 5

5 = 5

Thus, the sum of any two of these numbers is not greater than the third.

Hence, it is not possible to draw a triangle whose sides are 2 cm, 3 cm and 5 cm.

(ii) 3 cm, 6 cm, 7 cm

Solution:-

Clearly, we have:

(3 + 6) = 9 > 7

(6 + 7) = 13 > 3

(7 + 3) = 10 > 6

Thus, the sum of any two of these numbers is greater than the third.

Hence, it is possible to draw a triangle whose sides are 3 cm, 6 cm and 7 cm.

(iii) 6 cm, 3 cm, 2 cm

Solution:-

Clearly, we have:

(3 + 2) = 5 < 6

Thus, the sum of any two of these numbers is less than the third.

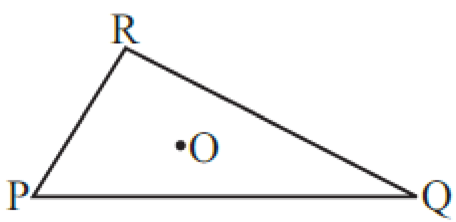
Hence, it is not possible to draw a triangle whose sides are 6 cm, 3 cm and 2 cm.

2. Take any point O in the interior of a triangle PQR. Is

(i) OP + OQ > PQ?

(ii) OQ + OR > QR?

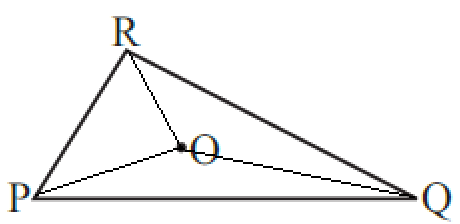
(iii) OR + OP > RP?



Solution:-

If we take any point O in the interior of a triangle PQR and join OR, OP, OQ.

Then, we get three triangles ΔOPQ, ΔOQR and ΔORP is shown in the figure below.



We know that,

The sum of the length of any two sides is always greater than the third side.

(i) Yes, ΔOPQ has sides OP, OQ and PQ.

So, OP + OQ > PQ

(ii) Yes, ΔOQR has sides OR, OQ and QR.

So, OQ + OR > QR

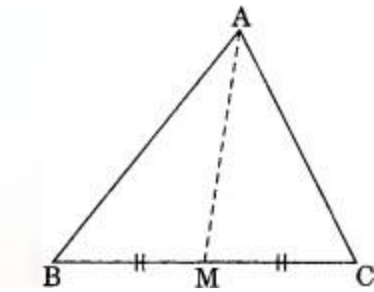
(iii) Yes, ΔORP has sides OR, OP and PR.

So, OR + OP > RP

3. AM is a median of a triangle ABC.

Is AB + BC + CA > 2 AM?

(Consider the sides of triangles ΔABM and ΔAMC.)



Solution:-

We know that,

The sum of the length of any two sides is always greater than the third side.

Now consider the ΔABM,

Here, AB + BM > AM … [equation i]

Then, consider the ΔACM

Here, AC + CM > AM … [equation ii]

By adding equation [i] and [ii] we get,

AB + BM + AC + CM > AM + AM

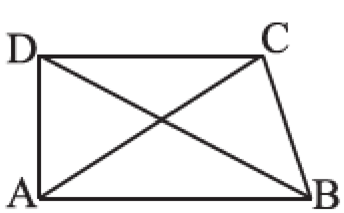
From the figure we have, BC = BM + CM

AB + BC + AC > 2 AM

Hence, the given expression is true.

4. ABCD is a quadrilateral.

Is AB + BC + CD + DA > AC + BD?



Solution:-

We know that,

The sum of the length of any two sides is always greater than the third side.

Now consider the ΔABC,

Here, AB + BC > CA … [equation i]

Then, consider the ΔBCD

Here, BC + CD > DB … [equation ii]

Consider the ΔCDA

Here, CD + DA > AC … [equation iii]

Consider the ΔDAB

Here, DA + AB > DB … [equation iv]

By adding equation [i], [ii], [iii] and [iv] we get,

AB + BC + BC + CD + CD + DA + DA + AB > CA + DB + AC + DB

2AB + 2BC + 2CD + 2DA > 2CA + 2DB

Take out 2 on both the side,

2(AB + BC + CA + DA) > 2(CA + DB)

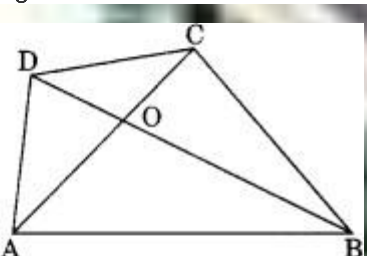
AB + BC + CA + DA > CA + DB

Hence, the given expression is true.

5. ABCD is quadrilateral. Is AB + BC + CD + DA < 2 (AC + BD)

Solution:-

Let us consider ABCD is quadrilateral and P is the point where the diagonals are intersect. As shown in the figure below.



We know that,

The sum of the length of any two sides is always greater than the third side.

Now consider the ΔPAB,

Here, PA + PB < AB … [equation i]

Then, consider the ΔPBC

Here, PB + PC < BC … [equation ii]

Consider the ΔPCD

Here, PC + PD < CD … [equation iii]

Consider the ΔPDA

Here, PD + PA < DA … [equation iv]

By adding equation [i], [ii], [iii] and [iv] we get,

PA + PB + PB + PC + PC + PD + PD + PA < AB + BC + CD + DA

2PA + 2PB + 2PC + 2PD < AB + BC + CD + DA

2PA + 2PC + 2PB + 2PD < AB + BC + CD + DA

2(PA + PC) + 2(PB + PD) < AB + BC + CD + DA

From the figure we have, AC = PA + PC and BD = PB + PD

Then,

2AC + 2BD < AB + BC + CD + DA

2(AC + BD) < AB + BC + CD + DA

Hence, the given expression is true.

