Chapter 8

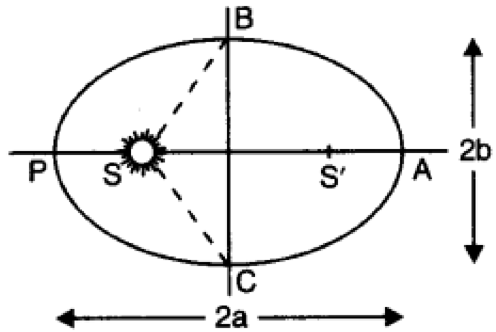
Gravitation

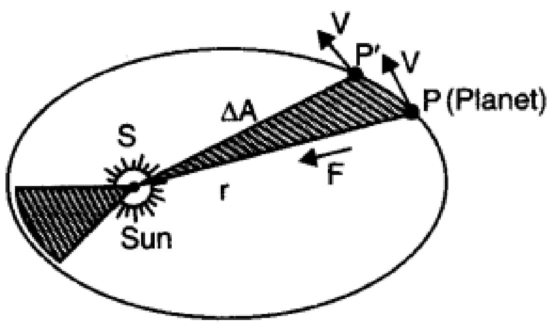
• Kepler's Laws of Planetary Motion

Johannes Kepler formulated three laws which describe planetary motion. They are as follows:

(i) Law of orbits. Each planet revolves around the sun in an elliptical orbit with the sun at one of the foci of the ellipse.

(ii) Law of areas. The speed of planet varies in such a way that the radius, vector drawn from the sun to planet sweeps out equal areas in equal times.





(*iii*) **Law of periods.** The square of the time period of revolution is proportional to the cube of the semi-major axis of the elliptical orbit. *i.e.,* $T^2 \propto r^3$.

If r_1 and r_2 are the shortest and the longest distances of the planet from the sun, the semi-major axis is given by $\left(\frac{r_1 + r_2}{2}\right)$.

• Newton's Law of Gravitation

Newton's law of gravitation states that every particle in the universe attracts every other particle with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them. The direction of the force is along the line joining the particles.

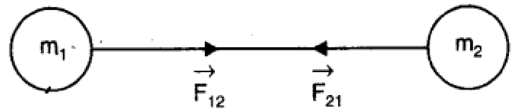
$$F \propto m_1 m_2 \quad \text{and} \quad F \propto \frac{1}{r^2}$$

$$\therefore \qquad F = G\frac{m_1 m_2}{r^2}$$

where G is a constant, called as the universal constant of gravitation.

- Vectorially the gravitation force is given as

$$\vec{F} = \frac{G\, m_1 m_2}{r^2}\,\hat{r}$$



$$\vec{F}_{12} + \vec{F}_{21} = 0$$

The gravitational force between two particles form an action-reaction pair.

- The value of 'G' has been experimentally determined *i.e.*, $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}. Dimensional formula of G is $[M^{-1} L^3 T^{-2}]$.

• Universal constant of gravitation G is numerically equal to the force of attraction between two particles of unit mass each separated by unit distance.
• Important Characteristics of Gravitational Force
(i) Gravitational force between two bodies is a central force i.e., it acts along the line joining the centres of the two interacting bodies.
(ii) Gravitational force between two bodies is independent of the nature of the intervening medium.
(iii) Gravitational force between two bodies does not depend upon the presence of other bodies.
(iv) It is valid for point objects and spherically symmetrical objects.
(v) Magnitude of force is extremely small.

• Principle of Superposition of Gravitation

It states that the resultant gravitational force $\vec{F}$ acting on a particle due to number of point masses is equal to the vector sum of the forces exerted by the individual masses on the given particle *i.e.*,

$$\vec{F} = \vec{F}_{01} + \vec{F}_{02} + \ldots\ldots + \vec{F}_{0n} = \sum_{i=1}^{n} \vec{F}_{0i}$$

where $\vec{F}_{01}, \vec{F}_{02}, \ldots\ldots \vec{F}_{0n}$ are the gravitational forces on a particle of mass m_0 due to particles of masses m_1, m_2, ... m_n respectively.

