



PUNA
INTERNATIONAL
SCHOOL

- **CLASS – 10**
- **SUBJECT - MATHS**
- **CHAPTER - 5**

SAMPLE
NOTE-BOOK

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Chapter - 5

Arithmetic Progressions

Exercise 5.1

1. In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

- (i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.
- (ii) The amount of air present in a cylinder when a vacuum pump removes one fourth of the air remaining in the cylinder at a time.
- (iii) The cost of digging a well after every meter of digging, when it costs Rs 150 for the first meter and rises by Rs 50 for each subsequent meter.
- (iv) The amount of money in the account every year, when Rs 10,000 is deposited at compound Interest at 8% per annum.

Ans.

(i) Taxi fare for 1st km = Rs 15, Taxi fare after 2 km = $15 + 8 = \text{Rs } 23$

Taxi fare after 3 km = $23 + 8 = \text{Rs } 31$

Taxi fare after 4 km = $31 + 8 = \text{Rs } 39$

Therefore, the sequence is 15, 23, 31, 39...

It is an arithmetic progression because difference between any two consecutive terms is equal which is 8. ($23 - 15 = 8$, $31 - 23 = 8$, $39 - 31 = 8$, ...)

(ii) Let amount of air initially present in a cylinder = V

Amount of air left after pumping out air by vacuum pump =

$$V - \frac{V}{4} = \frac{4V - V}{4} = \frac{3V}{4}$$

Amount of air left when vacuum pump again pumps out air

$$= \frac{3}{4}V - \left(\frac{1}{4} \times \frac{3}{4}V\right) = \frac{3}{4}V - \frac{3}{16}V = \frac{12V - 3V}{16} = \frac{9}{16}V$$

So, the sequence we get is like $V, \frac{3}{4}V, \frac{9}{16}V, \dots$

Checking for difference between consecutive terms ...

$$\frac{3}{4}V - V = -\frac{V}{4}, \quad \frac{9}{16}V - \frac{3}{4}V = \frac{9V - 12V}{16} = \frac{-3V}{16}$$

Difference between consecutive terms is not equal.

Therefore, it is not an arithmetic progression.

(iii) Cost of digging 1 meter of well = Rs 150

Cost of digging 2 meters of well = $150 + 50 = \text{Rs } 200$

Cost of digging 3 meters of well = $200 + 50 = \text{Rs } 250$

Therefore, we get a sequence of the form 150, 200, 250...

It is an arithmetic progression because difference between any two consecutive terms is equal. ($200 - 150 = 250 - 200 = 50 \dots$)

Here, difference between any two consecutive terms which is also called common difference is equal to 50.

(iv) Amount in bank after 1st year = $10000 \left(1 + \frac{8}{100}\right) \dots (1)$

Amount in bank after two years = $10000 \left(1 + \frac{8}{100}\right)^2 \dots (2)$

Amount in bank after three years = $10000 \left(1 + \frac{8}{100}\right)^3 \dots (3)$

Amount in bank after four years = $10000\left(1 + \frac{8}{100}\right)^4 \dots (4)$

It is not an arithmetic progression because $(2) - (1) \neq (3) - (2)$

(Difference between consecutive terms is not equal) Therefore, it is not an Arithmetic Progression.

2. Write first four terms of the AP, when the first term a and common difference d are given as follows:

(i) $a = 10, d = 10$

(ii) $a = -2, d = 0$

(iii) $a = 4, d = -3$

(iv) $a = -1, d = \frac{1}{2}$

(v) $a = -1.25, d = -0.25$

Ans. (i) First term = $a = 10, d = 10$

Second term = $a + d = 10 + 10 = 20$

Third term = second term + $d = 20 + 10 = 30$

Fourth term = third term + $d = 30 + 10 = 40$

Therefore, first four terms are: 10, 20, 30, 40

(ii) First term = $a = -2, d = 0$ Second

$$\text{term} = a + d = -2 + 0 = -2$$

Third term = second term + $d = -2 + 0 = -2$

Fourth term = third term + $d = -2 + 0 = -2$

Therefore, first four terms are: $-2, -2, -2, -2$

(iii) First term = $a = 4$, $d = -3$

Second term = $a + d = 4 - 3 = 1$

Third term = second term + $d = 1 - 3 = -2$

Fourth term = third term + $d = -2 - 3 = -5$

Therefore, first four terms are: $4, 1, -2, -5$

(a) First term = $a = -1$, $d = \frac{1}{2}$

Second term = $a + d = -1 + \frac{1}{2} = -\frac{1}{2}$

Third term = second term + $d = -\frac{1}{2} + \frac{1}{2} = 0$

Fourth term = third term + $d = 0 + \frac{1}{2} = \frac{1}{2}$

Therefore, first four terms are: $-1, -\frac{1}{2}, 0, \frac{1}{2}$

5. First term = $a = -1.25$, $d = -0.25$ Second

term = $a + d = -1.25 - 0.25 = -1.50$

Third term = second term + $d = -1.50 - 0.25 = -1.75$

Fourth term = third term + d

$= -1.75 - 0.25 = -2.00$

Therefore, first four terms are: $-1.25, -1.50, -1.75, -2.00$

3. For the following APs, write the first term and the common difference.

(i) 3, 1, -1, -3 ...

(ii) -5, -1, 3, 7...

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$

(iv) 0.6, 1.7, 2.8, 3.9 ...

Ans. (i) 3, 1, -1, -3...

First term = $a = 3$,

Common difference (d) = Second term – first term = Third term – second term and so on Therefore,

Common difference (d) = $1 - 3 = -2$

(ii) -5, -1, 3, 7...

First term = $a = -5$

Common difference (d) = Second term – First term = Third term – Second term and so on

Therefore, Common difference (d) = $-1 - (-5) = -1 + 5 = 4$

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \dots$

First term = $a = \frac{1}{3}$

Common difference (d) = Second term – First term

(iv) Third term – Second term and so on

Therefore, Common difference (d) = $\frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

8. 0.6, 1.7, 2.8, 3.9...

First term = $a = 0.6$

Common difference (d) = Second term – First term

= Third term – Second term and so on

Therefore, Common difference (d) = $1.7 - 0.6 = 1.1$

4. Which of the following are APs? If they form an AP, find the common difference d and write three more terms.

(i) 2, 4, 8, 16...

(ii) $\frac{5}{2}, 3, \frac{7}{2}$

(iii) -1.2, -3.2, -5.2, -7.2...

(iv) -10, -6, -2, 2...

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}$...

(vi) 0.2, 0.22, 0.222, 0.2222...

(vii) 0, -4, -8, -12...

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}$...

(IX) 1, 3, 9, 27...

(X) $a, 2a, 3a, 4a$...

(XI) a, a^2, a^3, a^4 ...

(XII) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$...

(XIII) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$...

(xiv) $1^2, 3^2, 5^2, 7^2$...

(xv) $1^2, 5^2, 7^2, 73...$

Ans. (i) 2, 4, 8, 16...

It is not an AP because difference between consecutive terms is not equal. As $4 - 2 \neq 8$

- 4

(II) 2, 3, $\frac{5}{2}$... $\frac{7}{2}$

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow \frac{5}{2} - 2 = 3 - \frac{5}{2} = \frac{1}{2}$$

Common difference (d) = $\frac{1}{2}$

Fifth term = $\frac{7}{2} + \frac{1}{2} = 4$ Sixth term = $4 + \frac{1}{2} = \frac{9}{2}$

Seventh term = $\frac{9}{2} + \frac{1}{2} = 5$

Therefore, next three terms are 4, $\frac{9}{2}$ and 5.

(iii) -1.2, -3.2, -5.2, -7.2...

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow -3.2 - (-1.2)$$

$$-5.2 - (-3.2)$$

$$-7.2 - (-5.2) = -2$$

Common difference (d) = -2

Fifth term = $-7.2 - 2 = -9.2$ Sixth term = $-9.2 - 2 = -11.2$

Seventh term = $-11.2 - 2 = -13.2$

Therefore, next three terms are -9.2, -11.2 and -13.2

(iv) -10, -6, -2, 2...

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow -6 - (-10) = -2 - (-6)$$

$$= -6 + 10 = -2 + 6 = 4$$

Common difference (d) = 4

Fifth term = $2 + 4 = 6$ Sixth term = $6 + 4 = 10$

Seventh term = $10 + 4 = 14$

Therefore, next three terms are 6, 10 and 14

(v) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \dots$

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow 3 + \sqrt{2} - 3$$

$$= \sqrt{2}, 3 + 2\sqrt{2} - (3 + \sqrt{2})$$

$$= 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$$

Common difference (d) = $\sqrt{2}$

Fifth term = $3 + 3\sqrt{2} + \sqrt{2} = 3 + 4\sqrt{2}$

Sixth term = $3 + 4\sqrt{2} + \sqrt{2} = 3 + 5\sqrt{2}$

Seventh term = $3 + 5\sqrt{2} + \sqrt{2} = 3 + 6\sqrt{2}$

Therefore, next three terms are $(3+4\sqrt{2}), (3+5\sqrt{2}), (3+6\sqrt{2})$

(vi) 0.2, 0.22, 0.222, 0.2222...

