# PUNA INTERNATIONAL SCHOOL

- **CLASS – 10**
- **SUBJECT - MATHS**
- **CHAPTER - 4**

# SAMPLE NOTE-BOOK

# Chapter - 4

## Quadratic Equations
## Exercise 4.1

**1. Check whether the following are Quadratic Equations.**

= $(x+1)^2 = 2(x-3)$

= $x^2 - 2x = (-2)(3-x)$

= $(x-2)(x+1) = (x-1)(x+3)$

= $(x-3)(2x+1) = x(x+5)$

= $(2x-1)(x-3) = (x+5)(x-1)$

= $x^2 + 3x + 1 = (x-2)^2$

= $(x+2)^3 = 2x(x^2-1)$

= $x^3 - 4x^2 - x + 1 = (x-2)^3$

**Ans. (i)** $(x+1)^2 = 2(x-3)$

$\{(a+b)^2 = a^2 + 2ab + b^2\}$

$\Rightarrow x^2 + 1 + 2x = 2x - 6$

$\Rightarrow x^2 + 7 = 0$

Here, degree of equation is 2.

Therefore, it is a Quadratic Equation.

**(ii)** $x^2 - 2x = (-2)(3-x)$

$\Rightarrow x^2 - 2x = -6 + 2x$

$\Rightarrow x^2 - 2x - 2x + 6 = 0$

$\Rightarrow x^2 - 4x + 6 = 0$

Here, degree of equation is 2.

Therefore, it is a Quadratic Equation.

= $(x - 2)(x + 1) = (x - 1)(x + 3)$

$\Rightarrow x^2 + x - 2x - 2 = x^2 + 3x - x - 3 = 0$

$\Rightarrow x^2 + x - 2x - 2 - x^2 - 3x + x + 3 = 0$

$12x - 2x - 2 - 3x + x + 3 = 0$

$13 - 3x + 1 = 0$

Here, degree of equation is 1.

Therefore, it is not a Quadratic Equation.

**4** $(x - 3)(2x + 1) = x(x + 5)$

$\Rightarrow 2x^2 + x - 6x - 3 = x^2 + 5x$

$\Rightarrow 2x^2 + x - 6x - 3 - x^2 - 5x = 0$

$\Rightarrow x^2 - 10x - 3 = 0$

Here, degree of equation is 2.

Therefore, it is a quadratic equation.

**(v)** $(2x - 1)(x - 3) = (x + 5)(x - 1)$

$\Rightarrow 2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$

$\Rightarrow 2x^2 - 7x + 3 - x^2 + x - 5x + 5 = 0$

(ii) $x^2 - 11x + 8 = 0$

Here, degree of Equation is 2.

Therefore, it is a Quadratic Equation.

**(vi)** $x^2 + 3x + 1 = (x-2)^2$

$\{(a-b)^2 = a^2 - 2ab + b^2\}$

$\Rightarrow x^2 + 3x + 1 = x^2 + 4 - 4x$

$\Rightarrow x^2 + 3x + 1 - x^2 + 4x - 4 = 0$

(a) $7x - 3 = 0$

Here, degree of equation is 1.

Therefore, it is not a Quadratic Equation.

**(vii)** $(x+2)^3 = 2x(x^2-1)$

$\{(a+b)^3 = a^3 + b^3 + 3ab(a+b)\}$

$\Rightarrow x^3 + 2^3 + 3(x)(2)(x+2) = 2x(x^2-1)$

$\Rightarrow x^3 + 8 + 6x(x+2) = 2x^3 - 2x$

$\Rightarrow 2x^3 - 2x - x^3 - 8 - 6x^2 - 12x = 0$

$\Rightarrow x^3 - 6x^2 - 14x - 8 = 0$

Here, degree of Equation is 3.

Therefore, it is not a quadratic Equation.

**(viii)** $x^3 - 4x^2 - x + 1 = (x-2)^3$

$\left\{(a-b)^3 = a^3 - b^3 - 3ab(a-b)\right\}$

$\Rightarrow x^3 - 4x^2 - x + 1 = x^3 - 2^3 - 3(x)(2)(x-2)$

$\Rightarrow -4x^2 - x + 1 = -8 - 6x^2 + 12x$

$\Rightarrow 2x^2 - 13x + 9 = 0$

Here, degree of Equation is 2.

Therefore, it is a Quadratic Equation.

**2. Represent the following situations in the form of Quadratic Equations:**

1. **The area of rectangular plot is $528 m^2$. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.**

2. **The product of two consecutive numbers is 306. We need to find the integers.**

3. **Rohan's mother is 26 years older than him. The product of their ages (in years) after 3 years will be 360. We would like to find Rohan's present age.**

4. **A train travels a distance of 480 km at uniform speed. If, the speed had been 8km/h less, then it would have taken 3 hours more to cover the same distance. We need to find speed of the train.**

**Ans. (i)** We are given that area of a rectangular plot is $528 m^2$.

Let breadth of rectangular plot be x metres

Length is one more than twice its breadth.

Therefore, length of rectangular plot is $(2x + 1)$ metres

Area of rectangle = length × breadth

7. $528 = x(2x + 1)$

8. $528 = 2x^2 + x$

$\Rightarrow 2x^2 + x - 528 = 0$

This is required Quadratic Equation.

