Activity 1

Objective

To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \perp m\}$ is symmetric but neither reflexive nor transitive.

Material Required

A piece of plywood, some pieces of wires (8), nails, white paper, glue etc.

Method of Construction

Take a piece of plywood and paste a white paper on it. Fix the wires randomly on the plywood with the help of nails such that some of them are parallel, some are perpendicular to each other and some are inclined as shown in Fig.1.

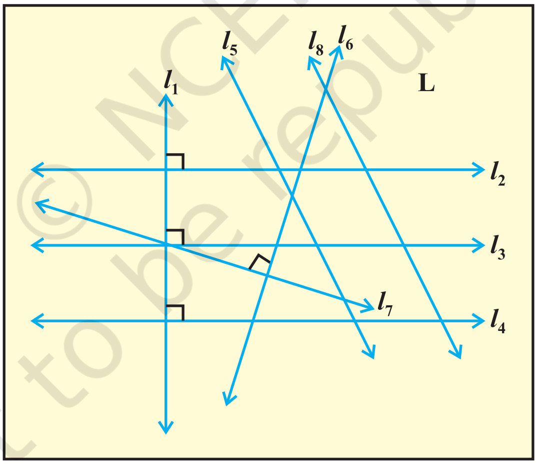

Fig. 1

Demonstration

1. Let the wires represent the lines $l_1, l_2, ..., l_8$.
2. l_1 is perpendicular to each of the lines l_2, l_3, l_4. [see Fig. 1]

3. l_6 is perpendicular to l_7.

4. l_2 is parallel to l_3, l_3 is parallel to l_4 and l_5 is parallel to l_8.

5. $(l_1, l_2), (l_1, l_3), (l_1, l_4), (l_6, l_7) \in$ R

Observation

1. In Fig. 1, no line is perpendicular to itself, so the relation R = $\{(l, m) : l \perp m\}$ _______ reflexive (is/is not).

2. In Fig. 1, $l_1 \perp l_2$. Is $l_2 \perp l_1$? _______ (Yes/No)

   $\therefore$ $(l_1, l_2) \in R \Rightarrow (l_2, l_1)$ _______ R $(\notin / \in)$

   Similarly, $l_3 \perp l_1$. Is $l_1 \perp l_3$? ________ (Yes/No)

   $\therefore$ $(l_3, l_1) \in R \Rightarrow (l_1, l_3)$ _______ R $(\notin / \in)$

   Also, $l_6 \perp l_7$. Is $l_7 \perp l_6$? ________ (Yes/No)

   $\therefore$ $(l_6, l_7) \in R \Rightarrow (l_7, l_6)$ _______ R $(\notin / \in)$

   $\therefore$ The relation R .... symmetric (is/is not)

3. In Fig. 1, $l_2 \perp l_1$ and $l_1 \perp l_3$. Is $l_2 \perp l_3$? ... (Yes/No)

   i.e., $(l_2, l_1) \in$ R and $(l_1, l_3) \in R \Rightarrow (l_2, l_3)$ _______ R $(\notin / \in)$

   $\therefore$ The relation R .... transitive (is/is not).

Application

This activity can be used to check whether a given relation is an equivalence relation or not.

Note

1. In this case, the relation is not an equivalence relation.
2. The activity can be repeated by taking some more wire in different positions.

Activity 2

Objective

To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \parallel m\}$ is an equivalence relation.

Material Required

A piece of plywood, some pieces of wire (8), plywood, nails, white paper, glue.

Method of Construction

Take a piece of plywood of convenient size and paste a white paper on it. Fix the wires randomly on the plywood with the help of nails such that some of them are parallel, some are perpendicular to each other and some are inclined as shown in Fig. 2.

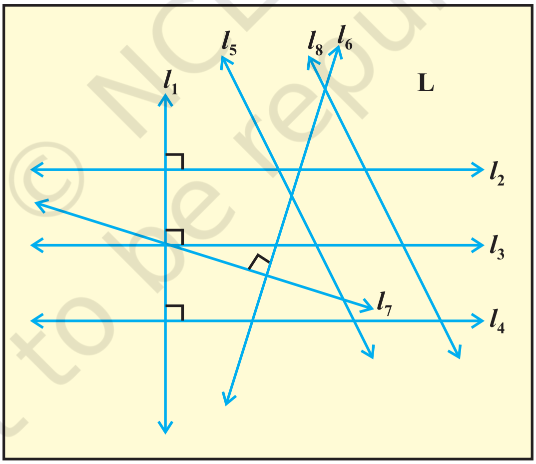

Fig. 2

Demonstration

1. Let the wires represent the lines $l_1, l_2, ..., l_8$.
2. l_1 is perpendicular to each of the lines l_2, l_3, l_4 (see Fig. 2).

