1. Units, Dimensions and Measurement

1.1 Physical Quantity

A quantity which can be measured and expressed in form of laws is called a physical quantity.

Physical quantity (Q) = Magnitude × Unit $= n \times u$

Where, n represents the numerical value and u represents the unit. as the unit(u) changes, the magnitude(n) will also change but product 'nu' will remain same.

i.e. $n\,u$ = constant, or $n_1u_1 = n_2u_2$ = constant;

1.2 Fundamental and Derived Units

Any unit of mass, length and time in mechanics is called a *fundamental, absolute or base unit.* Other units which can be expressed in terms of fundamental units, are called derived units

System of units: A complete set of units, both fundamental and derived for all kinds of physical quantities is called system of units.

(1) CGS system (2) MKS system (3) FPS system.

(4) ***S.I. system:*** It is known as International system of units. There are seven fundamental quantities in this system. These quantities and their units are given in the following table.

Quantity	Name of Unit	Symbol
Length	metre	*m*
Mass	kilogram	*kg*
Time	second	*s*
Electric Current	ampere	*A*
Temperature	Kelvin	*K*
Amount of Substance	mole	*mol*
Luminous Intensity	candela	*cd*

Besides the above seven fundamental units two supplementary units are also defined – Radian (*rad*) for plane angle and Steradian (*sr*) for solid angle.

1.3 Practical Units

(1) ***Length:***

(i) 1 fermi = 1 $fm = 10^{-15}\ m$

(ii) 1 X-ray unit = $1XU = 10^{-13}\ m$

(iii) 1 angstrom = $1Å = 10^{-10}\ m = 10^{-8}\ cm = 10^{-7}\ mm = 0.1\ \mu mm$

(iv) 1 micron = $\mu m = 10^{-6}\ m$

(v) 1 astronomical unit = 1 $A.U. = 1.49 \times 10^{11}\ m \approx 1.5 \times 10^{11}\ m \approx 10^8\ km$

(vi) 1 Light year = 1 $ly = 9.46 \times 10^{15}\ m$

(vii) 1 *Parsec* = 1*pc* = 3.26 light year

(2) ***Mass:***

(i) Chandra Shekhar unit: 1 CSU = 1.4 times the mass of sun = $2.8 \times 10^{30}\ kg$

(ii) Metric tonne: 1 Metric tonne = 1000 *kg*

(iii) Quintal: 1 Quintal = 100 *kg*

(iv) Atomic mass unit (amu): $amu = 1.67 \times 10^{-27}\ kg$ mass of proton or neutron is of the order of 1 amu

(3) ***Time:***

(i) Year: It is the time taken by earth to complete 1 revolution around the sun in its orbit.

(ii) Lunar month: It is the time taken by moon to complete 1 revolution around the earth in its orbit.

$1\ L.M. = 27.3$ days

(iii) Solar day: It is the time taken by earth to complete one rotation about its axis with respect to sun. Since this time varies from day to day, average solar day is calculated by taking average of the duration of all the days in a year and this is called Average Solar day.

1 Solar year = 365.25 average solar day

or average solar day $= \dfrac{1}{365.25}$ the part of solar year

(iv) Sedrial day: It is the time taken by earth to complete one rotation about its axis with respect to a distant star.

1 Solar year = 366.25 Sedrial day = 365.25 average solar day

Thus 1 Sedrial day is less than 1 solar day.

(v) Shake: It is an obsolete and practical unit of time.

1 Shake = $10^{-8}\ sec$

1.4 Dimensions of a Physical Quantity

When a derived quantity is expressed in terms of fundamental quantities, it is written as a product of different powers of the fundamental quantities. The powers to which fundamental quantities must be raised in order to express the given physical quantity are called its dimensions.