6. The lengths of two sides of a triangle are 12 cm and 15 cm. Between what two measures should the length of the third side fall?

Solution:-

We know that,

The sum of the length of any two sides is always greater than the third side.

From the question, it is given that two sides of triangle are 12 cm and 15 cm.

So, the third side length should be less than the sum of other two sides,

12 + 15 = 27 cm.

Then, it is given that the third side is cannot not be less than the difference of the two sides, 15 – 12 = 3 cm

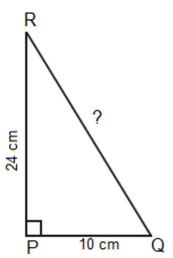
So, the length of the third side falls between 3 cm and 27 cm.

EXERCISE :6.5

1. PQR is a triangle, right-angled at P. If PQ = 10 cm and PR = 24 cm, find QR.

Solution:-

Let us draw a rough sketch of right-angled triangle



By the rule of Pythagoras Theorem,

Pythagoras theorem states that for any right angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of square on the legs.

In the above figure RQ is the hypotenuse,

$QR^2 = PQ^2 + PR^2$

$QR^2 = 10^2 + 24^2$

$QR^2 = 100 + 576$

$QR^2 = 676$

$QR = \sqrt{676}$

QR = 26 cm

Hence, the length of the hypotenuse QR = 26 cm.

2. ABC is a triangle, right-angled at C. If AB = 25 cm and AC = 7 cm, find BC.

Solution:-

Let us draw a rough sketch of right-angled triangle

By the rule of Pythagoras Theorem,

Pythagoras theorem states that for any right angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of square on the legs.

In the above figure RQ is the hypotenuse,

$AB^2 = AC^2 + BC^2$

$25^2 = 7^2 + BC^2$

$625 = 49 + BC^2$

By transposing 49 from RHS to LHS it becomes – 49

$BC^2 = 625 – 49$

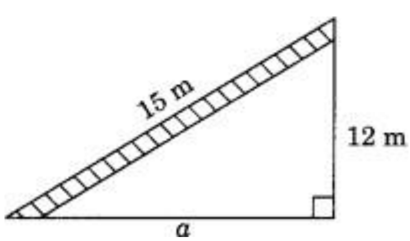
$BC^2 = 576$

$BC = \sqrt{576}$

BC = 24 cm

Hence, the length of the BC = 24 cm.

3. A 15 m long ladder reached a window 12 m high from the ground on placing it against a wall at a distance a. Find the distance of the foot of the ladder from the wall.



Solution:-

By the rule of Pythagoras Theorem,

Pythagoras theorem states that for any right angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of square on the legs.

In the above figure RQ is the hypotenuse,

$15^2 = 12^2 + a^2$

$225 = 144 + a^2$

By transposing 144 from RHS to LHS it becomes – 144

$a^2 = 225 – 144$

$a^2 = 81$

$a = \sqrt{81}$

a = 9 m

Hence, the length of a = 9 m.

4. Which of the following can be the sides of a right triangle?

(i) 2.5 cm, 6.5 cm, 6 cm.

(ii) 2 cm, 2 cm, 5 cm.

(iii) 1.5 cm, 2cm, 2.5 cm.

In the case of right-angled triangles, identify the right angles.

Solution:-

(i) Let a = 2.5 cm, b = 6.5 cm, c = 6 cm

Let us assume the largest value is the hypotenuse side i.e. b = 6.5 cm.

Then, by Pythagoras theorem,

$b^2 = a^2 + c^2$

$6.5^2 = 2.5^2 + 6^2$

42.25 = 6.25 + 36

42.25 = 42.25

The sum of square of two side of triangle is equal to the square of third side,

∴The given triangle is right-angled triangle.

Right angle lies on the opposite of the greater side 6.5 cm.

(ii) Let a = 2 cm, b = 2 cm, c = 5 cm

Let us assume the largest value is the hypotenuse side i.e. c = 5 cm.

Then, by Pythagoras theorem,

$c^2 = a^2 + b^2$

$5^2 = 2^2 + 2^2$

$25 = 4 + 4$

$25 \neq 8$

The sum of square of two side of triangle is not equal to the square of third side,

∴The given triangle is not right-angled triangle.

(iii) Let a = 1.5 cm, b = 2 cm, c = 2.5 cm

Let us assume the largest value is the hypotenuse side i.e. b = 2.5 cm.

Then, by Pythagoras theorem,

$b^2 = a^2 + c^2$

$2.5^2 = 1.5^2 + 2^2$

$6.25 = 2.25 + 4$

$6.25 = 6.25$

The sum of square of two side of triangle is equal to the square of third side,

∴The given triangle is right-angled triangle.

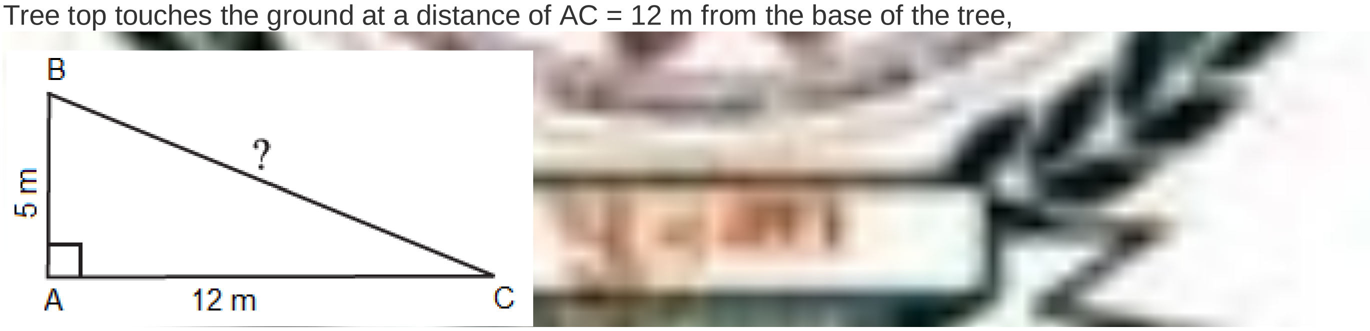
Right angle lies on the opposite of the greater side 2.5 cm.

