Notes
Chapter – 14
Factorisation

- **Factorisation:** Representation of an algebraic expression as the product of two or more expressions is called factorization. Each such expression is called a factor of the given algebraic expression.
- When we factorise an expression, we write it as a product of factors. These factors may be numbers, algebraic variables or algebraic expressions.
- An irreducible factor is a factor which cannot be expressed further as a product of factors.
- A systematic way of factorising an expression is the common factor method. It consists of three steps:
  (i) Write each term of the expression as a product of irreducible factors
  (ii) Look for and separate the common factors and
  (iii) Combine the remaining factors in each term in accordance with the distributive law.
- Sometimes, all the terms in a given expression do not have a common factor; but the terms can be grouped in such a way that all the terms in each group have a common factor. When we do this, there emerges a common factor across all the groups leading to the required factorisation of the expression. This is the method of regrouping.
- In factorisation by regrouping, we should remember that any regrouping (i.e., rearrangement) of the terms in the given expression may not lead to factorisation. We must observe the expression and come out with the desired regrouping by trial and error.
- A number of expressions to be factorised are of the form or can be put into the form: $a^2 + 2ab + b^2$, $a^2 – 2ab + b^2$, $a^2 – b^2$ and $x2 + (a + b)x + ab$. These expressions can be easily factorised using Identities I, II, III and IV
  $a^2 + 2ab + b^2 = (a + b)^2$
  $a^2 - 2ab + b^2 = (a - b)^2$
  $a^2 – b^2 = (a + b) (a - b)$
  $x2 + (a + b)x + ab = (x + a)(x + b)$
- In expressions which have factors of the type (x + a) (x + b), remember the numerical term gives ab. Its factors, a and b, should be so chosen that their sum, with signs taken care of, is the coefficient of x.
- We know that in the case of numbers, division is the inverse of multiplication. This idea is applicable also to the division of algebraic expressions.
- In the case of division of a polynomial by a monomial, we may carry out the division either by dividing each term of the polynomial by the monomial or by the common factor method.
- In the case of division of a polynomial by a polynomial, we cannot proceed by dividing each term in the dividend polynomial by the divisor polynomial. Instead, we factorise both the polynomials and cancel their common factors.
- In the case of divisions of algebraic expressions that we studied in this chapter, we have
- Dividend = Divisor × Quotient.
- In general, however, the relation is
- Dividend = Divisor × Quotient + Remainder
- Thus, we have considered in the present chapter only those divisions in which the remainder is zero.
- There are many errors students commonly make when solving algebra exercises. You should avoid making such errors.

Ex. 14.1

1(1). Find the common factors of the given term: 12x, 36

Sol. 12x, 36
$12x = \underline{2} \times 2 \times 3 \times x$
$36 = 2 \times 2 \times 3 \times 3$
Common prime factors are 2 (occurs twice) and 3.
∴ H.C.F. = $2 \times 2 \times 3 = 12$

1(2). Find the common factors of the given term: 2y, 22xy

Sol. 2y, 22xy
$2y = \underline{2} \times y$
$22 = \underline{2} \times 11 \times x \times y$
Common prime factors are 2 and y.
∴ H.C.F. = $2 \times y = 2y$

1(3). Find the common factors of the given term: 14pq, $28p^2q^2$

Sol. 14pq, $28p^2q^2$
14pq =2×7×po×qw
$28p^2q^2$ =2×2×7×p×p×qxq
Common prime factors are 2, 7, p and q.
∴ H.C.F. = $2 \times 7 \times p \times q = 14pq$

1(4). Find the common factors of the given term: 2x, $3x^2$, 4

Sol. 2x, $3x^2$, 4
2x=1×2×x
3x2=1×3×x×x
$4 = \underline{1} \times 2 \times 2$
Common factors are 1 and x.
∴ H.C.F. = $1 \times x = x$

1(5). Find the common factors of the given term: 6abc, $24ab^2$, $12a^2b$

Sol. 6abc =2×3×a×b×c
24ab2=2×2×2×3×a×b×b
12a2b=2×2×3×a×a×b
Common prime factors are 2, 3, a and b
∴ H.C.F. = $2 \times 3 \times a \times b$
= 6ab