It is not an AP because difference between consecutive terms is not equal. $\Rightarrow 0.22 -$

$$0.2 \neq 0.222 - 0.22$$

(vii) 0, -4, -8, -12...

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow -4 - 0 = -8 - (-4)$$

$$= -12 - (-8) = -4$$

Common difference (d) = -4

Fifth term = $-12 - 4 = -16$ Sixth term = $-16 - 4 = -20$

Seventh term = $-20 - 4 = -24$

Therefore, next three terms are -16, -20 and -24

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \dots$

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow -\frac{1}{2} - \left(-\frac{1}{2}\right) = -\frac{1}{2} - \left(-\frac{1}{2}\right) = 0$$

Common difference (d) = 0

Fifth term = $-\frac{1}{2} + 0 = -\frac{1}{2}$ Sixth term = $-\frac{1}{2} + 0 = -\frac{1}{2}$

Seventh term = $-\frac{1}{2} + 0 = -\frac{1}{2}$

Therefore, next three terms are $-\frac{1}{2}, -\frac{1}{2}$ and $-\frac{1}{2}$

(ix) 1, 3, 9, 27...

It is not an AP because difference between consecutive terms is not equal.

$$\Rightarrow 3-1 \neq 9-3$$

(x) $a, 2a, 3a, 4a...$

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow 2a - a = 3a - 2a = 4a - 3a = a$$

Common difference (d) = a

Fifth term = $4a + a = 5a$ Sixth term = $5a + a = 6a$

Seventh term = $6a + a = 7a$

Therefore, next three terms are $5a, 6a$ and $7a$

(ix) $a, a^2, a^3, a^4...$

It is not an AP because difference between consecutive terms is not equal.

$$\Rightarrow a^2 - a \neq a^3 - a^2$$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}...$

$$\Rightarrow \sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}$$

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow 2\sqrt{2} - \sqrt{2} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$$

Common difference (d) = $\sqrt{2}$

Fifth term = $4\sqrt{2} + \sqrt{2} = 5\sqrt{2}$ Sixth term = $5\sqrt{2} + \sqrt{2} = 6\sqrt{2}$

$$\text{Seventh term} = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2}$$

Therefore, next three terms are $5\sqrt{2}, 6\sqrt{2}, 7\sqrt{2}$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \dots$

It is not an AP because difference between consecutive terms is not equal.

$$\Rightarrow \sqrt{6} - \sqrt{3} \neq \sqrt{9} - \sqrt{6}$$

(xiv) $1^2, 3^2, 5^2, 7^2, \dots$

It is not an AP because difference between consecutive terms is not equal.

$$\Rightarrow 3^2 - 1^2 \neq 5^2 - 3^2$$

(xv) $1^2, 5^2, 7^2, 73, \dots$

$$\Rightarrow 1, 25, 49, 73, \dots$$

It is an AP because difference between consecutive terms is equal.

$$\Rightarrow 5^2 - 1^2$$

$$= 7^2 - 5^2 = 73 - 49 = 24$$

Common difference (d) = 24

$$\text{Fifth term} = 73 + 24 = 97 \quad \text{Sixth term} = 97 + 24 = 121$$

$$\text{Seventh term} = 121 + 24 = 145$$

Therefore, next three terms are 97, 121 and 145

Chapter - 5
Arithmetic Progressions
Exercise 5.2

= Find the missing variable from a, d, n and a_n , where a is the first term, d is the common difference and a_n is the nth term of AP.

= $a = 7, d = 3, n = 8$

= $a = -18, n = 10, a_n = 0$

= $d = -3, n = 18, a_n = -5$

= $a = -18.9, d = 2.5, a_n = 3.6$

= $a = 3.5, d = 0, n = 105$

Ans. (i) $a = 7, d = 3, n = 8$

We need to find a_n here.

Using formula $a_n = a + (n - 1)d$

Putting values of a, d and n,

$$= 7 + (8 - 1)3$$

$$= 7 + (7)3 = 7 + 21 = 28$$

12 $a = -18, n = 10, a_n = 0$

We need to find d here.

Using formula $a_n = a + (n - 1)d$

Putting values of a, a_n and n,

$$0 = -18 + (10 - 1)d$$

$$180 = -18 + 9d$$

$$198 = 9d \Rightarrow d = 2$$

$$(iv) \quad d = -3, n = 18, a_n = -5$$

We need to find a here.

Using formula $a_n = a + (n - 1)d$

Putting values of d, a_n and n,

$$-5 = a + (18 - 1)(-3)$$

$$(ii) \quad -5 = a + (17)(-3)$$

$$(iii) \quad -5 = a - 51 \Rightarrow a = 46$$

$$(b) \quad a = -18.9, d = 2.5, a_n = 3.6$$

We need to find n here.

Using formula $a_n = a + (n - 1)d$

Putting values of d, a_n and n,

$$3.6 = -18.9 + (n - 1)(2.5)$$

$$6. \quad 3.6 = -18.9 + 2.5n - 2.5$$

$$7. \quad 2.5n = 25 \Rightarrow n = 10$$

$$7. \quad a = 3.5, d = 0, n = 105$$

We need to find a_n here.

Using formula $a_n = a + (n - 1)d$

Putting values of d, n and a,

$$a_n = 3.5 + (105 - 1)(0)$$

$$\Rightarrow a_n = 3.5 + 104 \times 0$$

$$\Rightarrow a_n = 3.5 + 0 \Rightarrow a_n = 3.5$$

2. Choose the correct choice in the following and justify:

(iii) 30th term of the AP: 10, 7, 4... is

(A) 97

(B) 77

(C) -77

(D) -87

(iv) 11th term of the AP: -3, -12, 2... is

(A) 28

(B) 22

(C) -38

(D) $-48\frac{1}{2}$

Ans.(i) 10, 7, 4...

First term = a = 10, Common difference = d = 7 - 10 = 4 - 7 = -3

And n = 30 {Because, we need to find 30th term}

$$a_n = a + (n - 1)d$$

$$(ii) \quad a_{30} = 10 + (30-1)(-3) = 10 - 87 = -77$$

Therefore, the answer is (C).

$$(v) \quad -3, -\frac{1}{2}, 2, \dots$$

$$\text{First term} = a = -3, \text{ Common difference} = d = -\frac{1}{2} - (-3) = 2 - \left(-\frac{1}{2}\right) = \frac{5}{2}$$

And $n = 11$ (Because, we need to find 11th term)

$$a_n = -3 + (11-1) \frac{5}{2} = -3 + 25 = 22$$

Therefore 11th term is 22 which means answer is (B).

9. In the following AP's find the missing terms:

$$(i) \quad 2, _, 26 \quad (ii)$$

$$_, 13, _, 3$$

$$9. \quad 5, _, _, 9\frac{1}{2}$$

$$10. \quad -4, _, _, _, 6$$

$$11. \quad _, 38, _, _, _, -22$$

$$\text{Ans. (i) } 2, _, 26$$

We know that difference between consecutive terms is equal in any A.P. Let the missing term be x .

$$x - 2 = 26 - x$$

$$(i) \quad 2x = 28 \Rightarrow x = 14$$

Therefore, missing term is 14.

(ii) __, 13, __, 3

Let missing terms be x and y .

The sequence becomes $x, 13, y, 3$

We know that difference between consecutive terms is constant in any A.P. $y - 13 = 3 -$

y

(ii) $2y = 16 \Rightarrow y =$

8 And $13 - x = y -$

13

(iii) $x + y = 26$

But, we have $y = 8,$

10. $x + 8 = 26 \Rightarrow x = 18$

Therefore, missing terms are 18 and 8.

(ii) 5, __, __, $9\frac{1}{2}$

Here, first term = $a = 5$ And, 4th term = $a_4 = 9\frac{1}{2}$

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$$a_4 = 5 + (4 - 1)d$$

11. $\frac{19}{2} = 5 + 3d$

12. $19 = 2(5 + 3d)$

13. $19 = 10 + 6d$

14. $6d = 19 - 10$

$$12. \quad 6d = 9 \Rightarrow d = \frac{3}{2}$$

Therefore, we get common difference = $d = \frac{3}{2}$

$$\text{Second term} = a + d = 5 + \frac{3}{2} = \frac{13}{2}$$

$$\text{Third term} = \text{second term} + d = \frac{13}{2} + \frac{3}{2} = \frac{16}{2} = 8$$

Therefore, missing terms are $\frac{13}{2}$ and 8

(iv) 4. __, __, __, __, 6

Here, First term = $a = -4$ and 6th term = $a_6 = 6$

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$$a_6 = -4 + (6 - 1)d$$

$$13. \quad 6 = -4 + 5d$$

$$14. \quad 5d = 10 \Rightarrow d = 2$$

Therefore, common difference = $d = 2$

$$\text{Second term} = \text{first term} + d = a + d = -4 + 2 = -2$$

$$\text{Third term} = \text{second term} + d = -2 + 2 = 0$$

$$\text{Fourth term} = \text{third term} + d = 0 + 2 = 2$$

$$\text{Fifth term} = \text{fourth term} + d = 2 + 2 = 4$$

Therefore, missing terms are -2, 0, 2 and 4.

14. __, 38, __, __, __, -22

We are given 2nd and 6th term.