5. A tree is broken at a height of 5 m from the ground and its top touches the ground at a distance of 12 m from the base of the tree. Find the original height of the tree.

Solution:-

Let ABC is the triangle and B is the point where tree is broken at the height 5 m from the ground.

Tree top touches the ground at a distance of AC = 12 m from the base of the tree,



By observing the figure we came to conclude that right angle triangle is formed at A.

From the rule of Pythagoras theorem,

$BC^2 = AB^2 + AC^2$

$BC^2 = 5^2 + 12^2$

$BC^2 = 25 + 144$

$BC^2 = 169$

$BC = \sqrt{169}$

BC = 13 m

Then, the original height of the tree = AB + BC

= 5 + 13

= 18 m

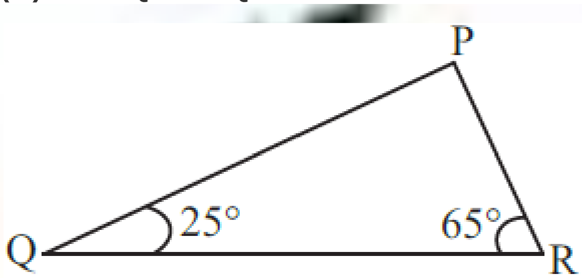
6. Angles Q and R of a ΔPQR are 25° and 65°.

Write which of the following is true:

(i) $PQ^2 + QR^2 = RP^2$

(ii) $PQ^2 + RP^2 = QR^2$

(iii) $RP^2 + QR^2 = PQ^2$



Solution:-

Given that $\angle Q = 25^\circ$, $\angle R = 65^\circ$

Then, $\angle P$ =?

We know that sum of the three interior angles of triangle is equal to 180°.

$\angle PQR + \angle QRP + \angle RPQ = 180^\circ$

$25^\circ + 65^\circ + \angle RPQ = 180^\circ$

$90^\circ + \angle RPQ = 180^\circ$

$\angle RPQ = 180 - 90$

$\angle RPQ = 90^\circ$

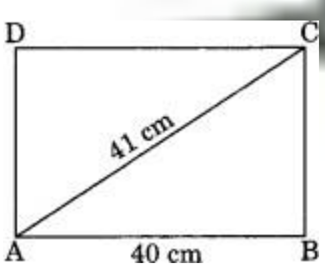
Also, we know that side opposite to the right angle is the hypotenuse.

$\therefore QR^2 = PQ^2 + PR^2$

Hence, (ii) is true

7. Find the perimeter of the rectangle whose length is 40 cm and a diagonal is 41 cm.

Solution:-



Let ABCD be the rectangular plot.

Then, AB = 40 cm and AC = 41 cm

BC =?

According to Pythagoras theorem,

From right angle triangle ABC, we have:

= $AC^2 = AB^2 + BC^2$

= $41^2 = 40^2 + BC^2$

= $BC^2 = 41^2 – 40^2$

= $BC^2 = 1681 – 1600$

= $BC^2 = 81$

= BC = $\sqrt{81}$

= BC = 9 cm

Hence, the perimeter of the rectangle plot = 2 (length + breadth)

Where, length = 40 cm, breadth = 9 cm

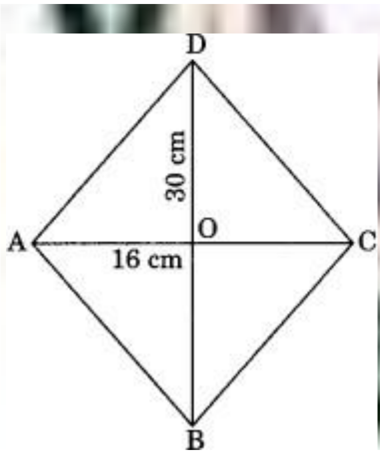
Then,

= 2(40 + 9)

= 2 × 49

= 98 cm

8. The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.



Solution:- Let PQRS be a rhombus, all sides of rhombus has equal length and its diagonal PR and SQ are intersecting each other at a point O. Diagonals in rhombus bisect each other at 90°.

So, PO = (PR/2)

= 16/2

= 8 cm

And, SO = (SQ/2)

= 30/2

= 15 cm

Then, consider the triangle POS and apply the Pythagoras theorem,

$PS^2 = PO^2 + SO^2$

$PS^2 = 8^2 + 15^2$

$PS^2 = 64 + 225$

$PS^2 = 289$

$PS = \sqrt{289}$

PS = 17 cm

Hence, the length of side of rhombus is 17 cm

Now,

Perimeter of rhombus = 4 × side of the rhombus

= 4 × 17

= 68 cm

∴ Perimeter of rhombus is 68 cm.