1(6). Find the common factors of the given term: 16x3,−4x2,32x

Sol. The given terms 16x3,−4x2,32x can be written as:

$16x^3 = 2\times2\times2\times2\times x\times x\times x$

$-4x^2 = -1\times2\times2\times x\times x$

$32x = 2\times2\times2\times2\times2\times x$

The common factors are 2, 2 and $x = 2\times2\times x = 4x$

1(7). Find the common factors of the given term: 10pq, 20qr, 30rp

Sol. $10pq = 2\times5\times p\times q$

$20qr = 2\times2\times5\times q\times r$

$30rp = 2\times3\times5\times r\times p$

Common prime factors are 2 and 5

∴ H.C.F. = 2 × 5 = 10

1(8). Find the common factors of the given term: $3x^2y^3$, $10x^3y^2$, $6x^2y^2z$

Sol. $3x^2y^3 = 3\times x\times x\times y\times y\times y$

$10x^3y^2 = 2\times5\times x\times x\times x\times y\times y$

$6x^2y^2z = 2\times3\times x\times x\times y\times y\times z$

Hence the common factors are x, x, y, y

and $x\times x\times y\times y = x^2y^2$

2(1). Factorise the expression: 7x – 42

Sol. $7x - 42 = 7(x - 6)$

2(2). Factorise the expression: 6p – 12q

Sol. $6p - 12q = 6(p - 2q)$

2(3). Factorise the expression: $7a^2 + 14a$

Sol. $7a^2 + 14a = 7a(a + 2)$

2(4). Factorise the expression: $-16z + 20z^3$

Sol. $-16z + 20z^3 = 4z(-4 + 5z^2)$

2(5). Factorise the expression: $20l^2m + 30alm$

Sol. $20l^2m + 30alm = 2 \times 2 \times 5 \times l \times l \times m + 2 \times 3 \times 5 \times a \times l \times m$
Taking common factors from each term,
$= 2 \times 5 \times l \times m(2 \times l + 3 \times a)$
$= 10lm(2l + 3a)$

2(6). Factorise the expression: $5x^2y - 15xy^2$

Sol. $5x^2y - 15xy^2 = 5xy\,(x - 3y)$

2(7). Factorise the expression: $10a^2 - 15b^2 + 20c^2$

Sol. $10a^2 - 15b^2 + 20c^2 = 2 \times 5 \times a \times a - 3 \times 5 \times b \times b + 2 \times 2 \times 5 \times c \times c$
Taking common factors from each term,
$= 5(2 \times a \times a - 3 \times b \times b + 2 \times 2 \times c \times c)$
$= 5(2a^2 - 3b^2 + 4c^2)$

2(8). Factorise the expression: $-4a^2 + 4ab - 4ca$

Sol. $-4a^2 + 4ab - 4ca = 4a\,(-a + b - c)$

2(9). Factorise the expression: $x^2yz + xy^2z + xyz^2$

Sol. $x^2yz + xyz + xyz^2 = x \times x \times y \times z + x \times y \times y \times z + x \times y \times z \times z$
Taking common factors from each term,
$= x \times y \times z(x + y + z)$
$= xyz(x + y + z)$

2(10). Factorise the expression: $ax^2y + bxy^2 + cxyz$

Sol. $ax^2y + bxy^2 + cxyz = a \times x \times x \times y + b \times x \times y \times y + c \times x \times y \times z$
Taking common factors from each term,
$= x \times y(a \times x + b \times y + c \times z)$
$= xy(ax + by + cz)$

3(1). Factorise :$x^2 + xy + 8x + 8y$

Sol. $x^2 + xy + 8x + 8y$
$= x\,(x + y) + 8\,(x + y)$
$= (x + y)\,(x + 8)$

3(2). Factorise :$15xy – 6x + 5y – 2$

Sol. $15xy – 6x + 5y – 2$
$= 3x (5y – 2) + 1 (5y – 2)$
$= (5y – 2) (3x + 1)$

3(3). Factorise : $ax + bx – ay – by$

Sol. $ax + bx – ay – by$
$= x (a + b) – y (a + b)$
$= (a + b)(x – y)$

3(4). Factorise : $15pq + 15 + 9q + 25p$

Sol. $15pq + 15 + 9q + 25p$
$= 15pq + 9q + 25p + 15$
$= 3q (5p + 3) + 5(5p + 3)$
$= (5p + 3) (3q + 5)$

3(5). Factorise : $z – 7 + 7xy – xyz$

Sol. $z – 7 + 7xy – xyz$
$= z – 7 – xyz + 7xy$
$= 1(z – 7) – xy (z – 7)$
$= (z – 7) (1 – xy)$.