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression, $a_2 = a + (2 - 1)d$ and $a_6 = a + (6 - 1)d$

$$15. \quad 38 = a + d \text{ and } -22 = a + 5d$$

These are equations in two variables, we can solve them using any method.

Using equation ($38 = a + d$), we can say that $a = 38 - d$.

Putting value of a in equation ($-22 = a + 5d$),

$$-22 = 38 - d + 5d$$

$$16. \quad 4d = -60$$

$$17. \quad d = -15$$

Using this value of d and putting this in equation $38 = a + d$, $38 = a - 15 \Rightarrow a = 53$

Therefore, we get $a = 53$ and $d = -15$

First term = $a = 53$

Third term = second term + $d = 38 - 15 = 23$

Fourth term = third term + $d = 23 - 15 = 8$

Fifth term = fourth term + $d = 8 - 15 = -7$

Therefore, missing terms are 53, 23, 8 and -7.

4. Which term of the AP: 3, 8, 13, 18 ... is 78?

Ans. First term = $a = 3$, Common difference = $d = 8 - 3 = 13 - 8 = 5$ and $a_n = 78$

Using formula

$$a_n = 3 + (n - 1) 5,$$

$$78 = 3 + (n-1) 5$$

$$75 = 5n - 5$$

$$80 = 5n \Rightarrow n = 16$$

It means 16th term of the given AP is equal to 78.

(A) Find the number of terms in each of the following APs:

(i) 7, 13, 19..., 205

(ii) 18, $15\frac{1}{2}$, 13..., -47

Ans. (i) 7, 13, 19 ..., 205

First term = $a = 7$, Common difference = $d = 13 - 7 = 19 - 13 = 6$ And

$$a_n = 205$$

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression, $205 = 7 +$

$$(n - 1) 6 = 7 + 6n - 6$$

$$(i) 205 = 6n + 1$$

$$(ii) 204 = 6n \Rightarrow n = 34$$

Therefore, there are 34 terms in the given arithmetic progression.

(ii) 18, $15\frac{1}{2}$, 13 ..., -47

First term = $a = 18$, Common difference = $d = 15\frac{1}{2} - 18 = \frac{31}{2} - 18 = \frac{31 - 36}{2} = \frac{-5}{2}$

And $a_n = -47$

Using formula $a_n = a + (n - 1)d$, to find n th term of arithmetic progression,

$$-47 = 18 + (n - 1) \left(-\frac{5}{2} \right)$$

$$= 36 - \frac{5}{2}n + \frac{5}{2}$$

(i) $-94 = 36 - 5n + 5$

(ii) $5n = 135 \Rightarrow n = 27$

Therefore, there are 27 terms in the given arithmetic progression.

6. Check whether -150 is a term of the AP: 11, 8, 5, 2...

Ans. Let -150 is the n^{th} of AP 11, 8, 5, 2... which means that $a_n = -150$ Here,

First term = $a = 11$, Common difference = $d = 8 - 11 = -3$

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$$-150 = 11 + (n - 1)(-3)$$

13. $-150 = 11 - 3n + 3$

14. $3n = 164 \Rightarrow n = \frac{164}{3}$

But, n cannot be in fraction.

Therefore, our supposition is wrong. -150 cannot be term in AP.

(i) Find the 31st term of an AP whose 11th term is 38 and 16th term is 73. Ans. Here

$$a_{11} = 38 \text{ and } a_{16} = 73$$

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

$$38 = a + (11 - 1)(d) \text{ And } 73 = a + (16 - 1)(d)$$

$$(ii) \quad 38 = a + 10d \text{ And } 73 = a + 15d$$

These are equations consisting of two variables.

$$\text{We have, } 38 = a + 10d$$

$$(iii) \quad a = 38 - 10d$$

Let us put value of a in equation ($73 = a + 15d$),

$$73 = 38 - 10d + 15d$$

$$14. \quad 35 = 5d$$

Therefore, Common difference = $d = 7$

Putting value of d in equation $38 = a + 10d$,

$$38 = a + 70$$

$$(i) \quad a = -32$$

Therefore, common difference = $d = 7$ and First term = $a = -32$

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

$$a_{31} = -32 + (31 - 1)(7)$$

$$= -32 + 210 = 178$$

Therefore, 31st term of AP is 178.

8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.

Ans. An AP consists of 50 terms and the 50th term is equal to 106 and $a_3 = 12$

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression, $a_{50} = a$

+ $(50 - 1)d$ And $a_3 = a + (3 - 1)d$

(ii) $106 = a + 49d$ And $12 = a + 2d$

$12 = 106 - 49d + 2d$

(iii) $47d = 94 \Rightarrow d = 2$

Putting value of d in the equation, $a = 106 - 49d$, $a =$

$106 - 49(2) = 106 - 98 = 8$

Therefore, First term = $a = 8$ and Common difference = $d = 2$

To find 29th term, we use formula $a_n = a + (n - 1)d$ which is used to find nth term of arithmetic progression,

$a_{29} = 8 + (29 - 1)2 = 8 + 56 = 64$

Therefore, 29th term of AP is equal to 64.

(iv) If the third and the ninth terms of an AP are 4 and -8 respectively, which term of this AP is zero?

Ans. It is given that 3rd and 9th term of AP are 4 and -8 respectively.

It means $a_3 = 4$ and $a_9 = -8$

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

$4 = a + (3 - 1)d$ And, $-8 = a + (9 - 1)d$

(v) $4 = a + 2d$ and $-8 = a + 8d$

These are equations in two variables.

Using equation $4 = a + 2d$, we can say that $a = 4 - 2d$

Putting value of a in other equation $-8 = a + 8d$, $-8 = 4 - 2d$

$$+ 8d$$

$$(vi) \quad -12 = 6d \Rightarrow d = -2$$

Putting value of d in equation $-8 = a + 8d$,

$$-8 = a + 8(-2)$$

$$15. \quad -8 = a - 16 \Rightarrow a = 8$$

Therefore, first term = $a = 8$ and Common Difference = $d = -2$ We

want to know which term is equal to zero.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$$0 = 8 + (n - 1)(-2)$$

$$0 = 8 - 2n + 2$$

$$0 = 10 - 2n$$

$$2n = 10 \Rightarrow n = 5$$

Therefore, 5^{th} term is equal to 0.

10. The 17th term of an AP exceeds its 10th term by 7. Find the common difference.

Ans. $a_{17} = a_{10} + 7 \dots (1)$

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$$a_{17} = a + 16d \dots (2)$$

$$a_{10} = a + 9d \dots (3)$$

Putting (2) and (3) in equation (1),

$$(ii) \quad + 16d = a + 9d + 7$$

$$(a) \quad 7d = 7$$

$$(b) \quad d = 1$$

11. Which term of the AP: 3, 15, 27, 39... will be 132 more than its 54th term?

Ans. Lets first calculate 54th of the given AP.

First term = $a = 3$, Common difference = $d = 15 - 3 = 12$

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression, $a_{54} = a + (54 - 1)d = 3 + 53(12) = 3 + 636 = 639$

We want to find which term is 132 more than its 54th term.

Let us suppose it is n^{th} term which is 132 more than 54th term.

$$a_n = a_{54} + 132$$

(b) $3 + (n - 1)12 = 639 + 132$

(c) $3 + 12n - 12 = 771$

(d) $12n - 9 = 771$

16. $12n = 780$

17. $n = 65$

Therefore, 65th term is 132 more than its 54th term.

12. Two AP's have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms.

Ans. Let first term of 1st AP = a

Let first term of 2nd AP = a'

It is given that their common difference is same.

Let their common difference be d .

It is given that difference between their 100th terms is 100.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

17. $+ (100 - 1)d - [a' + (100 -$

1) $d] = a + 99d - a' - 99d = 100$

$$(ii) \quad a - a' = 100 \dots (1)$$

We want to find difference between their 1000th terms which means we want to calculate:

$$(ii) \quad + (1000 - 1) d - [a' + (1000 - 1) d]$$
$$1) d] = a + 999d - a' - 999d = a - a'$$

Putting equation (1) in the above equation,

$$(iii) \quad + (1000 - 1) d - [a' + (1000 - 1) d]$$
$$(iv) \quad = a + 999d - a' + 999d = a - a' = 100$$

Therefore, difference between their 1000th terms would be equal to 100.

13. How many three digit numbers are divisible by 7?

Ans. We have AP starting from 105 because it is the first three digit number divisible by 7.

AP will end at 994 because it is the last three digit number divisible by 7.

Therefore, we have AP of the form 105, 112, 119..., 994

Let 994 is the nth term of AP.

We need to find n here.

First term = a = 105, Common difference = d = 112 - 105 = 7

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

$$994 = 105 + (n - 1)(7)$$

$$18. \quad 994 = 105 + 7n - 7$$

$$19. \quad 896 = 7n \Rightarrow n = 128$$

It means 994 is the 128th term of AP.

Therefore, there are 128 terms in AP.

(i) How many multiples of 4 lie between 10 and 250?

Ans. First multiple of 4 which lie between 10 and 250 is 12. The last

multiple of 4 which lie between 10 and 250 is 248. Therefore, AP is

of the form 12, 16, 20... ,248

First term = $a = 12$, Common difference = $d = 4$

Using formula $a_n = a + (n - 1) d$, to find n^{th} term of arithmetic progression,

$$248 = 12 + (n - 1) (4)$$

$$(ii) 248 = 12 + 4n - 4$$

$$(iii) 240 = 4n$$

$$(iv) n = 60$$

It means that 248 is the 60th term of AP.