3. l_6 is perpendicular to l_7.

4. l_2 is parallel to l_3, l_3 is parallel to l_4 and l_5 is parallel to l_8.

5. $(l_2, l_3), (l_3, l_4), (l_5, l_8), \in$ R

Observation

1. In Fig. 2, every line is parallel to itself. So the relation R = $\{(l, m) : l \parallel m\}$ .... reflexive relation (is/is not)

2. In Fig. 2, observe that $l_2 \parallel l_3$. Is l_3 ... l_2? ($\nparallel$ / $\parallel$)

   So, $(l_2, l_3) \in$ R $\Rightarrow (l_3, l_2)$ ... R ($\notin / \in$)

   Similarly, $l_3 \parallel l_4$. Is $l_4 \ldots l_3$? ($\nparallel$ / $\parallel$)

   So, $(l_3, l_4) \in$ R $\Rightarrow (l_4, l_3)$ ... R ($\notin / \in$)

   and $(l_5, l_8) \in$ R $\Rightarrow (l_8, l_5)$ ... R ($\notin / \in$)

$\therefore$ The relation R ... symmetric relation (is/is not)

3. In Fig. 2, observe that $l_2 \parallel l_3$ and $l_3 \parallel l_4$. Is l_2 ... l_4 ? ($\parallel$ / $\parallel$)

   So, $(l_2, l_3) \in$ R and $(l_3, l_4) \in$ R $\Rightarrow (l_2, l_4)$ ... R ($\in / \notin$)

   Similarly, $l_3 \parallel l_4$ and $l_4 \parallel l_2$. Is l_3 ... l_2 ? ($\nparallel$ / $\parallel$)

   So, $(l_3, l_4) \in$ R, $(l_4, l_2) \in$ R $\Rightarrow (l_3, l_2)$ ... R ($\in, \notin$)

Thus, the relation R ... transitive relation (is/is not)

Hence, the relation R is reflexive, symmetric and transitive. So, R is an equivalence relation.

Application

This activity is useful in understanding the concept of an equivalence relation.

Note

This activity can be repeated by taking some more wires in different positions.

Activity 3

OBJECTIVE

To demonstrate a function which is not one-one but is onto.

MATERIAL REQUIRED

Cardboard, nails, strings, adhesive and plastic strips.

METHOD OF CONSTRUCTION

1. Paste a plastic strip on the left hand side of the cardboard and fix three nails on it as shown in the Fig.3.1. Name the nails on the strip as 1, 2 and 3.
2. Paste another strip on the right hand side of the cardboard and fix two nails in the plastic strip as shown in Fig.3.2. Name the nails on the strip as *a* and *b*.
3. Join nails on the left strip to the nails on the right strip as shown in Fig. 3.3.

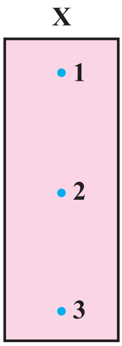

Fig. 3.1

Fig. 3.2

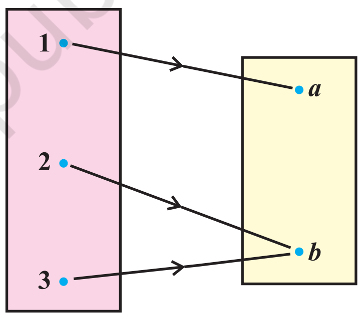

Fig. 3.3

DEMONSTRATION

1. Take the set X = {1, 2, 3}
2. Take the set Y = {*a*, *b*}
3. Join (correspondence) elements of X to the elements of Y as shown in Fig. 3.3

OBSERVATION

1. The image of the element 1 of X in Y is __________.

   The image of the element 2 of X in Y is __________.

The image of the element 3 of X in Y is __________.

So, Fig. 3.3 represents a __________ .

2. Every element in X has a _________ image in Y. So, the function is _________(one-one/not one-one).

3. The pre-image of each element of Y in X _________ (exists/does not exist). So, the function is ________ (onto/not onto).