Ex. 14.2

1(1). Factorise the expression: $a^2 + 8a + 16$

Sol. $a^2 + 8a + 16$
$= (a^2) + 2(4) (a) + (4)^2$
$= (a + 4)^2$. . . . [Applying Identity I]

1(2). Factorise the expression: $p^2 – 10p + 25$

Sol. $p^2 - 10p + 25 = p^2 + (-5 - 5)p + (-5)(-5)$
Using identity $x^2 + (a + b)x + ab = (x + a)(x + b)$,
Here $x = p$, $a = -5$ and $b = -5$
So, $p^2 - 10p + 25 = (p - 5)(p - 5)$

1(3). Factorise the expression : $25m^2 + 30m + 9$

Sol. $25m^2 + 30m + 9$
$= (5m)^2 + 2(5m)(3) + (3)^2$
$= (5m + 3)^2$ ..... [Applying Identity I

1(4). Factorise the expression :$49y^2 + 84yz + 36z^2$

Sol. $49y^2 + 84yz + 36z^2$
$= (7y)^2 + 2(7y)(6z) + (6z)^2$
$= (7y + 6z)^2$ . . . . [Using Identity I

1(5). Factorise the expression : $4x^2 – 8x + 4$

Sol. $4x^2 – 8x + 4$
$= 4(x^2 – 2x + 1)$
$= 4[(x)^2 – 2(x)(1) + (1)^2]$
$= 4(x – 1)^2$. . . . [Applying Identity II]

1(6). Factorise the expression$121b^2 – 88bc + 16c^2$

Sol. $121b^2 – 88bc + 16c^2$
$= (11b)^2 – 2(11b)(4c) + (4c)^2$
$= (11b – 4c)^2$. . . . [Using Identity II]

1(7). Factorise the expression: $(l + m)^2 – 4lm$
(Hint: Expand $(l + m)^2$ first)

Sol. $(l + m)^2 – 4lm$
$= (l^2 + 2lm + m^2) – 4lm$. . . . [Using Identity I]
$= l^2 + (2lm – 4lm) + m^2$. . . . [Combining the like terms]
$= l^2 – 2lm + m^2$
$= (l)^2 – 2(l)(m) + (m)^2$
$= (l – m)^2$. . . . [Applying Identity II]

1(8). Factorise the expression: $a^4 + 2a^2b^2 + b^4$

Sol. $a^4 + 2a^2b^2 + b^4$
$= (a^2)^2 + 2(a^2)(b^2) + (b^2)^2$
$= (a^2 + b^2)^2$. . . . [Using Identity I

2(1). Factorise: $4p^2 – 9q^2$

Sol. $4p^2 - 9q^2$
$= (2p)^2 - (3q)^2$
$= (2p - 3q)(2p + 3q)$. . . . [Using Identity III

2(2). Factorise: $63a^2 - 112b^2$

Sol. $63a^2 - 112b^2$
$= 7(9a^2 - 16b^2)$
$= 7\{(3a)^2 - (4b)^2\}$
$= 7(3a - 4b)(3a + 4b)$. . . . [Applying Identity III]

2(3). Factorise: $49x^2 - 36$

Sol. $49x^2 - 36$
$= (7x)^2 - (6)^2$
$= (7x - 6)(7x + 6)$. . . . [Using Identity III

2(4). Factorise: $16x^5 - 144x^3$

Sol. $16x^5 - 144x^3$
$= 16x^3(x^2 - 9)$
$= 16x^3\{(x)^2 - (3)^2\}$
$= 16x^3(x - 3)(x + 3)$

2(5). Factorise: $(l + m)^2 - (l - m)^2$

Sol. $(l + m)^2 - (l - m)^2$
$= l\{(l + m) - (l - m)\}\{(l + m) + (l - m)\}$ . . . . [Applying Identity III
$= (2m)(2l)$
$= 4lm$