So, we can say that there are 60 multiples of 4 which lie between 10 and 250.

(iii) For what value of n , are the n^{th} terms of two AP's: 63, 65, 67... and 3, 10, 17... equal? Ans. Lets

first consider AP 63, 65, 67...

First term = $a = 63$, Common difference = $d = 65 - 63 = 2$

Using formula $a_n = a + (n - 1) d$, to find n^{th} term of arithmetic progression,

$$a_n = 63 + (n - 1) (2) \dots (1)$$

Now, consider second AP 3, 10, 17...

First term = $a = 3$, Common difference = $d = 10 - 3 = 7$

Using formula $a_n = a + (n - 1) d$, to find n^{th} term of arithmetic progression,

$$a_n = 3 + (n - 1) (7) \dots (2)$$

According to the given condition:

$$(1) = (2)$$

$$19. 63 + (n - 1) (2) = 3 + (n - 1) (7)$$

$$20. 63 + 2n - 2 = 3 + 7n - 7$$

$$(i) 65 = 5n \Rightarrow n = 13$$

Therefore, 13th terms of both the AP's are equal.

20. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.

Ans. Let first term of AP = a

Let common difference of AP = d

It is given that its 3rd term is equal to 16.

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression,

$$16 = a + (3 - 1)(d)$$

$$(i) 16 = a + 2d \dots (1)$$

It is also given that 7th term exceeds 5th term by 12.

According to the given condition:

$$a_7 = a_5 + 12$$

$$(i) a + (7 - 1)d = a + (5 - 1)d + 12$$

$$(ii) 2d = 12 \Rightarrow d = 6$$

Putting value of d in equation $16 = a + 2d$,

$$16 = a + 2(6) \Rightarrow a = 4$$

Therefore, first term = a = 4

And, common difference = d = 6

Therefore, AP is 4, 10, 16, 22...

17. Find the 20th term from the last term of the AP: 3, 8, 13... , 253.

Ans. We want to find 20th term from the last term of given AP.

So, let us write given AP in this way: 253 ... 13, 8, 3

Here First term = $a = 253$, Common Difference = $d = 8 - 13 = -5$

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$$a_{20} = 253 + (20 - 1)(-5)$$

$$(ii) \quad a_{20} = 253 + 19(-5) = 253 - 95 = 158$$

Therefore, the 20th term from the last term of given AP is 158.

- (i) The sum of the 4th and 8th terms of an AP is 24 and the sum of 6th and 10th terms is 44.
(ii) Find the three terms of the AP.

Ans. The sum of 4th and 8th terms of an AP is 24 and sum of 6th and 10th terms is 44.

$$a_4 + a_8 = 24$$

$$\text{and } a_6 + a_{10} = 44$$

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression, $\Rightarrow a + (4 - 1)d + [a + (8 - 1)d] = 24$

$$\text{And, } a + (6 - 1)d + [a + (10 - 1)d] = 44$$

$$(i) \quad a + 3d + a + 7d = 24$$

$$\text{And } a + 5d + a + 9d = 44$$

$$(ii) \quad 2a + 10d = 24 \quad \text{And } 2a + 14d = 44$$

$$(iii) \quad a + 5d = 12 \quad \text{And } a + 7d = 22$$

These are equations in two variables.

Using equation, $a + 5d = 12$, we can say that $a = 12 - 5d...$ (1)

Putting (1) in equation $a + 7d = 22$,

$$12 - 5d + 7d = 22$$

$$23. \quad 12 + 2d = 22$$

$$24. \quad 2d = 10$$

$$25. \quad d = 5$$

Putting value of d in equation $a = 12 - 5d$,

$$a = 12 - 5(5) = 12 - 25 = -13$$

Therefore, first term = $a = -13$ and, Common difference = $d = 5$

Therefore, AP is $-13, -8, -3, 2...$

Its first three terms are $-13, -8$ and -3 .

24. Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Ans. Subba Rao's starting salary = Rs 5000

It means, first term = $a = 5000$

He gets an increment of Rs 200 after every year.

Therefore, common difference = $d = 200$

His salary after 1 year = $5000 + 200 = \text{Rs } 5200$

His salary after two years = $5200 + 200 = \text{Rs } 5400$

Therefore, it is an AP of the form $5000, 5200, 5400, 5600... , 7000$

We want to know in which year his income reaches Rs 7000.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression, $7000 =$

$$5000 + (n - 1)(200)$$

(ii) $7000 = 5000 + 200n - 200$

(iii) $7000 - 5000 + 200 = 200n$

(iv) $2200 = 200n$

(v) $n = 11$

It means after 11 years, Subba Rao's income would be Rs 7000.

= **Ramkali saved Rs. 5 in the first week of a year and then increased her weekly savings by Rs. 1.75.**

If in the n^{th} week, her weekly savings become Rs 20.75, find n .

Ans. Ramkali saved Rs. 5 in the first week of year. It means first term = $a = 5$

Ramkali increased her weekly savings by Rs 1.75.

Therefore, common difference = $d = \text{Rs } 1.75$

Money saved by Ramkali in the second week = $a + d = 5 + 1.75 = \text{Rs } 6.75$

Money saved by Ramkali in the third week = $6.75 + 1.75 = \text{Rs } 8.5$

Therefore, it is an AP of the form: 5, 6.75, 8.5 ... , 20.75

We want to know in which week her savings become 20.75.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression, $20.75 = 5 + (n - 1)(1.75)$

25. $20.75 = 5 + 1.75n - 1.75$

26. $17.5 = 1.75n$

27. $n = 10$

It means in the 10^{th} week her savings become Rs 20.75

Chapter - 5
Arithmetic Progressions
Exercise 5.3

1. Find the sum of the following AP's.

(i) 2, 7, 12... to 10 terms

(ii) -37, -33, -29... to 12 terms (iii)

0.6, 1.7, 2.8... to 100 terms

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$ to 11 terms

Ans. (i) 2, 7, 12... to 10 terms

Here First term = $a = 2$, Common difference = $d = 7 - 2 = 5$ and $n = 10$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$S_n = \frac{10}{2}[4 + (10-1)5] = 5(4 + 45) = 5 \times 49 = 245$$

= -37, -33, -29... to 12 terms

Here First term = $a = -37$, Common difference = $d = -33 - (-37) = 4$ And n

= 12

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP

$$S_n = \frac{12}{2}[-74 + (12-1)4] = 6(-74 + 44) = 6 \times (-30) = -180$$

13 0.6, 1.7, 2.8... to 100 terms

Here First term = $a = 0.6$, Common difference = $d = 1.7 - 0.6 = 1.1$ And $n =$

100

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$S_n = \frac{100}{2} [1.2 + (100 - 1) 1.1] = 50 (1.2 + 108.9) = 50 \times 110.1 = 5505$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}, \dots$ to 11 terms

Here First term = $a = \frac{1}{15}$ Common difference = $d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$S_n = \frac{11}{2} \left[\frac{2}{15} + (11-1) \frac{1}{60} \right] = \frac{11}{2} \left(\frac{2}{15} + \frac{1}{6} \right) = \frac{11}{2} \left(\frac{4+5}{30} \right) = \frac{11}{2} \times \frac{9}{30} = \frac{33}{20}$$

2. Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 + \dots + 84$

(ii) $34 + 32 + 30 + \dots + 10$

(i) $-5 + (-8) + (-11) + \dots + (-230)$

Ans. (i) $7 + 10\frac{1}{2} + 14 + \dots + 84$

Here First term = $a = 7$, Common difference = $d = \frac{21}{2} - 7 = \frac{21-14}{2} = \frac{7}{2} = 3.5$

And Last term = $l = 84$

We do not know how many terms are there in the given AP.

So, we need to find n first.

Using formula $a_n = a + (n-1)d$ to find n^{th} term of arithmetic progression,

$$= 7 + (3.5)n - 3.5 = 84$$

$$= 3.5n = 84 + 3.5 - 7$$

$$= 3.5n = 80.5$$

$$n = 23$$

Therefore, there are 23 terms in the given AP.

It means $n = 23$.

Applying formula, $S_n = \frac{n}{2}(a+l)$ to find sum of n terms of AP,

$$S_{23} = \frac{23}{2}(7 + 84)$$

$$\Rightarrow S_{23} = \frac{23}{2} \times 91 = 1046.5$$

$$(ii) 34 + 32 + 30 + \dots + 10$$

Here First term = $a = 34$, Common difference = $d = 32 - 34 = -2$

And Last term = $l = 10$

We do not know how many terms are there in the given AP.

So, we need to find n first.

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$$[34 + (n - 1)(-2)] = 10$$

$$1434 - 2n + 2 = 10$$

$$-2n = -26 \Rightarrow n = 13$$

Therefore, there are 13 terms in the given AP.

It means $n = 13$.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP,

$$S_{13} = \frac{13}{2}(34 + 10) = \frac{13}{2} \times 44 = 286$$

$$(iii) \quad -5 + (-8) + (-11) + \dots + (-230)$$

Here First term = $a = -5$, Common difference = $d = -8 - (-5) = -8 + 5 = -3$ And

Last term = $l = -230$

We do not know how many terms are there in the given AP.

