



Notes
Chapter – 12
Exponents and Powers

- **Numbers with exponents obey the following laws of exponents.**

(a) $a^m \times a^n = a^{m+n}$

(b) $a^m \div a^n = a^{m-n}$

(c) $(a^m)^n = a^{mn}$

(d) $a^m \times b^m = (ab)^m$

(e) $a^0 = 1$

(f) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

- Very small numbers can be expressed in standard form using negative exponents.
- Use of Exponents to Express Small Number in Standard form:
 - (i) Very large and very small numbers can be expressed in standard form.
 - (ii) Standard form is also called scientific notation form.
 - (iii) A number written as $m \times 10^n$ is said to be in standard form if m is a decimal number such that $1 \leq m < 10$ and n is either a positive or a negative integer.
- Examples: $150,000,000,000 = 1.5 \times 10^{11}$.
- Exponential notation is a powerful way to express repeated multiplication of the same number. For any non-zero rational number 'a' and a natural number n, the product $a \times a \times a \times \dots \times a$ (n times) $= a^n$. It is known as the nth power of 'a' and is read as '**a raised to the power n**'. The rational number a is called the base and n is called exponent.

CHAPTER - 12
Exponents and Powers

Ex. 12.1**I. Evaluate:**

(i) 3^{-2} (ii) $(-4)^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-5}$

Ans. (i) $3^{-2} = \frac{1}{3^2}$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= \frac{1}{9}$$

(ii) $(-4)^{-2} = \frac{1}{(-4)^2}$

$$\left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= \frac{1}{16}$$

(iii) $\left(\frac{1}{2}\right)^{-5} = \left(\frac{2}{1}\right)^5$

$$\left[\because (a^m)^n = \frac{1}{a^m} \right]$$

$$= 32$$

2. Simplify and express the result in power notation with positive exponent:

(i) $(-4)^5 \div (-4)^8$

(ii) $\left(\frac{1}{2^3}\right)^2$

(iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$

(iv) $(3^{-7} \div 3^{-10}) \times 3^{-5}$

(v) $2^{-3} \times (-7)^{-3}$

Ans. (i) $(-4)^5 \div (-4)^8 = (-4)^{5-8} \left[\because a^m \div a^n = a^{m-n} \right]$

$= (-4)^{-3} = \frac{1}{(-4)^3} \left[\because a^{-m} = \frac{1}{a^m} \right]$

(ii) $\left(\frac{1}{2^3}\right)^2 = \frac{1^2}{(2^3)^2}$

$\left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \right]$

$= \left[\because (a^m)^n = a^{m \times n} \right]$

(iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4 = (-3)^4 \times \frac{5^4}{3^4} \left[\because \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \right]$

$\left[\because (ab)^m = a^m b^m \right]$

$$= 3^{4+4} \times 5^4 \left[\because a^m \div a^n = a^{m-n} \right]$$

$$= 3^0 \times 5^4 = 5^4 \left[\because a^0 = 1 \right]$$

$$\text{(iv)} \quad (3^{-7} \div 3^{-10}) \times 3^{-5} = 3^{-7-(-10)} \times 3^{-5} \left[\because a^m \div a^n = a^{m-n} \right]$$

$$= 3^{-7+10} \times 3^{-5} = 3^3 \times 3^{-5} = 3^{3+(-5)} \left[\because a^m \times a^n = a^{m+n} \right]$$

$$= 3^{-2} = \frac{1}{3^2} \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$\text{(v)} \quad 2^{-3} \times (-7)^{-3} = \frac{1}{2^3} \times \frac{1}{(-7)^3} \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= \frac{1}{\{2 \times (-7)\}^3} = \frac{1}{(-14)^3} \left[\because (ab)^m = a^m b^m \right]$$

3. Find the value of:

$$\text{(i)} \quad (3^0 + 4^{-1}) \times 2^2$$

$$\text{(ii)} \quad (2^{-1} \times 4^{-1}) \div 2^{-2}$$

$$\text{(iii)} \quad \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$$

$$\text{(iv)} \quad (3^{-1} + 4^{-1} + 5^{-1})^0$$

$$\text{(v)} \quad \left\{ \left(\frac{-2}{3}\right)^{-2} \right\}^2$$

Ans.