1.5 Important Dimensions of Complete Physics

Mechanics

S. N.	Quantity	Unit	Dimension
(1)	Velocity or speed (v)	m/s	$[M^0L^1T^{-1}]$
(2)	Acceleration (a)	m/s^2	$[M^0LT^{-2}]$
(3)	Momentum (P)	$kg\text{-}m/s$	$[M^1L^1T^{-1}]$
(4)	Impulse (I)	$Newton\text{-}sec$ or $kg\text{-}m/s$	$[M^1L^1T^{-1}]$
(5)	Force (F)	$Newton$	$[M^1L^1T^{-2}]$
(6)	Pressure (P)	$Pascal$	$[M^1L^{-1}T^{-2}]$
(7)	Kinetic energy (E_K)	$Joule$	$[M^1L^2T^{-2}]$
(8)	Power (P)	$Watt$ or $Joule/s$	$[M^1L^2T^{-3}]$
(9)	Density (d)	kg/m^3	$[M^1L^{-3}T^0]$
(10)	Angular displacement (θ)	$Radian$ ($rad.$)	$[M^0L^0T^0]$
(11)	Angular velocity (ω)	$Radian$/sec	$[M^0L^0T^{-1}]$
(12)	Angular acceleration (α)	$Radian/sec^2$	$[M^0L^0T^{-2}]$
(13)	Moment of inertia (I)	$kg\text{-}m^2$	$[M^1L^2T^0]$
(14)	Torque (τ)	$Newton\text{-}meter$	$[M^1L^2T^{-2}]$

S. N.	Quantity	Unit	Dimension
(15)	Angular momentum (L)	*Joule-sec*	$[M^1L^2T^{-1}]$
(16)	Force constant or spring constant (k)	*Newton / m*	$[M^1L^0T^{-2}]$
(17)	Gravitational constant (G)	$N\text{-}m^2/kg^2$	$[M^{-1}L^3T^{-2}]$
(18)	Intensity of gravitational field (E_g)	N/kg	$[M^0L^1T^{-2}]$
(19)	Gravitational potential (V_g)	*Joule / kg*	$[M^0L^2T^{-2}]$
(20)	Surface tension (T)	N/m or $Joule/m^2$	$[M^1L^0T^{-2}]$
(21)	Velocity gradient (V_g)	$Second^{-1}$	$[M^0L^0T^{-1}]$
(22)	Coefficient of viscosity (η)	*kg / m-s*	$[M^1L^{-1}T^{-1}]$
(23)	Stress	N/m^2	$[M^1L^{-1}T^{-2}]$
(24)	Strain	No unit	$[M^0L^0T^0]$
(25)	Modulus of elasticity (E)	N/m^2	$[M^1L^{-1}T^{-2}]$
(26)	Poisson Ratio (σ)	No unit	$[M^0L^0T^0]$
(27)	Time period (T)	*Second*	$[M^0L^0T^1]$
(28)	Frequency (n)	*Hz*	$[M^0L^0T^{-1}]$

Heat

S. N.	Quantity	Unit	Dimension
(1)	Temperature (T)	*Kelvin*	$[M^0L^0T^0\theta^1]$
(2)	Heat (Q)	*Joule*	$[ML^2T^{-2}]$
(3)	Specific Heat (c)	*Joule / kg-K*	$[M^0L^2T^{-2}\theta^{-1}]$
(4)	Thermal capacity	*Joule / K*	$[M^1L^2T^{-2}\theta^{-1}]$
(5)	Latent heat (L)	*Joule / kg*	$[M^0L^2T^{-2}]$
(6)	Gas constant (R)	*Joule / mol-K*	$[M^1L^2T^{-2}\theta^{-1}]$
(7)	Boltzmann constant (k)	*Joule / K*	$[M^1L^2T^{-2}\theta^{-1}]$
(8)	Coefficient of thermal conductivity (K)	*Joule / m-s-K*	$[M^1L^1T^{-3}\theta^{-1}]$
(9)	Stefan's constant (σ)	$Watt/m^2\text{-}K^4$	$[M^1L^0T^{-3}\theta^{-4}]$
(10)	Wien's constant (b)	*Meter-K*	$[M^0L^1T^0\theta]$
(11)	Planck's constant (h)	*Joule-s*	$[M^1L^2T^{-1}]$
(12)	Coefficient of Linear Expansion (□)	$Kelvin^{-1}$	$[M^0L^0T^0\theta^{-1}]$
(13)	Mechanical eq. of Heat (J)	*Joule / Calorie*	$[M^0L^0T^0]$
(14)	Vander wall's constant (a)	$Newton\text{-}m^4$	$[ML^5T^{-2}]$
(15)	Vander wall's constant (b)	m^3	$[M^0L^3T^0]$