So, we need to find n first.

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$$[-5 + (n - 1)(-3)] = -230$$

$$-5 - 3n + 3 = -230$$

$$-3n = -228 \Rightarrow n = 76$$

Therefore, there are 76 terms in the given AP.

It means $n = 76$.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP,

$$S_{76} = \frac{76}{2}(-5 - 230) = 38 \times (-235) = -8930$$

3. In an AP

(iii) given $a = 5, d = 3, a_n = 50$, find n and S_n .

(iv) given $a = 7, a_{13} = 35$, find d and S_{13} .

(v) given $a_{12} = 37, d = 3$, find a and S_{12} .

(vi) given $a_3 = 15, S_{10} = 125$, find d and a_{10} .

(vii) given $d = 5, S_9 = 75$, find a and a_9 .

(viii) given $a = 2, d = 8, S_n = 90$, find n and a_n .

(ix) given $a = 8, a_n = 62, S_n = 210$, find n and d .

(x) given $a_n = 4, d = 2, S_n = -14$, find n and a .

(xi) given $a = 3, n = 8, S = 192$, find d .

(xii) given $l = 28, S = 144$, and there are total of 9 terms. Find a .

Ans. (i) Given $a = 5, d = 3, a_n = 50$, find n and S_n .

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$$a_n = 5 + (n - 1)(3)$$

$$50 = 5 + 3n - 3$$

$$48 = 3n \Rightarrow n = 16$$

Applying formula, $S_n = \frac{n}{2}[2a + (n - 1)d]$ to find sum of n terms of AP,

$$S_{16} = \frac{16}{2}[10 + (16-1)3] = 8(10 + 45) = 8 \times 55 = 440$$

$$S_n = 440$$

Therefore, $n = 16$ and

(ii) Given $a = 7$, $a_{13} = 35$, find d and S_{13} .

Using formula $a_n = a + (n-1)d$, to find n th term of arithmetic progression,

$$a_{13} = 7 + (13-1)(d)$$

$$35 = 7 + 12d$$

$$28 = 12d \Rightarrow d = \frac{7}{3}$$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$S_{13} = \frac{13}{2}\left[14 + (13-1)\frac{7}{3}\right] = \frac{13}{2}(14 + 28) = \frac{13}{2} \times 42 = 273$$

Therefore, $d = \frac{7}{3}$ and $S_{13} = 273$

(iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12} .

Using formula $a_n = a + (n-1)d$, to find n th term of arithmetic progression,

$$a_{12} = a + (12-1)3$$

$$37 = a + 33 \Rightarrow a = 4$$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$S_{12} = \frac{12}{2}[8 + (12-1)3] = 6(8 + 33) = 6 \times 41 = 246$$

Therefore, $a = 4$ and $S_{12} = 246$

(iv) Given $a_3 = 15, S_{10} = 125$, find d and a_{10} .

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$$a_3 = a + (3 - 1)(d)$$

$$15 = a + 2d$$

$$a = 15 - 2d \dots (1)$$

Applying formula, $a_n = a + (n - 1)d$ to find sum of n terms of AP,

$$S_{10} = \frac{10}{2} [2a + (10 - 1)d]$$

$$125 = 5(2a + 9d) = 10a + 45d$$

Putting (1) in the above equation,

$$125 = 5[2(15 - 2d) + 9d] = 5(30 - 4d + 9d)$$

$$125 = 150 + 25d$$

$$125 - 150 = 25d$$

$$-25 = 25d \Rightarrow d = -1$$

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression, $a_{10} = a + (10 - 1)d$

Putting value of d and equation (1) in the above equation, $a_{10} = 15$

$$-2d + 9d = 15 + 7d$$

$$= 15 + 7(-1) = 15 - 7 = 8$$

Therefore, $d = -1$ and $a_{10} = 8$

(v) Given $d = 5$, $S_9 = 75$, find a and a_9 .

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$S_9 = \frac{9}{2}[2a + (9-1)5]$$

$$\Rightarrow 75 = \frac{9}{2}[2a + 40]$$

$$150 = 18a + 360$$

$$-210 = 18a$$

$$a = \frac{-35}{3}$$

Using formula $a_n = a + (n-1)d$, to find n^{th} term of arithmetic progression,

$$a_9 = \frac{-35}{3} + (9-1)(5)$$

$$= \frac{-35}{3} + 40 = \frac{-35 + 120}{3} = \frac{85}{3}$$

$$\text{Therefore, } a = \frac{-35}{3} \text{ and } a_9 = \frac{85}{3}$$

(vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n .

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$90 = \frac{n}{2}[4 + (n-1)8]$$

$$\Rightarrow 90 = \frac{n}{2}[4 + 8n - 8]$$

$$\Rightarrow 90 = \frac{n}{2}[8n - 4]$$

$$\Rightarrow 8n^2 - 4n - 180 = 0$$

$$\Rightarrow 2n^2 - n + 45 = 0$$

$$\Rightarrow 2n^2 - 10n + 9n - 45 = 0$$

$$2n(n - 5) + 9(n - 5) = 0$$

$$(n - 5)(2n + 9) = 0$$

$$n = 5, -9/2$$

We discard negative value of n because here n cannot be in negative or fraction.

The value of n must be a positive integer.

Therefore, $n = 5$

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,
 $a_5 = 2 + (5 - 1)(8) = 2 + 32 = 34$

Therefore, $n = 5$ and $a_n = 34$

(vii) Given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d .

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$$62 = 8 + (n - 1)(d) = 8 + nd - d$$

$$62 = 8 + nd - d$$

$$nd - d = 54$$

$$nd = 54 + d \dots (1)$$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$210 = \frac{n}{2}[16 + (n-1)d] = \frac{n}{2}(16 + nd - d)$$

Putting equation (1) in the above equation,

$$210 = \frac{n}{2}[16 + 54 + d - d] = \frac{n}{2} \times 70$$

$$\Rightarrow n = \frac{210 \times 2}{70} = 6 \Rightarrow n = 6$$

Putting value of n in equation (1),

$$6d = 54 + d \Rightarrow d = \frac{54}{5}$$

Therefore, $n = 6$ and $d = \frac{54}{5}$

(viii) Given $a_n = 4, d = 2, S_n = -14$, find n and a.

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression,

$$4 = a + (n-1)(2) = a + 2n - 2$$

$$4 = a + 2n - 2$$

$$6 = a + 2n$$

$$a = 6 - 2n \dots (1)$$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP,

$$-14 = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(2a + 2n - 2)$$

$$\Rightarrow -14 = \frac{n}{2}(2a + 2n - 2)$$

Putting equation (1) in the above equation, we get $-28 = n$

$$[2(6 - 2n) + 2n - 2]$$

$$(ii) \quad -28 = n(12 - 4n + 2n - 2)$$

$$(iii) \quad -28 = n(10 - 2n)$$

$$\Rightarrow 2n^2 - 10n - 28 = 0$$

$$\Rightarrow n^2 - 5n - 14 = 0$$

$$\Rightarrow n^2 - 7n + 2n - 14 = 0$$

$$(iii) \quad n(n - 7) + 2(n - 7) = 0$$

$$(iv) \quad (n + 2)(n - 7) = 0$$

$$(v) \quad n = -2, 7$$

Here, we cannot have negative value of n .

Therefore, we discard negative value of n which means $n = 7$.

Putting value of n in equation (1), we get

$$a = 6 - 2n = 6 - 2(7) = 6 - 14 = -8$$

Therefore, $n = 7$ and $a = -8$

(ix) Given $a = 3$, $n = 8$, $S = 192$, find d .

Using formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$\frac{8}{2}$$

$$192 = [6 + (8 - 1) d] = 4 (6 + 7d)$$

$$(iv) \quad 192 = 24 + 28d$$

$$(v) \quad 168 = 28d \Rightarrow d = 6$$

(x) Given $l = 28$, $S = 144$, and there are total of 9 terms. Find a .

Applying formula, $S_n = \frac{n}{2} [a + l]$, to find sum of n terms, we get

$$144 = \frac{9}{2} [a + 28]$$

$$(ii) \quad 288 = 9 [a + 28]$$

$$(iii) \quad 32 = a + 28 \Rightarrow a = 4$$

4. How many terms of the AP: 9, 17, 25, ... must be taken to give a sum of 636? Ans. First

term = $a = 9$, Common difference = $d = 17 - 9 = 8$, $S_n = 636$

Applying formula, $S_n = \frac{n}{2} [2a + (n - 1)d]$ to find sum of n terms of AP, we get

$$636 = \frac{n}{2} [18 + (n - 1) (8)]$$

$$(i) \quad 1272 = n (18 + 8n - 8)$$

$$\Rightarrow 1272 = 18n + 8n^2 - 8n$$

$$\Rightarrow 8n^2 + 10n - 1272 = 0$$

$$\Rightarrow 4n^2 + 5n - 636 = 0$$

Comparing equation $4n^2 + 5n - 636 = 0$ with general form $an^2 + bn + c = 0$, we get

$$a = 4, b = 5 \text{ and } c = -636$$

Applying quadratic formula, $n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and putting values of a, b and c, we get

$$n = \frac{-5 \pm \sqrt{5^2 - 4(4)(-636)}}{8}$$

$$\Rightarrow n = \frac{-5 \pm \sqrt{25 + 10176}}{8}$$

$$\Rightarrow n = \frac{-5 \pm 101}{8}$$

$$\Rightarrow n = \frac{96}{8}, \frac{-106}{8} = 12, -\frac{106}{8}$$

We discard negative value of n here because n cannot be in negative, n can only be a positive integer.