APPLICATION

This activity can be used to demonstrate the concept of one-one and onto function.

NOTE

Demonstrate the same activity by changing the number of the elements of the sets X and Y.

Activity 4

Objective

To demonstrate a function which is one-one but not onto.

Material Required

Cardboard, nails, strings, adhesive and plastic strips.

Method of Construction

1. Paste a plastic strip on the left hand side of the cardboard and fix two nails in it as shown in the Fig. 4.1. Name the nails as *a* and *b*.
2. Paste another strip on the right hand side of the cardboard and fix three nails on it as shown in the Fig. 4.2. Name the nails on the right strip as 1, 2 and 3.
3. Join nails on the left strip to the nails on the right strip as shown in the Fig. 4.3.

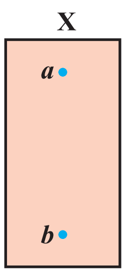

Fig. 4.1

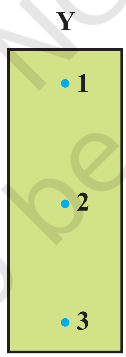

Fig. 4.2

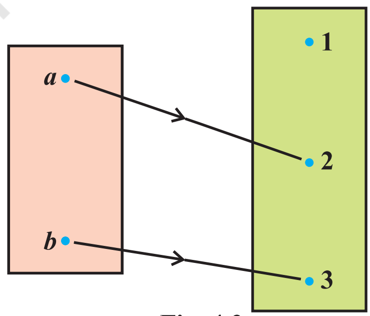

Fig. 4.3

Demonstration

1. Take the set X = $\{a, b\}$
2. Take the set Y = $\{1, 2, 3\}$.
3. Join elements of X to the elements of Y as shown in Fig. 4.3.

Observation

1. The image of the element *a* of X in Y is ______________.

   The image of the element *b* of X in Y is ______________.

   So, the Fig. 4.3 represents a _____________________.

2. Every element in X has a _________ image in Y. So, the function is _____________ (one-one/not one-one).

3. The pre-image of the element 1 of Y in X __________ (exists/does not exist). So, the function is __________ (onto/not onto).

   Thus, Fig. 4.3 represents a function which is _________ but not onto.

Application

This activity can be used to demonstrate the concept of one-one but not onto function.

Activity 19

Objective

To evaluate the definite integral $\int_a^b \sqrt{(1-x^2)}\, dx$ as the limit of a sum and verify it by actual integration.

Material Required

Cardboard, white paper, scale, pencil, graph paper

Method of Construction

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Draw two perpendicular lines to represent coordinate axes XOX′ and YOY′.
3. Draw a quadrant of a circle with O as centre and radius 1 unit (10 cm) as shown in Fig.19.

The curve in the 1st quadrant represents the graph of the function $\sqrt{1-x^2}$ in the interval [0, 1].

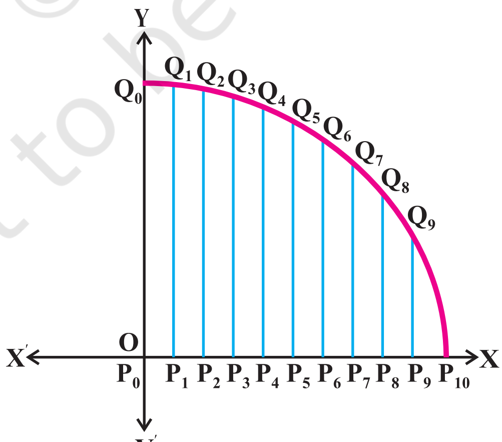

Fig. 19

DEMONSTRATION

1. Let origin O be denoted by P_0 and the points where the curve meets the x-axis and y-axis be denoted by P_{10} and Q, respectively.
2. Divide P_0P_{10} into 10 equal parts with points of division as, P_1, P_2, P_3, ..., P_9.
3. From each of the points, P_i , i = 1, 2, ..., 9 draw perpendiculars on the x-axis to meet the curve at the points, Q_1, Q_2, Q_3 ,..., Q_9. Measure the lengths of P_0Q_0, P_1 Q_1, ..., P_9Q_9 and call them as y_0, y_1 , ..., y_9 whereas width of each part, P_0P_1, P_1P_2, ..., is 0.1 units.
4. $y_0 = P_0Q_0 = 1$ units