**(ii)** Let two consecutive numbers be x and (x + 1). It is

given that $x(x + 1) = 306$

$\Rightarrow x^2 + x = 306$

$\Rightarrow x^2 + x - 306 = 0$

**(iii)** Let present age of Rohan = x years

$(x + 3)(x + 29) = 360$

$\Rightarrow x^2 + 29x + 3x + 87 = 360$

$\Rightarrow x^2 + 32x - 273 = 0$

**(iv)** Let speed of train be x km/h

Time taken by train to cover 480 km = $\frac{480}{x}$ hours

If speed had been 8km/h less then time taken would be $\frac{480}{x-8}$ hours

According to given condition, if speed had been 8km/h less then time taken is 3 hours less.

Therefore, 480/x-8 $-$ 480/x $= 3$

$480(x - x + 8) = 3x(x - 8)$

$\Rightarrow 3840 = 3x^2 - 24x$

$\Rightarrow 3x^2 - 24x - 3840 = 0$

Dividing equation by 3, we get

$\Rightarrow x^2 - 8x - 1280 = 0$

This is the required Quadratic Equation.

## Chapter - 4

## Quadratic Equations
## Exercise 4.2

1. **Find the roots of the following Quadratic Equations by factorization.**

**(i)** $x^2 - 3x - 10 = 0$

**(ii)** $2x^2 + x - 6 = 0$

**(iii)** $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

**(iv)** $2x^2 - x + \frac{1}{8} = 0$

**(v)** $100x^2 - 20x + 1 = 0$

**Ans. (i)** $x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x - 5) + 2(x - 5) = 0$

$\Rightarrow (x - 5)(x + 2) = 0$

$\Rightarrow x = 5, -2$

**(ii)** $2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x + 2) - 3(x + 2) = 0$

$\Rightarrow (2x - 3)(x + 2) = 0$

$\Rightarrow x = \frac{3}{2}, -2$

**(iii)** $\sqrt{2}x^2+7x+5\sqrt{2}=0$

$\Rightarrow \sqrt{2}x^2+2x+5x+5\sqrt{2}=0$

$\Rightarrow \sqrt{2}x(x+\sqrt{2})+5(x+\sqrt{2})=0$

$\Rightarrow$

$\Rightarrow x=\frac{-5}{\sqrt{2}}, -\sqrt{2}$

$\Rightarrow x=\frac{-5}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}, -\sqrt{2}$

$\Rightarrow x=\frac{-5\sqrt{2}}{2}, -\sqrt{2}$

**(iv)** $2x^2-x+\frac{1}{8}=0$

$\Rightarrow \frac{16x^2-8x+1}{8}=0$

$\Rightarrow 16x^2-8x+1=0$

$\Rightarrow 16x^2-4x-4x+1=0$

$\Rightarrow 4x(4x-1)-1(4x-1)=0$

$\Rightarrow (4x-1)(4x-1)=0$

$\Rightarrow x=\frac{1}{4}, \frac{1}{4}$

**(v)** $100x^2-20x+1=0$

$\Rightarrow 100x^2-10x-10x+1=0$

$\Rightarrow 10x(10x-1)-1(10x-1)=0$

$\Rightarrow (10x - 1)(10x - 1) = 0$

$\Rightarrow x = \frac{1}{10}, \frac{1}{10}$

2. **Solve the following problems given:**

**(i)** $x^2 - 45x + 324 = 0$

**(ii)** $x^2 - 55x + 750 = 0$ **Ans.**

**(i)** $x^2 - 45x + 324 = 0$

$\Rightarrow x^2 - 36x - 9x + 324 = 0$

$\Rightarrow x(x - 36) - 9(x - 36) = 0$

$\Rightarrow (x - 9)(x - 36) = 0$

$\Rightarrow x = 9, 36$

**(ii)** $x^2 - 55x + 750 = 0$

$\Rightarrow x^2 - 25x - 30x + 750 = 0$

$\Rightarrow x(x - 25) - 30(x - 25) = 0$

$\Rightarrow (x - 30)(x - 25) = 0$

$\Rightarrow x = 30, 25$

3. **Find two numbers whose sum is 27 and product is 182.**

**Ans.** Let first number be $x$ and let second number be $(27 - x)$

According to given condition, the product of two numbers is 182.

Therefore,

*14* $(27 - x) = 182$

$\Rightarrow 27x - x^2 = 182$

$\Rightarrow x^2 - 27x + 182 = 0$

$\Rightarrow x^2 - 14x - 13x + 182 = 0$

$\Rightarrow x(x - 14) - 13(x - 14) = 0$

$\Rightarrow (x - 14)(x - 13) = 0$

$\Rightarrow x = 14, 13$

Therefore, the first number is equal to 14 or 13

And, second number is $= 27 - x = 27 - 14 = 13$ or Second number $= 27 - 13 = 14$

Therefore, two numbers are 13 and 14.