2(6). Factorise: $9x^2y^2 - 16$

Sol. $9x^2y^2 - 16$
$= (3xy)^2 - (4)^2$
$= (3xy - 4)(3xy + 4)$. . . . [Using Identity]

2(7). Factorise: $(x^2 - 2xy + y^2) - z^2$

Sol. $(x^2 - 2xy + y^2) - z^2$
$= (x - y)^2 - z^2$. . . . [Using Identity II
$=(x - y - z)(x - y + z)$. . . . [Using Identity III

2(8). Factorise: $25a^2 - 4b^2 + 28bc - 49c^2$

Sol. $25a^2 - 4b^2 + 28bc - 49c^2$
$= 25a^2 - (4b^2 - 28bc + 49c^2)$
$= 25a^2 - \{(2b)^2 - 2(2b)(7c) + (7c)^2\}$
$= (5a)^2 - (2b - 7c)^2$. . . . [Using Identity II
$= \{5a - (2b - 7c)\}\{5a + (2b - 7c)\}$
$= (5a - 2b + 7c)(5a + 2b - 7c)$

3(1). Factorise the expressions: $ax^2 + bx$

Sol. $ax^2 + bx$
$ax^2 + bx = x(ax + b)$

3(2). Factorise the expressions: $7p^2 + 21q^2$

Sol. $7p^2 + 21q^2$
$7p^2 + 21q^2 = 7(p^2 + 3q^2)$

3(3). Factorise the expressions: $2x^3 + 2xy^2 + 2xz^2$

Sol. $2x^3 + 2xy^2 + 2xz^2$
$2x^3 + 2xy^2 + 2xz^2 = 2x(x^2 + y^2 + z^2)$

3(4). Factorise the expressions: $am^2 + bm^2 + bn^2 + an^2$

Sol. $am^2 + bm^2 + bn^2 + an^2$
$am^2 + bm^2 + bn^2 + an^2 = am^2 + bm^2 + an^2 + bn^2$
$= m^2(a + b) + n^2(a + b)$
$= (a + b)(m^2 + n^2)$

3(5). Factorise the expressions: $(1m + 1) + m + 1$

Sol. $(1m + 1) + m + 1$
$= 1(m + 1) + 1(m + 1)$
$= (m + 1)(1 + 1)$
$=2(m+1)$

3(6). Factorise the expressions: $y(y + z) + 9(y + z)$

Sol. $y(y + z) + 9(y + z)$
$= (y + z)(y + 9)$

3(7). Factorise the expressions: $5y^2 - 20y - 8z + 2yz$

Sol. $5y^2 - 20y - 8z + 2yz$
$= 5y^2 - 20y + 2yz - 8z$
$= (y - 4)(5y + 2z)$

3(8). Factorise the expressions: $10ab + 4a + 5b + 2$

Sol. $10ab + 4a + 5b + 2$
$= 2a(5b + 2) + 1(5b + 2)$
$= (5b + 2)(2a + 1)$

3(9). Factorise the expressions: $6xy - 4y + 6 - 9x$

Sol. $6xy - 4y + 6 - 9x$
$= 6xy - 4y - 9x + 6$
$= 2y(3x - 2) - 3(3x - 2)$
$= (3x - 2)(2y - 3)$

4(1). Factorise: $a^4 - b^4$

Sol. $a^4 - b^4$
$= (a^2)^2 - (b^2)^2$
$= (a^2 - b^2)(a^2 + b^2)$
$= (a - b)(a + b)(a^2 + b^2)$. . . . [Using Identity III

4(2). Factorise: $p^4 - 81$

Sol. $p^4 - 81$
$= (p^2)^2 - (9)^2$
$= (p^2 - 9)(p^2 + 9)$ . . . . [Using Identity III
$= \{(p)^2 - (3)^2\}(p^2 + 9)$
$= (p - 3)(p + 3)(p^2 + 9)$. . . . [Using Identity III