   $y_1 = P_1Q_1 = 0.99$ units

   $y_2 = P_2Q_2 = 0.97$ units

   $y_3 = P_3Q_3 = 0.95$ units

   $y_4 = P_4Q_4 = 0.92$ units

   $y_5 = P_5Q_5 = 0.87$ units

   $y_6 = P_6Q_6 = 0.8$ units

   $y_7 = P_7Q_7 = 0.71$ units

   $y_8 = P_8Q_8 = 0.6$ units

   $y_9 = P_9Q_9 = 0.43$ units

   $y_{10} = P_{10}Q_{10}$ = which is very small near to 0.
5. Area of the quadrant of the circle (area bounded by the curve and the two axis) = sum of the areas of trapeziums.

$$=\frac{1}{2}\times 0.1\begin{bmatrix}(1+0.99)+(0.99+0.97)+(0.97+0.95)+(0.95+0.92)\\ +(0.92+0.87)+(0.87+0.8)+(0.8+0.71)+(0.71+0.6)\\ +(0.6+0.43)+(0.43)\end{bmatrix}$$

$= 0.1\ [0.5 + 0.99 + 0.97 + 0.95 + 0.92 + 0.87 + 0.80 + 0.71 + 0.60 + 0.43]$

$= 0.1 \times 7.74 = 0.774$ sq. units.(approx.)

6. Definite integral $= \int_0^1 \sqrt{1-x^2}\, dx$

$$= \left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1} x\right]_0^1 = \frac{1}{2} \times \frac{\pi}{2} = \frac{3.14}{4} = 0.785 \text{ sq.units}$$

Thus, the area of the quadrant as a limit of a sum is nearly the same as area obtained by actual integration.

Observation

1. Function representing the arc of the quadrant of the circle is y = ______.
2. Area of the quadrant of a circle with radius 1 unit $= \int_0^1 \sqrt{1-x^2}\, dx =$ ________.
   sq. units
3. Area of the quadrant as a limit of a sum = _______ sq. units.
4. The two areas are nearly _________.

Application

This activity can be used to demonstrate the concept of area bounded by a curve. This activity can also be applied to find the approximate value of π.

Note

Demonstrate the same activity by drawing the circle $x^2 + y^2 = 9$ and find the area between $x = 1$ and $x = 2$.

Activity 21

Objective

To verify that angle in a semi-circle is a right angle, using vector method.

Material Required

Cardboard, white paper, adhesive, pens, geometry box, eraser, wires, paper arrow heads.

Method of Construction

1. Take a thick cardboard of size 30 cm × 30 cm.
2. On the cardboard, paste a white paper of the same size using an adhesive.
3. On this paper draw a circle, with centre O and radius 10 cm.

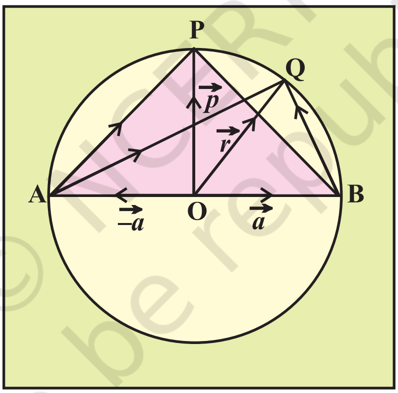

Fig. 21

4. Fix nails at the points O, A, B, P and Q. Join OP, OA, OB, AP, AQ, BQ, OQ and BP using wires.
5. Put arrows on OA, OB, OP, AP, BP, OQ, AQ and BQ to show them as vectors, using paper arrow heads, as shown in the figure.

Demonstration

1. Using a protractor, measure the angle between the vectors $\overrightarrow{AP}$ and $\overrightarrow{BP}$, i.e., $\angle APB = 90°$.

2. Similarly, the angle between the vectors $\overrightarrow{AQ}$ and $\overrightarrow{BQ}$, i.e., $\angle$ AQB = 90°.

3. Repeat the above process by taking some more points R, S, T, ... on the semi-circles, forming vectors AR, BR; AS, BS; AT, BT; ..., etc., i.e., angle formed between two vectors in a semi-circle is a right angle.