• Acceleration Due to Gravity

The acceleration produced in a body on account of the force of gravity is known as acceleration due to gravity. It is usually denoted by 'g'. It is always towards the centre of Earth.
If a body of mass 'm' lying on the surface of the earth, the gravitational force acting on the body is given by

$$F = G\frac{Mm}{R^2} \quad \text{where,} \quad M \rightarrow \text{mass of earth,} \quad R \rightarrow \text{radius of earth}$$

acceleration due to gravity, $g = \dfrac{GM}{R^2}$.

• Mass and Mean Density of Earth

Mass and Mean density of Earth is given in the following manner.

Mass of earth, $$M = \frac{gR^2}{G}$$

Mean density of earth, $$\rho = \frac{3g}{4\pi GR}.$$

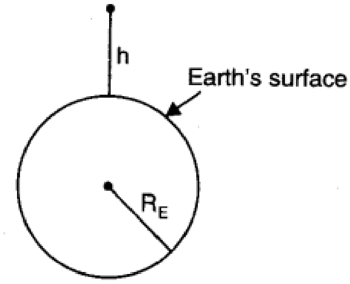
• Variation of Acceleration Due to Gravity

The value of acceleration due to gravity changes with height (i.e., altitude), depth, shape of the earth and rotation of earth about its own axis.
(a) Effect of Altitude. As one goes above the surface of Earth, value of acceleration due to gravity gradually goes on decreasing. If g_h be the value of acceleration due to gravity at a height h from the surface of Earth, then

$$g_h = \frac{g}{\left(1+\frac{h}{R}\right)^2}$$

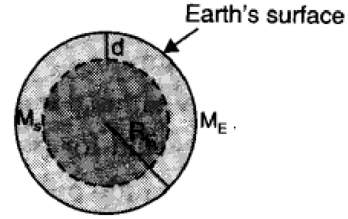
If $h << R$, then

$$g_h = g\left(1-\frac{2h}{R}\right). \text{ (using Binomial Theorem)}$$



(*b*) **Effect of Depth.** The value of acceleration due to gravity decreases with depth. It is given as

$$g_d = g\left(1-\frac{d}{R}\right) \quad \text{where} \quad d \to \text{depth.}$$



At the centre of the earth, the value of acceleration due to gravity becomes zero.

(*c*) **Effect due to rotation of earth.** The acceleration due to gravity (*i*) decreases due to rotation of earth. (*ii*) increases with the increase in lattitude. It is given by

$$g_\theta = g\left(1-\frac{R\omega^2}{g}\cos^2\theta\right), \quad \text{where,} \quad \theta \to \text{latitude of the point.}$$

(*d*) **Effect due to shape.** The equatorial radius of earth is longer than its polar radius. The value of g increases from equator to pole. It is given as:

$$g_{\text{pole}} > g_{\text{equator}}$$

• Gravitational Field
The space around a body within which its gravitational force of attraction is experienced by other bodies is called gravitational field.
• Intensity of Gravitational Field
The intensity of the gravitational field of a body at a point in the field is defined as the force experienced by a body of unit mass placed at that point provided the presence of unit mass does not disturb the original gravitational field.

Intensity of gravitational field at a point p is given by

$$I = \frac{GM}{x^2}$$

where M creates a field and the point p where the field is estimated is at a distance r.

In vector form, $\vec{I} = -\frac{GM}{x^2}\hat{x}$

Here $-ve$ sign shows that the gravitational intensity is of attractive nature.

• Unit of intensity of gravitational field in S.I. is Nkg^{-1} or ms^{-2} and in cgs system, is dyne g^{-1} or cm s^{-2}.
Dimensional formula of gravitational intensity is $[LT^{-2}]$.

• Gravitational Potential
The gravitational potential at a point in the gravitational field of a body is defined as the amount of work done in bringing a body of unit mass from infinity to that point.
Gravitational potential at a point situated at a distance r from a body or particle of mass M is given

$$V = -\frac{GM}{r}$$

Its unit is joule/kg and it is a scalar quantity. Its dimensional formula is $[L^2 T^{-3}]$.

• Gravitational Potential Energy

The work done in carrying a mass 'm' from infinity to a point at distance r is called gravitational potential energy.