4(3). Factorise: $x^4 - (y + z)^4$

Sol. $x^4 - (y + z)^4$
$= (x^2)^2 - \{(y + z)^2\}^2$. . . . [Using Identity III
$= \{x^2 - (y + z)^2\}\{x^2 + (y + z)^2\}$. . . . [Using Identity III
$= (x - y - z)(x + y + z)\{x^2 + (y + z)^2\}$

4(4). Factorise: $x^4 - (x - z)^4$

Sol. $x^4 – (x – z)^4$

$= (x^2)^2 – \{(x – z)^2\}^2$. . . . [Using Identity III

$= \{x^2 – (x – z)^2\} \{x^2 + (x – z)^2\}$ . . . . [Applying Identity III

$= (x – x + z) (x + x – z) \{x^2 + (x – z)^2\}$

$= z(2x – z) \{x^2 + (x – z)^2\}$

$= z(2x – z) (x^2 + x^2 – 2xz + z^2)$. . . . [Using Identity II

$= z (2x – z) (2x^2 – 2xz + z^2)$

4(5). Factorise: $a^4 – 2a^2b^2 + b^4$

Sol. $a^4 – 2a^2b^2 + b^4$

$= (a^2)^2 – 2(a^2) (b^2)+ (b^2)^2$

$= (a^2 – b^2)^2$ . . . . [Using Identity II

$= \{(a – b) (a + b)^2\}$. . . . [Using Identity III

$= (a – b)^2 (a + b)^2$.

5(1). Factorise the expressions: $p^2 + 6p + 8$

Sol. $p^2 + 6p + 8$

$= p^2 + 6p + 9 – 1$

$= \{(p)^2 + 2(p) (3) + (3)^2\} – (1)^2$

$= (p + 3)^2 – (1)^2$. . . . [Using Identity I

$= (p + 3 – 1) (p + 3 + 1)$. . . . [Using Identity III]

$= (p + 2) (p + 4)$

5(2). Factorise the expressions: $q^2 - 10q + 21$

Sol. $q^2 – 10q + 21$

$= q^2 – 10q + 25 – 4$

$= \{(q)^2 – 2(q) (5) + (5)^2\} – 4$

$= (q – 5)^2 – (2)^2$ . . . . [Using Identity II]

$= (q – 5 – 2) (q – 5 + 2)$. . . . [Using Identity III]

$= (q – 7) (q – 3)$

5(3). Factorise the expressions: $p^2 + 6p – 16$

Sol. $p^2 + 6p – 16$

$= p^2 + 6p + 9 – 25$

$= (p)^2 + 2(p) (3) + (3)^2 – (5)^2$

$= (p + 3)^2 – (5)^2$. . . . [Using Identity I

$= (p + 3 + – 5) (p + 3 + 5)$ . . . . [Applying Identity III]

$= (p – 2) (p + 8)$.

EX:14.3

1(1). Carry out the division: $28x^4 \div 56x$

Sol. $28x^4 \div 56x$
=28x456x
=x32

1(2). Carry out the division: $-36y^3 \div 9y^2$

Sol. $-36y^3 \div 9y^2$
=−36y39y2
= -4y

1(3). Carry out the division: $66pq^2r^3 \div 11qr^2$

Sol. $66pq^2r^3 \div 11qr^2$
=66pq2r311qr2
= 6pqr

1(4). Carry out the division: $34x^3y^3z^3 \div 51xy^2z^3$

Sol. $34x^3y^3z^3 \div 51xy^2z^3$
=34x3y3z351xy2z3
=23x2y

1(5). Carry out the division: $12a^8b^8 \div (-6a^6b^4)$

Sol. $12a^8b^8 \div (-6a^6b^4)$
=12a8b8−6a6b4
= $-2a^2b^4$

2(1). Divide the given polynomial by the given monomial: $(5x^2 - 6x) \div 3x$

Sol. $(5x^2 - 6x) \div 3x$
=$5x^2 - 6x$ / 3x
=x(5x−6) / 3x
=1 / 3 (5x−6)

2(2). Divide the given polynomial by the given monomial: $(3y^8 - 4y^6 + 5y^4) \div y^4$

Sol. $(3y^8 - 4y^6 + 5y^4) \div y^4$

=3y8−4y6+5y4y4
=y4(3y4−4y2+5)y4
= $3y^4 – 4y^2 + 5$

2(3). Divide the given polynomial by the given monomial: $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$