4. **Find two consecutive positive integers, sum of whose squares is 365. Ans.**

Let first number be $x$ and let second number be $(x + 1)$

According to given condition,

$x^2 + (x + 1)^2 = 365$

$\{(a+b)^2 = a^2 + b^2 + 2ab\}$

$\Rightarrow x^2 + x^2 + 1 + 2x = 365$

$\Rightarrow 2x^2 + 2x - 364 = 0$

Dividing equation by 2

$\Rightarrow x^2 + x - 182 = 0$

$\Rightarrow x^2 + 14x - 13x - 182 = 0$

$\Rightarrow x(x + 14) - 13(x + 14) = 0$

$(x + 14)(x - 13) = 0$

$\Rightarrow x = 13, -14$

Therefore, first number = 13 {We discard -14 because it is negative number) Second

number = $x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers are 13 and 14 whose sum of squares is equal to 365.

5. **.The altitude of right triangle is 7 cm less than its base. If, hypotenuse is 13 cm. Find the other two sides.**

**Ans.** Let base of triangle be $x$ cm and let altitude of triangle be $(x - 7)$ cm It is given

that hypotenuse of triangle is 13 cm According to Pythagoras Theorem,

$( \quad = x^2 + (x - 7)^2$ [Since, $(a+b)^2 = a^2 + b^2 + 2ab$]

$\Rightarrow 169 = x^2 + x^2 + 49 - 14x$

$\Rightarrow 169 = 2x^2 - 14x + 49$

$\Rightarrow 2x^2 - 14x - 120 = 0$

Dividing equation by 2

$\Rightarrow x^2 - 7x - 60 = 0$

$\Rightarrow x^2 - 12x + 5x - 60 = 0$

$\Rightarrow x(x - 12) + 5(x - 12) = 0$

$\Rightarrow (x - 12)(x + 5) = 0$

$\Rightarrow x = -5, 12$

We discard $x = -5$ because length of side of triangle cannot be negative.

Therefore, base of triangle = 12 cm

Altitude of triangle = $(x - 7) = 12 - 7 = 5$ cm

6. **A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If, the total cost of production on that day was Rs. 90, find the number of articles produced and the cost of each article.**

**Ans.** Let cost of production of each article be Rs x

We are given total cost of production on that particular day = Rs 90

Therefore, total number of articles produced that day = 90/x

According to the given conditions,

$$x = 2\left(\frac{90}{x}\right) + 3$$

$$\Rightarrow x = \frac{180}{x} + 3$$

$$\Rightarrow x = \frac{180 + 3x}{x}$$

$$\Rightarrow x^2 = 180 + 3x$$

$$\Rightarrow x^2 - 3x - 180 = 0$$

$$\Rightarrow x^2 - 15x + 12x - 180 = 0$$

$\Rightarrow x(x - 15) + 12(x - 15) = 0$

$\Rightarrow (x - 15)(x + 12) = 0 \Rightarrow x = 15, -12$

Cost cannot be in negative, therefore, we discard $x = -12$ Therefore, $x = Rs\ 15$

which is the cost of production of each article.

Number of articles produced on that particular day = $\frac{90}{15} = 6$

# Chapter - 4

## Quadratic Equations

## Exercise 4.3

1. **Find the roots of the following quadratic equations if they exist by the method of completing square.**

**(i)** $2x^2 - 7x + 3 = 0$

**(ii)** $2x^2 + x - 4 = 0$

**(iii)** $4x^2 + 4\sqrt{3}x + 3 = 0$

**(iv)** $2x^2 + x + 4 = 0$

**Ans. (i)** $2x^2 - 7x + 3 = 0$

First we divide equation by 2 to make coefficient of $x^2$ equal to 1,

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

We divide middle term of the equation by $2x$, we get $\frac{7}{2}x \times \frac{1}{2x} = \frac{7}{4}$

We add and subtract square of $\frac{7}{4}$ from the equation $x^2 - \frac{7}{2}x + \frac{3}{2} = 0$,

$$\Rightarrow x^2 - \frac{7}{2}x + \frac{3}{2} + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 = 0$$

$$\Rightarrow x^2 + \left(\frac{7}{4}\right)^2 - \frac{7}{2}x + \frac{3}{2} - \left(\frac{7}{4}\right)^2 = 0$$

$$\left\{(a-b)^2 = a^2 + b^2 - 2ab\right\}$$

$\Rightarrow \left(x-\frac{7}{4}\right)^2+\frac{24-49}{16}=0$

$\Rightarrow \left(x-\frac{7}{4}\right)^2=\frac{49-24}{16}$

=> $\left(x-\frac{7}{4}\right)^2=\frac{25}{16}$

Taking Square root on both sides,

$\Rightarrow x-\frac{7}{4}=\pm\frac{5}{4}$

$\Rightarrow x=\frac{5}{4}+\frac{7}{4}=\frac{12}{4}=3 \; and \; x=-\frac{5}{4}+\frac{7}{4}=\frac{2}{4}=\frac{1}{2}$

Therefore, $x=\frac{1}{2},3$

**(ii)** $2x^2+x-4=0$

Dividing equation by 2,

$x^2+\frac{x}{2}-2=0$

Following procedure of completing square,

=> $x^2+\frac{x}{2}-2+\left(\frac{1}{4}\right)^2-\left(\frac{1}{4}\right)^2=0$

$\Rightarrow x^2+\frac{x}{2}+\left(\frac{1}{4}\right)^2-2-\frac{1}{16}=0$

$\left\{(a+b)^2=a^2+b^2+2ab\right\}$

$$\left(x+\frac{1}{4}\right)^2-\frac{33}{16}=0$$

$$\Rightarrow\left(x+\frac{1}{4}\right)^2=\frac{33}{16}$$

Taking square root on both sides,

$$\Rightarrow x+\frac{1}{4}=\pm\frac{\sqrt{33}}{4}$$

$$\Rightarrow x=\frac{\sqrt{33}}{4}-\frac{1}{4}=\frac{\sqrt{33}-1}{4}\,and\;x=-\frac{\sqrt{33}}{4}-\frac{1}{4}=\frac{-\sqrt{33}-1}{4}$$