CHAPTER –7 Congruence of Triangles

KEY POINTS TO REMEMBER

If two figures have exactly the same shape and size, then they are said to be congruent.
For congruence, we use the symbol '='

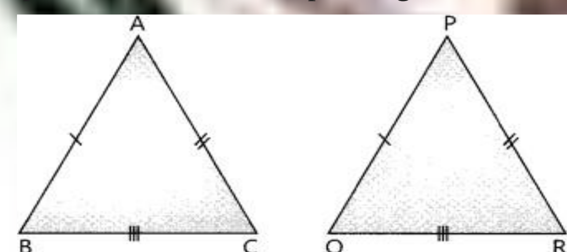
Two plane figures are congruent, if each when superposed on the other covers it exactly.

e.g. F_1 and F_2 are congruent if the trace copy of F_1 fits exactly on that F_2. We can write this as $F_1 = F_2$

Two line segments, $\overline{AB}$ and $\overline{CD}$ are congruent if they have equal lengths. We can write this as $\overline{AB} = \overline{CD}$. However, it is common to write it as $\overline{AB} = \overline{CD}$.

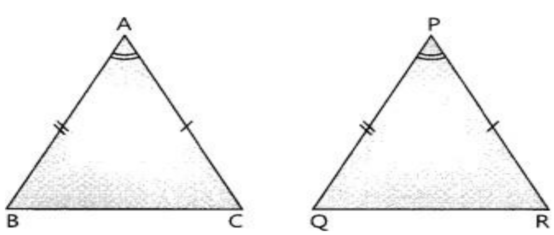
wo angles ∠ABC and ∠PQR, are congruent if their measures are equal. We can write this as ∠ABC = ∠PQR or m∠ABC = m∠PQR. Also, it is common to write it as ∠ABC = ∠PQR

SSS Congruence of two triangles: Two triangles are congruent if the three sides of the one are equal to the three corresponding sides of the other.



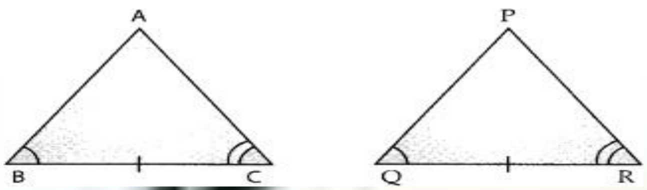
where, AB = PQ, AC = PR, BC = QR

SAS Congruence of two triangles: Two triangles are congruent if two sides and the angle included between them in one of the triangles are equal to the corresponding sides and the angle included between them of the other triangle

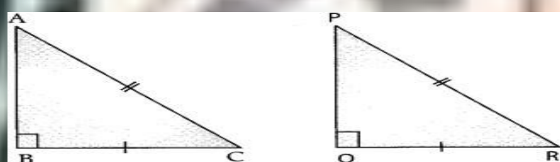


where, AB = PQ, AC = PR, ∠BAC = ∠QPR.

ASA congruence of two triangles: Two triangles are congruent if two angles and the side included between them in one of the triangles are equal to the corresponding angles and the side included between them of the other triangle.



RHS Congruence of two right-angled triangles: Two right-angled triangles are congruent if the hypotenuse and a leg of one of the triangles are equal to the hypotenuse and the corresponding leg of the other triangle



where, AC = PR, BC = QR, ∠ABC = ∠PQR = 90°.

Congruence of Plane Figures

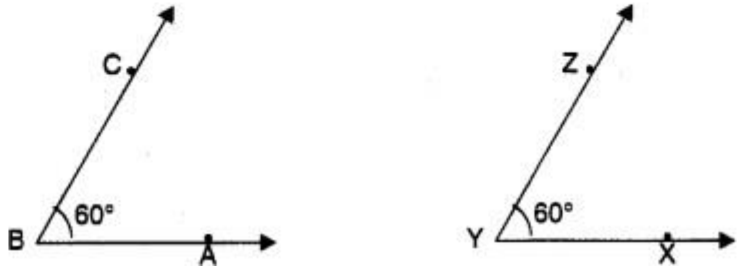
Two figures F_1 and F_2 are said to be congruent if they cover each other completely.
In this case, we write $F_1 = F_2$.

Congruence Among Line Segments

If two line segments have the same (i.e., equal) length, they are congruent. Conversely, if two line segments are congruent, they have the same length.
Line segments are congruent ⇔ their lengths are the same
If line segment AB is congruent to line segment CD, then we write AB = CD
Sometimes we also write AB = CD
and simply say that the line segments AB and CD are equal.

Congruence of Angles

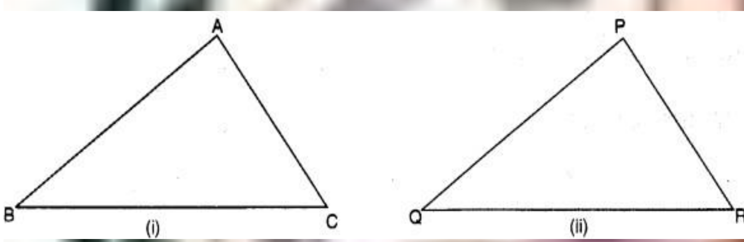
If two angles have the same measure, they are congruent.
Conversely, if two angles are congruent, their measures are the same.
Angles congruent ⇔ Angle measures same
or m∠ABC = m∠XYZ If ∠ABC is congruent to ∠XYZ, then we write ∠ABC = ∠XYZ
where m stands for the measure.



In the above case, the measure is 60°.

Congruence of Triangles

"Two triangles are congruent if they are copies of each other and when superposed, they cover each other exactly."



ΔABC and ΔPQR have the same size and shape. They are congruent. So we would express this as ΔABC = ΔPQR.

This means that, when we place ΔPQR on ΔABC, P falls on A, Q falls on B and R falls on C, also $\overline{PQ}$ falls along $\overline{AB}$, $\overline{QR}$ falls along $\overline{BC}$ and $\overline{PR}$ falls along $\overline{AC}$.

If under a given correspondence, two triangles are congruent, then their corresponding parts (i.e. angles and sides) that match one another are equal. Thus in these two congruent triangles, we have:
Corresponding vertices: A and P, B and Q; C and R.
Corresponding sides: $\overline{AB}$ and $\overline{PQ}$, $\overline{BC}$ and $\overline{QR}$; $\overline{AC}$ and $\overline{PR}$.
Corresponding angles: ∠A and ∠P, ∠B and ∠Q; ∠C and ∠R.