$$\left(1 + \frac{1}{4}\right) \times 2^2 \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$(i) (3^0 + 4^{-1}) \times 2^2 =$$

$$= \left(\frac{4+1}{4}\right) \times 2^2 = \frac{5}{4} \times 2^2 = \frac{5}{2^2} \times 2^2 = 5 \times 2^{2-2} \quad [\because a^m \div a^n = a^{m-n}]$$

$$= 5 \times 2^0 = 5 \times 1 = 5 \quad [\because a^0 = 1]$$

$$(ii) (2^{-1} \times 4^{-1}) \div 2^{-2} = \left(\frac{1}{2^1} \times \frac{1}{4^1}\right) \div 2^{-2} \quad [\because a^{-m} = \frac{1}{a^m}]$$

$$= \left(\frac{1}{2} \times \frac{1}{2^2}\right) \div 2^{-2} = \frac{1}{2^3} \div 2^{-2} \quad [\because a^m \times a^n = a^{m+n}]$$

$$= 2^{-3} \div 2^{-2} = 2^{-3-(-2)} = 2^{-3+2} = 2^{-1} \quad [\because a^m \div a^n = a^{m-n}]$$

$$= \frac{1}{2} \quad [\because a^{-m} = \frac{1}{a^m}]$$

$$(iii) \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$$

$$= (2^{-1})^{-2} + (3^{-1})^{-2} + (4^{-1})^{-2}$$

$$\left[\because a^{-m} = \frac{1}{a^m}\right]$$

$$= 2^{-1 \times (-2)} + 3^{-1 \times (-2)} + 4^{-1 \times (-2)} \quad [\because (a^m)^n = a^{m \times n}]$$

$$= 2^2 + 3^2 + 4^2 = 4 + 9 + 16 = 29$$

$$(iv) (3^{-1} + 4^{-1} + 5^{-1})^0 = \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right)^0 \quad [\because a^{-m} = \frac{1}{a^m}]$$
$$\left(\frac{20+15+12}{60}\right)^0 = \left(\frac{47}{60}\right)^0 = 1$$

=

$$[\because a^0 = 1]$$

$$(v) \left\{ \left(\frac{-2}{3} \right)^{-2} \right\}^2 = \left(\frac{-2}{3} \right)^{-2 \times 2} \quad [\because (a^m)^n = a^{m \times n}]$$

$$= \left(\frac{-2}{3} \right)^{-4} = \left(\frac{-3}{2} \right)^4 \quad [\because a^{-m} = \frac{1}{a^m}]$$

$$= \frac{81}{16}$$

4. Evaluate:

$$(i) \frac{8^{-1} \times 5^3}{2^{-4}} \quad (ii) (5^{-1} \times 2^{-1}) \times 6^{-1}$$

$$\text{Ans. (i)} \quad \frac{8^{-1} \times 5^3}{2^{-4}} = \frac{(2^3)^{-1} \times 5^3}{2^{-4}} = \frac{2^{-3} \times 5^3}{2^{-4}} \quad [\because (a^m)^n = a^{m \times n}]$$
$$= 2^{-3 - (-4)} \times 5^3 = 2^{-3+4} \times 5^3 \quad [\because a^m \div a^n = a^{m-n}]$$
$$= 2 \times 125 = 250$$

$$(ii) (5^{-1} \times 2^{-1}) \times 6^{-1} = \left(\frac{1}{5} \times \frac{1}{2} \right) \times \frac{1}{6} \quad [\because a^{-m} = \frac{1}{a^m}]$$
$$= \frac{1}{10} \times \frac{1}{6} = \frac{1}{60}$$

5. Find the value of m for which $5^m \div 5^{-3} = 5^5$. Ans.

$$5^m \div 5^{-3} = 5^5$$

$$\Rightarrow 5^{m-(-3)} = 5^5$$

$$[\because a^m \div a^n = a^{m-n}]$$

$$\Rightarrow 5^{m+3} = 5^5$$

Comparing exponents both sides, we get

$$\Rightarrow m+3=5$$

$$\Rightarrow m=5-3$$

$$\Rightarrow m=2$$

1. Evaluate:

$$(i) \left\{ \left(\frac{1}{3} \right)^{-1} - \left(\frac{1}{4} \right)^{-1} \right\}^{-1} \quad (ii) \left(\frac{5}{8} \right)^{-7} \times \left(\frac{8}{5} \right)^{-4}$$

Ans.

$$(i) \left\{ \left(\frac{1}{3} \right)^{-1} - \left(\frac{1}{4} \right)^{-1} \right\} = \left\{ \left(\frac{3}{1} \right)^1 - \left(\frac{4}{1} \right)^1 \right\} \quad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= \{3-4\} = -1$$

$$(ii) \left(\frac{5}{8} \right)^{-7} \times \left(\frac{8}{5} \right)^{-4} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-4}}{5^{-4}} \quad \left[\because \left(\frac{a}{b} \right)^m = \frac{a^m}{b^m} \right]$$

$$= 5^{-7-(-4)} \times 8^{-4-(-7)} \quad \left[\because a^m \div a^n = a^{m-n} \right]$$

$$= 5^{-7+4} \times 8^{-4+7} = 5^{-3} \times 8^3 = \frac{8^3}{5^3} \quad \left[\because a^{-m} = \frac{1}{a^m} \right]$$

$$= \frac{512}{125}$$

6. Simplify:

$$(i) \frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} \quad (t \neq 0)$$