Sol. $8(x^3y^2z^2 + x^2y^3z^2 + x^2y^2z^3) \div 4x^2y^2z^2$
=8(x3y2z2+x2y2z2+x2y2z2)4x2y2z2
=8x2y2z2(x+y+z)4x2y2z2
= 2(x + y + z)

2(4). Divide the given polynomial by the given monomial: $(x^3 + 2x^2 + 3x) \div 2x$

Sol. $(x^3 + 2x^2 + 3x) \div 2x$
=x3+2x2+3x2x
=x(x2+2x+3)2x
=x2+2x+32

2(5). Divide the given polynomial by the given monomial: $(p^3q^6 – p^6q^3) \div p^3q^3$

Sol. $(p^3q^6 – p^6q^3) \div p^3q^3$
=p3q6−p6q3p3q3
=p3q3(q3−p3)p3q3
= $q^3 – p^3$

3(1). Work out the division: (10x – 25) ÷ 5

Sol. (10x – 25) ÷ 5
=10x−255
=5(2x−5)5
= (2x – 5)

3(2). Work out the division: (10x – 25) ÷ (2x – 5)

Sol. (10x – 25) ÷ (2x – 5)
=10x−252x−5
=5(2x−5)(2x−5)
= 5

3(3). Work out the division : 10y(6y + 21) ÷ 5(2y + 7)

Sol. 10y (6y + 21) ÷ 5(2y + 7)
=10y(6y+21)5(2y+7)

$=10y\times3(2y+7)5(2y+7)$
$= 6y$

3(4). Work out the division: $9x^2y^2(3z – 24) \div 27xy (z – 8)$

Sol. $9x^2y^2(3z – 24) \div 27xy (z – 8)$
$=9x2y2(3z−24)27xy(z−8)$
$=9x2y23(z−8)27xy(z−8)$
$= xy$

3(5). Work out the division: $96abc (3a – 12) (5b – 30) \div 144 (a – 4) (b – 6)$

Sol. $96abc (3a – 12) (5b – 30) \div 144 (a – 4) (b – 6)$
$=96abc(3a−12)(5b−30)144(a−b)(b−6)$
$=96abc\times3(a−4)\times5(b−6)144(a−4)(b−6)$
$= 10abc$

4(1). Divide as directed: $5(2x + 1) (3x + 5) \div (2x + 1)$

Sol. $5(2x + 1) (3x + 5) \div (2x + 1)$
$=5(2x+1)(3x+5)2x+1$
$= 5(3x + 5)$

4(2). Divide as directed: $26xy (x + 5) (y – 4) \div 13x (y – 4)$

Sol. $26xy (x + 5) (y – 4) \div 13x (y – 4)$
$=26xy(x+5)(y−4)13x(y−4)$
$= 2y(x + 5)$

4(3). Divide as directed: $52pqr (p + q) (q + r) (r + p) \div 104pq (q + r) (r + p)$

Sol. We have $52pqr (p + q) (q + r)(r + p) \div 104pq (q + r) (r + p)$
$=52pqr(p+q)(q+r)(r+p)104pq(q+r)(r+p)$
$=r(p+q)2$

4(4). Divide as directed: $20(y + 4) (y^2 + 5y + 3) \div 5(y + 4)$

Sol. $20(y + 4) (y^2 + 5y + 3) \div 5(y + 4)$
$=20(y+4)(y2+5y+3)5(y+4)$
$= 4(y^2 + 5y + 3)$

4(5). Divide as directed: $x (x + 1) (x + 2) (x + 3) \div x (x + 1)$

Sol. $x (x + 1) (x + 2) (x + 3) \div x (x + 1)$

$=x(x+1)(x+2)(x+3)x(x+1)$

$= (x + 2) (x + 3)$

5(1). Factorise the expression and divide them as directed: $(y^2 + 7y + 10) \div (y + 5)$

Sol. $(y^2 + 7y + 10) \div (y + 5)$

$=y2+7y+10y+5$

$=y2+2y+5y+10y+5$ . . . .[Using Identity IV

$=y(y+2)+5(y+2)y+5$

$=(y+2)(y+5)y+5$

$= y + 2$

5(2). Factorise the expression and divide them as directed: $(m^2 – 14m – 32) \div (m + 2)$