Therefore, $x=\frac{\sqrt{33}-1}{4},\frac{-\sqrt{33}-1}{4}$

**(iii)** $4x^2+4\sqrt{3}x+3=0$

Dividing equation by 4,

$$x^2+\sqrt{3}x+\frac{3}{4}=0$$

Following the procedure of completing square,

$$\Rightarrow x^2+\sqrt{3}x+\frac{3}{4}+\left(\frac{\sqrt{3}}{2}\right)^2-\left(\frac{\sqrt{3}}{2}\right)^2=0$$

$$\Rightarrow x^2+\left(\frac{\sqrt{3}}{2}\right)^2+\sqrt{3}x+\frac{3}{4}-\frac{3}{4}=0$$

$\Rightarrow \quad \left\{(a+b)^2 = a^2 + b^2 + 2ab\right\}$

$\Rightarrow \left(x+\frac{\sqrt{3}}{2}\right)\left(x+\frac{\sqrt{3}}{2}\right) = 0$

$\Rightarrow x+\frac{\sqrt{3}}{2} = 0, x+\frac{\sqrt{3}}{2} = 0$

$\Rightarrow x = -\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$

**(iv)** $2x^2 + x + 4 = 0$

Dividing equation by 2,

$x^2 + \frac{x}{2} + 2 = 0$

Following the procedure of completing square,

$\Rightarrow x^2 + \frac{x}{2} + 2 + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 = 0$

$\Rightarrow x^2 + \left(\frac{1}{4}\right)^2 + \frac{x}{2} + 2 - \left(\frac{1}{4}\right)^2 = 0$

$\left\{(a+b)^2 = a^2 + b^2 + 2ab\right\}$

$\Rightarrow \left(x+\frac{1}{4}\right)^2 + 2 - \frac{1}{16} = 0$

$\Rightarrow \left(x+\frac{1}{4}\right)^2 = \frac{1}{16} - 2 = \frac{1-32}{16} = \frac{-31}{16}$

Right hand side does not exist because square root of negative number does not exist.

Therefore, there is no solution for quadratic equation $2x^2 + x + 4 = 0$

2. **Find the roots of the following Quadratic Equations by applying quadratic formula.**

**(i)** $2x^2 - 7x + 3 = 0$

**(ii)** $2x^2 + x - 4 = 0$

**(iii)** $4x^2 + 4\sqrt{3}x + 3 = 0$

**(iv)** $2x^2 + x + 4 = 0$

**Ans. (i)**

$2x^2 - 7x + 3 = 0$

Comparing quadratic equation $2x^2 - 7x + 3 = 0$ with general form $ax^2 + bx + c = 0$, we get a = 2, b = -7 and c = 3

Putting these values in quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(3)}}{2 \times 2}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$\Rightarrow x = \frac{7 \pm 5}{4}$$

$$\Rightarrow x = \frac{7+5}{4}, \frac{7-5}{4}$$

15$x$ = 3, ½

**(ii)** $2x^2 + x - 4 = 0$

Comparing quadratic equation $2x^2 + x - 4 = 0$ with the general form $ax^2 + bx + c = 0$, we get *a* = 2, *b* = 1 and *c* = −4

Putting these values in quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(-4)}}{2 \times 2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{33}}{4}$$

$$\Rightarrow x = \frac{-1 + \sqrt{33}}{4}, \frac{-1 - \sqrt{33}}{4}$$

**(iii)** $4x^2 + 4\sqrt{3}x + 3 = 0$

Comparing quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$ with the general form $ax^2 + bx + c = 0$, we get *a* = 4, *b* = $4\sqrt{3}$ and *c* = 3

Putting these values in quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-4\sqrt{3} \pm \sqrt{(4\sqrt{3})^2 - 4(4)(3)}}{2 \times 4}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm \sqrt{0}}{8}$$

$$\Rightarrow x = \frac{-\sqrt{3}}{2}$$

A quadratic equation has two roots. Here, both the roots are equal.

Therefore, $x = \frac{-\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$

**(iv)** $2x^2 + x + 4 = 0$

Comparing quadratic equation $2x^2 + x + 4 = 0$ with the general form $ax^2 + bx + c = 0$, we get $a = 2$, $b = 1$ and $c = 4$

Putting these values in quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(2)(4)}}{2 \times 2}$$

$\Rightarrow x = \frac{-1 \pm \sqrt{-31}}{4}$

But, square root of negative number is not defined.

Therefore, Quadratic Equation $2x^2 + x + 4 = 0$ has no solution.