$$(ii) \frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$$

$$\begin{aligned} \text{Ans. (i)} \quad & \frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} \\ &= \frac{5^2 \times t^{-4}}{5^{-3} \times 5 \times 2 \times t^{-8}} \\ &= \frac{5^{2-(-3)-1} \times t^{-4-(-8)}}{2} \end{aligned}$$

$$= \frac{5^{2+3-1} \times t^{-4+8}}{2} = \frac{5^4 \times t^4}{2} = \frac{625}{2} t^4$$

$$(ii) \frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$$

$$= \frac{3^{-5} \times (2 \times 5)^{-5} \times 5^3}{5^{-7} \times (2 \times 3)^{-5}}$$

$$= \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}}$$

=

$$\frac{3^{-5} \times 2^{-5} \times [5^{-5+3} (ab)^5 \times 2^{-5} \times 5^3]^2}{5^{-7} \times 2^{-5} \times 3^{-5}} \left[\because a^m \times a^n = a^{m+n} \right]$$

$$= 3^{-5-(-5)} \times 2^{-5-(-5)} \times 5^{-2-(-7)} \quad [\because a^m \div a^n = a^{m-n}]$$

$$= 3^{-5+5} \times 2^{-5+5} \times 5^{-2+7} = 3^0 \times 2^0 \times 5^5$$

$$= 1 \times 1 \times 3125 \quad [\because a^0 = 1]$$

$$= 3125$$

Exponents and Powers

Ex. 12.2

1. Express the following numbers in standard form:

(i) 0.0000000000085

(ii) 0.00000000000942

(iii) 6020000000000000

(iv) 0.00000000837

(v) 31860000000

Ans. (i) 0.0000000000085

$$= 0.0000000000085 \times \frac{10^{12}}{10^{12}} = 8.5 \times 10^{-12}$$

(ii) 0.00000000000942

$$= 0.00000000000942 \times \frac{10^{12}}{10^{12}} = 9.42 \times 10^{-12}$$

(iii) 6020000000000000

$$= 6020000000000000 \times \frac{10^{15}}{10^{15}} = 6.02 \times 10^{15}$$

(iv) 0.00000000837

$$= 0.00000000837 \times \frac{10^9}{10^9} = 8.37 \times 10^{-9}$$

(v) 31860000000

$$= 31860000000 \times \frac{10^{10}}{10^{10}} = 3.186 \times 10^{10}$$

2. Express the following numbers in usual form:

(i) 3.02×10^{-5}

(ii) 4.5×10^4

(iii) 3×10^{-8}

(iv) 1.0001×10^9

(v) 5.8×10^{12}

(vi) 3.61492×10^5

Ans. (i) $3.02 \times 10^{-5} = 302/100 \times 1 / 100000 = 302/10000000 = 0.0000302$.

(ii) $4.5 \times 10^4 = 4.5 \times 10000 = 45000$

(iii) $3 \times 10^{-8} = \frac{3}{10^8} = 0.00000003$

(iv) $1.0001 \times 10^9 = 1000100000$

$$(v) 5.8 \times 10^{12} = 5.8 \times 1000000000000$$

$$= 5800000000000$$

$$(vi) 3.61492 \times 10^5 = 3.61492 \times 1000000$$

$$= 3614920$$

3. Express the number appearing in the following statements in standard form:

(i) 1 micron is equal to $\frac{1}{1000000}$ m.

(ii) Charge of an electron is 0.000,000,000,000,000,16 coulomb.

(iii) Size of a bacteria is 0.0000005 m.

(iv) Size of a plant cell is 0.00001275 m.

(v) Thickness of a thick paper is 0.07 mm.

Ans. (i) 1 micron

$$= \frac{1}{1000000} = \frac{1}{10^6} = 1 \times 10^{-6} \text{ m}$$

(ii) Charge of an electron is 0.0000000000000000016 coulombs.

$$= 0.0000000000000000016 \times \frac{10^{19}}{10^{19}}$$

$$= 1.6 \times 10^{-19} \text{ coulomb}$$

(iii) Size of bacteria = 0.0000005

$$\frac{5}{10000000} = \frac{5}{10^7} = 5 \times 10^{-7} \text{ m}$$

(iv) Size of a plant cell is 0.00001275 m

$$= 0.00001275 \times \frac{10^5}{10^5} = 1.275 \times 10^{-5} \text{ m}$$

(v) Thickness of a thick paper = 0.07 mm

$$\frac{7}{100} = \frac{7}{10^2} = 7 \times 10^{-2}$$

= mm = mm

4. In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Ans. Thickness of one book = 20 mm Thickness

of 5 books = $20 \times 5 = 100$ mm Thickness of one

paper = 0.016 mm Thickness of 5 papers = 0.016

$\times 5$

= 0.08 mm

Total thickness of a stack = $100 + 0.08$

= 100.08 mm

= $100.08 \times \frac{10^2}{10^2}$

= 1.0008×10^2 mm

-----THE END-----