OBSERVATION

By actual measurement.

$$|\overrightarrow{OP}| = |\overrightarrow{OA}| = |\overrightarrow{OB}| = |\overrightarrow{OQ}| = r = a = p = \_\_\_\_\_\_\_ ,$$

$|\overrightarrow{AP}|$ = _______ , $|\overrightarrow{BP}|$ = _______, $|\overrightarrow{AB}|$ = ______

$|\overrightarrow{AQ}|$ = _______ , $|\overrightarrow{BQ}|$ = _______

$|\overrightarrow{AP}|^2 + |\overrightarrow{BP}|^2$ = ________, $|\overrightarrow{AQ}|^2 + |\overrightarrow{BQ}|^2$ = ________

So, $\angle$APB = ________ and $\overrightarrow{AP}.\overrightarrow{BP}$ ________ $\angle$AQB = ________ and $\overrightarrow{AQ}.\overrightarrow{BP}$ = ________

Similarly, for points R, S, T, ________

$\angle$ARB = ________, $\angle$ASB = ________, $\angle$ATB = ________, ________

i.e., angle in a semi-circle is a right angle.

APPLICATION

This activity can be used to explain the concepts of

(i) opposite vectors

(ii) vectors of equal magnitude

(iii) perpendicular vectors

(iv) Dot product of two vectors.

NOTE

Let OA = OB = a = OP = p

$\overrightarrow{OA} = -\vec{a}$, $\overrightarrow{OB} = \vec{a}$, $\overrightarrow{OP} = \vec{p}$

$\overrightarrow{AP} = -\overrightarrow{OA} + \overrightarrow{OP} = \vec{a} + \vec{p}$., $\overrightarrow{BP} = \vec{p} - \vec{a}$.

$\overrightarrow{AP}.\overrightarrow{BP} = (\vec{p}+\vec{a}).(\vec{p}-\vec{a}) = |\vec{p}|^2 - |\vec{a}|^2 = 0$.

$\left(\text{since} |\vec{p}|^2 = |\vec{a}|\right)$

So, the angle APB between the vectors $\overrightarrow{AP}$ and $\overrightarrow{BP}$ is a right angle.

Similarly, $\overrightarrow{AQ}.\overrightarrow{BQ} = 0$, so, $\angle AQB = 90°$ and so on.

Activity 22

Objective

To locate the points to given coordinates in space, measure the distance between two points in space and then to verify the distance using distance formula.

Material Required

Drawing board, geometry box, squared paper, nails of different lengths, paper arrows.