Electricity

S. N.	Quantity	Unit	Dimension
(1)	Electric charge (q)	*Coulomb*	$[M^0L^0T^1A^1]$
(2)	Electric current (I)	*Ampere*	$[M^0L^0T^0A^1]$
(3)	Capacitance (C)	*Coulomb / volt or Farad*	$[M^{-1}L^{-2}T^4A^2]$
(4)	Electric potential (V)	*Joule / coulomb*	$M^1L^2T^{-3}A^{-1}$

S. N.	Quantity	Unit	Dimension
(5)	Permittivity of free space (ε_0)	$\frac{Coulomb^2}{Newton-meter^2}$	$[M^{-1}L^{-3}T^4A^2]$
(6)	Dielectric constant (K)	Unitless	$[M^0L^0T^0]$
(7)	Resistance (R)	$Volt/Ampere$ or ohm	$[M^1L^2T^{-3}A^{-2}]$
(8)	Resistivity or Specific resistance (ρ)	$Ohm\text{-}meter$	$[M^1L^3T^{-3}A^{-2}]$
(9)	Coefficient of Self-induction (L)	$\frac{volt-second}{ampere}$ or $henery$ or $ohm\text{-}second$	$[M^1L^2T^{-2}A^{-2}]$
(10)	Magnetic flux (ϕ)	$Volt\text{-}second$ or $weber$	$[M^1L^2T^{-2}A^{-1}]$
(11)	Magnetic induction (B)	$\frac{newton}{ampere-meter}$ $\frac{Joule}{ampere-meter^2}$ $\frac{volt-\text{second}}{meter^2}$ or $Tesla$	$[M^1L^0T^{-2}A^{-1}]$
(12)	Magnetic Intensity (H)	$Ampere/meter$	$[M^0L^{-1}T^0A^1]$
(13)	Magnetic Dipole Moment (M)	$Ampere\text{-}meter^2$	$[M^0L^2T^0A^1]$
(14)	Permeability of Free Space (μ_0)	$\frac{Newton}{ampere^2}$ or $\frac{Joule}{ampere^2-meter}$ or $\frac{Volt-\text{second}}{ampere-meter}$ or $\frac{Ohm-second}{meter}$ or $\frac{henery}{meter}$	$[M^1L^1T^{-2}A^{-2}]$
(15)	Surface charge density (σ)	$Coulomb\,metre^{-2}$	$[M^0L^{-2}T^1A^1]$
(16)	Electric dipole moment (p)	$Coulomb-meter$	$[M^0L^1T^1A^1]$
(17)	Conductance (G) ($1/R$)	ohm^{-1}	$[M^{-1}L^{-2}T^3A^2]$
(18)	Conductivity (σ) ($1/\rho$)	$ohm^{-1}meter^{-1}$	$[M^{-1}L^{-3}T^3A^2]$
(19)	Current density (J)	$Ampere/m^2$	$M^0L^{-2}T^0A^1$
(20)	Intensity of electric field (E)	$Volt/meter$, $Newton/coulomb$	$M^1L^1T^{-3}A^{-1}$
(21)	Rydberg constant (R)	m^{-1}	$M^0L^{-1}T^0$