While talking about the congruence of triangles, not only the measures of angles and lengths of sides matter, but also the matching of vertices. In the above case, the correspondence is A ↔ P,

B ↔ Q, C ↔ R
We may write this as ABC ↔ PQR

Criteria for Congruence of Triangles
SSS Congruence Criterion
If under a given correspondence, the three sides of one triangle are respectively equal to the three sides of

another triangle, then the triangles are congruent.

SAS Congruence Criterion
If under a corresponding two sides and the angle included between them of a triangle are respectively equal to two sides and the angle included between them of another triangle, then the triangles are congruent.

ASA Congruence Criterion
If under a correspondence two angles and the included side of a triangle are equal to two angles and the included side of another triangle, then the triangles are congruent.

Congruence Among Right-Angled Triangles
RHS Congruence Criterion
If under a correspondence, the hypotenuse and one side of a right-angled triangle are respectively equal to the hypotenuse and one side of another right-angled triangle, then the triangles are congruent.
We call this "RHS" congruence because R stands for Right-angle, H stands for Hypotenuse and S stands for Side.

EXERCISE :7.1

1. Complete the following statements:

(a) Two line segments are congruent if ___________.

Solution:-

Two line segments are congruent if they have the same length.

(b) Among two congruent angles, one has a measure of 70°; the measure of the other angle is ___________.

Solution:-

Among two congruent angles, one has a measure of 70°; the measure of the other angle is 70°.

Because, if two angles have the same measure, they are congruent. Also, if two angles are congruent, their measure are same.

(c) When we write ∠A = ∠B, we actually mean .

Solution:-

When we write ∠A = ∠B, we actually mean m ∠A = m ∠B.

2. Give any two real-life examples for congruent shapes.

Solution:-

The two real-life example for congruent shapes are

(i) Fan feathers of same brand.

(ii) Size of chocolate in the same brand.

(iii) Size of pens in the same brand

3. If ΔABC ≅ ΔFED under the correspondence ABC ↔ FED, write all the corresponding congruent parts of the triangles.

Solution:-

Two triangles are congruent if pairs of corresponding sides and corresponding angles are equal.

All the corresponding congruent parts of the triangles are,

∠A ↔ ∠F, ∠B ↔ ∠E, ∠C ↔ ∠D

Correspondence between sides:

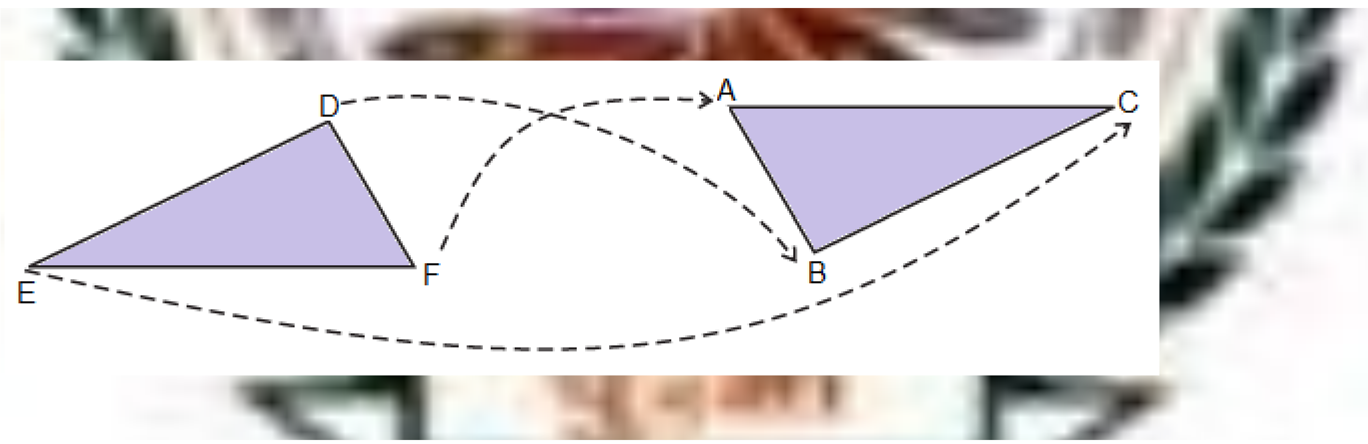
4. If ΔDEF ≅ ΔBCA, write the part(s) of ΔBCA that correspond to

(i) ∠E

(ii) $\overline{EF}$

(iii) ∠F

(iv) $\overline{DF}$



From above the figure we can say that,

The part(s) of ΔBCA that correspond to,

(i) ∠E ↔ ∠C

(ii) $\overline{EF} \leftrightarrow \overline{CA}$

(iii) ∠F ↔ ∠A

(iv) $\overline{DF} \leftrightarrow \overline{BA}$

EXERCISE :7.2

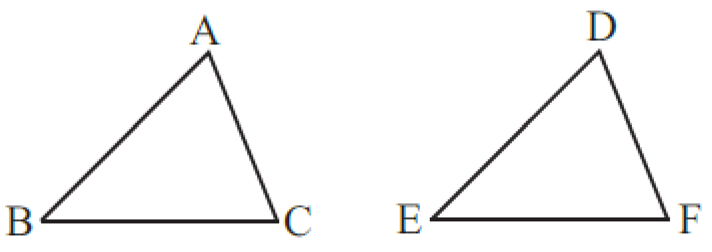
1. Which congruence criterion do you use in the following?