Sol. $(m^2 – 14m – 32) \div (m + 2)$

$=m2−14m−32m+2$

$=m2−16m+2m−32m+2$. . . . [Using Identity IV

$=m(m−16)+2(m−16)m+2$

$=m(m−16)(m+2)m+2$

$= m – 16$

5(3). Factorise the expression and divide them as directed: $(5p^2 – 25p + 20) \div (p – 1)$

Sol. $(5p^2 – 25p + 20) \div (p – 1)$

$=5(p2−5p+4)p−1$

$=5(p2−p−4p+4)p−1$. . . . [Applying Identity IV]

$=5\{p(p−1)−4(p−1)\}p−1$

$=5(p−1)(p−4)p−1$

$= 5 (p – 4)$

5(4). Factorise the expression and divide them as directed: $4yz (z^2 + 6z – 16) \div 2y (z + 8)$

Sol. $4yz (z^2 + 6z – 16) \div 2y (z + 8)$

$=4yz(z2+6z−16)2y(z+8)$

$=2z(z2+6z−16)z+8$

$=2z(z2+8z−2z−16)z+8$. . . . [Using Identity IV]

$=2z[z(z+8)−2(z+8)]z+8$

$=2z(z+8)(z−2)z+8$

$= 2z (z – 2)$

5(5). Factorise the expression and divide them as directed: $5pq (p^2 – q^2) \div 2p (p + q)$

Sol. $5pq\ (p^2 – q^2) \div 2p\ (p + q)$
=5pq(p2−q2)2p(p+q)
=5pq(p+q)(p−q)2p(p+q). . . . [Using Identity III]
=5q(p−q)2

5(6). Factorise the expression and divide them as directed:$12xy\ (9x^2 – 16y^2) \div 4xy\ (3x + 4y)$

Sol. $12xy\ (9x^2 – 16y^2) \div 4xy\ (3x + 4y)$
=12xy(9x2−16y2)4xy(3x+4y)
=3(9x2−16y2)3x+4y
=3{(3x)2−(4y2)}3x+4y
=3(3x+4y)(3x−4y)3x+4y
= 3(3x – 4y)

5(7). Factorise the expression and divide them as directed: $39y^3\ (50y^2 – 98) \div 26y^2(5y + 7)$

Sol. $39y^3\ (50y^2 – 98) \div 26y^2(5y + 7)$
=39y3(50y2−98)26y2(5y+7)
=39y3×2×(25y2−49)26y2(5y+7)
=39y3×2×{(5y)2−(7)2}26y2(5y+7)
=39y3×2×(5y+7)(5y−7)26y2(5y−7). . . . [Using Identity III
= 3y (5y – 7)

EX: 14.4

1. Find and correct the errors in the mathematical statement: 4(x – 5) = 4x – 5

Sol. L.H.S. 4(x−5)=4x−20≠ R.H.S.
Hence the correct mathematical statements is 4(x−5)=4x−20

2. Find and correct the errors in the mathematical statement: $x(3x + 2) = 3x^2 + 2$

Sol. $x(3x + 2) = 3x^2 + 2x$

3. Find and correct the errors in the mathematical statement: 2x + 3y = 5xy

Sol. 2x + 3y = 2x + 3y

4. Find and correct the errors in the mathematical statement: x + 2x + 3x = 5x

Sol. $x + 2x + 3x = 6x$

5. Find and correct the errors in the mathematical statement: $5y + 2y + y – 7y = 0$

Sol. $5y + 2y + y – 7y = y$

6. Find and correct the errors in the mathematical statement: $3x + 2x = 5x^2$

Sol. $3x + 2x = 5x$

7. Find and correct the errors in the mathematical statement: $(2x)^2 + 4(2x) + 7 = 2x^2 + 8x + 7$

Sol. $(2x)^2 + 4(2x) + 7 = 4x^2 + 8x + 7$

8. Find and correct the errors in the mathematical statement: $(2x)^2 + 5x = 4x + 5x = 9x$

Sol. $(2x)^2 + 5x = 4x^2 + 5x$

9. Find and correct the errors in the mathematical statement: $(3x + 2)^2 = 3x^2 + 6x + 4$

Sol. $(3x + 2)^2 = 9x^2 + 12x + 4$

10(1). Find and correct the errors in the mathematical statement. Substituting $x = –3$ in the given equation $x^2 + 5x + 4$ gives $(–3)^2 + 5(-3) + 4 = 9 + 2 + 4 = 15$.