The gravitational potential energy of the system is given by

$$U = -\frac{GMm}{r}$$

i.e., Gravitational potential energy = gravitational potential x mass of the body.

It is a scalar quantity and measured in joule.

• Escape Velocity

The minimum velocity required to project a body vertically upward from the surface of earth so that it comes out of the gravitational field of earth is called escape velocity.

It is given by $$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$ where $M \rightarrow$ Mass of Earth

$R \rightarrow$ radius of the Earth.

• Satellite

A satellite is a body which is revolving continuously in an orbit around a comparatively much larger body.

The orbit may be either circular or elliptical. A man-made object revolving in an orbit around a planet is called an artificial satellite.

• Orbital Velocity

Orbital velocity of a satellite is the minimum velocity required to put the satellite into a given orbit around earth.

The orbital velocity is given by

$$v_o = \sqrt{\frac{GM}{(R+h)}} = \sqrt{\frac{gR^2}{(R+h)}}$$

where, $h \rightarrow$ height of the satellite above the surface of earth

$M \rightarrow$ Mass of the earth

$R \rightarrow$ radius of the earth.

For a satellite orbiting just near earth (where $h << R$), we have

$$v_o = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

- Time taken by a satellite to complete one revolution around the earth is known as its time of revolution or time period T. It is given by

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}} = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}} = \sqrt{\frac{3\pi(R+h)^3}{G\rho R^3}}$$

where ρ is the mean density of the earth.

- For satellite orbiting close to the surface of earth, $h << R$

$$T = \sqrt{\frac{3\pi}{G\rho}}.$$

- For satellite orbiting close to the surface of earth, the relation between orbital and escape velocity is given by

$$v_e = \sqrt{2}\, v_o .$$

- Height of satellite above the earth's surface is given by

$$h = \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R$$

- A satellite revolving around the earth possesses kinetic energy as well as potential energy.

The P.E. of a satellite is

$$U = -\frac{GMm}{(R+h)}$$

The kinetic energy of satellite is

$$K = \frac{GMm}{2(R+h)}$$

Total mechanical energy of the satellite = $-\dfrac{GMm}{2(R+h)}$

- The energy required to remove the satellite from its orbit around the earth to infinity is called Binding energy of the satellite.

Binding energy of a satellite = $-E = \dfrac{GMm}{2(R+h)}$.

• Geostationary Satellite

The satellite having the same time period of revolution as that of the earth is called geostationary satellite. Such satellites should rotate in the equatorial plane from west to east.

The orbit of a geostationary satellite is called 'parking orbit'. These satellites are used for communication purposes.

A geostationary satellite revolves around the earth in a circular orbit at a height of about 36,000 km from the surface of earth.

• IMPORTANT TABLES

TABLE 8.1 Data from measurement of planetary motions given below confirm Kepler's Law of Periods

$a \equiv$ Semi-major axis in units of 10^{10} m.

$T \equiv$ Time period of revolution of the planet in years(y).

$Q \equiv$ The quotient (T^2/a^3) in units of 10^{-34} y^2 m^{-3}.)

Planet	a	T	Q
Mercury	5.79	0.24	2.95
Venus	10.8	0.615	3.00
Earth	15.0	1	2.96
Mars	22.8	1.88	2.98
Jupiter	77.8	11.9	3.01
Saturn	143	29.5	2.98
Uranus	287	84	2.98
Neptune	450	165	2.99
Pluto	590	248	2.99

TABLE 8.2.

Physical Quantity	*Symbol*	*Dimensions*	*Unit*	*Remarks*
Gravitational Constant	G	$[M^{-1} L^3 T^{-2}]$	N m^2 kg^{-2}	6.67×10^{-11}
Gravitational Potential Energy	$V(r)$	$[M L^2 T^{-2}]$	J	$-\frac{GMm}{r}$ (scalar)
Gravitational Potential	$U(r)$	$[L^2 T^{-2}]$	J kg^{-1}	$-\frac{GM}{r}$ (scalar)
Gravitational Intensity	E or g	$[LT^{-2}]$	m s^{-2}	$\frac{GM}{r^2}\hat{r}$ (vector)