1.6 Quantities Having Same Dimensions

S. N.	Dimension	Quantity
(1)	$[M^0L^0T^{-1}]$	Frequency, angular frequency, angular velocity, velocity gradient and decay constant
(2)	$[M^1L^2T^{-2}]$	Work, internal energy, potential energy, kinetic energy, torque, moment of force
(3)	$[M^1L^{-1}T^{-2}]$	Pressure, stress, Young's modulus, bulk modulus, modulus of rigidity, energy density
(4)	$[M^1L^1T^{-1}]$	Momentum, impulse
(5)	$[M^0L^1T^{-2}]$	Acceleration due to gravity, gravitational field intensity

(6)	$[M^1L^1T^{-2}]$	Thrust, force, weight, energy gradient
(7)	$[M^1L^2T^{-1}]$	Angular momentum and Planck's constant
(8)	$[M^1L^0T^{-2}]$	Surface tension, Surface energy (energy per unit area)
(9)	$[M^0L^0T^0]$	Strain, refractive index, relative density, angle, solid angle, distance gradient, relative permittivity (dielectric constant), relative permeability etc.
(10)	$[M^0L^2T^{-2}]$	Latent heat and gravitational potential
(11)	$[M^0L^2T^{-2}\theta^1]$	Thermal capacity, gas constant, Boltzmann constant and entropy
(12)	$[M^0L^0T^1]$	$\sqrt{l/g}, \sqrt{m/k}, \sqrt{R/g}$, where l = length g = acceleration due to gravity, m = mass, k = spring constant
(13)	$[M^0L^0T^1]$	L/R, $\sqrt{LC}$, RC where L = inductance, R = resistance, C = capacitance
(14)	$[ML^2T^{-2}]$	$I^2Rt, \frac{V^2}{R}t, \; VIt, \; qV, \; LI^2, \; \frac{q^2}{C}, CV^2$ where I = current, t = time, q = charge, L = inductance, C = capacitance, R = resistance

1.7 Application of Dimensional Analysis

(1) To find the unit of a physical quantity in a given system of units.
(2) To find dimensions of physical constant or coefficients.
(3) To convert a physical quantity from one system to the other.
(4) To check the dimensional correctness of a given physical relation: This is based on the '*principle of homogeneity*'. According to this principle the dimensions of each term on both sides of an equation must be the same.
(5) To to derive new relations.

1.8 Limitations of Dimensional Analysis

(1) If dimensions are given, physical quantity may not be unique.
(2) Numerical constant having no dimensions cannot be deduced by the methods of dimensions.
(3) The method of dimensions can not be used to derive relations other than product of power functions. For example,

$$s = ut + (1/2)at^2 \qquad \text{or} \qquad y = a\sin\omega t$$

(4) The method of dimensions cannot be applied to derive formula consist of more than 3 physical quantities.

1.9 Significant Figures

Significant figures in the measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is the accuracy of the measurement. The reverse is also true.

The following rules are observed in counting the number of significant figures in a given measured quantity.

(1) All non-zero digits are significant.
(2) A zero becomes significant figure if it appears between to non-zero digits.
(3) Leading zeros or the zeros placed to the left of the number are never significant.
Example: 0.543 has three significant figures.
0.006 has one significant figures.

(4) Trailing zeros or the zeros placed to the right of the number are significant.
Example: 4.330 has four significant figures.
343.000 has six significant figures.

(5) In exponential notation, the numerical portion gives the number of significant figures.
Example: 1.32×10^{-2} has three significant figures.

1.10 Rounding Off

(1) If the digit to be dropped is less than 5, then the preceding digit is left unchanged.
Example: $x = 7.82$ is rounded off to 7.8, again $x = 3.94$ is rounded off to 3.9.

(2) If the digit to be dropped is more than 5, then the preceding digit is raised by one.
Example: $x = 6.87$ is rounded off to 6.9, again $x = 12.78$ is rounded off to 12.8.

(3) If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one.
Example: $x = 16.351$ is rounded off to 16.4, again $x = 6.758$ is rounded off to 6.8.

(4) If digit to be dropped is 5 or 5 followed by zeros, then preceding digit is left unchanged, if it is even.
Example: $x = 3.250$ becomes 3.2 on rounding off, again $x = 12.650$ becomes 12.6 on rounding off.