3. **Find the roots of the following equations:**

**(i)** $\frac{x-1}{x} = 3, x \neq 0$

**(ii)** $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

**Ans. (i)** $x - \frac{1}{x} = 3 \; where \; x \neq 0$

$\Rightarrow \frac{x^2 - 1}{x} = 3$

$\Rightarrow x^2 - 1 = 3x$

$\Rightarrow x^2 - 3x - 1 = 0$

Comparing equation $x^2 - 3x - 1 = 0$ with general form $ax^2 + bx + c = 0$, We get $a = 1$, $b = -3$ and $c = -1$

Using quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve equation,

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{13}}{2}$$

$$\Rightarrow x = \frac{3 + \sqrt{13}}{2}, \frac{3 - \sqrt{13}}{2}$$

**(ii)** $\dfrac{1}{x+4} - \dfrac{1}{x-7} = \dfrac{11}{30}$ *where* $x \neq -4, 7$

$$\Rightarrow \frac{(x-7)-(x+4)}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow \frac{-11}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow -30 = x^2 - 7x + 4x - 28$$

$$\Rightarrow x^2 - 3x + 2 = 0$$

Comparing equation $x^2 - 3x + 2 = 0$ with general form $ax^2 + bx + c = 0$,

We get $a = 1$, $b = -3$ and $c = 2$

Using quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve equation,

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(2)}}{2 \times 1}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{1}}{2}$$

$\Rightarrow x = \frac{3+\sqrt{1}}{2}, \frac{3-\sqrt{1}}{2}$

(vi) $x = 2, 1$

4. **The sum of reciprocals of Rehman's ages (in years) 3 years ago and 5 years from now is 1/3 Find his present age.**

**Ans.** Let present age of Rehman = $x$ years

Age of Rehman 3 years ago = $(x - 3)$ years.

Age of Rehman after 5 years = $(x + 5)$ years

According to the given condition:

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\Rightarrow \frac{(x+5)+(x-3)}{(x-3)(x+5)} = \frac{1}{3}$$

$3(2x + 2) = (x - 3)(x + 5)$

$6x + 6 = x^2 - 3x + 5x - 15$

$\Rightarrow x^2 - 4x - 15 - 6 = 0$

Comparing quadratic equation $x^2 - 4x - 21 = 0$ with general form $ax^2 + bx + c = 0$, We get $a = 1$, $b = -4$ and $c = -21$

Using quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-21)}}{2 \times 1}$$

$\Rightarrow x = \frac{4 \pm \sqrt{16+84}}{2}$

$\Rightarrow x = \frac{4 \pm \sqrt{100}}{2} = \frac{4 \pm 10}{2}$

$\Rightarrow x = \frac{4+10}{2}, \frac{4-10}{2}$

$x = 7, -3$

We discard $x=-3$. Since age cannot be in negative.

Therefore, present age of Rehman is 7 years.

5. **In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.**

**Ans.** Let Shefali's marks in Mathematics = $x$

Let Shefali's marks in English = $30 - x$

If, she had got 2 marks more in Mathematics, her marks would be = $x + 2$

If, she had got 3 marks less in English, her marks in English would be = $30 - x - 3 = 27 - x$

According to given condition:

=> $(x + 2)(27 - x) = 210$

$\Rightarrow 27x - x^2 + 54 - 2x = 210$

$\Rightarrow x^2 - 25x + 156 = 0$

Comparing quadratic equation $x^2 - 25x + 156 = 0$ with general form $ax^2 + bx + c = 0$,

We get $a = 1$, $b = -25$ and $c = 156$

Applying Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{25 \pm \sqrt{(-25)^2 - 4(1)(156)}}{2 \times 1}$$

$\Rightarrow x = \frac{25 \pm \sqrt{625 - 624}}{2}$

$\Rightarrow x = \frac{25 \pm \sqrt{1}}{2}$

$\Rightarrow x = \frac{25 + 1}{2}, \frac{25 - 1}{2}$

$x = 13, 12$

Therefore, Shefali's marks in Mathematics = 13 or 12

Shefali's marks in English = 30 – $x$ = 30 – 13 = 17

Or Shefali's marks in English = 30 – $x$ = 30 – 12 = 18

Therefore, her marks in Mathematics and English are (13, 17) or (12, 18).

6. **The diagonal of a rectangular field is 60 metres more than the shorter side. If, the longer side is 30 metres more than the shorter side, find the sides of the field.**

**Ans.** Let shorter side of rectangle = $x$metres

Let diagonal of rectangle = $(x + 60)$ metres

Let longer side of rectangle = $(x + 30)$ metres

According to pythagoras theorem,

=> $(x + 60)^2 = (x + 30)^2 + x^2$

$\Rightarrow x^2 + 3600 + 120x = x^2 + 900 + 60x + x^2$

$\Rightarrow x^2 - 60x - 2700 = 0$

Comparing equation $x^2 - 60x - 2700 = 0$ with standard form $ax^2 + bx + c = 0$, We get

$a = 1$, $b = -60$ and $c = -2700$

Applying quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{60 \pm \sqrt{(-60)^2 - 4(1)(-2700)}}{2 \times 1}$$

$$\Rightarrow x = \frac{60 \pm \sqrt{3600 + 10800}}{2}$$

$$\Rightarrow x = \frac{60 \pm \sqrt{14400}}{2} = \frac{60 \pm 120}{2}$$

$$\Rightarrow x = \frac{60 + 120}{2}, \frac{60 - 120}{2}$$

$x = 90, -30$

We ignore –30. Since length cannot be in negative.