Therefore, $n = 12$

Therefore, 12 terms of the given sequence make sum equal to 636.

11. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Ans. First term = $a = 5$, Last term = $l = 45$, $S_n = 400$

Applying formula, $S_n = \frac{n}{2}[a + l]$ to find sum of n terms of AP, we get

$$400 = \frac{n}{2}[5 + 45]$$

$$\Rightarrow \frac{400}{50} = \frac{n}{2} \Rightarrow n = 16$$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP and putting value of

n, we get

$$400 = \frac{16}{2}[10 + (16-1)d]$$

(iv) $400 = 8(10 + 15d)$

(v) $400 = 80 + 120d$

(vi) $320 = 120d$

(vii) $\frac{320}{120} = \frac{8}{3}$

12. The first and the last terms of an AP are 17 and 350 respectively. If, the common difference is 9, how many terms are there and what is their sum?

Ans. First term = $a = 17$, Last term = $l = 350$ and Common difference = $d = 9$

Using formula $a_n = a + (n-1)d$, to find nth term of arithmetic progression, we get $350 = 17 + (n-1)(9)$

(vi) $350 = 17 + 9n - 9$

(vii) $342 = 9n \Rightarrow n = 38$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP and putting value of n, we get

$$S_{38} = \frac{38}{2}[34 + (38-1)9]$$

$$S_{38} = 19(34 + 333) = 6973$$

Therefore, there are 38 terms and their sum is equal to 6973.

13. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

Ans. It is given that 22nd term is equal to 149 $\Rightarrow a_{22} = 149$

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression, we get

$$149 = a + (22 - 1)(7)$$

$$149 = a + 147 \Rightarrow a = 2$$

Applying formula, $S_n = \frac{n}{2}[2a + (n - 1)d]$ to find sum of n terms of AP and putting value of a, we get

$$S_{22} = \frac{22}{2}[4 + (22 - 1)7]$$

$$S_{22} = 11(4 + 147)$$

$$S_{22} = 1661$$

Therefore, sum of first 22 terms of AP is equal to 1661.

14. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Ans. It is given that second and third term of AP are 14 and 18 respectively.

Using formula $a_n = a + (n - 1)d$, to find nth term of arithmetic progression, we get $14 = a + (2 - 1)d$

$$(c) \quad 14 = a + d \dots (1)$$

And, $18 = a + (3 - 1)d$

$$(d) \quad 18 = a + 2d \dots (2)$$

These are equations consisting of two variables.

Using equation (1), we get, $a = 14 - d$

Putting value of a in equation (2), we get

$$18 = 14 - d + 2d$$

(e) $d = 4$

Therefore, common difference $d = 4$

Putting value of d in equation (1), we get

$$18 = a + 2(4)$$

18. $a = 10$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_{51} = \frac{51}{2}[20 + (51-1)d] = \frac{51}{2}(20 + 200) = \frac{51}{2} \times 220 = 51 \times 110 = 5610$$

Therefore, sum of first 51 terms of an AP is equal to 5610.

15. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Ans. It is given that sum of first 7 terms of an AP is equal to 49 and sum of first 17 terms is equal to 289.

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$49 = \frac{7}{2}[2a + (7-1)d]$$

(iii) $98 = 7(2a + 6d)$

(iv) $7 = a + 3d \Rightarrow a = 7 - 3d \dots (1)$

$$\text{And, } 289 = \frac{17}{2} [2a + (17-1)d]$$

$$578 = 17(2a + 16d)$$

$$34 = 2a + 16d$$

$$17 = a + 8d$$

Putting equation (1) in the above equation, we get

$$17d = 7 - 3d + 8d$$

$$10 = 5d \Rightarrow d = 2$$

Putting value of d in equation (1), we get

$$a = 7 - 3d = 7 - 3(2) = 7 - 6 = 1$$

Again applying formula, $S_n = \frac{n}{2} [2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_n = \frac{n}{2} [2(1) + (n-1)2]$$

$$\Rightarrow S_n = \frac{n}{2} [2 + 2n - 2]$$

$$\Rightarrow S_n = \frac{n}{2} \times 2n \Rightarrow S_n = n^2$$

Therefore, sum of n terms of AP is equal to n^2 .

16. Show that a_1, a_2, \dots, a_n form an AP where a_n is defined as below:

(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Ans. (i) We need to show that a_1, a_2, \dots, a_n form an AP where $a_n = 3 + 4n$

Let us calculate values of a_1, a_2, a_3, \dots using

$$a_n = 3 + 4n \quad a_1 = 3 + 4(1) = 3 + 4 = 7$$

$$a_2 = 3 + 4(2) = 3 + 8 = 11$$

$$a_3 = 3 + 4(3) = 3 + 12 = 15$$

$$a_4 = 3 + 4(4) = 3 + 16 = 19$$

So, the sequence is of the form 7, 11, 15, 19 ...

Let us check difference between consecutive terms of this sequence.

$$11 - 7 = 4, 15 - 11 = 4, 19 - 15 = 4$$

Therefore, the difference between consecutive terms is constant which means terms a_1, a_2, \dots, a_n form an AP.

We have sequence 7, 11, 15, 19 ...

First term = $a = 7$ and Common difference = $d = 4$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_{15} = \frac{15}{2}[14 + (15-1)4] = \frac{15}{2}(14 + 56) = \frac{15}{2} \times 70 = 15 \times 35 = 525$$

Therefore, sum of first 15 terms of AP is equal to 525.

(ii) We need to show that a_1, a_2, \dots, a_n form an AP where $a_n = 9 - 5n$

Let us calculate values of a_1, a_2, a_3, \dots using $a_n = 9 - 5n$

$$a_1 = 9 - 5(1) = 9 - 5 = 4 \quad a_2 = 9 - 5(2) = 9 - 10 = -1$$

$$a_3 = 9 - 5(3) = 9 - 15 = -6 \quad a_4 = 9 - 5(4) = 9 - 20 = -11$$

So, the sequence is of the form 4, -1, -6, -11 ...

Let us check difference between consecutive terms of this sequence.

$$-1 - (4) = -5, -6 - (-1)$$

$$(v) -6 + 1 = -5, -11 - (-6)$$

$$(vi) -11 + 6 = -5$$

Therefore, the difference between consecutive terms is constant which means terms a_1, a_2, \dots, a_n form an AP.

We have sequence 4, -1, -6, -11 ...

First term = $a = 4$ and Common difference = $d = -5$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_{15} = \frac{15}{2}[8 + (15-1)(-5)] = \frac{15}{2}(8 - 70) = \frac{15}{2} \times (-62) = 15 \times (-31) = -465$$

Therefore, sum of first 15 terms of AP is equal to -465.

11. If the sum of the first n terms of an AP is $(4n - n^2)$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the n th terms.

Ans. It is given that the sum of n terms of an AP is equal to $(4n - n^2)$

It means $S_n = 4n - n^2$

Let us calculate S_1 and S_2 using $S_n = 4n - n^2$

$$S_1 = 4(1) - (1)^2 = 4 - 1 = 3$$

$$S_2 = 4(2) - (2)^2 = 8 - 4 = 4$$

First term = $a = 3 \dots (1)$

Let us find common difference now.

We can write any AP in the form of general terms like $a, a + d, a + 2d \dots$

We have calculated that sum of first two terms is equal to 4 i.e. $S_2 = 4$ Therefore,

we can say that $a + (a + d) = 4$

Putting value of a from equation (1), we get

$$2a + d = 4$$

$$(iv) 2(3) + d = 4$$

$$(v) 6 + d = 4$$

$$(vi) d = -2$$

Using formula $a_n = a + (n - 1)d$, to find n^{th} term of arithmetic progression,

$$\text{Second term of AP} = a_2 = a + (2 - 1)d = 3 + (2 - 1)(-2) = 3 - 2 = 1$$

$$\text{Third term of AP} = a_3 = a + (3 - 1)d = 3 + (3 - 1)(-2) = 3 - 4 = -1$$

$$\text{Tenth term of AP} = a_{10} = a + (10 - 1)d = 3 + (10 - 1)(-2) = 3 - 18 = -15$$

$$n^{\text{th}} \text{ term of AP} = a_n = a + (n - 1)d = 3 + (n - 1)(-2) = 3 - 2n + 2 = 5 - 2n$$

12. Find the sum of the first 40 positive integers divisible by 6.

Ans. The first 40 positive integers divisible by 6 are 6, 12, 18, 24 ... 40 terms.