(a) Given: AC = DF

AB = DE

BC = EF

So, ΔABC ≅ ΔDEF



Solution:-

By SSS congruence property:- Two triangles are congruent if the three sides of one triangle are respectively equal to the three sides of the other triangle.

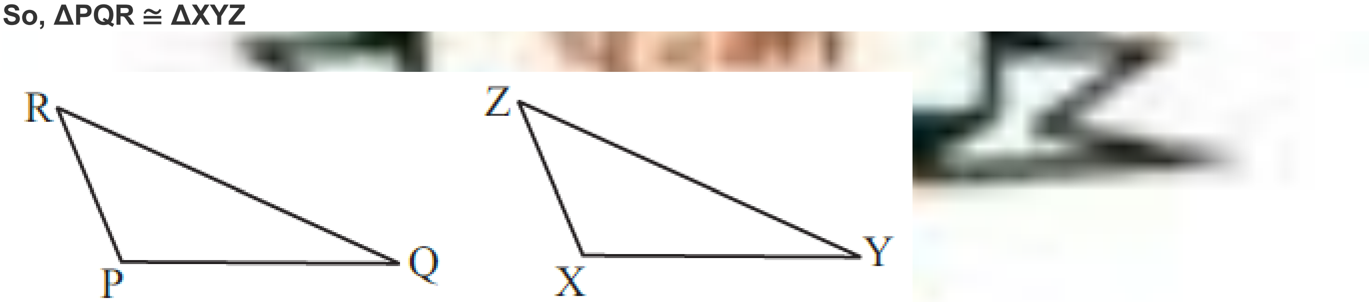
ΔABC ≅ ΔDEF

(b) Given: ZX = RP

RQ = ZY

∠PRQ = ∠XZY

So, ΔPQR ≅ ΔXYZ



Solution:-

By SAS congruence property:- Two triangles are congruent if the two sides and the included angle of one are respectively equal to the two sides and the included angle of the other.

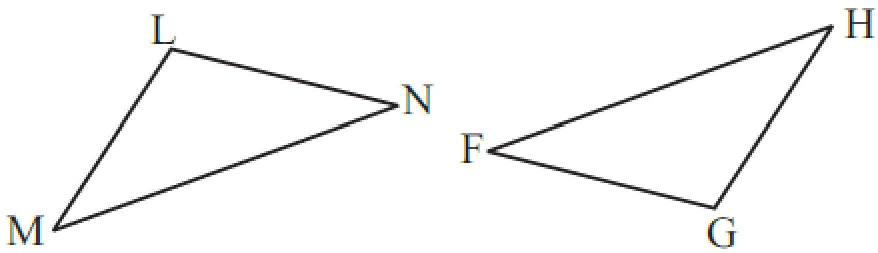
ΔACB ≅ ΔDEF

(c) Given: ∠MLN = ∠FGH

∠NML = ∠GFH

∠ML = ∠FG

So, ΔLMN ≅ ΔGFH



Solution:-

By ASA congruence property:- Two triangles are congruent if the two angles and the included side of one are respectively equal to the two angles and the included side of the other.

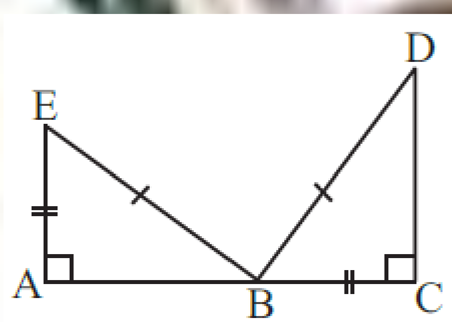
ΔLMN ≅ ΔGFH

(d) Given: EB = DB

AE = BC

∠A = ∠C = 90°

So, ΔABE ≅ ΔACD



Solution:-

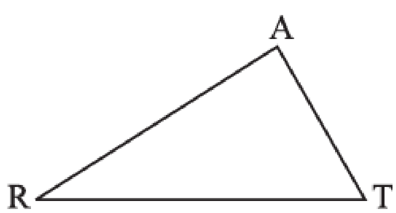
By RHS congruence property:- Two right triangles are congruent if the hypotenuse and one side of the first triangle are respectively equal to the hypotenuse and one side of the second.

ΔABE ≅ ΔACD

2. You want to show that ΔART ≅ ΔPEN,

(a) If you have to use SSS criterion, then you need to show

(i) AR = (ii) RT = (iii) AT =



Solution:-

We know that,

SSS criterion is defined as, two triangles are congruent if the three sides of one triangle are respectively equal to the three sides of the other triangle.

∴ (i) AR = PE

(ii) RT = EN

(iii) AT = PN

(b) If it is given that ∠T = ∠N and you are to use SAS criterion, you need to have

(i) RT = and (ii) PN =

Solution:-

We know that,

SAS criterion is defined as, two triangles are congruent if the two sides and the included angle of one are respectively equal to the two sides and the included angle of the other.

∴ (i) RT = EN

(ii) PN = AT

(c) If it is given that AT = PN and you are to use ASA criterion, you need to have

(i) ? (ii) ?

Solution:-

We know that,

ASA criterion is defined as, two triangles are congruent if the two angles and the included side of one are respectively equal to the two angles and the included side of the other.

Then,

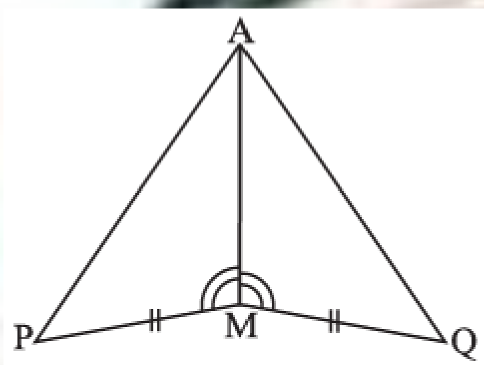
(i) ∠ATR = ∠PNE

(ii) ∠RAT = ∠EPN

3. You have to show that ΔAMP ≅ ΔAMQ.

In the following proof, supply the missing reasons.