Therefore, $x = 90$ which means length of shorter side = 90 metres

And length of longer side = $x + 30 = 90 + 30 = 120$ metres

Therefore, length of sides are 90 and 120 in metres.

7. **The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.**

**Ans.** Let the larger number be $x$, then square of smaller number be $8x$ and square of larger number be $x^2$.

According to condition:

$x^2 - 8x = 180$

$\Rightarrow \quad x^2 - 8x - 180 = 0$

$\Rightarrow \quad x^2 - 18x + 10x - 180 = 0$

$\Rightarrow \quad x(x - 18) + 10(x - 18) = 0$

$\Rightarrow \quad (x - 18)(x + 10) = 0$

$\Rightarrow \quad x - 18 = 0$ or $x + 10 = 0$

$\Rightarrow \quad x = 18 \quad x = -10$

When $x = 18$, then square of smaller number = 144

Then smaller number = $\pm 12$

Therefore, two numbers are (12, 18) or (−12, 18)

8. **A train travels 360 km at a uniform speed. If, the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.**

**Ans.** Let the speed of the train = x km/hr

If, speed had been 5 km/hr more, train would have taken 1 hour less.

So, according to this condition

=> $\frac{360}{x} = \frac{360}{x+5} + 1$

$\Rightarrow 360\left(\frac{1}{x} - \frac{1}{x+5}\right) = 1$

$\Rightarrow 360\left(\frac{x+5-x}{x(x+5)}\right) = 1$

$\Rightarrow 360 \times 5 = x^2 + 5x$

$\Rightarrow x^2 + 5x - 1800 = 0$

Comparing equation $x^2 + 5x - 1800 = 0$ with general equation $ax^2 + bx + c = 0$, We get

$a = 1$, $b = 5$ and $c = -1800$

Applying quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-1800)}}{2\times1}$$

$$\Rightarrow x = \frac{-5 \pm \sqrt{25 + 7200}}{2}$$

$$\Rightarrow x = \frac{-5 \pm \sqrt{7225}}{2} = \frac{-5 \pm 85}{2}$$

$$\Rightarrow x = \frac{-5 + 85}{2}, \frac{-5 - 85}{2}$$

(i) $x = 40, -45$

Since speed of train cannot be in negative. Therefore, we discard $x = -45$ Therefore,

speed of train = 40 km/hr

**9. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.**

**Ans.** Let time taken by tap of smaller diameter to fill the tank = $x$ hours

Let time taken by tap of larger diameter to fill the tank = $(x - 10)$ hours

It means that tap of smaller diameter fills $\frac{1}{x}^{th}$ part of tank in 1 hour.… (1)

And, tap of larger diameter fills $\frac{1}{x-10}^{th}$ part of tank in 1 hour. … (2)

When two taps are used together, they fill tank in $\frac{75}{8}$ hours.

In 1 hour, they fill $\frac{8}{75}^{th}$ part of tank $\left(\frac{1}{\frac{75}{8}}=\frac{8}{75}\right)$… (3)

From (1), (2) and (3),

=> $\frac{1}{x}+\frac{1}{x-10}=\frac{8}{75}$

⇒ $\frac{x-10+x}{x(x-10)}=\frac{8}{75}$

$75(2x-10) = 8(x^2-10x)$

$150x-750 = 8x^2-80x$

⇒ $8x^2-80x-150x+750=0$

⇒ $4x^2-115x+375=0$

Comparing equation $4x^2-115x+375=0$ with general equation $ax^2+bx+c=0$, We get

$a = 4$, $b = -115$ and $c = 375$

Applying quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{115\pm\sqrt{(-115)^2-4(4)(375)}}{2\times4}$$

$\Rightarrow x = \frac{115 \pm \sqrt{13225 - 6000}}{8}$

$\Rightarrow x = \frac{115 \pm \sqrt{7225}}{8}$

$\Rightarrow x = \frac{115 \pm 85}{8}$

$\Rightarrow x = \frac{115 + 85}{8}, \frac{115 - 85}{8}$

$x = 25, 3.75$

Time taken by larger tap = $x - 10 = 3.75 - 10 = -6.25$ hours

Time cannot be in negative. Therefore, we ignore this value.

Time taken by larger tap = $x - 10 = 25 - 10 = 15$ hours

Therefore, time taken by larger tap is 15 hours and time taken by smaller tap is 25 hours.