Therefore, we want to find sum of 40 terms of sequence of the form:

6, 12, 18, 24 ... 40 terms

Here, first term = $a = 6$ and Common difference = $d = 12 - 6 = 6$, $n = 40$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_{40} = \frac{40}{2}[12 + (40-1)6]$$

$$= 20(12 + 39 \times 6)$$

$$= 20(12 + 234)$$

$$= 20 \times 246 = 4920$$

13. Find the sum of the first 15 multiples of 8.

Ans. The first 15 multiples of 8 are 8, 16, 24, 32 ... 15 terms

First term = $a = 8$ and Common difference = $d = 16 - 8 = 8$, $n = 15$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_{15} = \frac{15}{2}[16 + (15-1)8] = \frac{15}{2}(16 + 14 \times 8) = \frac{15}{2}(16 + 112) = \frac{15}{2} \times 128 = 15 \times 64 = 960$$

14. Find the sum of the odd numbers between 0 and 50. Ans.

The odd numbers between 0 and 50 are 1, 3, 5, 7 ... 49

It is an arithmetic progression because the difference between consecutive terms is constant. First term = $a =$

1, Common difference = $3 - 1 = 2$, Last term = $l = 49$

We do not know how many odd numbers are present between 0 and 50. Therefore,

we need to find n first.

Using formula $a_n = a + (n - 1) d$, to find nth term of arithmetic progression, we get

$$49 = 1 + (n - 1) 2$$

$$(ii) \quad 49 = 1 + 2n - 2$$

$$(iii) \quad 50 = 2n \Rightarrow n = 25$$

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP, we get

$$S_{25} = \frac{25}{2}(1 + 49) = \frac{25}{2} \times 50 = 25 \times 25 = 625$$

15. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

Ans. Penalty for first day = Rs 200, Penalty for second day = Rs 250 Penalty

for third day = Rs 300

It is given that penalty for each succeeding day is Rs 50 more than the preceding day.

It makes it an arithmetic progression because the difference between consecutive terms is constant.

We want to know how much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

So, we have an AP of the form 200, 250, 300, 350 ... 30 terms First

term = $a = 200$, Common difference = $d = 50$, $n = 30$

Applying formula, $S_n = \frac{n}{2}[2a + (n - 1)d]$ to find sum of n terms of AP, we get

$$S_n = \frac{30}{2} [400 + (30 - 1)50]$$

$$\Rightarrow S_n = 15 (400 + 29 \times 50)$$

$$S_n = 15 (400 + 1450) = 27750$$

Therefore, penalty for 30 days is Rs. 27750.

16. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If, each prize is Rs 20 less than its preceding term, find the value of each of the prizes.

Ans. It is given that sum of seven cash prizes is equal to Rs 700.

And, each prize is Rs 20 less than its preceding term.

Let value of first prize = Rs. a

Let value of second prize = Rs (a - 20)

Let value of third prize = Rs (a - 40)

So, we have sequence of the form:

a, a - 20, a - 40, a - 60 ...

It is an arithmetic progression because the difference between consecutive terms is constant. First term = a,

Common difference = d = (a - 20) - a = -20 n = 7 (Because there are total of seven prizes)

$S_7 = \text{Rs } 700$ {given}

Applying formula, $S_n = \frac{n}{2} [2a + (n - 1)d]$ to find sum of n terms of AP, we get

$$S_7 = \frac{7}{2}[2a + (7-1)(-20)]$$

$$\Rightarrow 700 = \frac{7}{2}[2a - 120]$$

(iii) $200 = 2a - 120$

(iv) $320 = 2a \Rightarrow a = 160$

Therefore, value of first prize = Rs 160

Value of second prize = $160 - 20 =$ Rs 140

Value of third prize = $140 - 20 =$ Rs 120

Value of fourth prize = $120 - 20 =$ Rs 100

Value of fifth prize = $100 - 20 =$ Rs 80

Value of sixth prize = $80 - 20 =$ Rs 60

Value of seventh prize = $60 - 20 =$ Rs 40

17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g, a section of Class I will plant 1 tree, a section of class II will plant two trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

Ans. There are three sections of each class and it is given that the number of trees planted by any class is equal to class number.

The number of trees planted by class I = number of sections $\times 1 = 3 \times 1 = 3$ The number

of trees planted by class II = number of sections $\times 2 = 3 \times 2 = 6$

The number of trees planted by class III = number of sections $\times 3 = 3 \times 3 = 9$ Therefore, we have sequence of the form 3, 6, 9 ... 12 terms

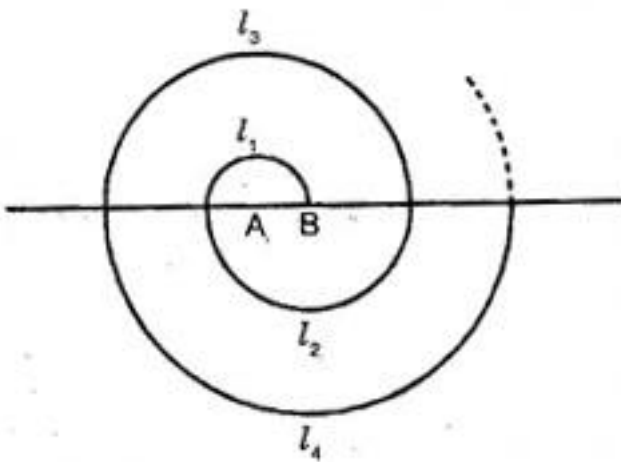
To find total number of trees planted by all the students, we need to find sum of the sequence 3, 6, 9, 12 ... 12 terms.

First term = $a = 3$, Common difference = $d = 6 - 3 = 3$ and $n = 12$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_{12} = \frac{12}{2}[6 + (12-1)3] = 6(6 + 33) = 6 \times 39 = 234$$

18. A spiral is made up of successive semicircles, with centers alternatively at A and B, starting with center at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... What is the total length of such a spiral made up of thirteen consecutive semicircles.



Ans. Length of semi-circle = $\frac{\text{Circumference of circle}}{2} = \frac{2\pi r}{2} = \pi r$

Length of semi-circle of radii 0.5 cm = $\pi (0.5)$ cm

Length of semi-circle of radii 1.0 cm = $\pi (1.0)$ cm

Length of semi-circle of radii 1.5 cm = $\pi (1.5)$ cm

Therefore, we have sequence of the form:

$\pi(0.5), \pi(1.0), \pi(1.5) \dots$ 13 terms {There are total of thirteen semi-circles}.

To find total length of the spiral, we need to find sum of the sequence $\pi(0.5), \pi(1.0), \pi(1.5) \dots$ 13 terms

Total length of spiral = $\pi(0.5) + \pi(1.0) + \pi(1.5) \dots$ 13 terms

(iii) Total length of spiral = $\pi(0.5 + 1.0 + 1.5) \dots$ 13 terms ... (1)

Sequence 0.5, 1.0, 1.5 ... 13 terms is an arithmetic progression. Let us find the sum of this sequence.

First term = $a = 0.5$, Common difference = $1.0 - 0.5 = 0.5$ and $n = 13$

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

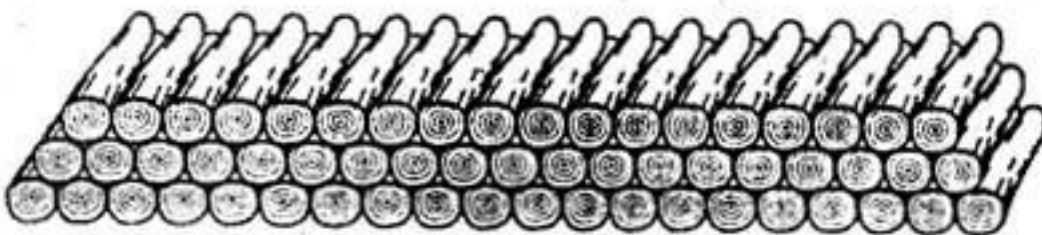
$$S_{13} = \frac{13}{2}[1 + (13-1)0.5] = 6.5(1+6) = 6.5 \times 7 = 45.5$$

Therefore, $0.5 + 1.0 + 1.5 + 2.0 \dots$ 13 terms = 45.5

Putting this in equation (1), we get

Total length of spiral = $\pi(0.5 + 1.5 + 2.0 + \dots$ 13 terms) = $\pi(45.5) = 143$ cm

19. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?



Ans. The number of logs in the bottom row = 20

The number of logs in the next row = 19

The number of logs in the next to next row = 18

Therefore, we have sequence of the form 20, 19, 18 ...

First term = $a = 20$, Common difference = $d = 19 - 20 = -1$

We need to find that how many rows make total of 200 logs.

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$$200 = \frac{n}{2}[40 + (n-1)(-1)]$$

$$26. \quad 400 = n(40 - n + 1)$$

$$27. \quad 400 = 40n - n^2 + n$$

$$\Rightarrow n^2 - 41n + 400 = 0$$

It is a quadratic equation, we can factorize to solve the equation.

$$n^2 - 25n - 16n + 400 = 0$$

$$n(n - 25) - 16(n - 25) = 0$$

$$(n - 25)(n - 16)$$

$$n = 25, 16$$

We discard $n = 25$ because we cannot have more than 20 rows in the sequence. The sequence is of the form: 20, 19, 18 ...

At most, we can have 20 or less number of rows.

Therefore, $n = 16$ which means 16 rows make total number of logs equal to 200.