(5) If digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd.
Example: $x = 3.750$ is rounded off to 3.8, again $x = 16.150$ is rounded off to 16.2.

1.11 Significant Figures in Calculation

The following two rules should be followed to obtain the proper number of significant figures in any calculation.

(1) The result of an addition or subtraction in the number having different precisions should be reported to the same number of decimal places as are present in the number having the least number of decimal places

(2) The answer to a multiplication or division is rounded off to the same number of significant figures as is possessed by the least precise term used in the calculation.

1.12 Order of Magnitude

Order of magnitude of quantity is the power of 10 required to represent the quantity. For determining this power, the value of the quantity has to be rounded off. While rounding off, we ignore the last digit which is less than 5. If the last digit is 5 or more than five, the preceding digit is increased by one. For example,

(1) Speed of light in vacuum = $3 \times 10^8\ ms^{-1} \approx 10^8$ m/s (ignoring 3 < 5) (ignoring 3 < 5)

(2) Mass of electron = 9.1×10^{-31} kg 10^{-30} kg (as 9.1 > 5).

1.13 Errors of Measurement

The measured value of a quantity is always somewhat different from its actual value, or true value. This difference in the true value of a quantity is called error of measurement.

(1) ***Absolute error:*** Absolute error in the measurement of a physical quantity is the magnitude of the difference between the true value and the measured value of the quantity.
Let a physical quantity be measured n times. Let the measured value be $a_1, a_2, a_3, \ldots.. a_n$.
The arithmetic mean of these value is $a_m = \dfrac{a_1 + a_2 + \ldots.a_n}{n}$

Usually, a_m is taken as the true value of the quantity, if the same is unknown otherwise. By definition, absolute errors in the measured values of the quantity are

$\Delta a_1 = a_m - a_1$

$\Delta a_2 = a_m - a_2$

…………

$\Delta a_n = a_m - a_n$

The absolute errors may be positive in certain cases and negative in certain other cases.

(2) ***Mean absolute error:*** It is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity. It is represented by $\overline{\Delta a}$. Thus

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + . | \quad \Delta a_n}{n}$$

Hence the final result of measurement may be written as $a = a_m \pm \overline{\Delta a}$

This implies that any measurement of the quantity is likely to lie between $(a_m + \overline{\Delta a})$ and $(a_m - \overline{\Delta a})$.

(3) ***Relative error or Fractional error:*** Relative error or Fractional error

$$= \frac{\text{mean absolute error}}{\text{mean value}} = \frac{\overline{\Delta a}}{a_m}$$

(4) ***Percentage error:*** Percentage error $= \dfrac{\overline{\Delta a}}{a_m} \times 100\%$

1.14 Propagation of Errors

(1) ***Error in sum of the quantities:*** Suppose $x = a + b$

Let Δa = absolute error in measurement of a

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x *i.e.* sum of a and b.

The maximum absolute error in x is $\Delta x = \pm(\Delta a + \Delta b)$

(2) ***Error in difference of the quantities:*** Suppose $x = a - b$

The maximum absolute error in x is $\Delta x = \pm(\Delta a + \Delta b)$

(3) ***Error in product of quantities:*** Suppose $x = a \times b$

The maximum fractional error in x is $\dfrac{\Delta x}{x} = \pm\left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right)$

(4) ***Error in division of quantities:*** Suppose $x = \dfrac{a}{b}$

The maximum fractional error in x is $\dfrac{\Delta x}{x} = \pm\left(\dfrac{\Delta a}{a} + \dfrac{\Delta b}{b}\right)$

(5) ***Error in quantity raised to some power:*** Suppose $x = \dfrac{a^n}{b^m}$

The maximum fractional error in x is $\dfrac{\Delta x}{x} = \pm\left(n\dfrac{\Delta a}{a} + m\dfrac{\Delta b}{b}\right)$

- ❑ The quantity which have maximum power must be measured carefully because it's contribution to error is maximum.