**10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If, the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of two trains.**

**Ans.** Let average speed of passenger train = $x$ km/h

Let average speed of express train = $(x + 11)$ km/h

Time taken by passenger train to cover 132 km = $\frac{132}{x}$ hours

Time taken by express train to cover 132 km = $\left(\frac{132}{x+11}\right)$ hours

According to the given condition,

=> $\frac{132}{x} = \frac{132}{x+11} + 1$

⇒ $132\left(\frac{1}{x} - \frac{1}{x+11}\right) = 1$

⇒ $132\left(\frac{x+11-x}{x(x+11)}\right) = 1$

132 (11) = $x$ ($x$ + 11)

⇒ $1452 = x^2 + 11x$

⇒ $x^2 + 11x - 1452 = 0$

Comparing equation $x^2 + 11x - 1452 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get $a = 1$, $b = 11$ and $c = -1452$

Applying Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-11 \pm \sqrt{(11)^2 - 4(1)(-1452)}}{2 \times 1}$$

⇒ $x = \frac{-11 \pm \sqrt{121 + 5808}}{2}$

⇒ $x = \frac{-11 \pm \sqrt{5929}}{2}$

⇒ $x = \frac{-11 \pm 77}{2}$

⇒ $x = \frac{-11 + 77}{2}, \frac{-11 - 77}{2}$

$x = 33, -44$

As speed cannot be in negative. Therefore, speed of passenger train = 33 km/h

And, speed of express train = $x$ + 11 = 33 + 11 = 44 km/h

11. **Sum of areas of two squares is 468 m$^2$. If, the difference of their perimeters is 24 metres, find the sides of the two squares.**

**Ans.** Let perimeter of first square = $x$ metres

Let perimeter of second square = ($x$ + 24) metres

Length of side of first square = $\frac{x}{4}$ metres {Perimeter of square = $4 \times length\ of\ side$}

Length of side of second square = $\left(\frac{x+24}{4}\right)$ metres

Area of first square = side × side = $\frac{x}{4} \times \frac{x}{4} = \frac{x^2}{16} m^2$

Area of second square = $\left(\frac{x+24}{4}\right)^2 m^2$

According to given condition:

=> $\frac{x^2}{16} + \left(\frac{x+24}{4}\right)^2 = 468$

⇒ $\frac{x^2}{16} + \frac{x^2 + 576 + 48x}{16} = 468$

⇒ $\frac{x^2 + x^2 + 576 + 48x}{16} = 468$

⇒ $2x^2 + 576 + 48x = 468 \times 16$

⇒ $2x^2 + 48x + 576 = 7488$

$\Rightarrow 2x^2 + 48x - 6912 = 0$

$\Rightarrow x^2 + 24x - 3456 = 0$

Comparing equation $x^2 + 24x - 3456 = 0$ with standard form $ax^2 + bx + c = 0$, We get

$a = 1$, $b = 24$ and $c = -3456$

Applying Quadratic Formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-24 \pm \sqrt{(24)^2 - 4(1)(-3456)}}{2 \times 1}$$

$$\Rightarrow x = \frac{-24 \pm \sqrt{576 + 13824}}{2}$$

$$\Rightarrow x = \frac{-24 \pm \sqrt{14400}}{2} = \frac{-24 \pm 120}{2}$$

$$\Rightarrow x = \frac{-24 + 120}{2}, \frac{-24 - 120}{2}$$

$x = 48, -72$

Perimeter of square cannot be in negative. Therefore, we discard $x = -72$.

Therefore, perimeter of first square = 48 metres

And, Perimeter of second square = $x + 24 = 48 + 24 = 72$ metres

Side of First square = $\dfrac{Perimeter}{4} = \dfrac{48}{4} = 12\ m$

And, Side of second Square = $\dfrac{Perimeter}{4} = \dfrac{72}{4} = 18\ m$

# Chapter - 4

# Quadratic Equations

## Exercise 4.4

1. **Find the nature of the roots of the following quadratic equations. If the real roots exist, find them.**

**(i)** $2x^2 - 3x + 5 = 0$

**(ii)** $3x^2 - 4\sqrt{3}x + 4 = 0$

**(iii)** $2x^2 - 6x + 3 = 0$

**Ans. (i)** $2x^2 - 3x + 5 = 0$

Comparing this equation with general equation $ax^2 + bx + c = 0$,

We get $a = 2$, $b = -3$ and $c = 5$

Discriminate = $b^2 - 4ac = (-3)^2 - 4$ (2) (5)

=9–40=−31

Discriminate is less than 0 which means equation has no real roots.

**(ii)** $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing this equation with general equation $ax^2 + bx + c = 0$,

We get $a = 3$, $b = -4\sqrt{3}$ and c = 4

Discriminate = $b^2 - 4ac = \left(-4\sqrt{3}\right)^2 - 4$ (3) (4)

=48–48=0

Discriminate is equal to zero which means equations have equal real roots
Applying quadratic formula to find roots,

$$x = \frac{4\sqrt{3} \pm \sqrt{0}}{6} = \frac{2\sqrt{3}}{3}$$

Because, equation has two equal roots, it means $x = \frac{2\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}$

**(iii)** $2x^2 - 6x + 3 = 0$

Comparing equation with general equation $ax^2 + bx + c = 0$,

We get $a = 2$, $b = -6$, and $c = 3$

Discriminate = $b^2 - 4ac = (-6)^2 - 4$ (2) (3)

=36–24=12

Value of discriminate is greater than zero.

Therefore, equation has distinct and real roots.

Applying quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to find roots,

$$x = \frac{6 \pm \sqrt{12}}{4} = \frac{6 \pm 2\sqrt{3}}{4}$$

$\Rightarrow x = \frac{3 \pm \sqrt{3}}{2}$

$\Rightarrow x = \frac{3 + \sqrt{3}}{2}, \frac{3 - \sqrt{3}}{2}$

2. **Find the value of k for each of the following quadratic equations, so that they have two equal roots.**

**(i)** $2x^2 + kx + 3 = 0$

**(ii)** ***kx*** **(*x* − 2) + 6 = 0**

**Ans. (i)** $2x^2 + kx + 3 = 0$

We know that quadratic equation has two equal roots only when the value of discriminant is equal to zero.