We also need to find number of logs in the 16th row.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP, we get

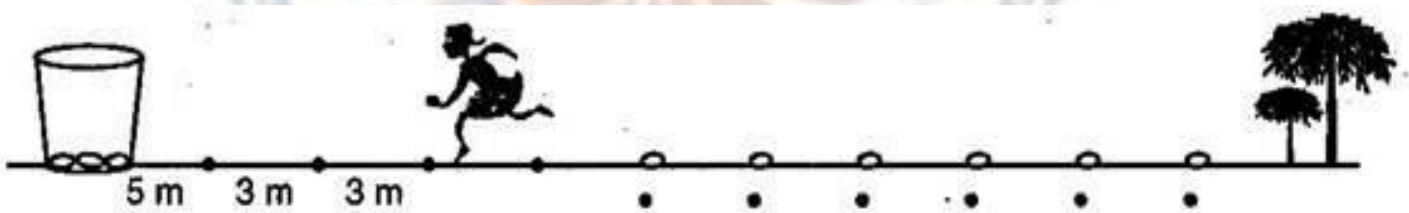
$$200 = 8(20 + l)$$

$$(vi) \quad 200 = 160 + 8l$$

$$(vii) \quad 40 = 8l \Rightarrow l = 5$$

Therefore, there are 5 logs in the top most row and there are total of 16 rows.

20. In a potato race, a bucket is placed at the starting point, which is 5 meters from the first potato, and the other potatoes are placed 3 meters apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?



Ans. The distance of first potato from the starting point = 5 meters

Therefore, the distance covered by competitor to pick up first potato and put it in bucket = $5 \times 2 = 10$ meters

The distance of Second potato from the starting point = $5 + 3 = 8$ meters {All the potatoes are 3 meters apart from each other}

Therefore, the distance covered by competitor to pick up 2nd potato and put it in bucket = $8 \times 2 = 16$ meters

The distance of third potato from the starting point = $8 + 3 = 11$ meters

Therefore, the distance covered by competitor to pick up 3rd potato and put it in bucket = $11 \times 2 = 22$ meters

Therefore, we have a sequence of the form 10, 16, 22 ... 10 terms (There are ten terms because there are ten potatoes)

To calculate the total distance covered by the competitor, we need to find:

$10 + 16 + 22 + \dots$ 10 terms

First term = $a = 10$, Common difference = $d = 16 - 10 = 6$ $n =$

10 {There are total of 10 terms in the sequence}

Applying formula, $S_n = \frac{n}{2} [2a + (n-1)d]$ to find sum of n terms of AP, we get

$$S_{n=10} = \frac{10}{2} [20 + (10-1)6] = 5(20 + 54) = 5 \times 74 = 370$$

Therefore, total distance covered by competitor is equal to 370 meters.

Chapter - 5
Arithmetic Progressions
Exercise 5.4

1. Which term of the AP: 121, 117, 113, is its first negative term? Ans.

Given: 121, 117, 113,

Here $a=121$, $d=117-121=-4$

Now, $a_n = a + (n-1)d$

$$= 121 + (n-1)(-4)$$

$$= 121 - 4n + 4 = 125 - 4n$$

For the first negative term, $a_n < 0$

$$\Rightarrow 125 - 4n < 0$$

$$\Rightarrow 125 < 4n$$

$$\Rightarrow \frac{125}{4} < n$$

$$\Rightarrow 31\frac{1}{4} < n$$

n is an integer and $n > 31\frac{1}{4}$.

Hence, the first negative term is 32nd term.

= The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of sixteen terms of the AP.

Ans. Let the AP be $a-4d, a-3d, a-2d, a-d, a, a+d, a+2d, a+3d, \dots$

Then, $a_3 = a - 2d, a_7 = a + 2d$

$$\Rightarrow a_3 + a_7 = a - 2d + a + 2d = 6$$

$$\Rightarrow 2a = 6$$

$$\Rightarrow a = 3 \dots \dots \dots (i)$$

Also $(a - 2d)(a + 2d) = 8$

$$\Rightarrow a^2 - 4d^2 = 8$$

$$\Rightarrow 4d^2 = a^2 - 8$$

$$\Rightarrow 4d^2 = 3^2 - 8$$

$$\Rightarrow 4d^2 = 1$$

$$\Rightarrow d^2 = \frac{1}{4} \Rightarrow d = \pm \frac{1}{2}$$

Taking $d = \frac{1}{2}$,

$$S_{16} = \frac{16}{2} [2 \times (a - 4d) + (16 - 1)d]$$

$$= 8 \left[2 \times \left(3 - 4 \times \frac{1}{2} \right) + 15 \times \frac{1}{2} \right]$$

$$= 8 \left[2 + \frac{15}{2} \right] = 8 \times \frac{19}{2} = 76$$

Taking $d = \frac{-1}{2}$,

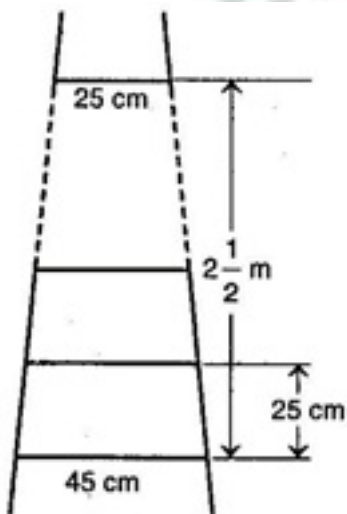
$$S_{16} = \frac{16}{2} [2 \times (a - 4d) + (16 - 1)d]$$

$$= 8 \left[2 \times \left(3 - 4 \times \frac{-1}{2} \right) + 15 \times \frac{-1}{2} \right]$$

$$= 8 \left[\frac{20 - 15}{2} \right] = 8 \times \frac{5}{2} = 20$$

$$\therefore S_{16} = 20 \text{ and } 76$$

15 A ladder has rungs 25 cm apart (see figure). The rungs decrease uniformly in length from 45 cm, at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?



Ans. Number of rungs $(n) = \frac{2\frac{1}{2} \times 100}{25} = 10$

The length of the wood required for rungs = sum of 10 rungs

$$= \frac{10}{2} [25 + 45] = 5 \times 70 = 350 \text{ cm}$$

20 The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x .

Ans. Here $a = 1$ and $d = 1$

$$\begin{aligned} \therefore S_{x-1} &= \frac{x-1}{2} [2 \times 1 + (x-1-1) \times 1] \\ &= \frac{x-1}{2} (2 + x - 2) \end{aligned}$$

$$\frac{(x-1)x}{2} = \frac{x^2 - x}{2}$$

$$\begin{aligned} S_x &= \frac{x}{2} [2 \times 1 + (x-1) \times 1] \\ &= \frac{x}{2} (x+1) = \frac{x^2 + x}{2} \end{aligned}$$

$$\begin{aligned} S_{49} &= \frac{49}{2} [2 \times 1 + (49-1) \times 1] \\ &= \frac{49}{2} (2 + 48) = 49 \times 25 \end{aligned}$$

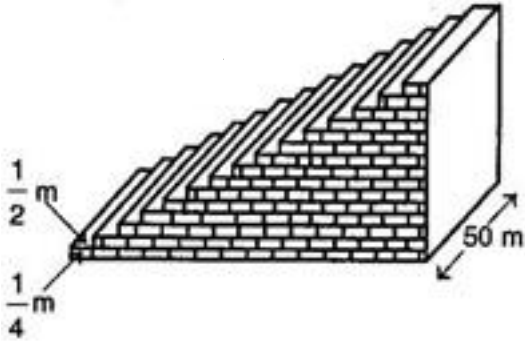
According to question,

$$\begin{aligned} S_{x-1} &= S_{49} - S_x \\ \Rightarrow \frac{x^2 - x}{2} &= 49 \times 25 - \frac{x^2 + x}{2} \\ \Rightarrow \frac{x^2 - x}{2} + \frac{x^2 + x}{2} &= 49 \times 25 \\ \Rightarrow \frac{x^2 - x + x^2 + x}{2} &= 49 \times 25 \\ \Rightarrow x^2 &= 49 \times 25 \\ \Rightarrow x &= \pm 35 \end{aligned}$$

Since, x is a counting number, so negative value will be neglected.

$$\therefore x = 35$$

(v) A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.



Each step has a rise of $\frac{1}{4}$ m and a tread of $\frac{1}{2}$ m (see figure). Calculate the total volume of concrete required to build the terrace.

Ans. Volume of concrete required to build the first step, second step, third step, (in m^2) are

$$\frac{1}{4} \times \frac{1}{2} \times 50, \left(2 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50, \left(3 \times \frac{1}{4}\right) \times \frac{1}{2} \times 50, \dots$$

$$\Rightarrow \frac{50}{8}, 2 \times \frac{50}{8}, 3 \times \frac{50}{8}, \dots$$

$$\therefore \text{Total volume of concrete required} = \frac{50}{8} + 2 \times \frac{50}{8} + 3 \times \frac{50}{8} + \dots$$

$$= \frac{50}{8} [1 + 2 + 3 + \dots]$$

$$= \frac{50}{8} \times \frac{15}{2} [2 \times 1 + (15 - 1) \times 1] [\because n = 15]$$

$$= \frac{50}{8} \times \frac{15}{2} \times 16$$

$$= 750 m^3$$