Steps	**Reasons**
(i) PM = QM	**(i) …**
(ii) ∠PMA = ∠QMA	**(ii) …**
(iii) AM = AM	**(iii) …**
(iv) ΔAMP ≅ ΔAMQ	



Solution:-

Steps	Reasons
(i) PM = QM	(i) From the given figure
(ii) ∠PMA = ∠QMA	(ii) From the given figure
(iii) AM = AM	(iii) Common side for the both triangles
(iv) ΔAMP ≅ ΔAMQ	(iv) By SAS congruence property:- Two triangles are congruent if the two sides and the included angle of one are respectively equal to the two sides and the included angle of the other.

4. In ΔABC, ∠A = 30°, ∠B = 40° and ∠C = 110°

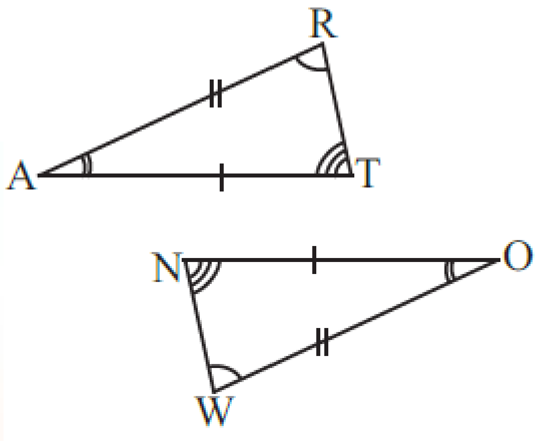
In ΔPQR, ∠P = 30°, ∠Q = 40° and ∠R = 110°

A student says that ΔABC ≅ ΔPQR by AAA congruence criterion. Is he justified? Why or Why not?

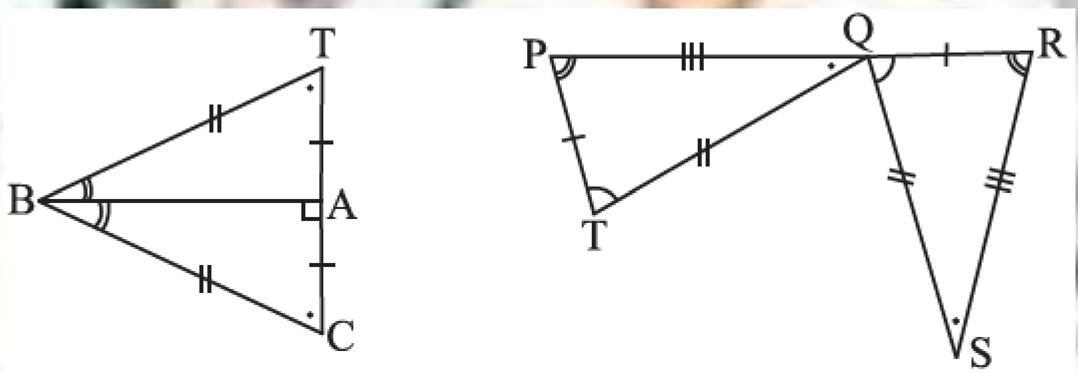
Solution:-

No, because the two triangles with equal corresponding angles need not be congruent. In such a correspondence, one of them can be enlarged copy of the other.

5. In the figure, the two triangles are congruent. The corresponding parts are marked. We can write ΔRAT ≅ ?



6. Complete the congruence statement:



ΔBCA ≅ ΔQRS ≅

Solution:-

First consider the ΔBCA and ΔBTA

From the figure, it is given that,

BT = BC

Then,

BA is common side for the ΔBCA and ΔBTA

Hence, ΔBCA ≅ ΔBTA

Similarly,

Consider the ΔQRS and ΔTPQ

From the figure, it is given that

PT = QR

TQ = QS

PQ = RS

Hence, ΔQRS ≅ ΔTPQ

7. In a squared sheet, draw two triangles of equal areas such that

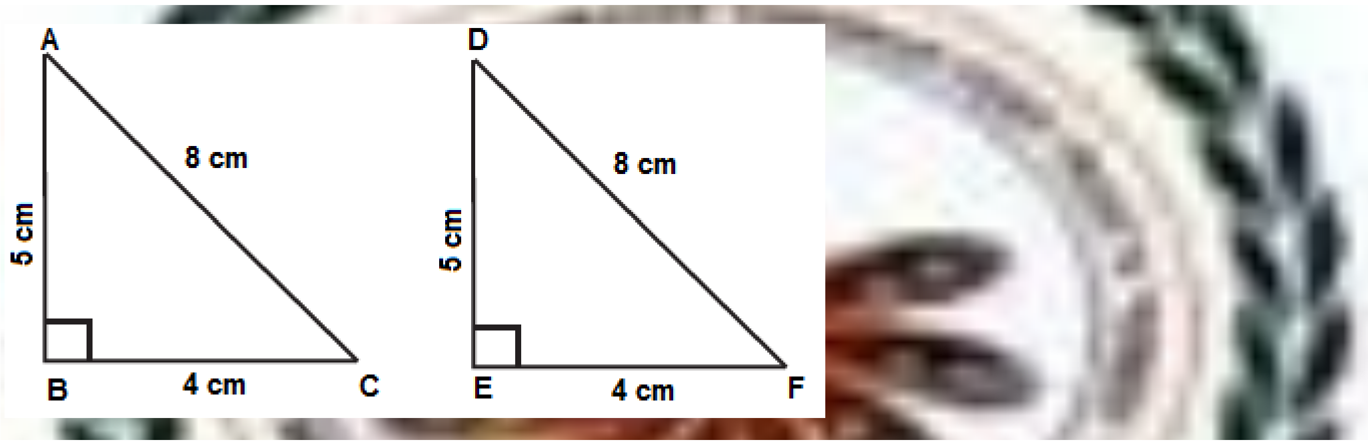
(i) The triangles are congruent.