Comparing equation $2x^2 + kx + 3 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get $a = 2$, $b = k$ and $c = 3$

Discriminant = $b^2 - 4ac = k^2 - 4$ (2) (3) = $k^2 - 24$

Putting discriminant equal to zero

$k^2 - 24 = 0 \Rightarrow k^2 = 24$

$\Rightarrow k = \pm\sqrt{24} = \pm 2\sqrt{6}$

$\Rightarrow k = 2\sqrt{6}, -2\sqrt{6}$

**16** $kx(x - 2) + 6 = 0$

$\Rightarrow kx^2 - 2kx + 6 = 0$

Comparing quadratic equation $kx^2 - 2kx + 6 = 0$ with general form $ax^2 + bx + c = 0$, we get $a = k$, $b = -2k$ and $c = 6$

Discriminate = $b^2 - 4ac = (-2k)^2 - 4$ (*k*) (6)

= $4k^2 - 24k$

We know that two roots of quadratic equation are equal only if discriminant is equal to zero.

Putting discriminant equal to zero

$4k^2 - 24k = 0$

5 $4k(k-6)=0 \Rightarrow k=0, 6$

The basic definition of quadratic equation says that quadratic equation is the equation of the form $ax^2+bx+c=0$, where $a \neq 0$.

Therefore, in equation $kx^2-2kx+6=0$, we cannot have k = 0.

Therefore, we discard k = 0.

Hence the answer is k = 6.

3. **Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is $800\,m^2$? If so, find its length and breadth.**

**Ans.** Let breadth of rectangular mango grove = $x$ metres Let

length of rectangular mango grove = $2x$ metres

Area of rectangle = $length \times breadth = x \times 2x = 2x^2\ m^2$

According to given condition:

=> $2x^2 = 800$

⇒ $2x^2 - 800 = 0$

= 0

Comparing equation $x^2 - 400 = 0$ with general form of quadratic equation $ax^2+bx+c=0$, we get $a = 1$, $b = 0$ and $c = -400$

Discriminate $= b^2 - 4ac = (0)^2 - 4$ (1) (−400) = 1600

Discriminate is greater than 0 means that equation has two disctinct real roots.

Therefore, it is possible to design a rectangular grove.

Applying quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve equation,

$$x = \frac{0 \pm \sqrt{1600}}{2 \times 1} = \frac{\pm 40}{2} = \pm 20$$

$x = 20, -20$

We discard negative value of $x$ because breadth of rectangle cannot be in negative. Therefore, $x$ =

breadth of rectangle = 20 metres

Length of rectangle = $2x$ = $2 \times 20$ = 40 metres

4. **. Is the following situation possible? If so, determine their present ages.**

**The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.**

**Ans.** Let age of first friend = $x$ years

then age of second friend = $(20 - x)$ years

Four years ago, age of first friend = $(x - 4)$ years

Four years ago, age of second friend = $(20 - x) - 4 = (16 - x)$ years

According to given condition,

=> $(x - 4)(16 - x) = 48$

$\Rightarrow 16x - x^2 - 64 + 4x = 48$

$\Rightarrow 20x - x^2 - 112 = 0$

$\Rightarrow x^2 - 20x + 112 = 0$

Comparing equation, $x^2 - 20x + 112 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get $a = 1$, $b = -20$ and $c = 112$

Discriminate = $b^2 - 4ac = (-20)^2 - 4$ (1) (112) = 400 – 448 = −48 < 0 Discriminate is less than zero which means we have no real roots for this equation. Therefore, the give situation is not possible.

5. **Is it possible to design a rectangular park of perimeter 80 metres and area 400 $m^2$. If so, find its length and breadth.**

**Ans.** Let length of park = $x$ metres

We are given area of rectangular park = $400m^2$

Therefore, breadth of park = $\frac{400}{x}$ metres{Area of rectangle = length × *breadth*}

Perimeter of rectangular park = 2 (*length* + *breath*) = $2\left(x+\frac{400}{x}\right)$ metres

We are given perimeter of rectangle = 80 metres

According to condition:

=> $2\left(x+\frac{400}{x}\right) = 80$

⇒ $2\left(\frac{x^2+400}{x}\right) = 80$

⇒ $2x^2 + 800 = 80x$

⇒ $2x^2 - 80x + 800 = 0$

⇒

Comparing equation, $x^2 - 40x + 400 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get $a = 1$, $b = -40$ and $c = 400$

Discriminate = $b^2 - 4ac = (-40)^2 - 4$ (1) (400) = 1600 – 1600 = 0

Discriminate is equal to 0.

Therefore, two roots of equation are real and equal which means that it is possible to design a rectangular park of perimeter 80 metres and area $400m^2$.

Using quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve equation,

$$x = \frac{40 \pm \sqrt{0}}{2} = \frac{40}{2} = 20$$

Here, both the roots are equal to 20.

Therefore, length of rectangular park = 20 metres

Breadth of rectangular park = $\frac{400}{x} = \frac{400}{20} = 20\,m$