Method of Construction

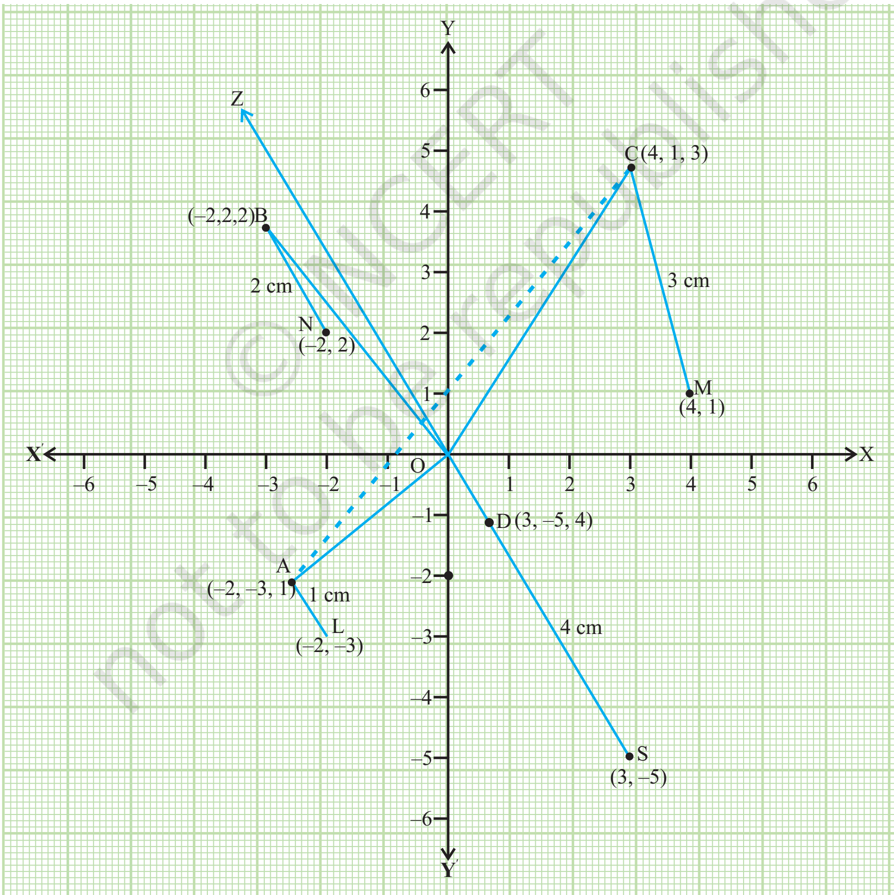

Fig 22

1. Take a drawing board and paste a squared paper on it.
2. Draw two lines X′OX and Y′OY to represent *x*-axis, *y*-axis respectively (see Fig. 22) and take 1 unit = 1 cm.
3. Fix a wire through O, in the vertical direction, representing the *z*-axis.
4. Fix nails of length 1 cm, 2 cm, 3 cm, 4 cm, etc. at different points on the squared paper (say at L (–2, –3), N (–2, 2), M (4, 1), S (3, –5)), etc.

Now the upper tips of these nails represent the points (say A, B, C, D) in the space.

Demonstration

1. Coordinates of the point A = (–2, –3, 1).
2. Coordinates of the point B = (–2, 2, 2).
3. Similarly find the coordinates of the point C and D.
4. By actual measurement (using a scale) distance AB = 5.1 cm.
5. By distance formula, $AB=\sqrt{(-2+2)^2+(-3-2)^2+(1-2)^2} = \sqrt{26} = 5.099$.

Thus, the distance AB, obtained by actual measurement is approximately same as the distance obtained by using the distance formula.

Same can be verified for other pairs of points A, C; B, C; A, D; C, D; B, D.

Observation

Coordinates of the point C = ________.

Coordinates of the point D = ________.

On actual measurement :

AC = ________, BC = _________.

AD = ________, CD = _________, BD = __________.

Using distance formula; AC = ________, BC = ________, AD = ________

CD = ________, BD = ________.

Thus, the distance between two points in space obtained on actual measurement and by using distance formula is approximately the same.

APPLICATION

1. This activity is useful in visualising the position of different points in space (coordinates of points).
2. The concept of position vectors can also be explained through this activity.

Activity 23

Objective

To demonstrate the equation of a plane in normal form.

Material Required

Two pieces of plywood of size 10 cm × 12 cm, a thin wooden rod with nuts and bolts fixed on both sides, 3 pieces of wires, pen/pencil.

Method of Construction

1. Fix the wooden rod in between two wooden pieces with nuts and bolts so that the rod is perpendicular to the two wooden pieces. So, it represents the normal to the plane.

2. Take three wires and fix, them as shown in Fig. 23 so that $\overrightarrow{OP}$ represents the vector $\vec{a}$ and $\overrightarrow{OA}$ represents $\vec{r}$. Then the wire $\overrightarrow{PA}$ represents the vector $\vec{r} - \vec{a}$.

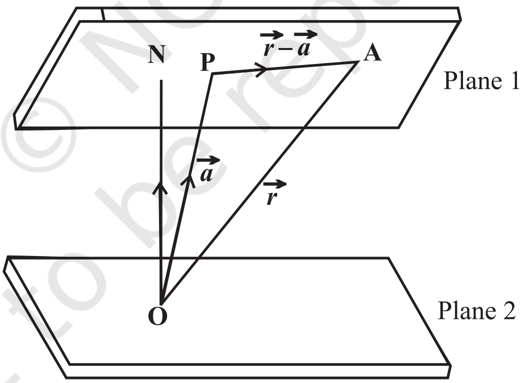

Fig. 23

Demonstration

1. The wire PA, i.e., the vector $(\vec{r} - \vec{a})$ lies on plane 1. On representing $\vec{n}$ as normal to plane 1, $\hat{n}$ is perpendicular to $(\vec{r} - \vec{a})$, normal to the plane.

2. Hence $\left(\vec{r} - \vec{a}\right) \cdot \vec{n} = 0$ which gives the equation of plane in the normal form.

Observation

1. $\vec{a}$ is the position vector of _________, $\vec{r}$ is the position vector of _________ vector $\vec{n}$ is perpendicular to the vector __________.

2. $(\vec{r}-\vec{a}).\ \hat{n}=0$, is the equation of the plane ________, in _______ form.

Application

This activity can also be utilised to show the position vector of a point in space (i.e., $\vec{a}$ as position vector of O, $\vec{r}$ the position vector of A).