(ii) The triangles are not congruent.

What can you say about their perimeters?

Solution:-

(ii)

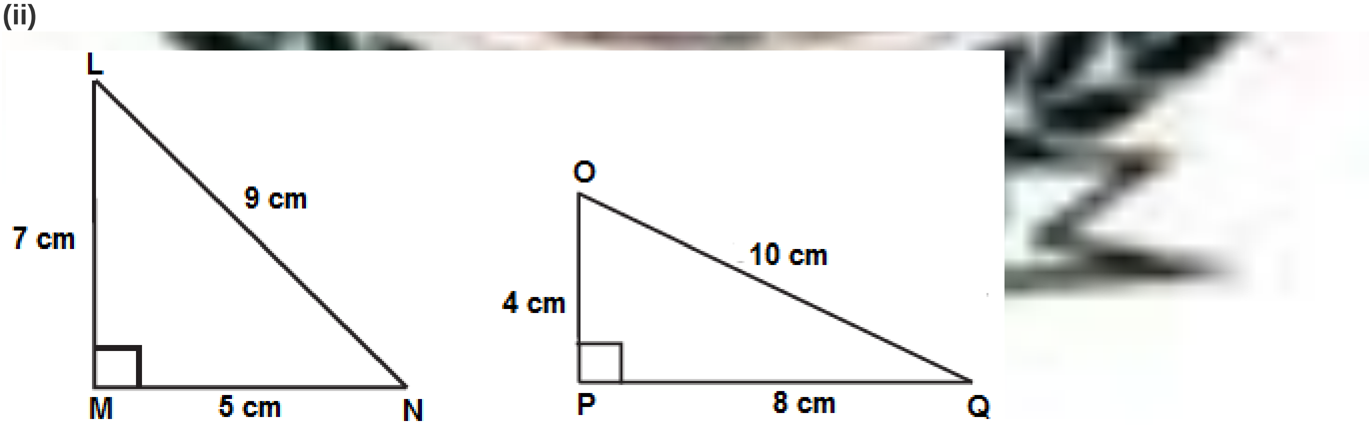


n the above figure, ΔABC and ΔDEF have equal areas.

And also, ΔABC ≅ ΔDEF

So, we can say that perimeters of ΔABC and ΔDEF are equal.

(ii)



In the above figure, ΔLMN and ΔOPQ

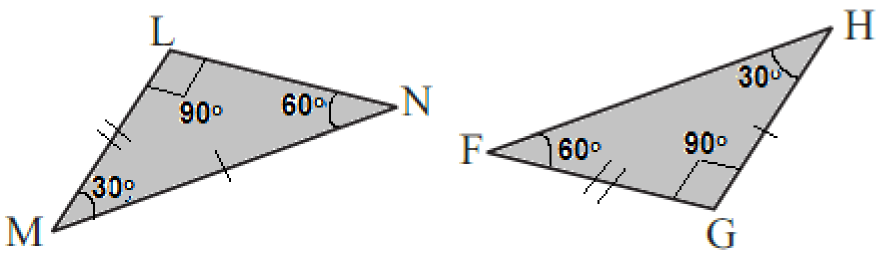
ΔLMN is not congruent to ΔOPQ

So, we can also say that their perimeters are not same.

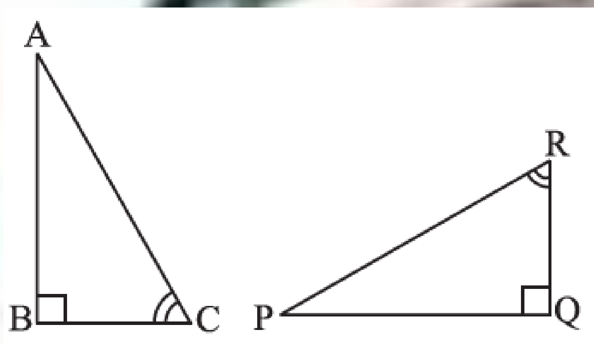
8. Draw a rough sketch of two triangles such that they have five pairs of congruent parts but still the triangles are not congruent.

Solution:-

Let us draw triangles LMN and FGH.



9. If ΔABC and ΔPQR are to be congruent, name one additional pair of corresponding parts. What criterion did you use?



Solution:-

By observing the given figure, we can say that

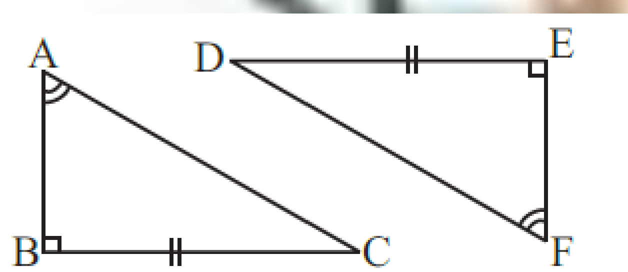
∠ABC = ∠PQR

∠BCA = ∠PRQ

The other additional pair of corresponding part is BC = QR

∴ ΔABC ≅ ΔPQR

10. Explain, why ΔABC ≅ ΔFED



Solution:-

From the figure, it is given that,

∠ABC = ∠DEF = 90°

∠BAC = ∠DFE

BC = DE

By ASA congruence property, two triangles are congruent if the two angles and the included side of one are respectively equal to the two angles and the included side of the other.

$\Delta ABC \cong \Delta FED$