Sol. $x^2 + 5x + 4 = (–3)^2 + 5(–3) + 4$
$= 9 – 15 + 4$
$= –2$

10(2). Find and correct the errors in the following mathematical statement. Substituting $x = – 3$ in the given equation $x^2 – 5x + 4$ gives $(– 3)^2 – 5(– 3) + 4 = 9 – 15 + 4 = – 2$.

Sol. $x^2 – 5x + 4 = (– 3)^2 – 5(– 3) + 4$
$= 9 + 15 + 4$
$= 28$

10(3). Find and correct the errors in the mathematical statement. Substituting $x = -3$ in the given equation $x^2 + 5x$ gives $(–3)^2 + 5(–3) = –9 – 15 = -24$.

Sol. $x^2 + 5x = (-3)^2 + 5(-3)$
$= 9 - 15 = -6$

11. Find and correct the error in the mathematical statement: $(y - 3)^2 = y^2 - 9$

Sol. LHS = $(y - 3)^2 = y^2 - 6y + 9$
RHS = $y^2 - 9$
LHS ≠ RHS
Correct statement would be
$(y - 3)^2 = y^2 - 6y + 9$

12. Find and correct the errors in the given mathematical statement: $(z + 5)^2 = z^2 + 25$

Sol. Taking L.H.S. =(z+5)2 =z2+2×z×5+(5)2
$= z^2 + 10z + 25$ [∵(a+b)2=a2+2ab+b2]
Hence the correct statement is
(z+5)2 = $z^2 + 10z + 25$.

13. Find and correct the error in the mathematical statement: $(2a + 3b)(a - b) = 2a^2 - 3b^2$

Sol. $(2a + 3b)(a - b) = 2a(a - b) + 3b(a - b)$
$= 2a^2 - 2ab + 3ba - 3b^2$
$= 2a^2 + ab - 3b^2$

14. Find and correct the error in the mathematical statement: $(a + 4)(a + 2) = a^2 + 8$.

Sol. $(a + 4)(a + 2) = a(a + 2) + 4(a + 2)$
$= a^2 + 2a + 4a + 8$
$= a^2 + 6a + 8$

15. Find and correct the error in the mathematical statement: $(a - 4)(a - 2) = a^2 - 8$

Sol. $(a - 4)(a - 2) = a(a - 2) - 4(a - 2)$
$= a^2 - 2a - 4a + 8$
$= a^2 - 6a + 8$

16. Find and correct the error in the mathematical statement: 3x23x2=0

Sol. 3x23x2=1

17. Find and correct the error in the mathematical statement: 3x2+13x2=1+1=2

Sol. $3x2+13x2=3x23x2+13x2$
$=1+13x2$
$=1+13(-3)2$
$=2827$

18. Find and correct the error in the mathematical statement: $3x3x+2=12$

Sol. We have $3x3x+2=12$
L.H.S. = $3x3x+2\neq 12$
so, L.H.S. $\neq$ R.H.S
Hence the correct statement is $3x3x+2 = 3x3x+2$

19. Find and correct the error in the mathematical statement: $34x+3=14x$.

Sol. $34x+3=3x4x+3$

20. Find and correct the errors in the given mathematical statements: $4x+54x=5$

Sol. Taking L.H.S.
$4x+54x=4x4x+54x=1+54x$
Hence, R.H.S. term is incorrect
Therefore, R.H.S. will be $1+54x$
Therefore, the correct mathematical statement is: $4x+54x=1+54x$

21. Find and correct the errors in the given mathematical statement: $7x+55=7x$

Sol. $7x+55=7x$
Taking L.H.S.
$7x+55=7x5+55=7x5+1$
Hence. R.H.S. term is incorrect
Therefore, R.H.S. will be $7x5+1$
Therefore, the correct mathematical statement is: $7x+55=7x5+1$